Einstein's Unified Field Theory Program

Tilman Sauer*

Status: Version of March 2, 2007

Einstein explicitly used the term 'unified field theory' in the title of a publication for the first time in 1925. Ten more papers appeared in which the term is used in the title, but Einstein had dealt with the topic already in half a dozen publications before 1925. In total he wrote more than forty technical papers on the subject. This work represents roughly a fourth of his overall oeuvre of original research articles, and about half of his scientific production published after 1920.

This contribution is an attempt to characterize Einstein's work on a unified field theory from four perspectives, by looking at its conceptual, representational, biographical, and philosophical dimensions. The space spanned by these four dimensions constitutes Einstein's unified field theory program. It is characterized by a conceptual understanding of physics that provides the foundational physical knowledge, the general problems, and the heuristic expectations. The mathematical representation both opens up and constrains the possibilities of elaborating the framework and exploring its inherent consequences. The biographical pathway, historically contingent to some extent, takes Einstein from the elaboration of one approach to the next. Finally, Einstein's most vivid philosophical concern was at the heart of this enterprise, and his insistence on the possibility and desirability of a unified field theory cannot be understood without fully acknowledging his philosophical outlook. Somewhat paradoxically, the philosophical dimension of his unification endeavors not only reveals him as being deeply rooted in the 19th century and its intellectual traditions, but also embodies an intellectual heritage that is perhaps just as important to acknowledge as his more widely celebrated achievements.

---

[*] To appear in *The Cambridge Companion to Einstein*, M. Janssen, C. Lehner (eds), Cambridge University Press, forthcoming. Please, ask before quoting or circulating.

Einstein's unification program was a program of reflection. It aimed at understanding a given content of knowledge in physics in a different way, at what Einstein considered an understanding of this content in an emphatic sense. This understanding was to see the whole of physics as an organic entity, where no part can be separated from any other without severe loss of meaning. The underlying motivation for this program of reflection, I want to suggest, was a conception of the task of human reasoning that would be adequate to a holistic understanding of a nature in which human beings live their lives. The explanation of isolated phenomena by subsuming them under general laws was a valid aspect of this task but it concerned, to pick up a terminological distinction of German idealism, our *Verstand* only. Human *Vernunft*, on the other hand, aimed at an understanding of nature in toto, with human beings living and acting in it. From this perspective, Einstein's political interventions flow out of the same philosophical world view as, in later years, a unified field theory would. Moreover, this philosophical outlook was also driving his creative productivity of his early years (Renn 1993, forthcoming). For an adequate historical and philosophical understanding of Einstein's intellectual contributions, one must take seriously Einstein's scientific activities that, although puzzling to many, he pursued with steadfast conviction despite many disappointments, in ever increasing scientific isolation, and without finding a solution.

An account of Einstein's work on a unified field theory that would go into technical detail is beyond the scope of the present contribution. We still lack fine grained historical investigations of his later work on which we could base such an account, i.e. investigations that would discuss his endeavors with technical understanding from a historical point of view and that would take into account his unpublished correspondence and manuscripts.[1] Any discussion of his work in this direction on a technical level also immediately requires a substantial amount of mathematical preliminaries, the exposition of which would either be lengthy or run the risk of being accessible only to a limited audience. Nevertheless, by discussing the dimensions of Einstein's unification program I hope to show that a full appreciation of these different dimensions is essential for a proper *historical* understanding of this substantial aspect of Einstein's intellectual heritage.

1 The conceptual dimension

It has been argued that Einstein's attempts at a unified field theory are to be seen as a continuation of heuristic aspects of his celebrated conceptual breakthroughs to special and general relativity (Bergmann 1979). With respect to special relativity, the reconsideration of the concept of simultaneity in frames of reference that are in relative motion to each other had indeed solved an outstanding conceptual contradiction among foundational aspects of classical physics. Specifically, the redefinition of simultaneity allowed Einstein to resolve the conflict between the universal validity of the principle of relativity of Newtonian mechanics and the principle of the constancy of the velocity of light by providing a conceptual justification for the Lorentz transformations. He thus successfully integrated two major fields of physics in what amounts to a conceptual unification.

But solving the conflict between the principles of relativity and of the constancy of the velocity of light in special relativity had created another problem, to be addressed by a relativistic theory of gravitation. The conceptual foundation of special relativity demanded the existence of an absolute and finite limit to the speed of any signal transmission. This fundamental aspect of special relativity was violated by Newtonian gravitation theory. A gravitational interaction between massive bodies that was an instantaneous action-at-a-distance posed a contradiction to the postulate of the non-existence of any signal transmission speed exceeding the speed of light. Closely related to this conceptual conflict was another inherent difference between the classical theories of gravitation and of electromagnetism. Newtonian gravitation theory conceptualizes the gravitational interaction as a static inter-particle interaction. Maxwellian electromagnetism, on the other hand, conceptualizes the electromagnetic interaction in terms of a dynamic field concept. The difference becomes most obvious by considering electromagnetic waves. In Maxwellian field theory, Coulomb's law is a very special case, and all dynamic effects propagate in electromagnetic waves with a finite speed. The Maxwell equations even allow for the existence of electromagnetic waves in vacuum without presupposing the existence of electric sources that would generate the field.

In addressing these issues, Einstein took a decisive turn very early in his investigations by linking these problems to the problem of generalizing the special theory of relativity to non-inertial, accelerated frames. By working out the implications of the equivalence hypotheses, he arrived at a general theory of relativity and a relativistic field theory of gravitation. This theory provided a resolution of the conceptual contradictions between classical gravitation theory and the Maxwellian theory of the electromagnetic field. Indeed, the general relativistic theory of gravitation conceptualized the gravitational force as a field, and one of the first elaborations of the new theory, after his breakthrough to the generally covariant field equations of gravitation in late 1915, concerned the existence of gravitational waves.

But, the success of this integration came at a price. The gravitational interaction was conceptualized as a dynamical field by a geometrization. Einstein now believed that his geometrized relativistic theory of the gravitational field demanded again a unification with the concept of the electromagnetic field. The latter had essentially been left unaffected by the redefinitions of the gravitational interaction brought about by the theory of general relativity.

Einstein thought that the new understanding of gravitation demanded further unification with classical Maxwellian theory of the electromagnetic field. But the conceptual differences between the geometrized gravitational field and the classical Maxwell field did not amount to an open or even only to a hidden inherent contradiction. There was no compelling reason why the electromagnetic field should be reconceptualized following the example of the relativistic gravitational field.

Nevertheless, the successful geometrization of the gravitational field in the general theory of relativity motivated many contemporaries to look for a geometrized theory that would also include the electromagnetic field. Unification at this level pertained to the conceptualization of the two known fundamental fields, the gravitational and the electromagnetic fields. Intrinsic criteria of validation or refutation for attempts at unification were not too sharply defined.[2] They included, first of all, the demand that the known equations for the gravitational and electromagnetic field could be obtained in some limiting case. The extent to which a unified description of the two fields then actually represented a conceptual unification was subject to different criteria. The mathematical representation should assign symmetric roles to the gravitational and the electromagnetic fields in some unspecified sense. In a stricter sense, the representation of the two fields should not be decomposable into two independent sets of equations governing the gravitational and the electromagnetic field. In other words, the unified description should inherently allow for some kind of non-trivial mixing of the two fields. But this mixing might occur on a purely representational level and thus might be principally unobservable. This latter postulate did not, therefore, also entail necessarily the even stronger condition that the unification must also, at least in principle, predict new physical effects.

The most desirable case of a unified description that would both yield the known laws of gravitation and electromagnetism and would also predict new effects, arising from a combination of the fields inherent in the unified description, and that would also be compatible with known empirical facts, has never been achieved. Nevertheless, many theoreticians would applaude progress toward this goal even if it only achieved a more restricted unification or if it only promised the prospect of achieving a true unification.

But the unified field theory program involved even more than these postulates. A unified conceptualization and mathematical representation of the gravitational and electromagnetic fields in the sense discussed so far was only one aspect of the conceptual dimension of the unification problem in physics at the time. A second aspect, independent of the unification of the fields, concerned the representation of matter.

In Newtonian mechanics as well as in Lorentz's interpretation of Maxwellian electromagnetism, the existence of masses or material points and of charges and currents was independent of the fields. Equations of motion were determined by the interaction of the charged particles with the fields as a set of independent, additional equations of the theory. The co-existence of particles and fields did not represent an inherent contradiction or inconsistency, but was commonly referred to as a dualism,[3] and as such was regarded as a violation of the ideal of unification.

