Fractal Territory Board Game

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***Abstract*-- A novel fractal board game is proposed by applying the concept of subdivision of fractals to an NxN grid of a continuous game board that consists of vertices and edges. The game board can be infinitely subdivided, thus generating an infinite number of sub-games. The application of the subdivision rule balances the dominance of the leading player by providing opportunities for the disadvantaged players to catch up, thus making gameplay more interesting. As additional subdivisions add to the complexity of the game, the gameplay can be maintained by a friendly GUI. This GUI provides camera control for regions of interest on the game board and hints for scoring. The proposed fractal territory board game can be played on fractal game boards where the subdivision of a shape keeps sub-games sharing edges and introduces new vertices.**

I. INTRODUCTION

Connection games are a family of abstract board games that define game play recursively [1][11]. Browne [2] introduced a recursive meta-rule to expand connection games to be fractal-like board games. While connection games are intuitive and easy to become familiarized with, the concept of a recursive game can enhance the depth of the game and make it more interesting. However, limiting the number of subdivisions on a discrete game board prevents connection games from being considered a truly fractal game.

In this paper, we introduce a Fractal Territory Board Game (FTBG) that is based on territory occupation as in the game of GO, Abalone [12], etc. The continuous game board consists of vertices and edges. By applying the concept of subdivision to the game board, sub-boards, also referenced in this paper as sub-games, are generated using the continuity of lines to provide infinite recursive subdivisions. Each of the two players are given a set of game pieces, one being black and one being white, and take turns placing one game piece on a vertex of the game board at any level to occupy a territory (area). The score is based on how much territory the player is able to occupy. This is determined by the number of game pieces on the boundary of a cell after all vertices of the cell are occupied. The basic concept of occupying territory is familiar to most players, making the proposed territory board game relatively intuitive. Also, as a Territory game, the rules of the game are simple making it easy to learn and play, but difficult to master.

The FTBG can be recursively subdivided into unlimited sub-games by the disadvantaged player; a mechanism that gives the losing player (usually white) an opportunity to catch up. The additional subdivisions of the board create sub-games that are concurrently played. It is for this reason that the FTBG is inherently a fractal game (recursive game). The following are also reasons the FTBG is an inherently fractal game:

- The FTBG has no special game pieces with different conditions. All game pieces are uniformly used in the same way for the same purpose.
- Because of the continuous nature of the game board, there are limitless possibilities for game play.
- The FTBG displays a self-similarity at all levels (sub-games) [6]; the game is played using the same rules at every level of the game board.
- The FTBG is a variable geometry game like connection games [2]. In other words, the FTBG can be played simultaneously on several levels of the game board.

Brown [2] applied the concept of fractal subdivision on Potential Y [3] to devise an imaginary game called Fractal Py. In Fractal Py, a player places point charges on a continuous potential field to affect a circular area instead of placing game pieces on a discrete game board. This made the game board a continuous game board allowing for unlimited subdivisions on any level making the game a truly fractal game. The game board can be subdivided infinitely, however the pixel limitations of the display used to play the game drastically limit the number of subdivisions possible. In order to overcome this challenge, a user friendly GUI can be created to act as a virtual camera to zoom-in on specific regions of the game board.

The added depth and complexity makes it difficult for players to plan moves on more than two or three levels and makes it difficult to calculate players' scores. In this paper, we utilize a computer to calculate the players' scores. In addition to computing the scores, the computer is also used in assisting players with game strategy by showing score hints.

The leading player in traditional abstract board games maintains dominance until the game is over. However, in our proposed FTBG, the player with the lower score has the option to subdivide the game board. This rule affects the fairness of gameplay, allowing the trailing player to potentially become the dominant player in the game. The rule also increases the length of gameplay. Moreover, the proposed FTBG can play on other fractal shape game boards, such as the Serpinski triangle, the Sierpinski carpet, and the Hexaflake game boards. This study found that playing the proposed FTBG on game boards of different geometries produce different gameplay experiences.

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II. Fractal Territory Board Game

A. Game Rule

- Players: The first player, who is referred to as the Black Player, uses black game pieces and the second player, who is referred to as the White Player, uses white game pieces.
- Rules

I. Before the game begins, both players must agree on a target score and a score difference.

II. The players take turns placing one game piece on an unoccupied vertex (cross-point of the board lines) of the game board at a time.

III. When the four vertices of a cell are covered by game pieces, the scores are updated according to the number of game pieces each player has on the vertices of all completed cells. All corresponding vertices contained within all sub-games of that cell must be taken into consideration when calculating the score.

IV. Once the scores are determined, the player with lower score has the option to subdivide the game board on his/her turn.

V. The game will be terminated when:

A、 All vertices of the game board are covered with a game piece,

B、 Any player reaches the target score, or

C、 The score difference between two players is larger than or equal to the set score difference.

