Assuming $\cos\theta \geq \cos\alpha$, two steps are required to compute $\ell \cap \mathcal{C}$: (1) computing $\mathcal{P} \cap \mathcal{C}$, and (2) computing $\ell \cap (\mathcal{P} \cap \mathcal{C})$. For the first step, a well-chosen coordinate system is vital. Since $\mathbf{n}$ and $\mathbf{v}$ are not parallel, $\mathbf{v} \times \mathbf{n}$ is well defined. Let vectors $\mathbf{u}$ and $\mathbf{w}$ be defined as follows:

$$\mathbf{u} = \mathbf{v} \otimes \mathbf{n},$$
$$\mathbf{w} = \mathbf{u} \otimes \mathbf{v} = (\mathbf{v} \otimes \mathbf{n}) \otimes \mathbf{v}.$$

Then $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are perpendicular to each other and form a right-handed u-v-w coordinate system with origin at $\mathbf{V}$ [Figure 2(a)]. Since $\mathbf{n} \perp \mathbf{u}$ and $\mathbf{V} \in \mathcal{P}$, $\mathcal{P}$ contains the u-axis and is perpendicular to the vw-plane.

Using this coordinate system, the direction vectors of $\mathcal{P} \cap \mathcal{C}$ are computed as follows. Consider a plane $\mathcal{H}$ with $v = 1$ in the u-v-w coordinate system. $\mathcal{H} \cap \mathcal{C}$ is a circle C, while $\mathcal{H} \cap \mathcal{P}$ is a line p. Let p and C intersect at A and B. Then the intersection of $\mathcal{P}$ and $\mathcal{C}$ consists of two lines, $\overleftrightarrow{VA}$ and $\overleftrightarrow{VB}$. Thus, if their direction vectors, $\delta_1 = \overrightarrow{VA}$ and $\delta_2 = \overrightarrow{VB}$, can be found, $\mathcal{P} \cap \mathcal{C}$ will be determined.

To compute A and B, first note that their w-coordinates are both equal to $\tan\theta$, and that $\frac{1}{2}\overline{AB} = (\tan^2\alpha - \tan^2\theta)^{1/2}$, where $\tan\alpha$ is the radius of circle C [Figure 2(b) and (c)]. Since $\overleftrightarrow{AB}$ is parallel to the u-axis, direction vectors $\delta_1 = \overrightarrow{VA}$ and $\delta_2 = \overrightarrow{VB}$ can be computed as follows:

$$\delta_1 = \mathbf{v} + (\tan\theta)\mathbf{w} + (\tan^2\alpha - \tan^2\theta)^{1/2}\mathbf{u},$$
$$\delta_2 = \mathbf{v} + (\tan\theta)\mathbf{w} - (\tan^2\alpha - \tan^2\theta)^{1/2}\mathbf{u}.$$