Philosophers of science have discussed some case studies of scientific unification,[2] focusing on its various virtues and drawbacks. The notorious forerunner to many unificatory attempts in string theory, the Kaluza-Klein theory,[3] is barely mentioned as a peculiar case of unification in the philosophical literature.[4] I claim that this unification by "geometrization" of the physical fields is a distinctive kind of unification[5] that offers insights into the relationship between unification and explanation. More precisely, I want to answer the following questions about unification in Kaluza and Klein:

I) What is specific to Kaluza-Klein unification and what does it teach us about unification in general?
II) Are unificatory mathematical structures in Kaluza-Klein equipped with explanatory power?
III) Where should we place Kaluza's and Klein's cases among other gauge unifications?
IV) What kind of brute facts do Kaluza and Klein rely upon?
V) In what sense is Klein's unification better than Kaluza's?
VI) What are the limitations of the Kaluza-Klein unification?

In order to better understand where my case study stands in the literature, in the first section I depict the philosophical approaches to scientific unification relevant for my case study. In the second and third sections, I describe in greater detail the steps toward unification taken by Kaluza and Klein respectively. In the last two sections I will directly address the above questions. My contribution is to clarify some aspects of unification in the Kaluza and Klein approaches to unification.

[2] The most comprehensive analysis of scientific unification is Morrison (2000) which contains an impressive number of illustrations. In the last years, the practice of scientific unification has been discussed in Plutynski (2005); Ducheyne (2005); Van Dongen (2002). My analysis is not a general approach to scientific unification, but a case study from which a limited number of general claims can be drawn.

[3] Kaluza-Klein arguably constitutes the starting point of a philosophical analysis of string theory, given its major implications for the treatment of quantum fields within N-dimensional manifolds and for its generalization to Yang-Mills fields. In this paper I focus only on this "classical" and inchoate stage of the pre-history of string theory.

[4] Aitchison (1991); van Dongen (2002); Weingard (nd, 1984) are among the few places where this mechanism of unification is raised.

[5] See for example Weingard (nd) who explains why Kaluza-Klein is a special case of unification.

1 Puzzles of scientific unification

Unification is a major virtue of a theory, but at the same time it is too vague and undetermined a philosophical concept. Philosophers and scientists likewise struggle to define it, to rank known cases or at least to describe or deal with some of its aspects. Despite many efforts, scientific unification remains a conundrum.[6] It is vague in the sense that there is no general definition or criterion available; when defined, it is often vulnerable to charges of triviality, spuriousness or *adhocness*. Examples of trivial or spurious unifications are often provided in the literature: unification as mere conjunction of child psychology and fluid dynamics is for Maudlin[7] trivial, whereas a conjunction of Kepler's law and Boyle's law is for Kitcher spurious.[8] Feynman's clever example[9] in which all laws have the form $A_i = 0$ (for example $(F - ma)^2 = A_1$, $\left(F - G\frac{m_1 m_2}{r^2}\right)^2 = A_2 \ldots$, etc. and "the theory of everything" (sic) is $\sum_i A_i = 0$ is frequently quoted against hyperbolized attempts to unification. In these mock cases, the new unificatory theory makes contribution (explanatory, confirmatory, interpretative on free parameters, etc.) in addition to the previous theories. A derivation of a law from the conjunction is a pointless "self-explanation" or "self-confirmation". A general criterion for what makes a unification a compelling one and what makes the other trivial or spurious is not available. In the present analysis, I prefer a relative notion of spuriousness and ad-hocness. Although I want to avoid the conclusion that Kaluza's and Klein's theories are in any major sense trivial, spurious or ad-hoc, my main target is to show that Klein improved significantly upon Kaluza's theory.

Even when unification is not trivial, it may or may not be relevant or related to major topics in philosophy of science such as the realism/antirealism debate, confirmation, success, intertheoretical reductionism, causation, etc. As if this were not enough, even if unification is not trivial and it is allegedly relevant to some other more stringent commitments like realism or empirical confirmation, the price to achieve it in some cases has turned out to be too high (the most notable case discussed by philosophers is the electroweak unification). Here I weasel out of providing a *general* answer to all these issues because I accept that unification is too vague a concept to qualify for a comprehensive approach

[6] Some would say that we have an "intuition" of it like "you know it when you see it". Looking at intuitively "borderline" cases surely helps but the this does not suffice. P. Teller expresses this uncertainty in a concise way: "I agree that unifications [and reductions] show something important about how our theories bear on the world. But I take the worries to show that we are very far from understanding what that 'something' is."(Teller, 2004, 443).

[7] (Maudlin, 1996, 131)

[8] (Kitcher, 1981, 526)

[9] (Feynman et al., 1993, 25-10-11)

at a general level. I prefer a more "pluralistic" talk about degrees of unification, stages or levels of unification or successful or unsuccessful unifications. Each and every case of unification can reveal unexpected and relevant aspects and a different mechanism of unification at work.[10] I plan to compare the relevant aspects of unification in my case studies.

1.1 The strengths and weaknesses of unification qua *explanation*

A working definition, inspired by the D-N model, would look like this: a unificatory theory (T_0) describes a set of phenomena previously described by two different theories (T_1 and T_2) using fewer sentences (or "covering" laws). In the mid 20^{th} century, the unity of science had been thought to operate in a reductionistic way: a new theory T_0, more general and more abstract, reduces the heretofore T_1 and T_2 theories. But even in the field of theoretical physics unification cannot be confined to reduction, as anti-reductionism and unification can coexist.[11] If not reduction, then what is the key concept of unification? At the acme of the D-N model, it has been suggested that the aim of explanation was *unification* i.e., "the comprehending of a maximum of facts and regularities in terms of a minimum of theoretical concepts and assumptions".[12] In Friedman's view, phenomena are represented by law-like sentences by the means of *explanation*. "I claim that this is the crucial property of scientific theories we are looking for; this is the essence of scientific explanation— science increases our understanding of the world by reducing the total number of independent phenomena that we have to accept as ultimate or given."[13] T_0 proceeds by providing fewer types of brute facts than T_1 and T_2 do. Hence its brute facts are more fundamental than the brute facts of T_1 and T_2 altogether. Consequently, unificatory theories are simpler (and maybe more beautiful) by explaining the world with less brute facts.

[10] Notwithstanding some approach to unification as a cognitive pattern present in the scientific research, few philosophers have approached a general theory of unification: for details see Maxwell (2004).

[11] Crystallography and solid-state physics "emerged" from the quantum theory without being reducible to it, but in both cases the unity of science remained a desideratum. See for details (Cat, 1998, 273). M. Polanyi, Ph. Anderson and Max Dresden have expressed various anti-reductionistic views during the last century, but have remained sympathetic to the unity of science. Also, general relativity (**GR**) provides a unification of spacetime and the Newtonian gravitational field is not a reduction as neither of them survived "unscathed" in **GR** (Maudlin, 1996, 133).

[12] Feigl (1970) quoted in (Kitcher, 1981, 508). William Kneale in *Probability and Induction* (1949) and Hempel in *Aspects of Scientific Explanation* (1965) had expressed similar views (see (Friedman, 1974, 15). Kitcher juxtaposed this "unofficial model of explanation" to the official D-N model.

[13] (Friedman, 1974, 15)

However, even if I rely on Friedman, I want to caveat his doctrine. Firstly, positing in an *a priori* way brute facts or trying to reduce their number by pseudo-explanations are both signs of weakness of a theory. What is important for a theory is not the sheer number of brute facts, but to get the *right* sorts of facts as brute.[14] Secondly, giving the difficulties of counting such brute facts,[15] reducing the number of *types* of facts generally is a better choice: "Science advances our understanding of nature by showing us how to derive descriptions of many phenomena, using the same patterns of derivations again and again and, in demonstrating this, it teaches us how to reduce the number of types of facts we have to accept as ultimate (or brute)".[16] Thirdly, as my case study will illustrate, the unificatory theory T_0 has to act as a "problem solver" for T_1 and T_2 without generating it own baggage of troubles. This issue has not been directly addressed in the literature on unification.

