Request Date: 12-JAN-2010
Expiration Date: 20-JAN-2010
ILL Number:
TGQ or OCLC #:

ILL Number: 3628700
TGQ or OCLC #: 3628697
Call Number: UCSB:Main Lib T385 .C558 Sciences Engineering Library
ID: UI3
Format: Article Printed
ISBN/ISSN: 0271-4159
Title: Computer Graphics World
Article Author: Armstrong, William W
Article Title: Perception Cues for n Dimensions
Part Pub. Date: 1985-03
Pages: 11-28
Requester: UCI Science Library

v. 8 pt. 2
v. 8 pt. 1

Publisher: PennWell Publishing Corp.
Address: University of California - Irvine
Document Access & Delivery
Science Library
P.O. Box 19557
Ariel: 128.200.102.110, Fax: 949-824-3695

Patron Name: Black,Donald Vaughn (Graduate [])
Patron e-mail: DBLACK@UCI.EDU
Service Level: Normal - Full Search
Delivery Method: Electronic Mail
Request Note: Digital Format (PDF) preferred
Need by Date:
Verification Source: MELVYL-UCLinks-sfx:citation
Supplier Reference:
Owned By: UCSB Library

Service Type: Copy non returnable
Max Cost: USD50
Payment Type: IFM
Copyright Compliance: CCG
Requester Symbol:
Return To: UCSB Interlibrary Loan/Davidson Library/525 UCEN Road/Santa Barbara, CA, 93106/U.S.A./ UCSB Ariel IP 128.111.96.251; Email ill@library.ucsb.edu

Best copy - tightly bound

pages 13-15, 17, 19, 21-23, 25 & 27 are ads

Perception Cues for n Dimensions

By William P. Armstrong, Evans & Sutherland, and Robert P. Burton, Brigham Young University

Objects in n-space ($n \geqslant 4$) must inevitably be depicted in the two or three dimensions of a given display. To convey a feeling of the objects' true dimensionality, projections must encode decipherable cues to the values of n-coordinates in 2- or 3-D. Visual depth perception cues are a natural encoding of 3-D information in our 2-D retinal images that our minds readily decode to infer 3-D information. Can depth cues be used to encode information concerning more than three dimensions?

Mathematicians seek to understand the development of functions with graphs in n-space; statisticians analyze multivariate data for relationships; physicists base explanations of natural phenomena on n-space dimensions; engineers attempt to cope with multiple simultaneous parameters. The development of models capable of conveying a visual impression of n-dimensionality holds enormous promise for their work. The computer has greatly facilitated their creation.

Models and depictions created prior to the advent of computer graphics had some success in conveying an understanding of four or more dimensions. Michael Noll, one of the first to use computer graphics to depict four dimensions, made stereo movies of rotating hyperobjects, but found that the resulting images only looked like objects distorting in 3-D space.

This is due to the way the depth cues were used. In hyperobject depictions, careful attention has not been paid to the way n-dimensional information is encoded in depth cues, resulting in decreased comprehension. In addition, many potential uses and combinations of depth cues to display n dimensions have not been explored. By examining the difficulties in comprehending dimensionality in such depictions, it is hoped that a less confusing method of conveying hyperdimensional objects will result.

Depth Perception Cues Extrapolated to n Dimensions

Psychologists have identified numerous visual cues by which our minds interpret 3-D space. A typical list of depth cues includes:

☐ *Motion Parallax:* Changes in the retinal images caused by movements of either objects or their observer.

☐ *Size:* The absolute, or the relative, size of the retinal images of objects.

☐ *Linear Perspective:* The convergence of parallel lines in the distance.

☐ *Angle of Regard:* The angle above or below an object from which it is seen (distant objects tend to be closer to the center of the field of vision).

☐ *Interposition:* The partial overlapping of the retinal images of objects.

☐ *Aerial Perspective:* Atmospheric effects causing a bluish color, blurriness or indistinctness in distant objects.

☐ *Light and Shade:* Reflections, highlights, shadows and other effects of light (especially those due to the almost constant presence of overhead light).

☐ *Texture:* The visibility and regularity of surface detail on objects.
☐ *Disparity:* Differences in the positions of the images of objects on each retina.
☐ *Convergence:* The different angles through which each eye must turn to locate an object.
☐ *Accommodation:* The difference in focusing on near and distant objects.

In 3-D graphics, each of these cues may be applied to the z-coordinate to convey information about depth in the displayed image. Cues to convey other information about coordinates could be applied equally well; some cues could be applied to multiple coordinates, conveying information about all of them.

The analysis of perception cues for n dimensions is begun by extrapolating the implementations of depth cues. Aerial perspective, effects of light and shade, and texture will not be discussed because full use of these cues requires the generation of shaded images—a subject that has not yet been fully explored. Convergence and accommodation are also excluded because they would seem to have little potential to convey information about n dimensions.

Motion Parallax: Motion cues in n-space extrapolate directly from the translation (changes in position) and rotation (changes in orientation) primitives in three dimensions. While rotations in 3-space are generally thought of as occurring about an axis, rotations in n-space are thought of as occurring in a set of parallel planes.

Rotation about an arbitrary plane is defined as the concatenation of primitive rotations. These transformations are implemented by translation and yw-rotation matrices in 4-space. Even though motion cues invoke a feeling of depth, in the absence of other cues, true depth is ambiguous.

Perspective Cues: Retinal images are perspective projections in which equivalent lengths become smaller as their distances from the observer increase. Perspectively projecting n dimensions to $n-1$ dimensions affects the cues of size, linear perspective and angle of regard.

Perspective projections from n dimensions are directly extrapolated from three dimensions. A viewing transformation is applied to place the viewpoint at the origin and to align the projection plane with the plane $x_n = d$. The projection is then calculated by dividing each vertex by x_n.