More specifically, it had been Gustav Mie who, in 1912, had proposed a way to overcome this dualism of particles and fields that was attractive to many physicists at the time (Mie 1912). Mie's idea was to look for nonlinear modifications or generalizations of Maxwell's equations that would allow for particle-like solutions of the electromagnetic field. We should be able to interpret a solution as a particle if it is spherically symmetric, if the field intensity is very high only in a finite region of space, and if the field equations imply sensible equations of motion for those localized maxima of the fields. Unfortunately, Mie's only explicit example had not provided a viable model for this program. It implied that two such particle-like solutions would necessarily move towards each other and merge. Hence no stable configuration of matter was possible for this special case. Nevertheless, as a research program Mie's idea had much appeal for many physicists and mathematicians, among them David Hilbert (Sauer 1999, Corry 2004, ch. 6), Hermann Weyl (Scholz 2004), and Einstein.

The basic properties of matter to be accounted for were the existence of two elementary particles, the electron and the proton. Each carried an elementary charge whose absolute value was the same, the electron as a negative charge, the proton as a positive charge. Although debated for a while in the early 1920's, there was no evidence for the existence of fractions of the elementary charge. Also, the electron has a definite rest mass, as has the proton, and the mass ratio between the proton and the electron is numerically of the order of 2000.

In the early thirties, this situation changed with the discovery of additional elementary particles (Kragh 1999, chap. 13). In 1932, both the neutron and the positron were discovered experimentally. The existence of an electrically neutral particle with the same mass as the proton had been discussed as early as 1920 by Rutherford, and the existence of a positively charged electron had been predicted by Dirac in 1931. But since the neutron has roughly the same mass as the proton but does not carry an electric charge, and since the positron has exactly the same rest mass as the electron but has positive elementary charge, the existence of these two elementary particles did not seriously challenge the matter aspect of Einstein's unified field theory program. The only consequence was that the unified theory no longer had to account for the *non*-existence of neutral particles with proton mass or of a positively charged electron. The postulate and existence of the positron also raised the question of antiparticles to the proton and the neutron. The antiproton and antineutron were only postulated in 1931 and 1935, respectively. They were actually discovered only after Einstein's death in 1955 resp. 1956.

More serious from the conceptual point of view of the unified field theory program was the postulate and discovery of mesons. The muon was discovered in 1937. The charged pion was postulated in 1935 and discovered in 1947, and the neutral pion was postulated in 1938 and discovered in 1950. It is with the discovery of these particles that the parameters for a unified field theory actually do change. By the mid-fities, at the time of Einstein's death, a dozen elementary particles were known. Ten years later the list of experimentally confirmed subatomic particles ran to about a hundred. With the discovery of elementary particles beyond the electron and proton, twentieth century physics also had to deal with two new kinds of fundamental interactions. The weak interaction which plays its most prominent role in the explanation of radioactive decay, and the strong interaction which came with the discovery of mesons and baryons other than protons and neutrons and their decays.

To the extent that the existence of mesons can be regarded an experimentally well-established fact during Einstein's lifetime, the conceptual dimension of a unified field theory no longer encompasses only two distinct elementary masses and a single elementary charge. To be sure, the history of the discovery of elementary particles is complex and what is identified in hindsight as the discovery of an elementary particle may have appeared less convincingly so to Einstein and his contemporaries. In any case, Einstein's unified field theory program was committed to the 'two-particle paradigm' (Kragh 1999, p. 190) of having to account for the existence of only a proton and an electron, or, we may say, to a generalized two-particle paradigm which would recognize only two distinct elementary masses, i.e. proton mass and electron mass, and one elementary charge, if one allows for the existence of neutrons and positrons as well.

The postulates of Einstein's unified field theory programs are still not exhausted with the conceptual unification of the gravitational and electromagnetic field and the accounting for two fundamental matter particles. The existence of an elementary charge as well as of elementary electron and proton masses already represented elements of discontinuity that had to be incorporated and justified in a field theoretical, i.e. essentially continuous conceptualization of matter. The development of quantum theory had made it clear that further elements of discreteness had to be represented in the field theoretic framework. Most prominently, these quantum aspects were evident in processes of emission and absorption of electromagnetic radiation by matter. With the development of relativistic quantum mechanics and quantum electrodynamics, the fields were being quantized, too, and many theoreticians at this point gave up looking for a classical unified field theory of the gravitational and electromagnetic fields.

The third aspect of Einstein's unification program therefore concerned quantum theory. At least on a programmatic level, Einstein did not ignore the theoretical advances in quantum field theory. But for Einstein, the empirical success of non-relativistic quantum mechanics did not demand that the unified description of the fundamental fields itself should be a quantized one. Rather the foundational unified conceptualization should somehow provide a conceptual justification for the stochastic aspects of quantum theory and for its violations of classical determinism. Also, whereas others began working on bringing together special relativity and electrodynamics with quantum theory, leaving aside the question of how the gravitational interaction would fit into the scheme, Einstein did not alter his priorities. For him, unification efforts had to start from a theory of the gravitational field and hence be general relativistic. Again, the early conceptual problems of interpreting and dealing with intrinsic problems of quantum field theory (negative energy solutions, etc.) and of quantum electrodynamics (infinities and renormalization procecedures) may have given him good reason to do so.

The fact that he considered quantum theory only a preliminary theory, highly sucessful phenomenologically but unjustifed on a foundational level, allowed him to concentrate on attempts of a classical theory without putting himself to the task of explaining quantum theory in the first step. To account for the characteristic consequences of quantum theory remained a goal of his unification endeavours. But in most cases no specifics of the quantum aspects of matter and radiation would or could be accounted for. The explanation of quantum mechanics within a unified field theory remained a programmatic desideratum in Einstein's work.

2 The representational dimension

The crucial mathematical concept that enabled the original formulation of general relativity is the metric tensor field $g_{\mu\nu}$. In general relativity, it plays a double role. On the one hand, it is at the heart of general relativity as a physical theory, by virtue of its meaning for space-time measurements. On the other hand, it is the mathematical object that is determined by the field equations of gravitation. In four-dimensional space-time, the field of a real, symmetric, second-rank metric tensor is a ten-component tensor, i.e. its definition in a given coordinate chart requires specification of ten independent functions, or ten numbers $g_{\mu\nu}(x^\mu)$ at each point x^μ of some open set of the space-time manifold. The tensor character then demands that these ten functions transform in a specific way when we change the coordinates.

The gravitational field equations at the heart of general relativity are 10 coupled partial differential equations for the components of the metric tensor. General covariance accounts for four identities among them, but the complexity of the solutions for such a system of coupled partial differential equations is such that the space of solutions cannot be surveyed analytically in its full extension. In certain special situations of high symmetry, or other restrictive conditions, analytical solutions are available. Schwarzschild's solution for spatial spherical symmetry is the most prominent among them. But even today most interesting cases can only be handled by extensive numerical calculations, if at all.

The field equations of general relativity are almost completely determined by the postulate of general covariance, the demand that second derivatives of the metric occur at most linearly, and by some boundary conditions, e.g. the demand of regular behavior at infinity. The only freedom that was left within those conditions was the addition of a cosmological term which Einstein introduced in 1917. The framework of the original theory of general relativity, however, did not allow inclusion of the electromagnetic field in any non-trivial sense of unification. In order to achieve unification, the representation therefore unavoidably had to be transcended.

Several ways of extending the representational framework for a unified field theory are possible and have been considered by Einstein and other researchers. Briefly, these extensions either consist in relaxing the conditions imposed on the original formulation, or in introducing other mathematical objects into the theory. Along the first option, one can explore the consequences of giving up the restriction to the requirements of reality or symmetry of the metric tensor components. One may also relax the restriction to four-dimensional space-time by considering higher-dimensional representations. Finally, one may relax the condition of locality of a field theory by introducing non-local dependencies into the theory.

Another possibility that opens up new ways to achieve a unification is the introduction of other mathematical objects into the mathematical framework, specifically by using other quantities as the dynamical variables of the theory. The most prominent example arises from the concept of the affine connection that was introduced into the theory of general relativity shortly after its initial formulations mainly through the work of Tullio Levi-Civita and Hermann Weyl. Another possibility that was used to explore ways of unification was to conceive of the field of tetrads, i.e. fields of point-dependent orthonormal bases of the tangent spaces at each point of the space-time manifold, as the fundamental dynamical variables.