- Game Board: Initially, the game board is a 3x3 grid, i.e. there are 3x3 vertices. On the first level, there is one 3x3 board. The second level has four 2x2 sub-boards (2x2 cells), as shown in
- Figure 1. Each game will produce game boards of varying shapes depending on the subdivisions created during gameplay.
- Figure 1(b) shows two levels of a game board using the quad-tree structure.
- Board Subdivision: Subdividing one cell generates four (2x2) cells on the next level cell (smaller squares).
- Winning Condition: When one of the agreed upon termination conditions is satisfied, the player with higher score is the winner.

B. Sample Play

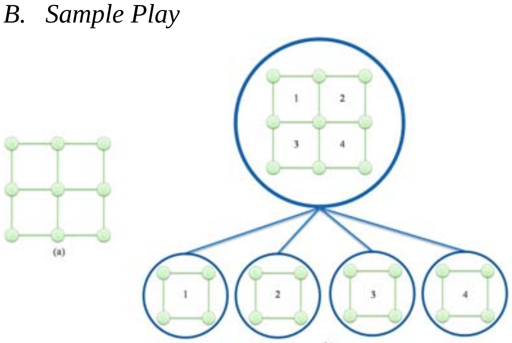

Figure 1. (a) The game board starts as a 3x3 grid. (b) Two levels of subgames are represented by the quadtree structure: the first level is the 3x3 board, consisting of four corner vertices, and the second level consists four 2x2 boards (quadrants, cells).

The Black and White players take turns to placing game pieces on the game board. In **Error! Reference source not found.**, White chooses to create a subdivision on the 8th move of the game in cell 1, the upper left cell of the game board, and places a white game piece on the newly created vertex. After Black places a game piece on the game board during the 13th move of the game, all vertices on the game board are occupied. Because all vertices of the game board are covered, the game ends according to termination rule A stated above. Both Black and White end with a score of 18, resulting in a tie.

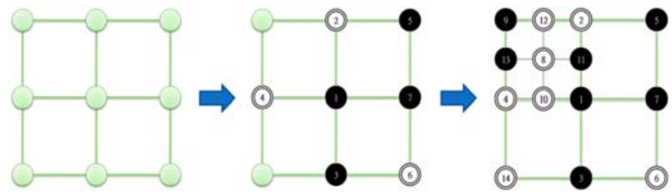

Figure 2. This is a sample game of FTBG. At the beginning of the game, the game board is a 3x3 grid. The number on each game piece represents the sequential order of how the game pieces were placed on the game board; The 8th move of the game shows White choosing to subdivide the upper left cell of the game board by placing a game piece at the center of that cell.

Error! Reference source not found. bellow shows the three different sub-boards (subgames) created during this game in a quadtree diagram. On each level of the game board, only the four corner vertices are counted when tallying up each player's score as written in Game Rule III. For example, level 1 of Figure 3 shows two corners occupied by Black and two corners occupied by White, resulting in a score of 2-2 for that level of the game. When the game is examined on level 2, we see that a total of ten corner vertices are occupied by Black and six are occupied by White, bringing the total score to 12-8 in favor of Black. Scoring is continued in this manner until all sub-boards have been tallied.

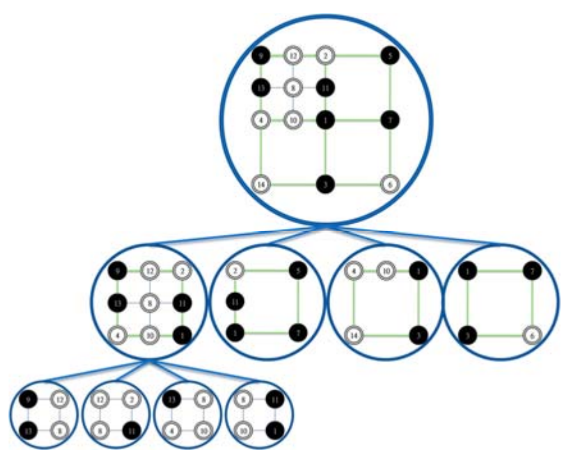

Figure 3 is a continuation of the sample game started in Figure 4. Here, three levels of the game are shown in quadtree form. Note that the 4 corner vertices of every cell are counted in the score.

III. Discussion

A. Advantage of Game Board Subdivision

When a player choses to create a subdivision in a cell on the game board, four sub-boards (cells) are generated. Subdivision only increases the importance of game play on the resulting sub-boards as the score considers the original game board and all sub-boards created during gameplay. Thus, players may opt to choose subdivision in order to have their game pieces occupy vertices shared by the resulting sub-boards. Players must consider the possible advantages and disadvantages of creating sub-boards.