The process is repeated to project $n-1$ dimensions to $n-2$ dimensions. However, a problem arises in that the original viewpoint (the

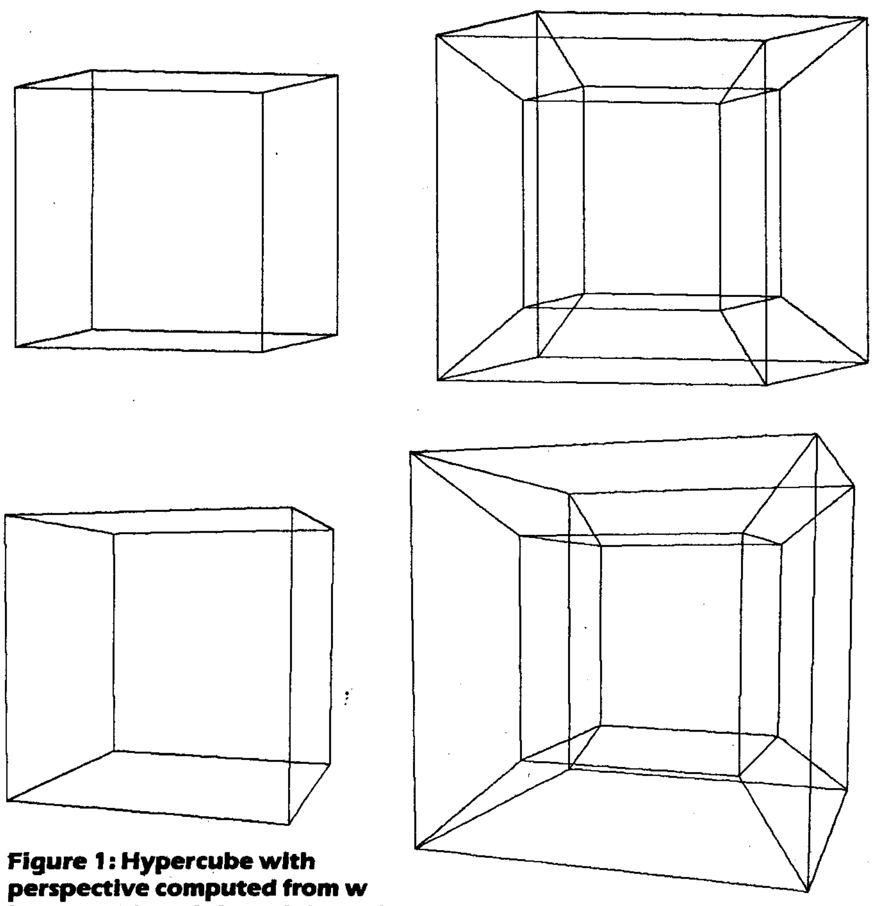

Figure 1: Hypercube with perspective computed from w increased from left to right and perspective computed from z increased from top to bottom.

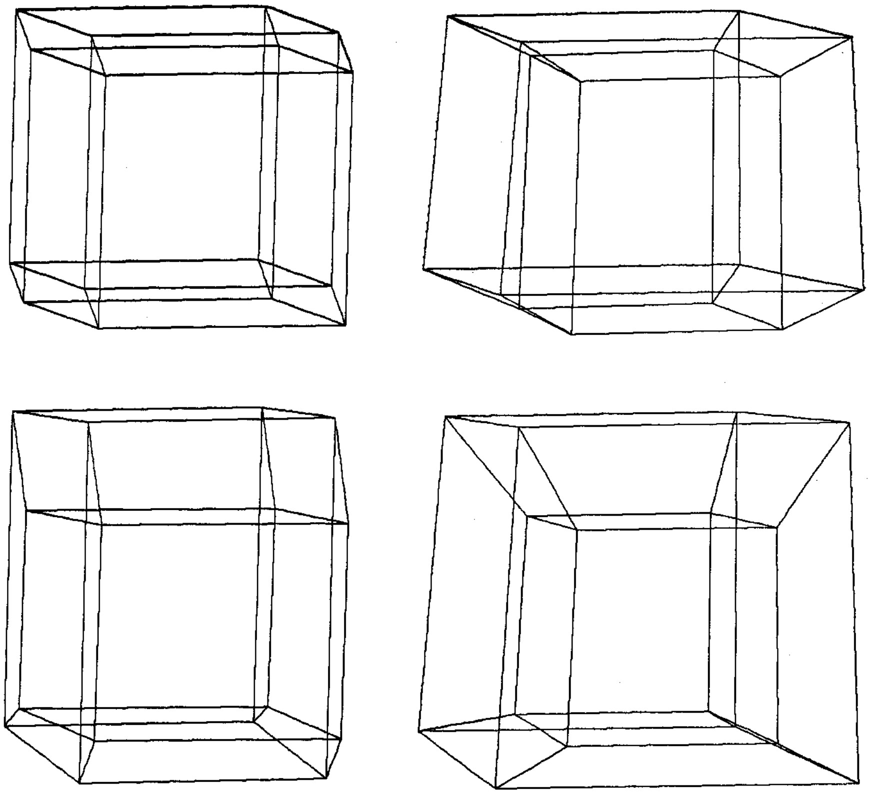

Figure 2: Perspective computed from w applied to the x-coordinate increased from left to right and applied to the y-coordinate increased from top to bottom.