In the last decade many have argued against this alleged connection between unification and explanatory power of theories. Most notably, Margaret Morrison claimed that unification and explanation are "decoupled".[17] Rather than being a special case of explanatory power, unification is independent of explanation such that "they have little to do with each other and in many cases are actually at odds."[18] Using examples of unified theories, Morrison argued that "the mechanisms crucial to the *unifying* process often supply little or no theoretical *explanation* of the physical dynamics of the unified theory."[19] Many of her case studies against unification *qua* explanation are taken from theoretical physics where unity is usually understood in terms of derivability from a mathematical structure.[20] The mathematical structure, for example the tensor calculus in special relativity (**SR**), most often bestows many theories with a higher degree of generality making it applicable in a variety of contexts and suited to unifying different domains. However, for Morrison this unificatory mechanism of quantitative laws does not provide any explanation of the "machinery" or the mechanism of the phenomena and she would answer in negative question .[21] The mark of a *truly* unified theory is "a specific mechanism or theoretical quantity/parameter that is not present in a simple conjunction, a parameter that represents the theory's ability to reduce,

[14] (Lange, 2002, 99).
[15] Barnes (1994).
[16] (Kitcher, 1989, 432).
[17] See especially Morrison (2000) but also Morrison (1995, 1992).
[18] (Morrison, 2000, 1-2) and (Morrison, 2000, 64).
[19] (Morrison, 2000, 4).
[20] Standard examples include Lagrangian formalism, Lorentz transformations and the symmetry group of a theory.
[21] "The machinery is what gives us the mechanism that explains why, but more importantly *how a* certain process takes place." (Morrison, 2000, 3). One example of "machinery" quoted in Morrison (Morrison, 2000, ch. 3) is Maxwell's explanation of the electrodynamics in terms of ether.

identify or synthesize two or more processes within the confines of a single theoretical framework".[22] Maxwell used a "substantial identification" of the optical aether with the electric ether on the base of the numerical identification of their velocity of transmission[23], although the real unificatory element in Maxwell was the "displacement current".

Even if this factor is present, other troubles linger for unification. Weinberg's current in the electroweak unification is the parameter that unifies the parameters of the electromagnetic theory and those of the weak interaction, but for Morrison it has no explanatory power (here the Higgs mechanism explains the phenomena) and it is as arbitrary as the previous ones.[24] Morrison argues that in this case (as well as in **SR** and to some extent in the biological synthesis) we have a *suspect* unity. Accordingly, many unification cases are less exemplary than believed and the mathematical structure alone does not imply true unification.

1.2 Unification in theoretical physics

Physics is replete with claimed instances of unification: in seeking new theories not yet empirically confirmed, physicists often espouse a desire for theoretical virtues like unification and strive to reach it for reasons ranging from aesthetic considerations like simplicity and harmony, to more pragmatic reasons like the paucity of language or computability restrictions.[25] Morrison's conclusion raises a question about unification in theoretical physics: how explanatory is a physical theory? Philosophers of physics prefer to directly relate unification not to explanation, but to the way in which different forces can be captured within one and the same mathematical formalism. It is not uncommon to relate unification in physics not to explanation, but to the gauge symmetries and group of the unified theory T_0. R. Weingard[26] proposed two major goals of unification of classical fields: unifying different force fields and respectively unifying a force field with its source.[27] In **SR** the first goal can be achieved by identifying the electric and magnetic fields with components of the tensor field $F_{\mu\nu}$ such that a Lorentz transformation transforms the components of one into the other. We will see that such a mechanism of unification is only *partially* present in Kaluza-Klein theory.

[22] (Morrison, 2000, 64).
[23] (Morrison, 2000, 98).
[24] (Morrison, 2000, 139).
[25] The so-called **GUT** (Grand Unified Theories), the standard model and string theory are examples of theories having unification as a primary motivation.
[26] (Weingard, nd, 1)
[27] As Kaluza-Klein is a vacuum theory, we will not discuss here the second goal.

In an attempt to rank the varieties of unification in theoretical physics, Tim Maudlin impose three conditions on any non-trivial unification of two theories (T_1 and T_2): a) T_1 and T_2 have to be consistent, b) the field force in T_1 has to obey the same dynamics as the field force in T_2 and c) there is a nomic correlation among the forces described by T_1 and T_2. The necessary conditions a)-c) constitute the lower limit of unification and I discuss in details whether Kaluza and Klein theories obey a)-c) in section 2 and 3. At the other end of the unification spectrum Maudlin situated two cases of "perfect unification": the unification of electric and magnetic field within relativistic electrodynamics as well as the unification of inertial and gravitational masses in **GR**. For example, the distinction between electric and magnetic fields disappears in relativistic electrodynamics: electric and magnetic fields are eliminated from the ontology by being replaced by the field tensor which is frame-independent. [28] **GR** provides novel explanations to phenomena discovered only later. Therefore, at the level of the perfect unification one can make a commitment to realism by believing that the entities postulated by **GR** are real.

While this upper limit of unification cannot be surpassed, what lays below the level of perfect unification? Maudlin accepts several discernible levels of unification below the perfect cases. [29] We have to face the fact that many gauge theories, praised as embodying unification, do not qualify as perfect. For Maudlin there are three levels of unification of gauge theories: [30]

- *(Level I)* Two gauge theories with their symmetry groups G_1 and neutral particle X_1 and respectively G_2 and neutral particle X_2 are "pasted" into a product group $G_1 \otimes G_2$ without any further ado. The "standard model" was build up as the product group: $SU(3) \otimes SU(2) \otimes U(1)$. It is barely any sort of unification and echoes the trivial unification by simply taking the conjunction of their dynamics.
- *(Level II)* The same procedure as in *Level I* is applied here, but the product gives rise to observable forces created from mixing the groups G_1 and G_2. The particles are given by a "mixing angle" (a free parameter of the new theory) between X_1 and X_2 . In the case of the electroweak unification the group is $SU(2) \otimes U(1)$). In this case we need to explicitly impose the condition of observability of these particles upon the theory and this reduces its explanatory power. This is dubbed by physicists "a partial unification, at best" [31] and
- *(Level III)* is premised on the simple gauge group whose instances are presumably the SU(5) of **GUT**. All the forces may be derived from this

[28] It is debatable whether the inertial and gravitational masses are eliminated by the structure of space-time, see (Maudlin, 1996, 135).
[29] (Maudlin, 1996, 132).
[30] (Maudlin, 1996, 139).
[31] H. M. Georgi quoted in (Maudlin, 1996, 138) and (Moriyasu, 1983, 110).

simple group. There is no product group and no mixing angle involved.

Even if neither Kaluza nor Klein are perfect unifications or 'Level III' unifications, I intend to locate them on Maudlin's schema. For this, I provide more details about the Kaluza-Klein particles involved in the unification in the following two sections.

2 Kaluza's unification on a linear fifth dimension (1921)

Theodor Kaluza[32] thought purely geometrically about the electromagnetic fields. The structure of spacetime should explain the **EM** equations, as it does explain gravity. As a field theory, Kaluza's formalism attempted to unify structures of fields without sources by embedding them into the geometry of spacetime. By this "geometrization", the fields become aspects of the same entity, the metric tensor, such that geometry and physics are no longer distinct ways of describing the world.[33] In its intention, Kaluza's approach was more metaphysical than computational or empirical as it aimed to remove the *duality* of gravity and electricity,"while not lessening the theory's [of gravity] enthralling beauty"[34] by directly envisaging the simplicity and the beauty of the theory.

Kaluza tried to provide the unification by "geometrization" by exploiting some formal similarities between **EM** and **GR** which were apparent to Einstein, G. Nördstrom and H. Weyl.[35] Both **EM** fields and gravitation are described by Poisson equations. For a Newtonian potential Φ, the Poisson equation is: $\nabla^2\Phi = 4\pi G\rho$ similar to the **EM** potential: $\nabla^2 V = -\frac{\rho}{\epsilon_0}$. On one hand, Maxwell equations successfully describe how electric and magnetic *fields* respond to charges and currents (all captured by J_μ). The "field strength tensor":[36]

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{1}$$

where A_μ is the 4-vector potential, is used to derive all Maxwell equations from the covariant equations:

[32] (Kaluza, 1921, 860)
[33] (Weingard, nd, 3)
[34] (Kaluza, 1921, 865).
[35] There are major differences between the two theories, too. For an excellent philosophical discussion about the role of dissimilarities with **EM** in the genesis of **GR** see Norton (1992).
[36] Time is the zeroth component of a 4-vector and $x^1 \ldots x^3$ are the Cartesian spatial coordinates.

$$\text{inhomogeneous Maxwell equations: } \partial_\nu F^{\mu\nu} = \mu_0 \; J^\mu \tag{2}$$

$$\text{homogeneous Maxwell equations: } \partial_\mu F_{\nu\kappa} + \partial_\nu F_{\kappa\mu} + \partial_\kappa F_{\mu\nu} = 0 \tag{3}$$

On the other hand, Einstein field equations were intended to show how *the metric* $g_{\mu\nu}$ responds to the presence of energy and momentum represented by T:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} \tag{4}$$

where $\kappa = \frac{8\pi G}{c^4}$ is a constant related to Newton's gravitational constant G, the Ricci curvature:

$$R_{\mu\nu} = R^\lambda_{\mu\lambda\nu} = \partial_\lambda \Gamma^\lambda_{\nu\mu} - \partial_\nu \Gamma^\lambda_{\lambda\mu} + \Gamma^\rho_{\lambda\sigma}\Gamma^\sigma_{\nu\sigma} - \Gamma^\rho_{\nu\sigma}\Gamma^\sigma_{\lambda\mu} \tag{5}$$

and the Ricci scalar (i.e., its contraction): $R = g^{\mu\nu} R_{\mu\nu}$ are defined as first derivatives of the "Christoffel symbols":

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} (\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}) \tag{6}$$

Notwithstanding the other difference in the nature of gravitation and of the **EM** interaction, on the left hand side of both (2) and (4) we encounter second order derivatives of fields depending on matter or charges codified in the right hand terms. So there should be a generalization of these types of dependencies which would encode g and A in the same mathematical structure. In the light of these similarities, both fields could stem from one and the same universal tensor. However, Einstein field equations (4) already have **EM** encoded in the right hand term $T_{\mu\nu}$: gravity was geometrized while electromagnetic fields were not. In the 20s, a physical field had been considered *geometrized* if its potential was to be found exclusively as part of the metric.[37] All other fields, as well as matter and charges, appeared only in the stress-energy tensor ($T_{\mu\nu}$). The "geometrization" program was intended to move all the non-material fields to G.