Each of the alternatives opens up a horizon of possibilities inherent in the mathematical representation that can be explored for its suitability for the unification program. In principle, any kind of combination of those extensions is possible, too. The problem then arises in each and every case to secure the possibility of a physical interpretation, to identify a set of field equations that replaces the gravitational and electromagnetic field equations of conventional general relativity, and to get a hold on the manifold of solutions for those field equations. Even before the discovery of further elementary particles the

1919–1922	Weyl, Kaluza, Eddington
1923	affine theory (Eddington)
1923	overdetermination
1925	metric-affine approach
1927	trace-free equations
1927	five-dimensional approach
1928–1931	distant parallelism
1931	semivector theory
1938-1941	five-dimensional approach
1944	bivectors
1945–1955	asymmetric theory

Table 1: Einstein's different approaches along the unified field theory program as discussed in the text.

unified field theory program was a research problem that easily transcended the intellectual capacities of a single researcher.

3 The biographical dimension

In this section, we will indicate, in rough strokes, the major approaches and main steps of Einstein's engagement along his unification program. Einstein's early work on the unification program after the completion of the theory of general relativity was, by and large, a reaction to approaches advanced by others. This is the case for the first geometrization of the electromagnetic field, proposed in 1918 by Hermann Weyl; for the first exploration of a five-dimensional theory suggested by Theodor Kaluza in 1919; and for the first attempt to base a unified field theory on the concept of the affine connection, rather than on the metric field, as advanced by Arthur Eddington in 1921.

Weyl's approach was motivated by a more general observation concerning the conceptual foundation of a field theory (Scholz 2001). In Euclidean geometry, we can compare vectors at different points both with respect to their lengths and with respect to their direction. In Riemannian geometry of general relativity, a distant comparison of vector direction is no longer possible. On parallel transport, a vector's direction at a distant point depends on the path along which parallel transport took place, and if transported along a closed loop in a curved manifold, the direction of a vector differs before and after parallel transport along the loop. The length of a vector, on the other hand, remains unchanged in Riemannian geometry. In Weyl's understanding, this was an inconsistency in the conceptual foundations of a true field theory, which should be represented by a truly infinitesimal geometry, in the sense that only assertions about neighboring tangent spaces should be meaningful.

Motivated by these considerations, Weyl introduced another geometrical object into the theory that he aptly called a "length connection." It determined the length of a vector on parallel transport just as the Levi-Civita connection determined the direction of a vector on parallel transport. The surprising thing was that on the level of the mathematical representation, Weyl could link the length connection to the electromagnetic vector potential and thus establish a link between the geometrical structure, given by the length connection, and the electromagnetic field.

Einstein's reaction to Weyl's approach was highly ambivalent. Weyl had sent him a manuscript expounding these ideas, asking him to submit it for publication in the Prussian Academy's *Sitzungsberichte*. On receiving the manuscript, Einstein called it "a first-class stroke of genius" (CPAE 8, Doc. 498), "wonderfully self-contained" (ibid., Doc. 499), and "very ingenious" (ibid., Doc. 500). Nevertheless he had also quickly found a serious objection to Weyl's theory.

The non-integrability of the vector length on parallel transport implied that the wavelength, for example, of light emitted by a radiating atom would depend on the prehistory of that atom. Empirical evidence, however, suggested that the wavelength of light emitted by atoms is only determined by the constitution of the atom, and not by its prehistory. When Einstein presented Weyl's paper to the Prussian Academy and mentioned his own criticism, fellow Academy members vetoed publication. As a compromise solution, the paper was published with an added paragraph in which Einstein put forth his measuring rod objection. As a theory of physical reality, Einstein henceforth considered Weyl's approach "fanciful nonsense" (CPAE 9, Doc. 294). And when Weyl included an exposition of his idea into the third edition of his textbook *Space—Time—Matter*, Einstein judged that Weyl had "messed up" the theory of general relativity (CPAE 9, Doc. 332).

Nevertheless, in March 1921, Einstein picked up on Weyl's theory in print and elaborated on the idea, as a "logical possibility," of giving up the postulate of the existence of parallel-transportable measuring rods as a fundamental assumption of general relativity (Einstein 1921e).

In April 1919, Einstein was confronted with another idea of fundamentally modifying the foundations of general relativity with a view of unifying the gravitational and electromagnetic fields. Theodor Kaluza, at the time *Privatdozent* in mathematics at the University of Königsberg, sent him a manuscript in which he introduced the concept of a fifth dimension to the underlying space-time manifold of general relativity (CPAE 9, Goenner and Wünsch 2003).

The basic idea was to represent the electromagnetic vector potential or the electromagnetic field components in terms of the additional components of the metric tensor resp. of the affine connection that came with the introduction of the additional fifth dimension.

Although physical meaningfulness suggests that we look only at restricted subgroups of the full diffeomorphism group, the demand of general covariance in five dimensions immediately implies that the components of the electromagnetic four-potential and the gravitational components of the metric tensor are mixed up into each other by means of pure coordinate transformations. In this respect, a unification of the two fields was achieved.

The approach is burdened with a number of difficulties on different levels. From an epistemological point of view, the introduction of another spacelike dimension raises the question of the ontological status of this dimension. A somewhat pragmatic answer to this issue was suggested by Einstein himself in his initial reaction to Kaluza's manuscript. If the trajectories of material particles that follow geodesics in five dimensions are projected down to non-geodesic trajectories in four dimensions, one might be able to interpret the deviations from the four-dimensional geodesic path as a direct effect of the electromagnetic field.

On a representational level the following diificulty arose. The five-dimensional metric tensor has 15 independent components. Of these, 10 are interpreted in the usual sense as components of the gravito-inertial field, while four more are associated with the four components of the electromagnetic vector potential. The problem then arises as to the physical interpretation of the fifteenth independent component, g_{55}. In his original paper, Kaluza tried to ignore this problem by arguing that the metric component g_{55} is not truly independent, but rather implicitly determined through the field equations in conjunction with the equation of motions. In later works along the five-dimensional approach, the problem of finding a meaning of the g_{55} component was interpreted differently. One tried to turn this difficulty into an advantage by assuming that the five-dimensional approach inherently introduced a new scalar field, that may be given an independent physical meaning.

A third difficulty was pointed out by Einstein in his initial reading of Kaluza's manuscript and turned out to be a fatal obstacle for Kaluza's plans to publish his paper. An order of magnitude consideration showed that for the equation of motion of an electron, the influence of the gravitational field turned out to be larger by many orders of magnitude than any reasonable physical interpretation would allow for.

Overall, Einstein's reaction to Kaluza's manuscript was just as ambivalent as was his response to Weyl's geometrization. Initially, he was much impressed by Kaluza's idea and considered it physically much more promising than Weyl's "mathematically deeply probing approach" (CPAE 9, Doc. 26). But on pointing out the difficulty of interpreting the g_{55}-term in the equation of motion for the electron, he asked Kaluza for understanding since "due to my existing substantive reservations" he could not himself present it for publication, notwithstanding his "great respect for the beauty and boldness" of the idea (CPAE 9, Doc. 48).

Thus ended their initial correspondence in May 1919, and Kaluza, unable to rebut Einstein's criticism, apparantly did not try to publish his manuscript elsewhere. But Einstein must have continued to think about it and, more than two years later, sent a postcard to Kaluza, expressing second thoughts about his previous rejection of Kaluza's manuscript. On Einstein's invitation, Kaluza now sent a manuscript in which he also expounded the difficulty of finding an interpretation for the different terms in the equation of motion, crediting Einstein with having alerted him to this problem. Einstein in turn now submitted Kaluza's manuscript to the Prussian Academy for publication in its proceedings (Kaluza 1921).

A few weeks later, Einstein and Grommer finished a paper investigating the problem of solutions to Kaluza's five-dimensional theory that are everywhere regular and centrally symmetric (Einstein and Grommer 1923). Their result was that "Kaluza's theory does not contain a solution that only depends on the $g_{\mu\nu}$ and is centrally symmetric and could then be interpreted as a (singularity free) electron" ([Einstein and Grommer 1923, p. VII).