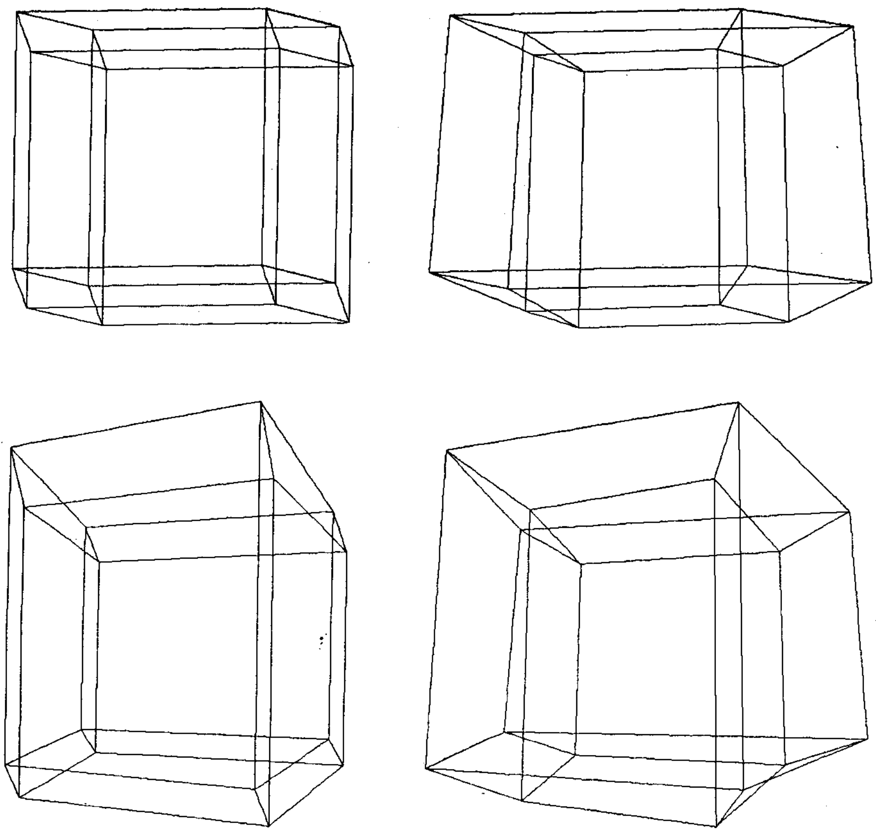

Figure 3: Perspective computed from w applied to the x-coordinate increased from left to right and perspective computed from z applied to the y-coordinate increased from top to bottom.

origin) is no longer in the proje tion. One way to accommodate th is to let the new viewpoint be t origin in the projection. Projecti to the hyperplane $x_{n-1} = d_2$ elim nates all information concerni the x_n-coordinate. This problem circumvented by choosing a poi other than the origin as the ne viewpoint, which is easily accor plished by applying a differe viewing transformation that alig the projection plane with the pla $x_n = 0$ and places the viewpoint the origin.

After projecting to n − 1 dime sions, the process can be repeate to project n − 1 dimensions to n − dimensions and for successive pr jections. *Figure 1* shows how pe spectives from z and w affect t depiction of a hypercube.

Clipping, to remove parts of o jects that are beyond the edges the display, is usually included perspective projection. Clippin ensures that each normalized c ordinate will be between −1 a 1. Lines and polygons are clipp using 3-D techniques; clippin higher-dimensional entities is co siderably more difficult.

Perspective projections need no be applied uniformly to all coord nates. It is feasible to apply a pe spective projection to certain coordinates to create a horizontal displacement and to apply another perspective projection to other co-ordinates to create a vertical dis-placement. *Figure 2* contrasts the perspective from w as applied to the x-coordinate (horizontal per-spective) with that applied to the y-coordinate (vertical perspective). *Figure 3* shows horizontal perspec-tive computed from w and vertical perspective computed from z. This dissociation of the effects of the two projections may aid in their discernment.

Interposition: The interposition cue is implemented in 3-D by using hidden-line or hidden-surface al-gorithms to remove obscured por-tions of scenes. When extrapolated to n dimensions, the problem of oc-clusion changes, because 2-D poly-gons no longer obscure n-dimen-sional objects. To generate a pic-ture of n-space with hidden ele-ments removed, n-dimensional

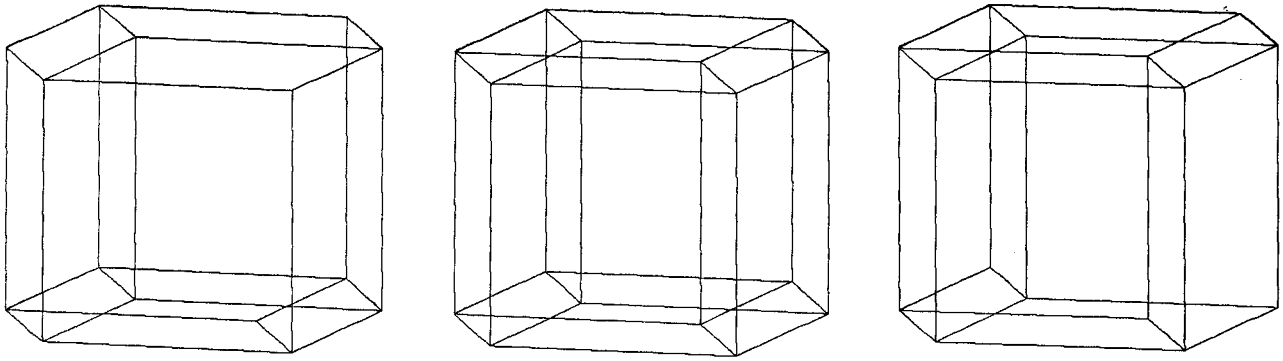

Figure 4: Hidden lines removed based on w, no hidden lines removed, and hidden lines removed based on z.

objects must be defined by the (n − 1)-dimensional objects that bound them; it is against these that objects must be tested for occlusion. These (n − 1)-dimensional bounding faces are also defined by their (n − 2)-dimensional bounding faces, and the process is repeated until 0-dimensional bounding faces (points) are defined by their coordinates.