As the rules state, the player with the lower score has the option to subdivide the game board. If this option is chosen, the player's game piece may occupy territory on all subsequent sub-boards created within that cell. This gives the trailing player a chance to tie or even overtake the other player resulting in a more interesting game play than other traditional abstract board games.

Figure 4(a) below shows the first 7 moves of a game. Here, each player places a game piece on the board strategically to maximize his/her own chances of being able to subdivide the game board. The 7th move results in Black completing the two cells on the right side of the game board. At this point in the game, Black has the higher score of 6 while White has a score of 2 allowing White the opportunity to subdivide the game board.

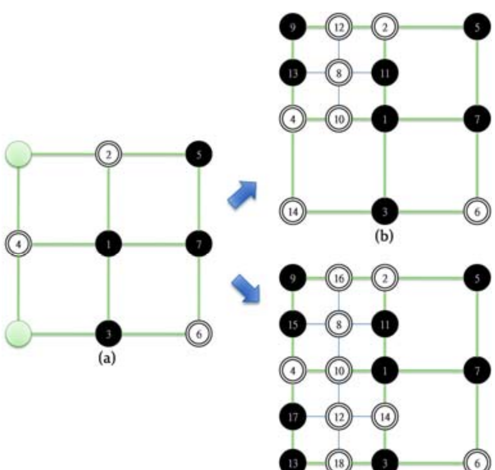

Figure 4. Advantage of game board subdivision. (a) Black has the advantage on the 7th move of the game (Black 6 – White 2). (b) The game is a draw (Black 18 – White 18) after White chooses to subdivide the top left cell during the 8th move of the game. (c) White chooses subdivision for both the 8th and 12th move of the game resulting in a score of 25 to 27 in favor of White.

Figure 4(b) above shows White performing a subdivision of the upper left cell as the 8th move of the game. The 9th, 11th, and 13th moves of the game show Black placing game pieces on the board in an effort to achieve the highest score possible. However, after the 13th move, all vertices of the game board are occupied terminating the game. The game play in Figure 4(b) yields a tie with both Black and White at a score of 18 each.

Figure 4(c) above illustrates yet an alternative gameplay possibility. Here White also performs a subdivision of the upper left cell on the 8th move, as in Figure 4(b), however White also performs a subdivision of the lower left cell on the 12th move of the game. Both Black and White utilize the best strategy possible to maximize their individual scores with each turn until all vertices are occupied after the 18th move of the game. At this point the game is terminated and a score count shows White as the winner (Black 25 – White 27).

B. Gameplay

Termination condition B, where any player reaches the agreed upon target score, and termination condition C, where the score difference between the two players is larger than or equal to the agreed upon score difference, add additional game play possibilities. These conditions provide alternative ways to win, increasing the number of gameplay strategies that can be utilized by the players.

Players can also agree to change the rule governing subdivisions during gameplay. For example, players can agree to roll a dice or draw a card in order to perform a subdivision, thus adding an element of luck to the game. This kind of change transforms the abstract strategy FTBG into a stochastic game, adding to the gameplay experience.

C. Fairness

Herik et al. [4] defines the fairness of a game as follows: in a draw game, if there is an equal possibility for all players in a game to make mistakes, it is a fair game. However, it is difficult to build a mathematical model to show that the possibility for the two players of a game to make mistakes is equal. This is because the continual addition of new strategies changes the number of possibilities that need to be considered within the model, thus skewing the results of that model. The infinite number of gameplay possibilities in the FTBG gives way to an infinite number of gameplay strategies that cannot be modeled.

D. Game UI

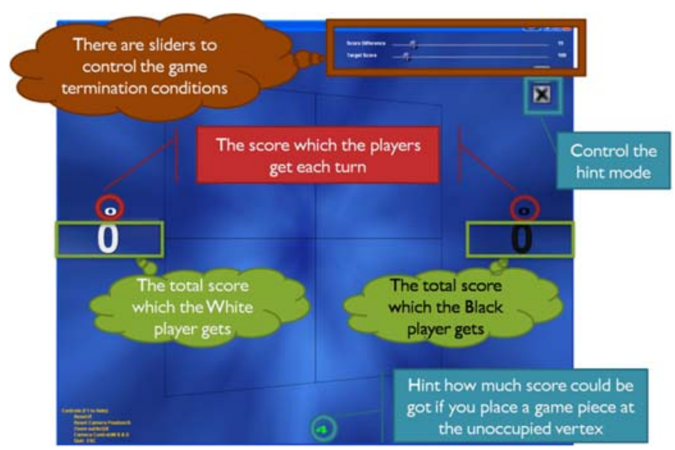

Figure 5 A screenshot of the GUI implemented to overcome the hardware challenge posed by pixel limitations.