A third approach toward a unified field theory may be called the affine approach. It was advancd most notably by Eddington in the early twenties (Eddington 1921, 1923, ch. VII), and was taken up also by Einstein. The idea was to base the theory on the concept of an affine connection as the fundamental mathematical quantity, rather than on the metric tensor. The starting point is the observation that a manifold that is equipped with a linear affine connection allows the definition of a Riemann curvature tensor and of a Ricci tensor. The latter is, in general, not a symmetric tensor, even if the connection is assumed to be symmetric. Eddington then suggested to identify the anti-symmetric part of the Ricci tensor as the electromagnetic field tensor, and to interpret the symmetric part of the Ricci tensor—after some rescaling—as the usual metric tensor field.

The problem that Eddington had left open was to provide field equations that would determine the affine connection. It is this question that Einstein addressed in a series of three brief notes published in 1923 in the Proceedings of the Prussian Academy (Einstein 1923a, 1923b, 1923c). He proposed to obtain the field equations from a variational principle, and also suggested a Lagrangian function for the field action. But in the course of trying to touch base with familiar general relativistic gravitation theory and Maxwellian electromagnetism, he also performed some transformation of variables and ended up with a variational formulation that, in fact, was almost equivalent to the variational formulation of the general theory of relativity with a Maxwellian field provided by Hilbert in 1915. This eqivalence was highlighted by Hilbert in lectures held in Hamburg and Zurich in the summer and fall of 1923, and prompted his republication of a merged version of his two communications on the "Foundations of Physics" in the *Mathematische Annalen* in 1924 (Hilbert 1924, Majer and Sauer 2005).

A problem in the approach of the affine theory was the proper interpretation of the fundamental variables of the dynamical theory. But also, and more importantly, Einstein found that the theory did not account for the electron-proton mass asymmetry and that no singularity-free electron solution seemed possible.

With Einstein's response to Weyl, Kaluza, and Eddington in the early twenties we find him reacting to approaches that had been advanced by others. At the end of 1923, after his work on the affine theory, Einstein published a paper that is again more original. It is entitled: "Does the field theory provide possibilities for a solution of the quantum problem?" (Einstein 1923d).

The point of this paper was to argue that there is, after all, a way to account for quantum phenomena by means of differential equations. Given that partial differential equations had been so successful in classical field theory, Einstein found it "hard to believe that partial differential equations would not be, as a last resort, suitable for an explanation of the facts."[4] The principal solution that he advanced in this paper was to generate discontinuities by overdetermining the classical variables with more differential equations than field variables. Although he gave some more technical details as to how this approach was to be understood, Einstein admitted that he was still unable to show concretely how it would solve the quantum problem. In fact, the idea had been on his mind for some years. Indeed, we find Einstein contemplating this issue as early as 1920:

> I do not believe that one must abandon the continuum in order to solve the [problem of] quanta. In analogy, one might have thought it possible to force general relativity from abandoning the coordinate system. In principle, of course, the continuum could be abandoned. But how in the world should one describe the relative motion of n points without a continuum? [...] I still believe as before that an overdetermination ought to be sought with differential equations for which the *solutions* no longer have any continuum properties. But how?[5]

The first original approach put forward by Einstein himself was published in a paper of 1925 in which also the term 'unified field theory' appeared for the first time in a title (Einstein 1925). In that paper, he explored a metric-affine approach, i.e. he took both a metric tensor field and a linear affine connection at the same time as fundamental variables. Both connection and metric were assumed to be asymmetric. Parallel transport then again defines a Ricci tensor and a Riemann curvature scalar, and Einstein defined tentative field equations in terms of a variational principle, taking the Riemann scalar as a Lagrangian just as in standard general relativity. As regards the interpretation of the mathematical objects, he tried to associate the gravitational and electromagnetic fields with the symmetric and antisymmetric parts of the metric field. In his attempt to recover the known cases, he could show that the metric was symmetric for the purely gravitational case and the usual compatibility condition for the Levi-Civita connection can be recovered. Maxwell's equations could be recovered, in the limit of weak gravitational fields, but only in a slightly different form that is not entirely equivalent to the original equations.

The basic problem of this approach seems to have been that Einstein did not know how to go on from here. Dealing with both an asymmetric metric tensor and an asymmetric connection opened up a vast field of possibilitites inherent in the mathematical framework, and many familiar results of the theory of Riemannian geometry no longer held. In particular, verifying the existence of non-singular, spherically symmetric charge distributions posed a formidable challenge. It was also unclear how to explicitly investigate the non-vacuum case beyond the first order approximation of weak gravitational fields. Einstein did not pursue this approach any longer in print but he did take it up once more, twenty years later, as his final approach toward a unified field theory, working on it until his death.

In 1927, Einstein published a mathematical note on the geometric interpretation of a modification of the original gravitational field equations of general relativity that he had investigated earlier in 1919 (Einstein 1919a, 1927a). The modification consisted in demanding that the equations be trace-free rather than have vanishing covariant divergence. In 1919, the modification had been motivated by considerations concerning the constitution of matter. In 1927, he argued in reaction to a short note by the mathematician George Yuri Rainich, that this modification could be given a geometric interpretation.

That same year, he also published two papers on Kaluza's five-dimensional theory (Einstein 1927b, 1927c) in which he showed that Kaluza's original results can also be obtained without the restriction to weak gravitational fields and slow velocities. It so happened that Einstein reproduced results that had been published only a year before by Oskar Klein, as he acknowledged in a note to his second paper. But whereas Klein's interest in Kaluza's theory was motivated by the wish to account for quantum phenomena within a unified field theory, Einstein does not mention this concern in his two notes.

The sequence of approaches mentioned here as well as Einstein's publication pace is determined not only by an internal logic. It is also determined, to some extent, by contingent external factors. Two such external factors that influenced his productivity in the twenties were his travelling and his health. In spring 1928, while on a visit to Switzerland, Einstein suffered a circulatory collapse. An enlargement of the heart was diagnosed and, back in Berlin, he was ordered strict bed rest. At the end of May he wrote to a friend: "In the tranquility of my sickness, I have laid a wonderful egg in the area of general relativity. Whether the bird that will hatch from it will be healthy and long-lived only the Gods know."[6] A few days later, he indeed presented a short note to the Prussian Academy on a mathematical structure that he called *Riemannian geometry, maintaining the concept of distant parallelism* (Einstein 1928a). This first note was of a purely mathematical nature. It was followed a week later by a second note (Einstein 1928b), in which Einstein explored the possibility of formulating a unified field theory within the new geometrical framework.

In this new approach a spacetime is characterized by a connection with vanishing Riemann-curvature in conjunction with a metric tensor field (Sauer 2006). The crucial mathematical construct that enabled Einstein to formulate a flat space-time that is nonetheless non-Euclidean was the concept of a tetrad field. Tetrads are orthonormal bases of the tangent spaces. Given a field of tetrads, parallel transport is then defined in a natural way, but this connection differs from the usual Levi-Civita connection. Parallel transport along a smooth tetrad field is curvature-free but the manifold is, in general, still non-Euclidean, since it allows for non-vanishing torsion. Torsion of a manifold is characterized by the absence of parallelograms, i.e. if a vector is parallel transported along a closed loop it will coincide with the original vector but if it is parallel transported along four legs of a parallelogram the torsion of the manifold will result in a displacement of the resulting vector from its original position.

Einstein soon was to learn that the mathematical concept of distant parallelism was by no means new and had already been explored by mathematicians, notably by Roland Weitzenböck and Elie Cartan. While immediately acknowledging the priority of others as far as the mathematics was concerned, Einstein nevertheless held high hopes for his idea of formulating a unified field theory within this structure. For him, the critical question was to find a field equation for the components of the dynamical tetrad fields. Each field of tetrads defines a metric tensor field. But the converse is not true, since the metric tensor components can only fix ten of the sixteen components of a tetrad. The additional six degrees of freedom are just what would be needed, so he thought, to accomodate the six degress of freedom of the Maxwell field in a unified description of gravitation and electromagnetism.