The process of removing obscured elements from a scene is simplified by further restricting the bounding faces. One restriction requires all points in each (n − 1)-dimensional bounding face to lie in the same (n − 1)-dimensional hyperplane in n-space. This is the "flatness" that makes polygons convenient for defining solid objects. This requirement forces all points in a bounding face to have a common normal. Normals facilitate both removal of back-faces and testing to determine on which side of a face a point lies. A hidden-line algorithm must determine which portions of each line are obscured by any bounding face. Various types of sorting, invisibility or minimax tests can simplify the testing of each line against each bounding face. The actual intersection of a line and bounding face is determined from the equation of the bounding face (given by its normal and a vertex) and that of the line. Research into hidden-volume algorithms is currently being pursued by Victor Steiner.

Interposition projects an n-dimensional object into an image in n − 1 dimensions. It could theoretically project those objects to n − 2 or higher dimensions, but the obscuring that takes place with each projection destroys information from the previous projection. For this reason, occlusion should usually be applied to a single dimension. This limits the usefulness of interposition to projecting four dimensions into three, requiring some form of 3-D transparency in the resulting display to prevent occlusion in 3-space. *Figure 4* shows a hypercube with hidden lines removed according to their z or w coordinates.

Hidden-element algorithms, particularly in n dimensions, require large amounts of computation. This makes interposition difficult to apply during realtime display of hyperdimensional objects, hampering its effective use as a perception cue. The computations can be reduced by requiring hyperobjects to be convex and non-interpenetrating, permitting the realtime display of simple, convex objects such as the hypercube.

Disparity: Stereo vision is such a powerful cue that it is difficult to believe that depth can be perceived with a single eye. Computer-generated images from slightly different viewpoints can be presented to each eye by a variety of means to preserve the effects of binocular vision. Extrapolating stereo to n dimensions, two slightly displaced viewpoints in n-space are computed from two different (n − 1)-dimensional projections. Rather than compute two viewing transformations, it is possible to implement stereo as a single transformation corresponding to a point midway between the two eyes. After this viewing transformation has been applied, stereo images are computed by two rotations in the x_1x_n-plane. The stereo effect arises due to the discrepancies between the x-coordinates in the two images, detected when they are presented t[o] each eye. Taking advantage of th[e] fact that x_n is otherwise ignored the stereo projection is expresse[d] by a matrix in 4-space. Adding an[d] then subtracting w′ from x′ com[]putes the two stereo images. *Fig[]ure 5b* shows the hypercube wit[h] disparity computed from the w-co[]ordinates, while *5c* shows the sam[e] hypercube with disparity com[]puted from the z-coordinates (som[e] people can binocularly fuse the tw[o] images by crossing their eyes an[d] focusing on the center image).

Each of the stereo images from [n] dimensions has n − 1 dimensions. Stereo projection can also be ap[]plied to these images to generat[e] four (n − 2)-dimensional images which can generate eight (n − 3)-dimensional images. The proces[s] repeats, doubling the number o[f] images with each projection, unti[l] displayable images are obtained.

Having only two eyes makes the presentation of these multiple images to separate eyes impossible. Remembering that stereo is a depth cue because of discrepancies between the images, it may be possible to achieve a feeling for dimensionality by placing the images next to each other. This would be like trying to understand depth in two stereo images placed side by side. A better solution is to alternately display the images in the same position—spatially superimposing the images should make discrepancies between them more easily noticed. When Bela Julesz displayed an ordinary random-dot stereogram of an elevated square in this manner, the square was perceived to oscillate back and forth in front of a background. This technique could be especially effective in projections of 4-space, alter-

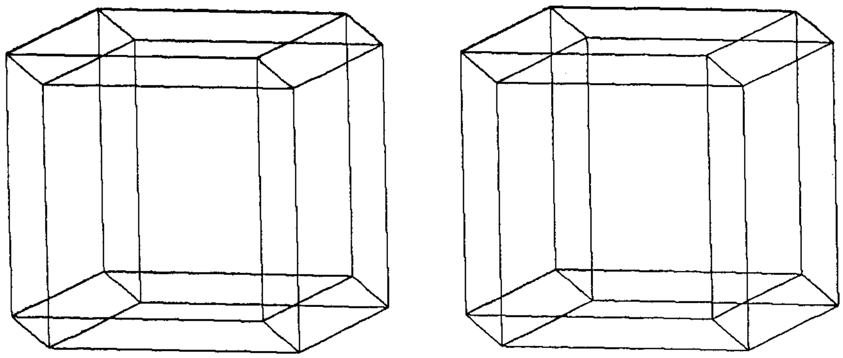

Figure 5a: Increased disparity.

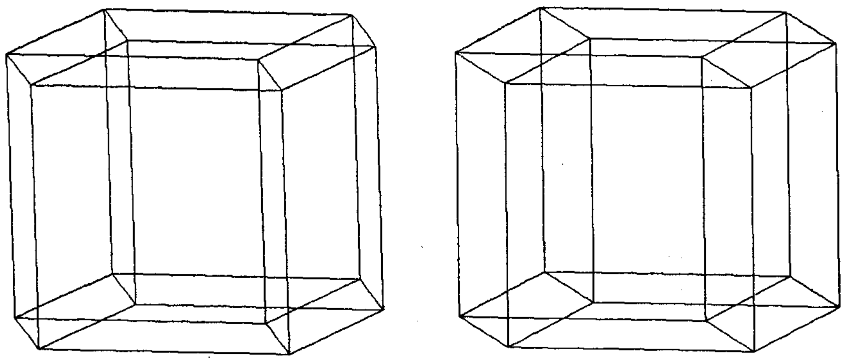

Figure 5b: Disparity computed from w.