Partially inspired by Einstein's "marble and wood" metaphor,[38] Kaluza thought that the universe is, strictly speaking, empty of matter and the only real en-

[37] (Pasini, 1988, 291).

[38] Einstein contemplated the possibility to turn the "wood" of $T^{\mu\nu}$ (the matter) into the "marble" of $G_{\mu\nu}$ (the spacetime) in (4). For him matter was a term that 'infested' the pure and clean structure of $G^{\mu\nu}$. By turning "wood" into "marble", Einstein intended to geometrize matter by providing its fully geometrical origin. As Kaku remarks, this was impossible without more physical clues and without a physical understanding of the "wood": "By analogy, think of a magnificent, gnarled tree growing in middle of a park. Architects have surrounded this grizzled tree with a plaza made of beautiful pieces of the purest marble. The architects have carefully assembled the marble pieces to resemble a dazzling floral pattern with vines and roots emanating from the tree. To paraphrase Mach's principle: The presence of the tree determines the pattern of the marble surrounding it. But Einstein hated this

tity is g. His starting point is the "vacuum hypothesis" (no matter, no charges present):

`VACUUM`: $T_{\mu\nu} = 0$

The Einstein equation for vacuum is:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \tag{7}$$

which implies also that, for vacuum, both R and $R_{\mu\nu}$ vanish:

$$R_{\mu\nu} = 0 \text{ and } R = 0 \tag{8}$$

and the vacuum Maxwell equation is:

$$\partial_\nu F^{\mu\nu} = 0 \tag{9}$$

Giving `VACUUM`, where is the place for the **EM** field? The intuitive answer is: somewhere in the expression of g itself. But even if both theories have their own vacuum solutions, if one tries to describe gravitation and electromagnetism by the same equations, the perturbation of gravitation due to electromagnetism cannot be qualified anymore as "gravitational vacuum". The attempt to unify the two vacuum solutions fails for various reasons. One of them is that in 4-D there is no way to add the field tensor $F_{\mu\nu}$ to the Christoffel symbols $\Gamma^\lambda_{\mu\nu}$, to preserve their properties and to impose later on $R_{\mu\nu} = 0$. Christoffel symbols are defined only up to the first derivatives of a single field and they represent the "displacement" of a vector. [39] Therefore, the "geometrization" of $F_{\mu\nu}$ is not possible in a *four-dimensional Riemannian* manifold. Weyl's solution was to alter the Riemannian form of the metric: his geometry is weaker than Riemann's in the sense that it is only conformal (only the angle between vectors is preserved by their parallel transport along a closed curve, not their lengths, nor their orientation). In the Riemann geometry, angles and lengths are conserved in a parallel transport of a vector. In Kaluza, the metric stays pseudo-Riemannian: what is changed is the dimensionality of the g, R and Γ tensors. [40]

dichotomy between wood, which seemed to be ugly and complicated, and marble, which was simple and pure. His dream was to turn the tree into marble; he would have liked to have a plaza completely made of marble, with a beautiful, symmetrical marble statue of a tree at its center."(Kaku, 1994, 99).

[39] If in a system of coordinate x^ν a vector A^ν at a point P is displaced in a neighbor point P' with coordinates $x^\nu + dx^\nu$, then the value at P' is $A^\nu - \Gamma^\nu_{\alpha\beta} A^\alpha dx_\beta$. $\Gamma^\nu_{\alpha\beta}$ gives the amount by which the component ν of the original vector depends on its own component α when it is displaced on the direction β with an infinite small displacement dx.

[40] Parenthetically, we need to mention that as early as 1914, G. Nördstrom expressed

By "calling a fifth dimension to the rescue",[41] Kaluza managed to express the **EM** field as part of the metric g. There is room for **EM** within g_{mn} and only matter and electrical charges (if any) are present in T_{mn}. He added to the Riemmanian $g_{\mu\nu}$ one row and one column:[42]

$$ds^2 = g^{(5)}_{mn} dx^m dx^n \tag{10}$$

All the expressions of tensors and the relations between them, as well as the Christoffel symbols, are simply generalized from four to five dimensions:

$$\text{Second type: } \Gamma^i_{rs} = \frac{1}{2} g^{il} \left(\partial_s g_{lr} + \partial_r g_{ls} - \partial_l g_{rs} \right) \tag{11}$$

$$\text{First type: } \begin{bmatrix} m\ n \\ r \end{bmatrix} = \Gamma_{mnr} = -\frac{1}{2} \left(\partial_m g_{nr} + \partial_n g_{rm} - \partial_r g_{mn} \right) \tag{12}$$

Kaluza speculated a formal similarity between the above forms of Christoffel symbols in 5-D (12) and the 4-D expression for $g_{\mu\nu}$ and $F_{\mu\nu}$. The 4×4 part of $g^{(5)}_{mn}$ can simply equate the $g_{\mu\nu}$. So where is the $F_{\mu\nu}$ to be placed? The simplest way is to divide $g^{(5)}$ in three sectors as follows:

$$g^{(5)}_{mn} = \left(\begin{array}{c|c} g_{\mu\nu}\text{= `G' sector} & g_{4\nu}\text{=`EM' sector} \\ \hline g_{\nu 4}\text{=`EM' sector} & g_{44} = \phi = ? \end{array} \right) \tag{13}$$

which can accommodate the $g_{\mu\nu}$ tensor in the 'G' sector as well as the A_μ vector in the 'EM' sector. More information about these sectors can be gathered from the Christoffel symbols which are provided by (12):

$$-2\Gamma_{4\mu\nu} = \partial_4 g_{\mu\nu} + \partial_\mu g_{\nu 4} - \partial_\nu g_{4\mu} \tag{14}$$

$$-2\Gamma_{\mu\nu 4} = \partial_\mu g_{\nu 4} + \partial_\nu g_{4\mu} - \partial_4 g_{\mu\nu} \tag{15}$$

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the metric as a 5×5 matrix. His paper, translated in (Appelquist et al., 1987, 50-60), is less known than Kaluza's and has had only a slight impact on the scientific community. He added another spatial dimension to the four existing ones in order to obtain an Abelian five-vector gauge field for which a Maxwell-like equation can be written, including a conserved 5D current. He was the first to explicitly claim that "we are entitled to regard the four-dimensional space-time as a surface in a five-dimensional world." The major difference between Nördstrom and Kaluza is that the former found gravity by applying **EM** to the 5D world, whereas the latter applied **GR** to it (Smolin, 2006, 47). From my point of view, Kaluza scores better than Nördstrom in respect of unification.

[41] (Kaluza, 1921, 967).

[42] Latin indices are numbers from 0 to 4 and Greek indices are from 0 to 3; vectors or tensors with Latin indices are 5-dimensional. Here x^0 is the time coordinate.

`CYLINDER CONDITION:` It is easy to see why there is a surplus structure in the 5-D metric and much of this has to be stripped away. Here is Kaluza's suggestion: in order to take (1) in, one term out of three is always set to zero in (14) and (15) such that $\Gamma_{4\mu\nu}$ and $\Gamma_{\mu\nu 4}$ will contain *only* **EM** terms. The best option is to hypothesize that $\partial_4 g_{\mu\nu}$ vanishes. This is formally the origin of the so called "cylinder" condition, the core of the Kaluza-Klein unification:

$$\texttt{CYL:}\ \partial_4 g_{mn} = 0 \tag{16}$$

We experience three dimensions of space and one of time because there are fields in these four 'directions' which are *not* constant. Small or null variations of the fields on the fifth dimension means that the world is "cylindrical": every point $P(x^0, ...x^4)$ can be identified with another point P' having the coordinates $P(x^0, ...x^4 + \delta x^4)$ if *all* fields and *all* derivatives are smooth on the fifth direction.[43] P and P' are still distinct, notwithstanding the values of all possible physical fields being equal or have close values at these points.