Playing the FTBG requires the assistance of a graphical user interface (GUI). **Error! Reference source not found.** is a screen shot of a game that utilizes a simple GUI. Two sliders at the upper right corner are used to set the score difference and target score for the game. As a player moves the cursor across the open vertices of the game board, the number at the center bottom of the screen displays the score that can be obtained by placing a game piece on that vertex. This feature can be disabled by checking the box in the top right corner of the screen. The score for White is on the left and the score for Black is on the right side of the screen. Two scores are displayed for each player. The number in larger font reflects the player's total score and the number in smaller font shows the number of points achieved during that player's last turn. The yellow text in the lower left corner of the screen is the game rules.

The fractal nature of the game allows for an infinite number of subdivisions. Each subdivision adds detail to the game board that must be displayed to the players. However, the pixel limitations of the displays used to show gameplay pose a challenge. The insufficient resolution of display devices limits the amount of detail that can be displayed on the screen. To resolve these display issues, the FTBG employs a GUI that acts as a three dimensional virtual camera. This camera is controlled by the players and can be used to focus in on any part of the game board. Figure 8 shows the virtual camera being used to zoom in on a dense area of the game board.

E. Diverse Game Boards for FTBG

The FTBG can be played on game boards of different geometries that have vertices connecting the different cells on the board. The Sierpinski triangle, shown in Figure 5, and the Sierpinski carpet, shown in Figure 6, are two game board possibilities. A game board of a different geometry produces a different number of sub-games when subdivision occurs as shown in Figure 5 and Figure 6. Alternative geometries also result in a different state-space complexity [1] and game tree size. Thus, different game board geometries produce different gameplay experiences.

Figure 6. Sierpinski triangle game board. A subdivision has been applied to each game board starting from the game board on the left.

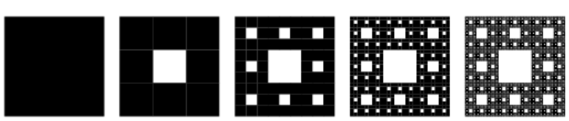

Figure 7. Sierpinski carpet game board. A subdivision has been applied to each game board beginning with the game board on the left.

IV. CONCLUSION

In this paper, the FTBG has been proposed in which the player with the lower score can choose to subdivide the game board. The concept of subdividing the game board reduces the possibility for sustained dominance by any one player as seen in traditional abstract board games. The concept of subdivision also improves the complexity of the game and produces a different gameplay experience. The complexity of the game increases as players choose to subdivide the game board. In order to display these additional sub-boards a GUI incorporates a virtual camera that is controlled by the players. The GUI also provides score hints to the players allowing the players to make better strategic gameplay decisions. The proposed continuous game board can be applied to a number of traditional connection board games such as Quadrant Y, Quadrant Hex, etc. In doing so, these games can be considered fractal board games. The game continuous game can also be applied to game boards of different geometries for a different gameplay experience.

The next step in furthering this idea is to explore the fairness aspect of gameplay in more depth. Designing an online version of the FTBG for individuals to play will generate gameplay data that can be analyzed to see if empirical unfairness exists. Also, the Strategy-Stealing argument [8] can be used to explore the existence of monotonic unfairness in this game.

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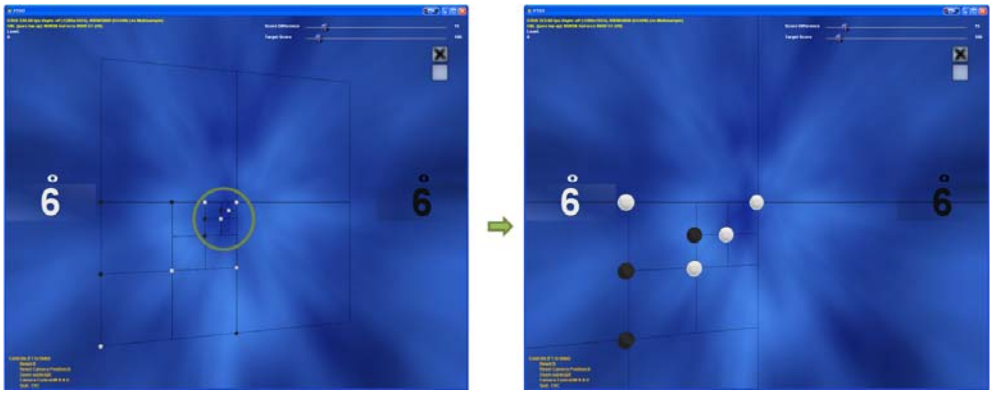

Figure 8. Players can control the camera to zoom-in on dense regions of the game board. The green circle in the left screenshot shows the region of the game board that will be magnified. The screenshot on the right shows that same region in more detail after being magnified.