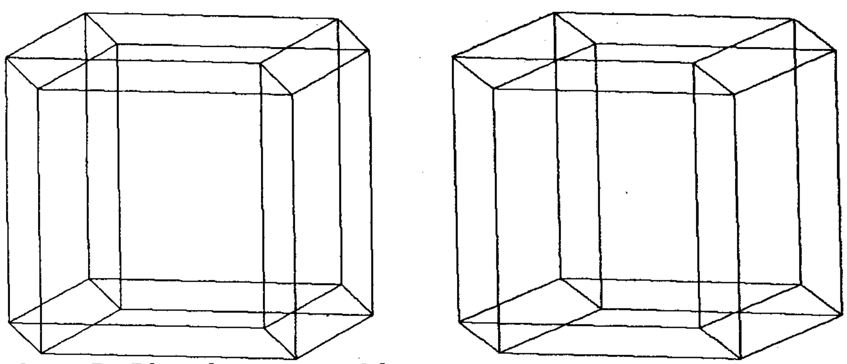

Figure 5c: Disparity computed from z.

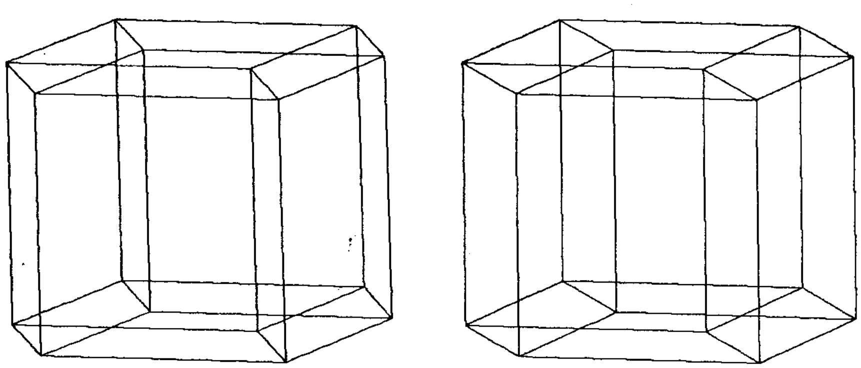

Figure 5d: Disparity computed from both w and z.

nately depicting two stereo images. Still another solution is to create only two images by first adding all the displacements, then subtracting them and viewing the resulting images with a stereo display. The effects of disparity from both w and z are shown in *Figure 5d.*

Disparity need not be restricted to the x-coordinate. *Figure 5e* shows disparity from w in y-coordinates (vertical disparity). These images do not naturally fuse binocularly because our minds are accustomed to decoding only the horizontal disparity as seen by our eyes. Disparity can also be applied from more than one dimension. *Figure 6* uses vertical disparity to encode information from w, then applies horizontal disparity, again from w, in the two resulting images to yield four "stereo" images. *Figure 7* is similar, but uses z to compute horizontal disparity. The same possibilities exist for displaying these images as before.

David Brisson created depictions of 4-D objects which he called "hyperstereopticons" by adding displacements in yw and xz to one image and subtracting them in another. The two resulting images were displayed in a stereoscope. Both pairs of diagonal images in *Figure 6* are "hyperstereopticons."

Multiple Cues for Depicting n Dimensions

In the extrapolation of depth cues, it has been shown how perspective or disparity can project multiple dimensions to fewer dimensions. Interposition was extrapolated to n dimensions, but could not be applied to multiple dimensions.

Because they are implemented by separate techniques, computer-generated images allow a dissociation of cues not possible in 3-D models. Interposition is applied from w and perspective from z in *Figure 8a*, perspective from z and disparity from w in *8b*, and disparity from z and interposition from w in *8c*. This shows the wide variety of different depictions possible through different uses of depth cues. Other cues would make possible the display of still more dimensions. The dissociation of cues should be particularly effective since infor-

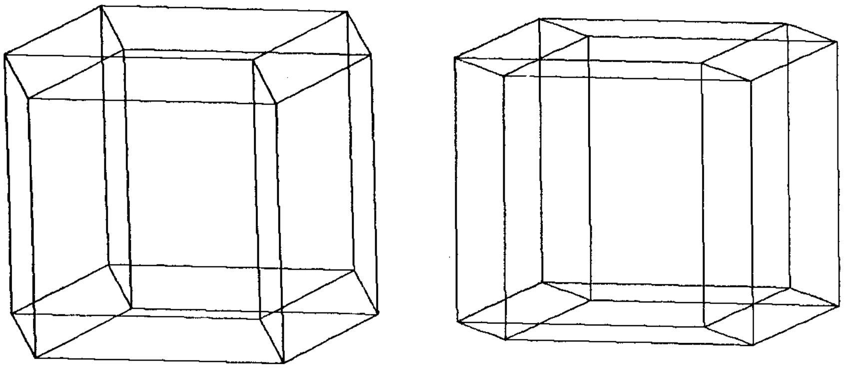

Figure 5e: Disparity computed from w and applied to y instead of x.

mation concerning different coordinates is encoded in completely different ways.

In natural vision, cues occur together and reinforce a feeling of depth. From higher dimensions, all cues may be applied using a single coordinate to project n dimensions to n − 1 dimensions (i.e., perspective is calculated in each of two stereo images, and obscured elements are then removed and other cues provided). The process may be repeated until displayable images are obtained *(Figure 9)*.