After postulating `CYL`, Kaluza suggested that $F_{\mu\nu}$ is a "degenerate" (*verstümmelte*) form of the Christoffel symbols in (12) and proceeded to the following identification:

$\texttt{ID}_1$:

$$\Gamma_{4\mu\nu} = \alpha F_{\mu\nu} \tag{17}$$

$$\Gamma_{\mu\nu 4} = -\alpha(\partial_\nu A_\mu + \partial_\mu A_\nu) \tag{18}$$

$$\Gamma_{44\mu} = \partial_\mu \phi \tag{19}$$

where $F_{\mu\nu}$ and A_μ are the **EM** quantities defined in (1) and satisfying (2) and (3), and ϕ is an arbitrary scalar field, not yet interpreted.

`WEAK FIELD:` In order to provide analytical solutions to the field equations, it is commonly assumed the perturbation formulation **GR** in which the metric differs only a little from its Euclidian value $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ (where $\eta_{\mu\nu}$ is a Minkowskian metric and "the perturbation" h is taken such that $|h_{\mu\nu}| \ll 1$). The linearized gravity can be expressed as: $g_{mn} \simeq \eta_{mn}$. In order to conduct his analysis, Kaluza assumed that the third and fourth terms in the Ricci curvature in 5-D:

$$R^m_{ijk} = \partial_j \Gamma^m_{ik} - \partial_k \Gamma^m_{ij} + \Gamma^n_{ik}\Gamma^m_{nj} - \Gamma^n_{ij}\Gamma^m_{nk} \tag{20}$$

are of the form Γ^2, and since Γ is of first-order, these contribute only to second order and can be discarded.

$$\texttt{WEAK:}\ R^m_{ijk} \cong \partial_j \Gamma^m_{ik} - \partial_k \Gamma^m_{ij} \tag{21}$$

[43] This analogy is from Einstein and Bergmann (1938).

It is easy to see that by contracting the Riemann tensor further, the Ricci tensor has a simpler form, too:

$$R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} \tag{22}$$
$$R_{4\nu} = \alpha \partial_\lambda F^\lambda_\nu \tag{23}$$
$$R_{44} = -\partial_\mu \partial^\mu \phi \tag{24}$$

Kaluza employed another well-known relation between the Christoffel symbols which can be particularized giving `CYL` to:

$$\frac{\partial \Gamma_{4ln}}{\partial n} + \frac{\partial \Gamma_{4mn}}{\partial l} + \frac{\partial \Gamma_{4nl}}{\partial m} = 0 \tag{25}$$

Kaluza supposed that the 5-D world is empty, so both the Ricci scalar and the curvature tensor vanish:

$$R_{mn} = 0 \text{ and } R = 0 \tag{26}$$

Then, what does the 5-D vacuum generate? The assumptions are not only consistent, but they provide an unexpected number of direct results. One wants to infer the vacuum solutions in 4-D from the 5-D vacuum solution. By mimicking some of the **GR** techniques , Kaluza was able to infer equations:

- The 5D metric:
$$g^{(5)}_{mn} = \begin{pmatrix} g_{\mu\nu} & 2\alpha A_\mu \\ 2\alpha A_\nu & 2\phi \end{pmatrix} \tag{27}$$
- Homogeneous Maxwell equations (3) from (25), (17) and (18).
- Einstein field equations 4-D from (22).
- A Poisson-like equation for ϕ from (24).
- The components of the energy momentum tensor in 5-D. In the `WEAK` approximation, the Ricci scalar is of higher order in h and the Einstein equations in 5-D are:
$$R_{mn} = \kappa\, T_{mn} \tag{28}$$
Again, from (23) and the inhomogeneous Maxwell equation (2), one can identify the components of T_{mn} as:
$$\texttt{ID}_2 : T_{\mu 4} = J_\mu \tag{29}$$
so Kaluza has bordered the 4-D energy momentum tensor $T_{\mu\nu}$ with a vector representing the currents and densities of charges. It is easy to show that $T_{55} = 0$ and then $T_{\mu\nu}$ is:
$$T^{(5)}_{mn} = \left(\begin{array}{c|c} \text{matter and densities: } T_{\mu\nu} & J^\mu \\ \hline \text{currents and charges: } J_\mu = \begin{pmatrix} c\rho & j_1 & j_2 & j_3 \end{pmatrix} & 0 \end{array} \right) \tag{30}$$

- Maxwell inhomogeneous equation (2) from `WEAK`, `CYL`, (17) and (23).

Even if Kaluza accomplished the intended unification program, two major aspects of **GR**—the geodesics and the definition of energy have to be explicitly analyzed.

2.2 Geodesics in 5-D

The first important test of Kaluza's new unified theory was the analysis of geodesics in 5-D. In the vacuum theory, $T_{\mu\nu}$ encodes the kinematic energy of test particles. The ideal situation would be like this: a small, charged test particle in 5-D falls on a geodesic in 5-D and its projection in 4-D is the expected trajectory of a charged particle (typically *not* a geodesic). But computation with relativistic test probes is almost impossible to carry, so Kaluza assumed a "slow motion approximation" (commonly used in **GR**) in which the 5-velocities $U^m = \frac{dx^m}{ds}$ are such that $U^m \cong \left(1, \vec{0}, U^4\right)$ and $ds^2 \cong d\tau^2$, where τ is the proper time. In this case:

$$T^{mn} = \mu_0 U^m U^n \tag{31}$$

and in order to estimate the geodesics, terms U^4 are needed. By generalizing the 4-D geodesic equation parameterized by λ :

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma}\frac{x^\rho}{d\lambda}\frac{x^\sigma}{d\lambda} = 0 \tag{32}$$

and by employing (22)-(24) and

$$\frac{d^2x^m}{d\lambda^2} + \Gamma^m_{ab}\frac{dx^a}{d\lambda}\frac{dx^b}{d\lambda} = -\sqrt{2\kappa}\ \ F^m_{\ n}\frac{dx^n}{d\lambda}\frac{dx^4}{d\lambda} - \partial^m\phi\frac{dx^4}{d\lambda}\frac{dx^4}{d\lambda} \tag{33}$$

Kaluza intended to infer the equation of a particle with mass M and charge q in curved spacetime in which an electric field tensor $F_{\mu\nu}$ is present:

$$\frac{d^2x^\rho}{dt^2} + \Gamma^\rho_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt} = -\frac{q}{Mc}F^\rho_{\ \mu}\frac{dx^\mu}{dt} \tag{34}$$

In order to identify the two equations, he chose the parametrization such that $\lambda = \tau \cong t$ and a vanishing term in ϕ in (33), such that he needed to assume that:

$$\phi = constant \tag{35}$$

The third identification is:

$$\mathtt{ID}_3 : U^4 = \frac{dx^4}{dt} = \frac{q}{Mc\sqrt{2\kappa}} \tag{36}$$

The interpretation of (36) can raise difficulties but also it constitutes a powerful tool for explaining electromagnetism. Two particles in 4-D which have the same mass and the same initial conditions and differ only in respect of their charge will follow two trajectories which are both projections of a geodesic in 5-D. This is explained by the fact that their U^4 is different.[44] Had we started with the small velocity approximation, we would want U^4 to be close to zero. The formalism applies only to relatively small velocities *and* to charges of $\rho_0/\mu_0 \ll 1$, which seems kosher for all practical purposes. But this second approximation is unsatisfactory for atomic dimensions where U^4 is not at all small for a given density of charge of electron or proton. In this case, the slow motion is no longer met and the motion of an electron is *not* a geodesics in $\mathbb{R}^5$ as U^4 is enormously large. This means that Kaluza's theory would not work for subatomic particles.