The story of the distant parallelism approach can be told largely as a story of attempts to find and justify a uniquely determined set of field equations for the tetrad components, with the demand that solutions of those field equations be given a sensible physical interpretation. The distant parallelism approach in this respect shows a number of marked similarities with Einstein's search for general relativistic field equations of gravitation in the years 1912–1915 (Sauer 2006). In 1912, it had been the introduction of the metric tensor into the theory that had started Einstein's research, and existing mathematical theorems had to be adapted to the theory. In 1928, it was the tetrad fields that allowed the investigation of a non-Euclidean geometry of vanishing curvature and, similarly, Einstein was made aware of existing mathematical results by mathematician colleagues. In both cases, Einstein's research quickly focussed on finding a set of field equations for the dynamical variables and, in both cases, it was difficult to satisfy all heuristic requirements. In response to these difficulties, Einstein changed back and forth between two different and complementary strategies, each starting from one particular set of heuristic postulates. In both episodes, Einstein at one point settled on a set of field equations that was justified more by physical considerations rather than by mathematical soundness. In both cases, Einstein continued to work out consequences of the field equations as well as continued to find a satisfactory mathematical justification for these equations. And finally, the demise of both theories came about by a combination of realizing more and more shortcomings of the theory and by discovering that an alternative approach promised to be more successful. However, while in 1915 the more successful theory that Einstein substituted for his "Entwurf"-theory was the final version of general relativity, the successor approach to the distant parallelism episode turned out to be yet another attempt at a unified field theory.

Einstein abandoned the distant parallelism approach when he realized that the tetrad formalism also allowed a different and new perspective on the Kaluza-Klein five-dimensional approach. Together with Walther Mayer, with whom he had collaborated already during the final stages of the distant parallelism approach, Einstein now explored a variant of the five-dimensional idea that seemed sufficiently new in order to justify again taking up the Kaluza-Klein approach (Einstein and Mayer 1931, 1932a). The novelty of the approach was that it was no longer the space-time manifold which was enlarged by a fifth space-like dimension. Rather Einstein and Mayer constructed a five-dimensional vector space at each point of four-dimensional space-time. The tetrad formalism allowed for an easy generalization to five dimensions, simply by adding another linearly independent vector to the tetrads. The five-dimensional vector spaces obviously could no longer be identified with tangent spaces of the underlying manifold, but Einstein and Mayer gave a projective mapping from the five-dimensional vector spaces to the four-dimensional tangent spaces.

While Einstein and Mayer succeeded to derive the gravitational and electromagnetic field equations from this new five-dimensional approach, they could not account for the structure of matter. In their first paper, they concluded that the existence of charged particles or currents was incompatible with the field equations. They also remarked that an understanding of quantum theory was not yet conceivable in this approach (Einstein and Mayer 1931). In order to allow for the existence of charged material particles, Einstein and Mayer investigated a generalization of their initial framework. The generalization resulted in a new set of field equations. In a subsequent publication, they investigated mathematical properties of these new equations, specifically the problem of compatibity without, however, commenting on a possible physical interpretation of those equations (Einstein and Mayer 1932a).

Since the five-dimensional vector space approach again ran into difficulties, Einstein and Mayer once more tried another approach (van Dongen 2003). Among all of Einstein's investigations into a unified field theory this would be the one that most directly addressed the problem of quantum theory. This time, the incentive came from Paul Ehrenfest, Einstein's Leyden colleague and one of his closest personal friends. Ehrenfest had closely studied recent investigations of a relativistic quantum theory by Wolfgang Pauli and Paul Dirac, and had introduced the term "spinor" for the two-component complex vector representation of the Lorentz group. Since spinors have somewhat counterintuitive transformation properties, e.g. a full rotation of 360^o changes the sign of the spinor, Ehrenfest was uncomfortable with the formalism and urged his colleagues to provide a more natural and intuitive mathematical representation. Einstein and Mayer picked up on this problem in four papers, published between November 1932 and January 1934 (Einstein and Mayer 1932b, 1933a, 1933b, 1934).

In essence, what Einstein and Mayer investigated in these papers was the Dirac equation in a different representation. They introduced what they called "semi-vectors," essentially a four-dimensional real vector representation of the Lorentz group. They argued that semi-vectors were a more natural concept than the suspicious spinors, most likely because of their similarity to ordinary four-dimensional space-time vectors. As it turned out, however, the field equations for semi-vectors turned out to be decomposable into equations that were equivalent to field equations using the spinors, which in hindisght is not surprising since semi-vectors are not an irreducible representation of the Lorentz group, whereas spinors are.

The publication history of the semi-vector approach also reflects the drastic changes in Einstein's life that took place during these months. The first paper (Einstein and Mayer 1932b) was published in the Proceedings of the Prussian Academy of Sciences, as were most of his technical papers on general relativity and unified field theory until this point. But while Einstein spent his third winter as a visiting scientist at the California Institute of Technology in Pasadena in 1932-33, the Nazis came to power and Einstein resigned his membership in the Prussian Academy and never returned to Germany. After his return to Europe in the spring of 1933, Einstein went to Belgium, travelled to Switzerland, and in effect spent most of the year in transit. The second and third notes on the semi-vector approach (Einstein and Mayer 1933a, 1933b) were published in the Proceedings of the Amsterdam Academy, and the fourth (Einstein and Mayer 1934) was published in *Annals of Mathematics*, a journal published in Princeton, where Einstein accepted a permanent position after returning to the United States in late 1933. Einstein would spend the rest of his life in Princeton, and a number of his later papers on general relativity and unified field theory were published in the *Annals of Mathematics.*

This change in publication record not only reflects Einstein's geographical move but also the fact that his investigations into a unified field theory increasingly took on the character of purely mathematical investigations. The change in publication venues may also be an indication of a change in the audience he was addressing, symptomatic of an increasing isolation from modern physics. However, the latter interpretation is at odds with the fact that, at least in the thirties, Einstein published a number of investigations in conventional general relativity that were of substantial physical significance: equations of motion, gravitational waves, and gravitational lensing.

During the mid-thirties, Einstein published little on unified field theory. Some of his investigations may simply never have been published, and there is some hope of interesting historical findings in his scientific manuscripts (Sauer 2004). But he also was spending much time and effort on behalf of other scientists and intellectuals who were trying to escape Nazi Germany. In any case, he published again on unified field theory in 1938, in a paper co-authored with Peter Bergmann (Einstein and Bergmann 1938). They reconsidered the ontological status of the fifth dimension. It had been a scandalon of Kaluza's idea that the extra spatial dimension was a complete mathematical artefact without any physical meaning. In their paper, Einstein and Bergmann entertained the possibility that the fifth dimension was to be regarded as real. Technically, they investigated consequences of substituting the so-called cylinder condition, which demands that all derivatives with respect to the fifth dimension vanish and makes the physical interpretation of the fifth dimension difficult, with the assumption that space was closed resp. periodic in the direction of the fifth dimension.

The problem with this new investigation along the Kaluza-Klein approach was that it led to integro-differential equations that were hard to solve. The problem was adressed in a follow-up paper, published three years later, that presented a way of turning those equations into differential equations (Einstein, Bargmann, and Bergmann 1941). But in analyzing these differential field equations, Einstein and his coworkers found it impossible to describe particles by non-singular solutions. They also found that the gravitational and electromagnetic field equations would be given by the same order of magnitude. This latter characteristic made it impossible to account for the quantitative difference in the strength of the gravitational and electrostatic forces between material particles.

The systematic difficulty of accounting for the existence of matter in a unified field theory was the subject of a little note by Einstein published the same year (Einstein 1941). Einstein proved that the vacuum gravitational field equations do not admit a stationary, singularity-free solution that is embedded in flat space and whose metric tensor would allow a classical limit for large spatial distances from the center that was of the form of a Newtonian gravitational potential for a finite mass.

The proof was reconsidered and generalized two years later in a joint paper with Wolfgang Pauli (Einstein and Pauli 1943), in which the authors prove the non-existence of regular solutions to the vacuum field equations that would asymptotically behave like the Newtonian gravitational potential, regardless of symmetry conditions for the field in regions of finite field strength. Moreover, the proof showed that this result was valid not only in four dimensions, but also for the Kaluza-Klein five-dimensional theory. In effect, this result indicated that, under very general conditions, any attempt to base a unified theory on the Riemann tensor would necessarily involve singularities in particle-like solutions.[7]

Clearly, the latter result was at odds with core requirements of Einstein's program, and it therefore does not come as a surprise that he was willing to reconsider key assumptions of his earlier efforts. An immediate outcome of such reconsiderations was a new approach that he pursued in two publications in 1943, one of them coauthored with his collaborator Valentin Bargmann (Einstein 1944, Einstein and Bargmann 1944).

The starting point now was to keep the four-dimensionality of the theory and also the requirement of general covariance, but to give up the postulate that a generalized theory of gravitation should necessarily be based on the existence of a Riemannian metric. What Einstein and Bargmann proposed instead was, in effect, an attempt at a non-local relativistic theory of gravitation. They investigated the properties of a new kind of mathematical object that they called "bivectors." In contrast to modern usage of the term, these are not asymmetric, second-rank tensors, but rather second-rank tensors that depend on two distinct points of the four-dimensional manifold. The transformation properties of these bivectors depend on the two distinct points of the manifold, each index of the bivector being associated with a different base point.