Deciphering Cues

A principal problem in deciphering information about n dimensions encoded using depth cues is the potential for ambiguous interpretation. Whenever n dimensions are projected into n − 1 dimensions, a common point is made to correspond to an infinite number of points. Because of this, a projection is always ambiguous—it can represent an infinite number of different objects equally well. This is evident in the Necker cube shown in *Figure 10* where cube A can be interpreted as either B or C. Depth cues clarify the interpretation; removing hidden lines makes cubes B and C readily identifiable. But it should be noted that the resulting figures are still ambiguous—for example, the interior corners could be either protruding or receding. A rotating image (particularly one with hidden lines removed) would result in a stronger conviction of the nature of the cube, but it could still represent a nonrigid object distorting. Psychologists do not fully understand why we "see" only one of the many possible interpretations. It is known that as more cues reinforce one interpretation, the tendency to choose that interpretation increases. But even if all cues correspond to a single interpretation, the projection remains ambiguous.

Though ambiguity can never be eliminated in projections, some uses of cues introduce more ambiguity than others. This is true when a single cue conveys information concerning multiple coordinates. For example, if identical viewing distances are used to compute perspectives from four dimensions to three and from three dimensions to two—which is particularly true in parallel projections—information concerning w and z is completely ambiguous. One effect is to cause rotations in the xw-plane to appear identical to rotations in the xz-plane. Therefore, understanding the location of points in z and w is difficult. Varying the viewing distances used in each projection makes the w- and z-coordinates differ in the degree of perspective caused by each. This distinction is subtle but discernable in rotating objects. A similar problem is evident in disparity, although the physical presence of distinct images may lessen the confusion.

Using a different cue to encode each coordinate avoids problems caused by using a single cue to en-

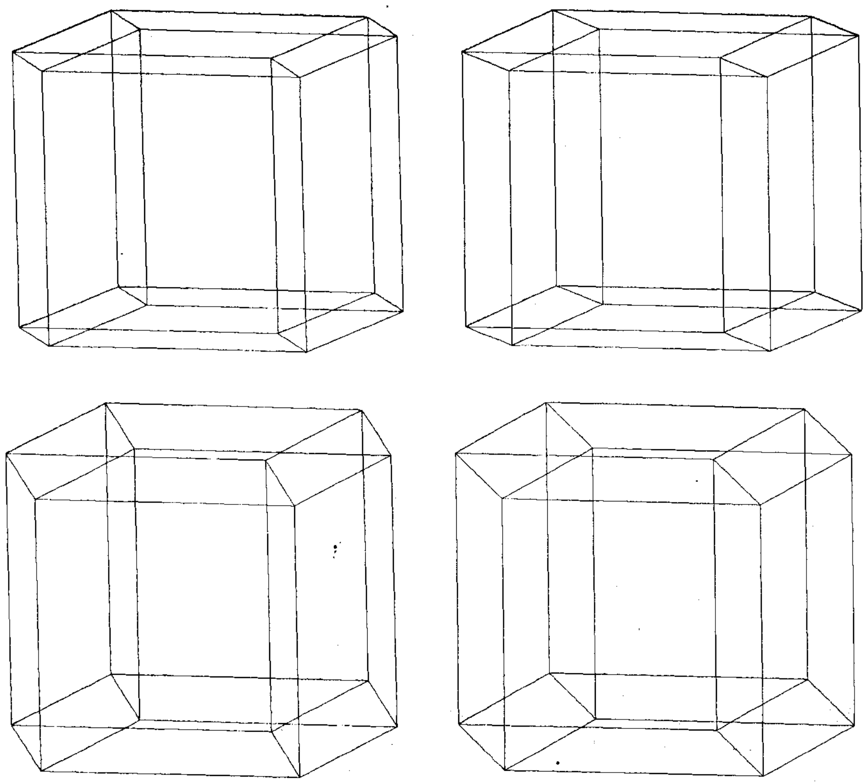

Figure 6: Vertical disparity computed from w, then horizontal disparity computed from w.

code multiple coordinates. This creates discernable differences in the effects of coordinates in the image of a hyperobject; however, visually deciphering the information still may not be possible. Our minds interpret all cues as depth information and seem to ignore information that contradicts what is perceived as depth. This confusion is evident in the projection of a hypercube *(Figure 8b)* where we see a cube within a cube. When rotated in the xz-plane, the cube-within-a-cube appears to spin about the y-axis. But when rotated in the xw-plane, our depth perception interprets the depiction as a cube in front of another cube spinning about an axis between them. It comes as no surprise that the hypercube appears to distort when rotated in both the xz- and xw-planes. Whether our minds can be taught to interpret information from cues as pertaining to a spatial dimension other then depth is a subject of continuing research.