3 Oskar Klein's compactification of x^4 (1926)

Five years after Kaluza's paper was published, Oskar Klein wrote a paper[45] and a note in *Nature*[46] in which he dealt with the idea of unification of **EM** and **GR** by analyzing not only the $g^{(5)}$ field, but also the wavefunction on a 5-D manifold.[47] The first part of Klein's 1926 paper is inspired by Kaluza and his treatment of the $g^{(5)}$ field, although the legend has it that Klein carefully read Kaluza only after he had finished writing it.[48] He himself started from the aforementioned similarities between **GR** and **EM**,[49] and postulated in 5-D the Riemannian metric (10), the forms of Ricci tensors and Christoffel

[44] I'll offer a more comprehensive discussion of this issue in section 4.
[45] Klein (1926a).
[46] Klein (1926b).
[47] In Klein (1928) he came back to the problem of the unification and restated the main idea of compactification in direct relation to conservation laws.
[48] In his autobiographical note Klein recalls: "When Pauli came to Copenhagen [in 1925], I showed him my manuscript on five-dimensional theory and after reading it he told me that Kaluza some years before had published a similar idea in a paper I had missed. So I looked it up [. . .] I read it rather carelessly but quoted, of course, in the paper I then wrote in a spirit of resignation. [. . .] In the paper I tried, however, to rescue what I could from the shipwreck."(Ekspong, 1991, 111)
[49] Witness Klein's confession again: "The similarity struck me between the ways the electromagnetic potentials and the Einstein gravitational potentials enter the [relativistic Hamilton-Jacobi equation for an electric particle], the electric charge in appropriate units appearing as the analogue to a [fifth] momentum component, the whole looking like a wave front equation in a space of [five] dimensions. This led me into a whirlpool of speculation, from which I did not detach myself for several years and which still has a certain attraction for me.". (Klein recollecting in 1989 the early 20s)(Ekspong, 1991, 108).

symbols from **GR**. Klein assumed that the 15 quantities of the symmetric tensor g_{mn} would accommodate the 12 components of both $g_{\mu\nu}$ and $A_{\mu\nu}$. In order to fit these into g_{mn} and by echoing Kaluza's `CYL` (16) Klein imposed some conditions on the coordinate system of the 5-D space:

- The first four coordinates are identical to the ordinary spacetime coordinates;
- The cylinder condition (`CYL`): the fields do not depend on x^4;
- $g_{44} = a$ where a is a constant.

These are all present under various guises in Kaluza (Kaluza imposed $\mathtt{g}_{44}$ only in order to derive the geodesics). In Klein, this condition becomes central. It is very important to mention here that `CYL` is just a working hypothesis: in the note to *Nature*, Klein would replace it with the compactification (`COMP`).[50] It can be proven that the only infinitesimal coordinate transformation which satisfies these conditions is:[51]

$$x^\mu \to x^\mu + \xi^\mu(x^\nu) \tag{37}$$
$$x^4 \to x^4 + \xi^4(x^\nu) \tag{38}$$

where ξ are smooth functions of only the first four coordinates $x^0 \dots x^3$. For such a transformation, the only metric tensor that preserves the line element ds^2 (see (10)) needs to have the form:

$$g^{(5)}_{mn} = \begin{pmatrix} g_{\mu\nu} + \mathbf{A}^{(5)}_\mu \mathbf{A}^{(5)}_\nu & \mathbf{A}^{(5)}_\mu \\ \mathbf{A}^{(5)}_\nu & 1 \end{pmatrix} \tag{39}$$

where $\mathbf{A}^{(5)}$ is a 5-vector of which all first four components transform[52] like the covariant components of the **EM** field and $\mathbf{A}^{(5)}_5 = 1$. The simplest way is to *identify* again the four components of this 5-D vector with the **EM** vector potential A_μ:

$$\mathtt{ID}_4 : \mathbf{A}^{(5)}_\mu = A_\mu \tag{40}$$

[50] I will come back on this issue later (p. 23sqq.)

[51] See (Klein, 1926a, 896), but the hereby terminology is from Bergmann (1942).

[52] The vector **A** is a vector field employed in projective geometry, see (Bergmann, 1942, 274).

The constant field ϕ is plugged into the expression of the metric in order to replace the $g_{55} = 1$:[53]

$$g^{(5)}_{mn} = \begin{pmatrix} g_{\mu\nu} + \phi A_\mu A_\nu & \phi A_\mu \\ \phi A_\nu & \phi \end{pmatrix} \tag{41}$$

3.1 Klein's metric

Despite these similarities, there are some important differences between Klein's and Kaluza's assumptions regarding the topology of the fifth dimension. Klein's metric is:

$$ds^2 = \left(g_{\mu\nu} + \kappa^2\phi^2 A_\mu A_\nu\right) dx^\mu dx^\nu + 2\kappa\phi A_\mu dx^\mu dx^4 + \phi^2 dx^4 dx^4 \tag{42}$$

In order to show that $\mathtt{ID}_4$ is not arbitrary, Klein inferred (4) and (3) from a variational principle (instead of guessing an expression for the Ricci tensor like Kaluza did) by requiring the minimization of the Hilbert action under the variation of the metric $\delta g^{(5)}_{mn}$ and of its first derivative $\partial_l g^{(5)}_{mn}$:

$$S_H = \int \mathcal{L}_1 d^5 x \tag{43}$$

where $\mathcal{L}_1 = R^{(5)}\sqrt{-g^{(5)}}$ is a Lagrange density of fields and $R^{(5)}$ is a Ricci-like invariant scalar defined by:

$$R^{(5)} = g^{mn} R_{mn} = g^{ik}\left(\partial_k \Gamma^{i\mu}_{\mu} - \partial_\mu \Gamma^{ik}_{\mu} + \Gamma^{i\mu}_{\nu}\Gamma^{k\mu}_{\mu} - \Gamma^{ik}_{\mu}\Gamma^{\mu\nu}_{\nu}\right) \tag{44}$$

similar to (5). By accepting Kaluza's `WEAK`, Klein disregarded the contribution of the last two terms and proceeded by applying the `CYL`. As $R^{(5)}\sqrt{-g^{(5)}}$ does not depend on x^4, the integral in ((43)) splits into two integrals like:

$$\int dx^4 \int R^{(5)}\sqrt{-|g^{(5)}|}d^\mu x$$

The action is an integral in 4-D only:

$$S = -\int d^4 x \sqrt{-g}\ \left(\frac{R}{\kappa^2} + \frac{1}{4}\phi F_{\mu\nu}F^{\mu\nu} + \frac{1}{6\kappa^2}\frac{\partial^\mu \phi\, \partial_\mu \phi}{\phi^2}\right) \tag{45}$$

[53] Klein did not provide a matrix form of the metric. I do not adopt the exponentiation of ϕ from Duff (1994). In the "projective geometry" formulation of Veblen and Hoffman the metric suffers an extra coordinate transformation $x^4 \to e^{x^4}$. The importance of the scalar field ϕ will be discussed later. See for details (O'Raifeartaigh and Straumann, 2000, 9), (Bergmann, 1942, 269). For a form similar to mine, see (van Dongen, 2002, 4). Witness the presence of the $\mathbf{A}^{(5)}$ in the 4×4 part of $g^{(5)}$.

The first integral in (45) is simply the action for gravity in 4-D, [54] while the second is an action of the electromagnetic field of a stress-energy tensor given by Maxwell equations and the third is the Klein-Gordon equation of the scalar field ϕ. [55] By minimizing the action $\delta S_H = 0$, the result is a system of two equations:

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \kappa T^{\mu\nu} \tag{46}$$

$$\partial_m \sqrt{-|g|}F^{\mu m} = 0 \tag{47}$$

where $R^{\mu\nu}$ is the contravariant component of the Ricci tensor and $T^{\mu\nu}$ the contravariant component of electromagnetic energy-momentum tensor. This result is strikingly close to Kaluza's. Through a minimization of the action of the $g^{(5)}$ field in 5-D, Klein recovered the gravitation field of Einstein field equations (4) and both Maxwell equations for vacuum.

3.2 Charges and matter on geodesics

The first good news for Klein was that the metric (41) provides the form of the geodesics in 5-D. [56] Indeed, Klein added to the action (43) a Lagrange density for the motion of n free charged particles. The total Lagrange density in the presence of fields and n probe particles is:

$$\mathcal{L} = \mathcal{L}_1 + \sqrt{-g^{(5)}}\kappa \sum_{i=1}^{n} g^{mn}\frac{dx_i^m}{d\lambda}\frac{dx_i^n}{d\lambda}$$

Similar to Kaluza's $\mathtt{ID}_3$, in order to derive the geodesics in 5-D, Klein interpreted the velocity on the fifth axis as proportional to the charge of the particle:

$$U^4 = \frac{e}{c}\frac{1}{\frac{d\tau}{d\lambda}} \tag{48}$$

where as usual $d\tau = \frac{1}{c}\sqrt{-ds^2}$ is the proper time in 5-D, λ is a parameter of the geodesics and e is the electrical charge of the electron. By taking the divergence of the field equations (46), one can prove that charged particles follow the

[54] (O'Raifeartaigh and Straumann, 2000, 9).
[55] (Overduin and Wesson, 1998, 15).
[56] "I became immediately very eager... to find out whether the Maxwell equations for the electromagnetic field together with Einstein's gravitational equations would fit into a formalism of five-dimensional Riemann geometry (corresponding to four space dimensions plus time) like the four-dimensional formalism of Einstein. It did not take me a long time to prove this in the linear approximation, assuming a five-equation, according to which an electric particle describes a five-dimensional geodesic."(Ekspong, 1991, 109-110)

geodesics in 5-D.[57] On such geodesics, the Lagrange function $L = \frac{1}{2}\left(\frac{ds}{d\tau}\right)^2$ provides the definition of the 5-D momentum:

$$p_i = \frac{\partial L}{\partial\left(\frac{dx^i}{d\lambda}\right)} \tag{49}$$

As there is no explicit dependence of L on x^4, we will always have a constant momentum on the fifth axis. The calculations render for an electron:

$$p_4 = \frac{e\sqrt{a}}{c\sqrt{2\kappa}} \tag{50}$$

where a is the constant value of the scalar field ϕ, so p_4 has the same value at any point of spacetime *if* the field ϕ is kept constant.