Although explicitly articulated in the context of the unified field theory program the two papers do not discuss any physical interpretation. Rather, they discuss properties of the mathematical structure that derives from the bivectors. They also discuss field equations for the bivectors, which turn out to be algebraic rather than differential equations. The difficulties of the bivector approach again came with finding and interpreting non-trivial solutions of the fundamental equations. The published papers indicate only preliminary results, partly credited to Einstein colleagues, Bargmann and Pauli, as well as to the Princeton mathematican Carl Ludwig Siegel, and explicitly mention ongoing research with Pauli. It is again possible that a closer scrutiny of Einstein's later research manuscripts might shed further light on Einstein's elaboration of this approach.

Judging by the published record, the bivector episode represents Einstein's penultimate distinct approach in the sequence of attempts to arrive at a unified field theory. Einstein devoted the last ten years of his life to the investigation of a framework that he had already worked on in the mid-twenties, to which he now returned in a first publication of 1945 (Einstein 1945). This last approach of Einstein's work along his unified field theory program was again based on a local Riemannian metric but on an asymmetric one.

Initially, Einstein took the metric tensor to have complex components and demanded Hermitian symmetry. Pauli, however, quickly pointed out to Einstein that the restriction to Hermitian symmetry was not necessary. The subsequent investigations were then following the pattern of earlier ones (Einstein 1946, 1948, 1950a, 1950b, 1950c, App. II, 1953, App. II, 1955, App. II, Einstein and Straus 1946, Einstein and Kaufmann 1954, 1955). Tentative field equations were tested for their mathematical properties, and it was checked whether the criteria for a physical interpretation could be applied. Along with the mathematical properties, Einstein worried very much about the problem of compatibility as he had with other, earlier approaches. Sincee the mathematics of a framework based on an asymmetric metric tensor is exceedingly more complex and less well-known than the standard formalism of (semi-)Riemannian differential geometry underlying conventional general relativity, Einstein spent the rest of his life elaborating the asymmetric theory. Einstein's very last considerations in this final approach were presented by his last assistant, Bruria Kaufmann, at the 50th anniversary of the relativity theory in Bern in July 1955 a few weeks after Einstein's death (Kaufmann 1956).

4 The philosophical dimension

For several reasons, Einstein's unified field theory investigations during the last twenty years of his life, and their broader scientific context, remain largely unexplored. The urgency of his political and social concerns increases dramatically with the Nazis' rise to power, and the ever increasing cruelty of their persecution of Jews, the outbreak of the Second World War, the holocaust, and the explosion of the first atomic bombs. His professional activities were more and more dominated by acts of solidarity with fellow emigrés, public statements and interviews, and other activities that he considered necessary to counteract developments that were against any rational organization of the human world. The political turmoil of the years before, during, and after the war forced him to be more and more active in non-scientific matters, and left him less and less occasion to delve into mathematical calculations. This shift in the balance of the theoretical and practical aspects of his world view is also reflected in our image of the later Einstein.

Nevertheless, as we have seen, Einstein did publish a number of papers on unified field theory program in the last two decades of his lifes. These are highly abstract and esoteric theoretical investigations, mostly of a mathematical character, exploring consequences of a generalized mathematics very much like venturing into an uncharted terrain. Many unpublished scientific manuscripts of those years are extant that give an idea of his struggle with the technical difficulties involved in interpreting a generalized mathematical framework along his program. A closer analysis of these manuscripts still needs to be done and will give a more detailed picture of the rationale of these attempts (Sauer 2004).

The striking contrast between Einstein's perseverance in his scientific program and the detached tone of his scientific publications of the forties and fifties and the urgent and sometimes desperate tone of his political statements need not be seen as a contradiction. Rather, it may be seen as arising from the same source of confidence in the ability and responsibility of the human race for a rational understanding and organization of their world. Despite the political chaos of the world that he lived in during the last two decades of his life, this confidence never failed him in all his political engagement. Despite all new developments in nuclear physics, the discovery of new elementary particles, the successes and paradoxes of early attempts at a quantum field theory and a quantum electrodynamics, this confidence did not fail him either in his attempt to arrive at a unified theory of gravitation and electromagnetism.

Two aspects of Einstein's later scientific work bear witness to this overarching confidence and its specific appearance. A first comment concerns the sincerity of Einstein's willingness to explain the motivation, content, and problems of his scientific research to a lay audience. In 1952 he gave a characteristic response to the request of a reader of *Popular Science Monthly* as to why the promise of his "great scientific achievement" of finding "one all-embracing formula" that would "solve the secret of the universe" seemed not be coming along as expected (Einstein 1952). Einstein replied by first passing a shot at the public media and, in particular, at "newspaper correspondents" for the fact that "laymen obtain an exaggerated impression of the significance of my efforts." But he then went on to explain with sober sincerity what he saw as the main problem of his present work. He had been proceeding to generalize relativistic equations of gravitation "by a purely mathematical procedure [...] and was hoping that the equations obtained in this measure should hold in the real world." But due to the "complexity of the mathematical problem." so far neither he nor anybody else had been able to find solutions to the equations that would allow the representation of empirically known facts, "so that it is completely impossible to tell whether the theory is 'true'."

To illustrate this point he drew the analogy to Newton's theory of gravitation. This theory had only been confirmed since Newton had been able to derive Kepler's laws from his general equations, which in turn had only been possible because the mass of the sun is so much heavier than that of the planets, allowing for approximately valid solutions of Newton's equations. If the bodies of the solar system on the other hand were of equal mass, such approximate solutions may not be obtained and "we might perhaps never know whether or not Newton's theory holds."

The response is characteristic not only for its sincerity, but also illustrates Einstein confidence in a program that admittedly lacked any empirical support. It is also characteristic in its appeal to the history of physics. The comparison with Newton clearly has a rhetorical dimension but it also points to a significant motivation for Einstein's confidence in the eventual success of his program, notwithstanding agnostic overtones. At other places, when explaining his research into a unified field theory to a wider audience, he explicitly linked it to a progressive continuity in the history of science.

In 1950, Einstein gave such an account of his latest efforts in a contribution to *Scientific American* (Einstein 1950a). The question as to why we "devise theory after theory," or why we "devise theories at all," Einstein wrote, has a somewhat trivial answer when we encounter new facts that cannot be explained by knonw theories. Yet, "there is another, more subtle motive of no less importance. This is the striving toward unification and simplification of the premises of the theory as a whole." This latter motive is fed by our in-born "passion for comprehension:"

> I believe that every true theorist is a kind of tamed metaphysicist, no matter how pure a "positivist" he may fany himself. The metaphysicist believes that the logically simple is also the real. The tamed metaphysicist believes that not all that is logically simple is embodied in experienced reality, but that the totality of all sensory experience can be "comprehended" on the basis of a conceptual system built on premises of great simplicity. (Einstein 1950, p. 13)

And in response to a fictitious objection by a sceptic that this would only be a "miracle creed," Einstein admitted that much "but it is a miracle creed which has been borne out to an amzing extent by the development of science."

Here then we see history of science as an underpinning of Einstein's metaphysical creed. Indeed, Einstein goes back to the atomistic conceptions of Leucippos that were borne out only in the kinetic theory of heat of the nineteenth century, or to Faraday's introduction of the field concept that was theoretically justified in Maxwell's theory and then experimentally proven to be real only after the fact, as it were, by Hertz's discovery of electromagnetic waves. Einstein's "train of thought which can lead to endeavors of such a speculative nature" as his own unified field theory maps onto a conceptual history of physics from Newtonian mechanics via Faraday-Maxwell field theory of electromagnetism and Lorentz's theory to the special theories of special general relativity. What emerges from this historical perspective is the significance of the field concept. It is the field concept that embodies the concepts of space and time. And it is the theory of the gravitational field as a special case that gives the "most important clue." And this historical perspective also justified Einstein's steadfast conviction that a unified field theory needs to be generally relativistic, i.e. in accordance with the principle of general relativity from the outset. He explicitly opposed an approach where "the rest of physics can be dealt with separately on the basis of special relativity, with the hope that later on the whole may be fitted consistently into a general relativistic scheme." Historical continuity placed the endeavor of finding a unified field theory above the theory of gravitation implied by general relativity:

> I do not believe that it is justifiable to ask: What would physics look like without gravitation?