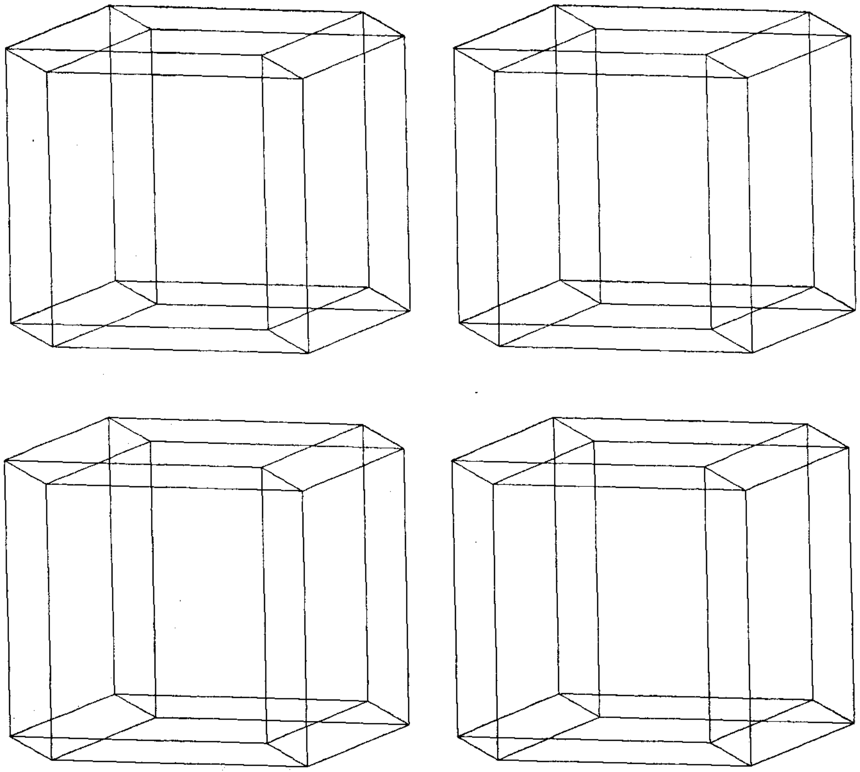

Figure 7: Vertical disparity computed from w, then horizontal disparity computed from z.

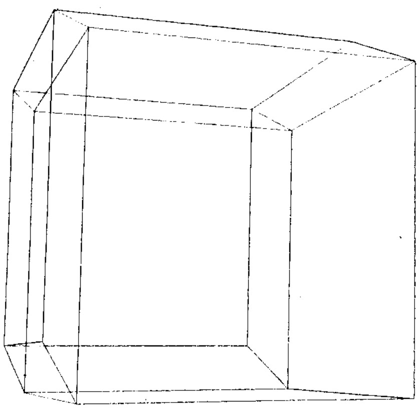

Figure 8a: Interposition to encode w and perspective to encode z.

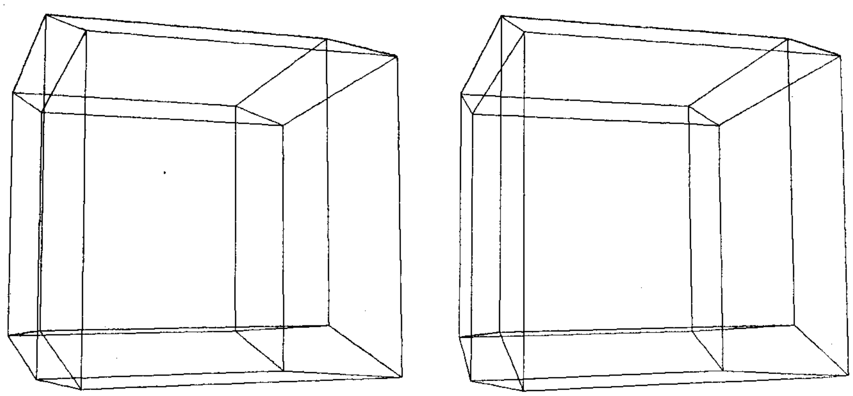

Figure 8b: Disparity computed from w and perspective from z.

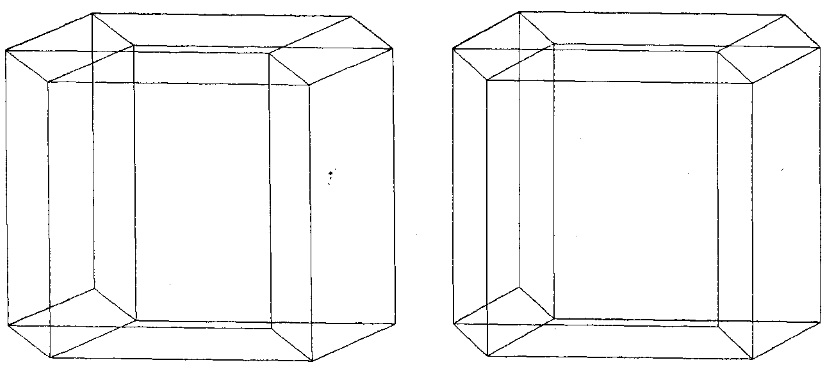

Figure 8c: Disparity computed from z and interposition from w.

Conclusion

Numerous techniques for creating displayable images of hyperdimensional objects have been presented. Confusion may arise in understanding the images because of ambiguities due to the encoding process and because our minds attempt to interpret cues as depth information. We conclude that by using depth cues it is possible to encode n-coordinates in fewer coordinates while retaining clues to the values of n-coordinates. Some of the techniques outlined appear