3.3 The 5-D wavefunction

The second part of the 1926 paper and the note in *Nature* are directly connected with two major developments of both relativity and quantum mechanics. Here Klein inferred for the first time the form of the relativistic wavefunction for a spinless particle.[58] Klein endeavored to connect quantum results with the analysis of geodesics in 5-D. Instead of describing only particles on the manifold, Klein explicitly relayed to de Broglie's treatment of quantum phenomena by analogy with mechanics.[59] Klein studied the differential form

[57] See (Klein, 1926a, 899).

[58] This equation was published in the same year by Klein, V. Fock and Gordon (allegedly Schrödinger had first discovered and immediately rejected it in 1925 because it could not explain spin). Klein's manuscript was submitted to the editors of *Zeitschrift für Physik* in April 1926, whereas Fock's and Gordon in July, respectively in September. Fock (1926) also used a 5-D formalism, very similar to Klein's. Not much attention has been paid to the fact that the Klein-Gordon equation originated in an explicit 5-D formalism.

[59] "[I tried] to learn as much as possible from Schrödinger and also from de Broglie, whose beautiful group velocity consideration impressed me very much even if by and by I saw that it did not essentially differ from my own way by means of the Hamilton-Jacobi equation. From Schrödinger I learnt in the first place his definition of the non-relativistic expressions for the current-density vector, which it was then easy to generalize to that belonging to the general relativistic wave equation. In this, after Schrödinger's success with the hydrogen atom, I definitely made up my mind to drop the possible non-linear terms, although I was still far from certain that this was more than a linear approximation. Also I derived the energy-momentum components, which in the five-dimensional formalism belonged to the current-density vector. These I published much later, due to the appearance in the meantime of a paper by Schrödinger containing the corresponding non-relativistic expressions."(Ekspong,

of a "ray" of a wave and then tried to identify it with the equation of the trajectory of a charged particle. The central point of the wave-particle analogy of de Broglie is the definition of the momentum operator by the operator "nabla" $\hat{\mathbf{p}} = -i\hbar\nabla$: [60]

$$\hat{P}_m = \frac{\partial}{\partial x^m} \tag{51}$$

Klein took a generalized form of a wave in 5-D:

$$\Box_g \psi = a_{ij}\left(\frac{\partial^2}{\partial x^i \partial x^j} - \Gamma^k_{ij}\frac{\partial}{\partial x^k}\right)\psi = 0 \tag{52}$$

where $\Box_g$ is a wave operator in 5D and a_{ij} are some functions of the coordinates only. He started with a harmonic wave in 5-D (very similar to that used in geometrical optics): [61]

$$\Psi = \Psi_0 e^{i\omega\Phi(x^m)} \tag{53}$$

and after replacing (53) into (52), he analyzed its behavior in two cases. [62] For ω large enough, the wave operator will have terms only in ω^2. The remainder is an equation of the phase ϕ:

$$a_{ik}\left(\frac{\partial\phi}{\partial x^i}\frac{\partial\phi}{\partial x^j}\right) = 0 \tag{54}$$

The Hamiltonian of the propagation of the wave can be written as:

$$H = \frac{1}{2}a_{ik}P_i P_j = 0 \tag{55}$$

which is similar to the one in the Hamilton-Jacobi formalism. Rays are geodesics of the differential form:

$$a_{ik}dx^i dx^k = 0 \tag{56}$$

The equation of motion of a charged particles in the Lagrange formalism is:

$$\mathcal{L} = \frac{1}{2}\frac{d\theta}{d\lambda} + \frac{ds}{d\lambda} \tag{57}$$

and in accordance to the duality postulated by de Broglie, the particle is represented by the wave so the rays coincide with the particle's trajectory. [63] The results are:

1991, 111-112)

[60] (van Dongen, 2002, 5).

[61] (Klein, 1926a, 900).

[62] I do not discuss here the case of small ω in which the Klein-Gordon equation originated.

[63] In 1924 de Broglie's associated to each bit of energy with mass m_0 a periodic wave with a wavelength: $\nu_0 = m_0c^2/h$(de Broglie, 1924, 11). The group velocity of this wave is the same as the velocity of the mass. Sommerfeld's condition for stability on hydrogen orbit can be inferred as conservation of phase. Schrödinger had anticipated in 1922 de Broglie's result that Weyl's scale factor (the exponential factor ϕ that

$$p_i = \frac{\partial \mathcal{L}}{\partial \frac{dx^i}{d\lambda}} \tag{58}$$

$$\texttt{and:}\ \ p_4 = \beta\left(\pm\frac{e}{c}\right) \tag{59}$$

where β is a constant. Because of $\Phi = -x^4 + S(x^0, x^1, x^2, x^3)$, (53) can be separated into:

$$\Psi = \exp{(i\omega x^4)}\Psi(x^\mu) \tag{60}$$

The conservation of phase along a closed trajectory in the fifth dimension is:

$$\omega \oint p_4 dx^4 = 2\pi n \tag{61}$$

and as the Hamiltonian of this wave is zero, the phase is conserved.

3.4 Compactification on x^4 and the new argument restated

In *Nature*, Klein proposed a major turnover. "The charge q, so far as our knowledge goes, is always a multiple of the electronic charge e, so that we may write $p_4 = n\frac{e}{k}$ with $n \in \mathbb{Z}$. This formula suggests that the atomicity of electricity may be interpreted as a quantum theory law." [64] He hinted toward the idea that the momenta on the x^4 is always quantized. Wave mechanics provided Klein with a clear form of a momentum on the fifth axis. But moving along x^4 is not simply a mechanical change of coordinates. This can be troublesome because it was for the first time when momentum had a non-dynamical interpretation. Though it is not a "quantity of motion", it has some properties of a momentum (always associated to moving particles or to waves). In an hydrogen atom for example, the momentum has a discrete spectrum, i.e. it is *quantized*. Because p^4 in (50) depends linearly on e, which is quantized, one may ask whether it is quantized, too. In polar coordinates, $\dot{\phi}$ or $\dot{\theta}$ are velocity-like quantities (they are actually angular velocities and there is an "angular momentum"), whereas p^4 is different. The analogy used by Klein has a pure heuristic role, as he has been inspired by early quantum results on closed orbits. The mathematical structure in both cases is of a periodic function: *ergo* the idea of a Fourier expansion. However, while the hydrogen atom can be represented in a coordinates in which $\phi = \phi + 2n\pi$, the atom itself does not live in a compactified space. The analysis of the wave in 5-D provided the idea

relates the lengths of a rod parallel transported from P to P': $l_{P'} = l_P exp \int \phi_i dx^i$) for closed orbits was an integral power of some universal constant. See Schrödinger (1923), (Vizgin, 1994, 99). Klein took inspiration from de Broglie's thesis.

[64] Klein (1926b).

of *compactification*. By taking into account de Broglie's hypothesis, one can infer:

$$p_4 = ne/c\sqrt{2\kappa} = n\hbar/\lambda_4 \tag{62}$$

where λ_4 is the radius of the closed circle on x^4. If one knows the quanta of electrical charge, from (62) one can deduce the compactification factor $\lambda_4 = 0.8 \cdot 10^{-30} cm$. Klein identified geometrically the points P and P' separated by $2\pi\lambda_4$ and rejected the linear topology of x^4 by the compactification hypothesis:

`COMP`: The x^4 axis is closed with a period of λ_4.

The new form of Klein's argument, the one usually cited, is obtained by replacing `CYL` with `COMP`. Instead of postulating the same values of fields on x^4, Klein took a topological stance: he supposed that the axis is curled with a very small radius. The consequence of the initial argument (`COMP`) was promoted to a hypothesis of the new argument and the hypothesis of the old argument (the quantization of charge) became a consequence of the new one. The new hypothesis `COMP` is then used to infer the quantization of charge and the new symmetry group of the theory. The smallness of λ_4, which is less than the Planck length, is the only reason as to why extensions on x^4 cannot be observed by macroscopic observers. Klein realizes that the discreteness of the charge spectrum, via the de Broglie relation, leads to a discrete wavelength in the fifth direction. So now in the new argument, given the value of λ_4, `COMP` explains `CYL`.