In the course of events, however, it was this conviction that separated Einstein from the majority of his contemporaries.

The Einstein that may emerge from seriously acknowledging the philosophical dimension of his unified field theory program may well be that of an intellectual whose belief in the viability of a unified theory of the gravitational and electromagnetic field was intimately connected to a historically outdated belief in the ability of a single human mind to grasp the mysteries of nature in simple terms. From this belief in the power of the human mind sprang the sincerity of his attempts to explain the motif and rationale of his investigations to a lay audience, and this same belief fueled his political interventions in a barbaric world. The futility of his scientific unification endeavours in the face of developments in theoretical physics, both during and after his life, suggests that the specific form of this belief in the power of the human mind belongs to a tradition of enlightenment whose days are gone. We may recognize the same belief in his early undisputed intellectual achievements, in his popular writings that we still recommend to students, in his humanitarian efforts that we still esteem, and in his explicit contentions against quantum theory that we consider deep and insightful. To the extent that Einstein's work on a unified field theory thus reveals a unity in his intellectual endeavours, we should be aware that we can learn from his intellectual achievements only beroy transforming his legacy to the more complex parameters of our time.[8]

Notes

[1] For first steps in this direction, see (Vizgin 1994, van Dongen 2002a, 2002b, 2003, Goenner 2004, Goldstein and Ritter 2003, Majer and Sauer 2005, Sauer 2006) upon which much of the following is based.

[2] For the following discussion, see also (Bergia 1993).

[3] See, e.g., Einstein to Kaluza, 29 May 1919, (CPAE 9, Doc. 48).

[4] " ... schwer zu glauben, daß die partielle Differentialgleichung in letzter Instanz ungeeignet sei, den Tatsachen gerecht zu werden." (Einstein 1923d, p. 360).

[5] "Daran dass man die Quanten lösen müsse durch Aufgeben des Kontinuums glaube ich nicht. Analog hätte man denken können, die allgemeine Relativität durch Aufgeben des Koordinatensystems zu erzwingen. Prinzipiell könnte ja das Kontinuum aufgegeben werden. Wie soll man aber die relative Bewegung von n Punkten irgendwie beschreiben ohne Kontinuum? [...] Ich glaube nach wie vor, man muss eine solche Überbestimmung durch Differentialgleichungen suchen, dass die *Lösungen* nicht mehr Kontinuumscharakter haben. Aber wie?" Einstein to Hedwig and Max Born, 27 January 1920, (CPAE 9, Doc. 284).

[6] "Ich habe in der Ruhe der Krankheit ein wundervolles Ei gelegt auf dem Gebiete der allgemeinen Relativität. Ob der daraus schlüpfende Vogel vital und langlebig sein wird, liegt noch im Schosse der Götter." Einstein to Heinrich Zangger, EA 40-069, quoted in (Sauer 2006).

[7] For a critical discussion of this paper, see also (van Dongen 2002a).

[8] I wish to thank Diana Buchwald, Christoph Lehner, and Tom Ryckman for reading and commenting on earlier versions of this paper.

References

[Bergia 1993] Bergia, Silvio. "Attempts at Unified Field Theories (191–1955). Alleged Failure and Intrinsic Validation/Refutation Criteria." in J. Earman et al. (eds.) *The Attraction of Gravitation. New Studies in the History of General Relativity.* Boston, Basel, Berlin: Birkhäuser, 1993 (Einstein Studies 5), pp. 274–307.

[Bergmann 1979] Bergmann, Peter. “The Quest for Unity: General Relativity and Unitary Field Theories.” in: Holton, G. and Elkana, Y., eds. *Albert Einstein. Historical and Cultural Perspectives.* Princeton: Princeton University Press, 1982, pp. 27–38.

[CPAE 8] Schulmann, Robert, et al. (eds.) *The Collected Papers of Albert Einstein, Vol. 8: The Berlin Years: Correspondence, 1914–1918*, Princeton: Princeton University Press, 1998.

[CPAE 9] Buchwald, Diana K., et al. (eds.) *The Collected Papers of Albert Einstein, Vol. 9: The Berlin Years: Correspondence, January 1919–April 1920*, Princeton: Princeton University Press, 2004.

[Corry 2004] Corry, Leo. *David Hilbert and the Axiomatization of Physics (1898–1918).* Dordrecht: Kluwer, 2004.

[van Dongen 2002a] van Dongen, Jeroen. “Einstein and the Kaluza-Klein particle.” *Studies in History and Philosophy of Modern Physics* 33 (2002) 185–210.

[van Dongen 2002b] van Dongen, Jeroen. *Einstein's Unification: General Relativity and the Quest for Mathematical Naturalness.* Ph.D. thesis, University of Amsterdam 2002.

[van Dongen 2003] van Dongen, Jeroen. “Einstein's Methodology, Semivectors and the Unification of Electrons and Protons.” *Archive for History of the Exact Sciences* 58 (2004) 219–254.

[Eddington 1921] Eddington, Arthur S. “A Generalisation of Weyl's Theory of the Electromagnetic and Gravitational Field.” *Royal Society of London. Proceedings* A99 (1921) 104–122.

[Eddington 1923] Eddington, Arthur S. *The Mathematical Theory of Relativity.* Cambridge: Cambridge University Press, 1923.

[Einstein 1919a] Einstein, Albert. “Spielen Gravitationsfelder im Aufbau der materiellen Elementarteilchen eine wesentliche Rolle?” *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1919, 349–356.

[Einstein 1921e] Einstein, Albert. “Über eine naheliegende Ergänzung des Fundamentes der allgemeinen Relativitätstheorie.” *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1921, 261–264.

[Einstein 1923a] Einstein, Albert. “Zur allgemeinen Relativitätstheorie.” *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1923, 32–38.

[Einstein 1923b] Einstein, Albert. “Bemerkung zu meiner Arbeit ‘Zur allgemeinen Relativitätstheorie’.” *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1923, 76–77.

[Einstein 1923c] Einstein, Albert. “Zur affinen Feldtheorie.” *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1923, 137–140.

[Einstein 1923d] Einstein, Albert. “Bietet die Feldtheorie Möglichkeiten für die Lösung des Quantenproblems?” *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1923, 359–364.

[Einstein 1925] Einstein, Albert. “Einheitliche Feldtheorie von Gravitation und Elektrizität.” *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1925, 414–419.

[Einstein 1927a] Einstein, Albert. “Über die formale Beziehung des Riemannschen Krümmungstensors zu den Feldgleichungen der Gravitation.” *Mathematische Annalen* 97 (1927) 99–103.

[Einstein 1927b] Einstein, Albert. “Zu Kaluzas Theorie des Zusammenhangs von Gravitation und Elektrizität. Erste Mitteilung.” *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1927, 23–25.

[Einstein 1927c] Einstein, Albert. “Zu Kaluzas Theorie des Zusammenhangs von Gravitation und Elektrizität. Zweite Mitteilung.” *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1927, 26–30.

[Einstein 1928a] Einstein, Albert. "Riemanngeometrie mit Aufrechterhaltung des Begriffes des Fern-Parallelismus." *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* (1928), 217–221.

[Einstein 1928b] Einstein, Albert. "Neue Möglichkeit für eine einheitliche Feldtheorie von Gravitation und Elektrizität." *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1928, 224–227.

[Einstein 1941] Einstein, Albert. "Demonstration of the Non-Existence of Gravitational Fields with a Non-Vanishing Total Mass Free of Singularities." *Tucumán Universidad Nacional, Revista* A2 (1941), 11-15.

[Einstein 1943] Einstein, Albert. "Bivector Fields, II." *Annals of Mathematics* 45 (1943), 15–23.

[Einstein 1945] Einstein, Albert. "Generalization of the Relativistic Theory of Gravitation." *Annals of Mathematics* 46 (1945), 578–584.

[Einstein 1948] Einstein, Albert. "Generalized Theory of Gravitation." *Reviews of Modern Physics* 20 (1948), 35–39.

[Einstein 1950a] Einstein, Albert. "On the Generalized Theory of Gravitation." *Scientific American* 182 (1950), 13–17.

[Einstein 1950b] Einstein, Albert. "The Bianchi Identities in the Generalized Theory of Gravitation." *Canadian Journal of Mathematics* 2 (1950), 120–128.