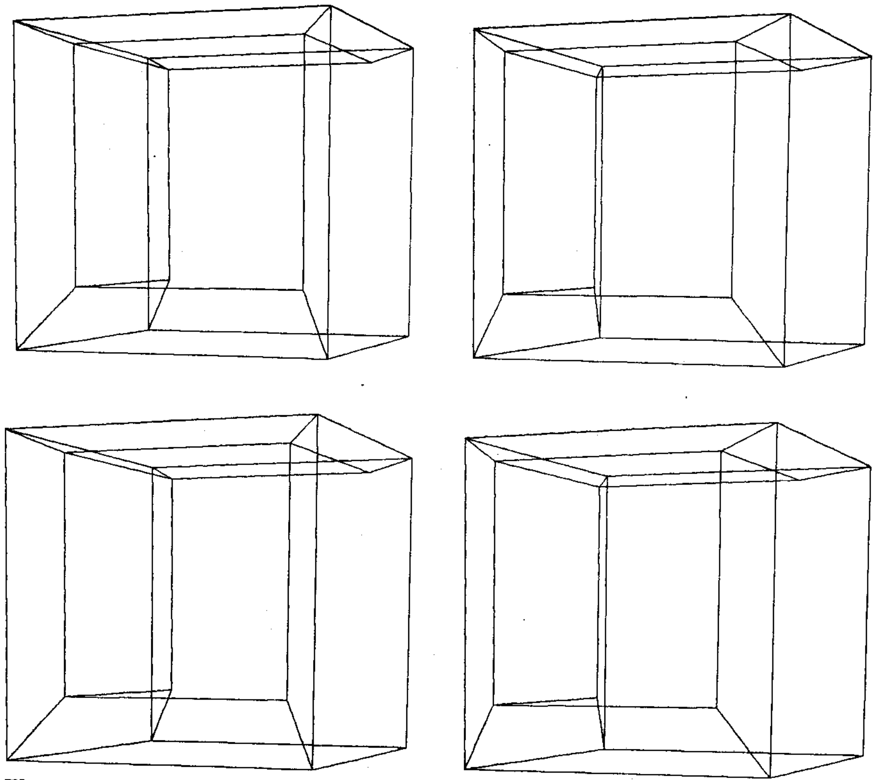

Figure 9: Perspective, interposition, and disparity to project 4-D to 3-D and perspective and disparity to project from 3-D to 2-D.

to retain those clues in a discernable fashion. Further research will determine if the information thus encoded is indeed decipherable. With increased investigation, such techniques may prove useful in helping to comprehend hyperdimensional objects. ■

References

Armstrong, William P. "Hyperdimensional Graphics: The Computer Display of Objects in n Dimensions." Master's thesis, Brigham Young University, 1981.

Banchoff, Thomas F. "Realtime Computer Graphics Analysis of Figures in Four-space." *Hypergraphics*. Boulder, CO: Westview Press, 1978.

———, and Charles M. Strauss. "Real-time Computer Graphics Techniques in Geometry." *Proceedings of Symposia in Applied Mathematics* 20, 1974.

Brisson, David W. *Hypergraphics*. Boulder, CO: Westview Press, 1978.

Burton, Robert P. and David R. Smith. "A Hidden-Line Algorithm for Hyperspace." *SIAM Journal on Computing*, February 1982.

Epstein, William. *Varieties of Perceptual Learning*. New York, NY: McGraw-Hill, 1967.

Isaacson, Paul L. "Hypothesized 4-D Phenomena." (Film) Brigham Young University, 1984.

———, Robert P. Burton and Douglas M. Campbell. "Presentation of Hypothesized 4-D Phenomena." *Computer Graphics World*, August 1984.

Julesz, Bela. *Foundations of Cyclopean Perception*. Chicago, IL: University of Chicago Press, 1971.

Liu, Mei-chi, Robert P. Burton and Douglas M. Campbell. "A Shadow Algorithm for Hyperspace". *Computer Graphics World*, July 1984.

Montgomery, Kent M. "A Constructive Solid Geometry Scheme for Representing Multidimensional Graphical Information." Master's thesis, Brigham Young University, 1984.

Murch, Gerald M. *Visual and Auditory Perception*. Indianapolis, IN: The Bobbs-Merrill Company, 1973.

Noll, A. Michael. "Computer Animation and the Fourth Dimension." AFIPS Conference Proceedings, December 1968.

——— "Computer Technique for Displaying n-dimensional Hyperobjects." *Communications of the ACM*, August 1967.

Reeder, Robert D. "Hypercircumvolution." (Film) Brigham Young University, 1980.

——— "The Presentation of Hyperdimensional Scenes: A Depth Cue Approach." Master's thesis, Brigham Young University, 1980.

Steiner, K. Victor. "A Hidden-volume Removal Algorithm for Hyperspace." Master's thesis, Brigham Young University, 1984.

William Armstrong, currently a software engineer at Evans & Sutherland Computer Corp., received an M.S. in Computer Science from Brigham Young University in 1981.

Robert Burton, a Professor of Computer Science at Brigham Young, received a Ph.D. in Computer Science from the University of Utah in 1973.

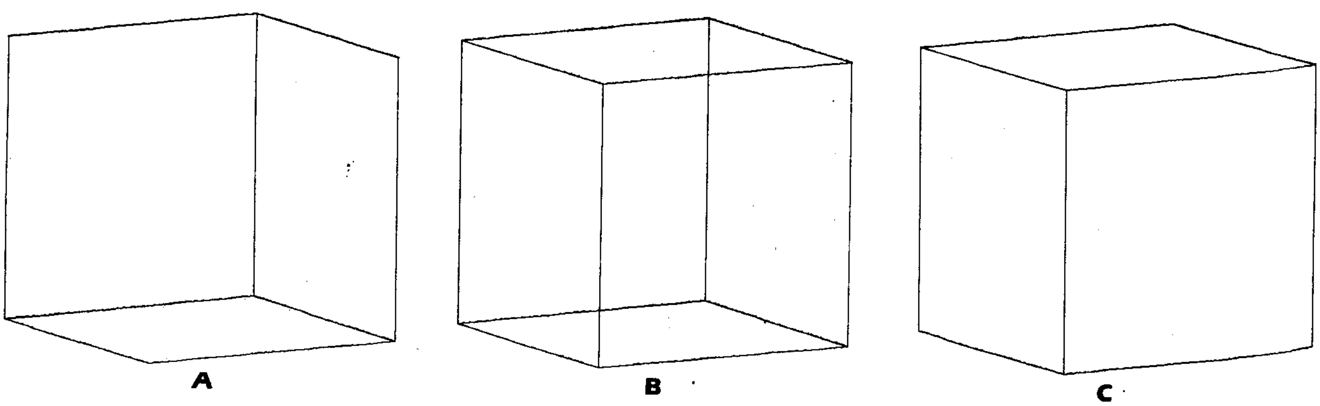

Figure 10: Necker cube A can be perceived as cube B or C.