What are the transformation allowed by `COMP`? Two points P and P' are *identical* iff $x'^4 = x^4 + 2\pi\lambda_4$. The new manifold is invariant under the group $GL(4) \bigotimes S(1)$, where $S(1)$ is the group of the translation (38). `COMP` is a topological invariant and not a coordinate variant of the theory. The new topology is not a mere coordinate system of *representation*, but it reflects the structure of x^4. The main reason to argue for this more realistic interpretation of topology is the fact that unlike for example the case of polar coordinates (which is nothing more than an alternative to Cartesian coordinates), there are no transformations that remove the symmetry $S(1)$ and linearize x^4.

If two particles have the same initial condition in 4-D x_0^μ but different ratios q/M, they will fall under the *same* geometrical shape in 5-D by following the geodesics. Obviously, this is an improvement over Kaluza's geodesics. Klein's metric rules out the small velocity approximation needed by Kaluza and solves the problem of geodesics. From this we can infer the quantization of the charged particle as being imposed by (61). This means that if the fifth dimension is compactified with a period of $2\pi\lambda_4$, then the electrical charge appears *quantized* in 4-D.

Only the first mode of Fourier expansion of fields is relevant. In Klein's days the fields $g_{\mu\nu}(x)$, $A_\mu(x)$ and $\phi(x)$ were thought to be mathematical objects which transform under four-dimensional general coordinate transformations. What Klein did not notice, but was used by Einstein in 1927, is that if the fifth axis is compactified, then all fields are periodical on x^4 and consequently they can be Fourier expanded having all other 4-D fields as coefficients. [65] This means that there is a ambiguity between the "real" 5-D tensor (or vector or scalar) and its 4-d "representation" ($g(x^\mu)$). The value of the 4-D field is reducible to an infinite number of values such that the first one is independent of the fifth coordinate:

$$g_{\mu\nu} = \sum_{n=0}^{n=\infty} g_{\mu\nu}^{(n)}(x^0, x^\mu) e^{inx^4/\lambda_4} = \tag{63}$$

$$g_{\mu\nu}^{(0)}(x^\mu) + g_{\mu\nu}^{(1)}(x^\mu) e^{ix^4/\lambda_4} + g_{\mu\nu}^{(2)}(x^\mu) e^{i2x^4/\lambda_4} + \ldots \tag{64}$$

Decades later, the expectation values of these fields:$\langle g_{\mu\nu}\rangle, \langle A_\mu\rangle, \langle\phi\rangle$, given by the first terms in the Fourier series, were interpreted as masses of particles. As Duff remarks, in today's parlance, the Fourier coefficient of order zero describes a graviton (spin 2), a photon (spin 1) and a dilaton (spin 0). Indeed, the masslessness of graviton $\langle g_{\mu\nu}\rangle = 0$ is due to the general covariance of *GR* (which can be interpreted as a gauge invariance); the masslessness of photon $\langle A_{\mu\nu}\rangle = 0$ is due to the gauge invariance; the masslessness of the dilaton $\langle\phi\rangle = 0$ is due to it being a Goldstone boson. [66]

4 Kaluza's unification: its limits and its promises

I intend to address the questions asked at the beginning of the paper and to link the Kaluza agenda to the literature on scientific unification.

Kaluza's identifications and explanations. There is no "real unificatory element" or "machinery" (such as the "displacement current" in Maxwell) in Kaluza. Instead of a "theoretical parameter", Kaluza depicts a mathematical operation that unifies. Similar to Maxwell's case, Kaluza used $\mathtt{ID}_1$-$\mathtt{ID}_3$ to explain why we have the illusion of **EM** and **GR** as disparate realms. Under some approximations, the IDs have helped Kaluza to represent the **EM** and **GR** interaction under one and the same formalism and to infer a geodesic equation. By using $\mathtt{ID}_1$ he inferred the form of the metric tensor g_{mn} and by using $\mathtt{ID}_2$, the geodesic equation. $\mathtt{ID}_3$ helped him to give an interpretation for p^4. Kaluza's IDs provide answers to "why" questions such as: Why is it apparent that **EM** phenomena are independent of gravitational phenomena?

[65] It can be shown that Klein's wavefield in 5-D is equivalent to the Fourier expansion. See (van Dongen, 2002, 190).
[66] (Duff, 1994, 6).

Why do macroscopic charged particles not move on geodesics in 4-D? Why do **GR** and **EM** obey Poisson equations? I conclude that question II) can be answered in affirmative because Kaluza provides explanatory power along unification, *pace* Morrison's claim.

Lack of coupling. One would like to have a **SR**-type of unification where the "electric field" by itself and the "magnetic field" by itself were doomed to fade away. This is not the case with Kaluza. He intended to provide that sort of unification but his formalism is not powerful enough to provide a correlation between **EM** and **GR**. Without it, his formalism is a conjunction of **GR** and **EM** without mutual interactions between them. In fact, by this decoupling, the strength tensor does not affect the metric in four dimensions, which is a drawback of the theory. His metric does not meet Mauldin's condition c), i.e. the coupling terms between the unified interactions or an explanation of their mutual effects as a law-like correlation between the two. We shall see that interaction terms do appear in Klein's metric, so for Klein, **EM** does add something to **GR**, and condition c) is met. Question III) can't be answered satisfactorily for Kaluza: his theory does not rank high on Maudlin's list as it does not meet condition c), i.e it is not a unification à la Maudlin. The mathematical operation that brings in unification does not come with a coupling term.

Adhocness of identifications. A common counterargument against Kaluza would be to suspect that the `ID`s are *ad-hoc* because they are designed to produce the sought after unification. What if the `ID`s and the dynamics on x^4 are concocted in order to reflect **EM**? Both **GR** and **EM** are represented by second order PDEs and they are both high-range forces. Apparently, Kaluza's method would work for any pairs of forces satisfying such formal conditions. In the prototypical case of unification of electric and magnetic forces within **SR**, the theory proves that they are descriptions of one and the same physical entity.

Another complaint came from Einstein. As Kaluza's theory minimally extended gravity from 4-D to 5-D, the new dimension seems to have been added to 4-D in the "letter and spirit" of relativity and thus unification is obvious. Einstein complained about this extension in a paper he wrote with Grommer in 1923.[67] In **GR**, the covariance of ds^2 was associated with the direct measurability of a 4-D distance. However, there are no measures of length or duration in 5-D because "length" here does not have the same meaning. Why should one preserve the covariance of ds^2 in 5-D? In one sense x^4, is special because *all* fields have the same values along it (which is not the case with $x^0 \dots x^3$). There are ways to dismiss Einstein's "suspect asymmetry" objection. Firstly, **SR**, a very successful theory, is based on the *asymmetry* between $x^1 \dots x^3$ and x^0. So there are no logical reasons to refute the fifth dimension

[67] Einstein and Grommer (1923)

on the basis of an asymmetry of the 5-D manifold, as the 4-D manifold is already asymmetric in this sense. Time is already a non-spatial dimension, as the metric along the fourth axis is not a "distance" with a '+' signature.[68] Secondly, the very fact that there is a "measurement operation" associated to four axes does not constitute a necessary condition to be imposed upon other axes. The fifth axis simply acts as a theoretical parameter which has major mathematical significance but which is difficult to measure (physics abounds with such theoretical entities).

The cylinder condition as a 'physical' brute fact. By imposing CYL, Kaluza tried to include in his formalism the fact that we do not experience the fifth direction of spacetime. According to **SR**, we have only an illusion of the "flow of time". Here things seem similar: the universe has a hidden dimension and the illusion or the indication of its existence is the whole spectra of electromagnetic phenomena. Being aware of this outlandish new "extra-world parameter", Kaluza imposed the CYL in order to account for its un-observability.[69] By CYL the topology of the fifth direction is not affected, it is still the linear topology of $\mathbb{R}$ and the external symmetries of spacetime are the same with those of **GR+EM**[70] CYL as a brute fact is difficult to tolerate and naturally seems ad-hoc. C. Callender proposes a way to see which facts are brute and which are not: "What we do not want to do is posit substantive truths about the world a priori to meet some unmotivated explanatory demand — as Hegel did when he notoriously said there must be six planets in the solar system."[71] CYL is an unexplained explainer, but a very uncomfortable one. For the sake of the beauty and simplicity of his theory, Kaluza committed the same kinf of *fiat* that Hegel did.