[Einstein 1950c] Einstein, Albert. *The Meaning of Relativity.* Third edition. Princeton: Princeton University Press, 1950.

[Einstein 1953] Einstein, Albert. *The Meaning of Relativity.* Fourth edition. Princeton: Princeton University Press, 1953.

[Einstein 1955] Einstein, Albert. *The Meaning of Relativity.* Fifth edition. Princeton: Princeton University Press, 1955.

[Einstein and Bargmann 1943] Einstein, Albert, and Bargmann, Valentin. "Bivector Fields, I." *Annals of Mathematics* 45 (1943), 1–14.

[Einstein, Bargmann, and Bergmann 1941] Einstein, Albert, Bargmann, Valentin, and Bergmann, Peter. “five-Dimensional Representation of Gravitation and Electricity.” In: *Theodore von Karman Anniversary Volume*, Pasadena: California Institute of Technology, 1941.

[Einstein and Bergmann 1938] Einstein, Albert, and Bergmann, Peter. “Generalization of Kaluza’s Theory of Electricity.” *Annals of Mathematics* 39 (1938), 683–701.

[Einstein and Grommer 1923] Einstein, Albert and Grommer, Jakob. “Beweis der Nichtexistenz ines überall regulären zentrisch symmetrischen Feldes nach der Feld-Theorie von Kaluza.” *Scripta Universitatis atque Bibliothecae Hierosolymitanarum. Mathematica et Physica*, Vol. I, Hierosolymis (Jerusalem) 1923.

[Einstein and Kaufmann 1954] Einstein, Albert, and Kaufmann, Bruria. “Algebraic Properties of the Field in the Relativistic Theory of the Asymmetric Field.” *Annals of Mathematics* 59 (1954), 230–244.

[Einstein and Kaufmann 1955] Einstein, Albert, and Kaufmann, Bruria. “A New Form of the General Relativistic Field Equations.” *Annals of Mathematics* 62 (1955), 128–138.

[Einstein and Mayer 1931] Einstein, Albert, and Mayer, Walther. “Einheitliche Theorie von Gravitation und Elektrizität.” *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1931, 541–557.

[Einstein and Mayer 1932a] Einstein, Albert, and Mayer, Walther. “Einheitliche Theorie von Gravitation und Elektrizität, 2. Abhandlung.” *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1932, 130–137.

[Einstein and Mayer 1932b] Einstein, Albert, and Mayer, Walther. “Semi-Vektoren und Spinoren.” *Preussische Akademie der Wissenschaften, Phys.-math. Klasse, Sitzungsberichte* 1932, 522–550.

[Einstein and Mayer 1932b] Einstein, Albert, and Mayer, Walther. “Die Dirac Gleichungen für Semi-Vektoren.” *Akademie van Wetenschappen* (Amsterdam) *Proceedings* 36.2 (1933), 497–516.

[Einstein and Mayer 1933b] Einstein, Albert, and Mayer, Walther. "Spaltung der natürlichsten Feldgleichungen für Semi-Vektoren in Spinor-Gleichungen vom Diracschen Typus." *Akademie van Wetenschappen* (Amsterdam) *Proceedings* 36.2 (1933), 615–619.

[Einstein and Mayer 1934] Einstein, Albert, and Mayer, Walther. "Darstellung der Semi-Vektoren als gewöhnliche Vektoren von besonderem Differentiations Charakter." *Annals of Mathematics* 35 (1934), 104–110.

[Einstein and Pauli 1943] Einstein, Albert, and Pauli, Wolfgang. "Non-Existence of Regular Solutions of Relativistic Field Equations." *Annals of Mathematics* 44 (1943), 131–137.

[Einstein and Straus 1946] Einstein, Albert, and Straus, E.G. "Generalization of the Relativistic Theory of Gravitation, II." *Annals of Mathematics* 47 (1946), 731–741.

[Goenner 2004] Goenner, Hubert. "On the History of Unified Field Theories." *Living Reviews in Relativity* 7 (2004), No. 2 [Online article]: http://www.livingreviews.org/lrr-2004-2.

[Goenner and Wünsch 2003] Goenner, Hubert, and Wünsch, Daniela, "Kaluza's and Klein's contributions to Kaluza-Klein-theory." Max Planck Insitute for the History of Science (Berlin), preprint 235 (2003).

[Goldstein and Ritter 2003] Goldstein, Catherine, and Ritter, Jim. "The Varieties of Unity: Sounding Unified Field Theories 1920–1930." In Ashtekar, A. *et al.* (eds.). *Revisiting the Foundations of Relativistic Physics. Festschrift in Honor of John Stachel*, Dordrecht, Boston, London: Kluwer, 2003, pp. 93–149.

[Hilbert 1924] Hilbert, David. "Die Grundlagen der Physik." *Mathematische Annalen* 92 (1924) 1–32.

[Kaluza 1921] Kaluza, Theodor. "Zum Unitätsproblem der Physik." *Preussische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte* (1921): 966–972.

[Kaufmann 1956] Kaufmann, Bruria. "Mathematical structure of the non-symmetric field theory." In: *Fünfzig Jahre Relativitätstheorie. Cinquantenaire de la Théorie de la Relativité. Jubilee of Relativity Theory.*

Mercier, A. and Kervaire, M. (eds), Basel: Birkhäuser, 1956 (Helvetica Physica Acta Supplementum IV), 227–238.

[Kragh 1999] Kragh, Helge. *Quantum Generations. A History of Physics in the Twentieth Century.* Princeton: Princeton University Press, 1999.

[Majer and Sauer 2005] Majer, Ulrich, and Sauer, Tilman. "Hilbert's 'World Equations' and His Vision of a Unified Science." In: A.J. Kox and J.Eisenstaedt (eds.) *The Universe of General Relativity* Bost: Birkhäuser, 2005 (Einstein Studies 11), 259–276. [www.arXiv.org/physics/0405110].

[Mie 1913] Mie, Gustav. "Grundlagen einer Theorie der Materie." *Annalen der Physik* 37 (1912), 511–534, 39 (1912), 1–40, 40 (1913), 1–65.

[O'Raifeartaigh and Straumann 2000] O'Raifeartaigh, Lochlainn, and Straumann, Norbert. "Gauge Theory: Historical Origins and Some Modern Developments." *Reviews of Modern Physics* 72 (2000): 1–23.

[Pais 1982] Pais, Abraham. *'Subtle is the Lord...' The Science and the Life of Albert Einstein.* Oxford University Press, 1982.

[Renn 1993] Renn, Jürgen. "Einstein as a Disciple of Galileo: A Comparative Study of Concept Development in Physics." *Science in Context* 6 (1993), 311–341.

[Renn forthcoming] Renn, Jürgen. "The Third Way to General Relativity: Einstein and Mach in Context]." In (Renn and Schemmel 2007a, pp. 21–76)

[Renn and Schemmel 2007a] Renn, Jürgen, and Schemmel, Matthias (eds.). *Gravitation in the Twilight of Classical Physics, Between Mechanics, Field Theory, and Astronomy* Springer 2007.

[Sauer 1999] Sauer, Tilman. "The Relativity of Discovery: Hilbert's First Note on the Foundations of Physics." *Archive for History of the Exact Sciences* 53 (1999), 529–575.

[Sauer 2004] Sauer, Tilman. "The Challenge of Editing Einstein's Scientific Manuscripts." *Documentary Editing* 26 (2004), 145–165.

[Sauer 2006] Sauer, Tilman. "Field Equations in Teleparallel Spacetime: Einstein's *Fernparallelismus* Approach toward Unified Field Theory." *Historia Mathematica* 33 (2006), 399–439.

[Scholz 2001] Scholz, Erhard (ed.). "Hermann Weyl's *Raum–Zeit–Materie* and a General Introduction to His Scientific Work." Basel: Birkhäuser, 2001 (DMV Seminar Band 30).

[Scholz 2004] Scholz, Erhard. "The changing concept of matter in H. Weyl's thought, 1918–1930." Preprint Wuppertal 2004.

[Stachel 1993] Stachel, John. "The Other Einstein: Einstein Contra Field Theory." *Science in Context* 6 (1993), pp. 275–290. Reprinted in: Stachel, John. *Einstein from 'B' to'Z'.* Boston: Birkhäuser, 2002, pp. 141–154.

[Vizgin 1994] Vizgin, Vladimir P. *Unified field theories in the first third of the 20th century.* Basel, Boston, Berlin: Birkhäuser, 1994.