Predictions and consequences of Kaluza's unification. One has to acknowledge that Kaluza's theory is a conjunction of **GR** and **EM** *only if* ϕ and its density R_{44} are left uninterpreted. In fact, he explicitly refrained from any predictive desideratum when he speculated that his theory did not surpass "mere capricious accident". However, he took the liberty to say more about ϕ. Four years before Schrödinger would discover the wave function, Kaluza speculated that, in the future, ϕ could act as a statistical quantity that can explain quantum fluctuations.[72] and it *could* get to predictions in the future. Likewise, in that case, ϕ would provide explanations to a plethora of phenomena such as the apparent indeterminacy of quantum facts in 4-D. The aim

[68] (Overduin and Wesson, 1998, 3)

[69] "One then has to take into account the fact that we are only aware of the space time variation of quantities, by making their derivatives with respect to the new parameter vanish or by considering them to be small as they are of a higher order."(Kaluza, 1921, 968)

[70] For other details see (Duff, 1994, 3).

[71] (Callender, 2004, 206).

[72] (Kaluza, 1921, 865).

of explaining quantum indeterminism as the appearance of fields existing in extra-dimensions was the Grail of many unified field theories: even Bohr and Einstein coquetted with this idea. But this was mere speculation. What is the scalar ϕ: surplus structure or would-be *explanadum*? For the time being, aside from the approximations of Kaluza's theory, ϕ can be taken as an arbitrary parameter without empirical consequences whatsoever and as a sign of ad-hocness which cannot be excluded at this stage of the theory. This entails another problem related to the reality of its new elements: the fifth dimension and the field ϕ. Notwithstanding any interpretation of ϕ, Kaluza's theory seems to be a notation variant of the **GR** and **EM**, acting more as a formalism than as a theory. Although at the beginning of the paper Kaluza exhorted us "[...] to consider *our* space-time to be a four-dimensional part of a $\mathbb{R}^5$" [73], at the end of the paper he became less convincing, downgrading his formalism to a mere computational trickery [74]

Other than his hope for a future quantum role of ϕ, Kaluza lacks a robust commitment to realism. So one may ask if we do need the fifth dimension more than we need phase space. By analogy, even if phase space is helpful in analytical mechanics, nobody has ever claimed that we really live in a $(q, \dot{q})$ space or (q, p) space. Phase space and configuration space are purely representational spaces that do not produce extra structures such as ϕ. However, x^4 is similar to such "useful fictions" unless the scalar field ϕ signals the existence of a particle, or it is related to the quantum fluctuations or it is related to the cosmological constant. Einstein and Bergmann claimed in 1938 that Kaluza's theory was equivalent to a "projective geometry" in which the 4-D manifold was enough and x^4 was projected back to 4-D to which they added "vectorial fields" by a "Four-Transformation". Alternatively, one may ask why we do not get rid of the third spatial dimension and just use two dimensions plus a vectorial "height" field. Even if equivalent from a formal point of view to its 3-D counterpart, such a theory may have hard time in describing everything in 3D. [75]

Novel predictions and observations were at that time the sole respected virtues of a scientific theory and Kaluza simply did not provide any. Unlike Weyl's theory, whose unrealistic predictions had scared away Einstein and Pauli, Kaluza's theory did not have blatantly bad predictions. Actually, we will see that the new element brought in, ϕ, has predictive and explanatory virtues. In short,

[73] (Kaluza, 1921, 859, my emphasis)
[74] "[...] it is difficult to think that the derived relations, which could scarcely be rejected at the level of theory, represent something more than the enticing game of a capricious chance. If one can establish that the presupposed connections are more than an empty theory, this would be nothing else than a new triumph for Einstein's General Theory of Relativity whose appropriate application to five dimensions has been our concern here."(Kaluza, 1921, 865)
[75] Thanks to _reference removed_ for this suggestion.

Kaluza's theory illustrates a weak form of unification because it is trivial and ad-hoc and because it does not provide a coupling term between **EM** and **GR**.

5 Extrinsic element of unification and novel explanations in Klein

Klein's new argument and the unification he achieved were more powerful than those of Kaluza. Klein employed `ID`s as Kaluza did, but he surpassed this procedure. In addressing question I), I claim that there are two aspects specific to Klein's unification: A) the extrinsic element of unification and B) the reduction of types of symmetries of the theory. Both are crucial to understand the improvements upon Kaluza.

A. The wavefunction as the unification element. Klein's unification element is the behavior of the wavefunction in 5-D which is an extrinsic element to both **GR** and **EM**. It plays the role of the displacement current in Maxwell and it is associated to a mathematical structure, i.e. the Sommerfeld condition of stationarity on a closed orbit. This mathematical condition afterward plays the heuristic role for the discovery of compactification which, as a topological condition, is compatible with **GR** and **EM**. I want to stress that the wavefunction in 5-D, undoubtedly inspired by de Broglie's Ansatz, is *not* an electromagnetic wave or a gravitational wave *per se*. As the hypothesis of the new argument, `COMP` is the unificatory structure equipped with explanatory powers. It comes from wave mechanics or, in a modern parlance, from the formalism of quantum mechanics in de Broglie's interpretation.

In Klein's case, the unificatory element is part of neither T_1 nor T_2. In trying to answer the second part of question I), one may ask whether the "extrinsic element of unification" is specific only to Klein's unification. It is worth knowing in general whether the element that generates the theory T_0 is intrinsic to T_1 or to T_2. A more general question can be asked: can the operation of unification always be performed completely within two theories? I have in mind a related case of unification. Without further details, I suggest here that in string theory "string" and "brane" are extrinsic elements to both the standard model and to the theory of gravity, although they play a major unificatory role.[76]

B. Klein's reduction of internal symmetries. Klein was able to explain internal symmetries of **EM** as external symmetries of 5-D. Because of `COMP`, the symmetry of the **EM** theory is recovered from the symmetry of spacetime

[76] I do not claim that an "extrinsic element" of unification characterizes *any* case of unification.

manifold $\mathbb{R}^4 \bigotimes S_1$ and the theory needs only "external symmetries". On one hand, **EM** theory has its internal symmetry of gauge invariance in 4-D called $U(1)$. On the other hand, a wave-function invariance solely demands "geometrical" transformations associated to the coordinates in 5-D (37) and (38). The metric transforms like this:

$$g'_{mn} \rightarrow g_{mn} - \partial_\mu \lambda_\nu - \partial_\nu \xi_\mu \tag{65}$$

and given (41), this corresponds to the gauge invariance symmetry of **EM**:

$$A'^\mu \rightarrow A^\mu - \frac{\partial \xi_\mu}{\partial x^\mu} \tag{66}$$

What is exciting is that $U(1)$ coincides with the invariance on a compactified topology. The *internal* symmetry of **EM** is reduced to the *external* symmetry of a one-dimensional manifold $S(1)$ (a geometrical consequence of the translation with a multiple of 2π on x^4) which reflects in letter and in spirit the creed of the "geometrization" program. The number of *types* of symmetry is then reduced, and not the sheer *number* of symmetries. Aspect B) of Klein's theory nicely echoes Kitcher's critique to Friedman's account of unification *qua* explanation.[77] The preference for external symmetries and the reduction or elimination of internal symmetries are manifest in the generalization of Kaluza-Klein to Yang-Mills field as well as in string theory: "our spacetime may have extra dimensions and spacetime symmetries in those dimensions are seen as internal (gauge) symmetries from the 4-D point of view. All symmetries could then be unified."[78]

Brute facts and explanations. In answering questions II) and IV), I claim that the power of explanation in Klein is greatly improved when compared to Kaluza. Klein's reversed argument, in which `COMP` becomes a brute fact that explains `CYL`, provided Klein with a powerful unificatory mechanism able to provide *novel* and unintended explanations. Klein's original intention had been to unify **EM** and **GR** by assuming `COMP` and he succeeded. The result surpassed his original expectation by explaining in addition the quantization of the electrical charge, the internal symmetry of **EM** as external symmetry of $S(1)$ and eventually the existence of particles. In addition, there was another "intended", albeit less successful, explanation in Klein's theory. Klein showed how Schrödinger's equation can be derived from the wave equation in 5-D in which "$\hbar$ does not originally appear, but is introduced in connection with the periodicity in x^4."[79] Does the Planck's constant originate in the periodicity of the fifth dimension? Unfortunately, this is only a partial outcome, at best. One can infer some quantum numbers, especially the quanta of charge, from the

[77] Kitcher (1976).
[78] This is the so called "KK symmetry principle" (Ortín, 2004, 291). Among other meanings, string theorists use unification as reduction of the *types* of symmetries.
[79] Klein (1926b).

symmetries of x^4, but not *all* of them. How much of quantum theory can be explained by this geometrization program? Not much. Quantum theory in its Hilbert space formulation is not captured by the topology of the fifth dimension,[80] so one should have serious doubts about whether the whole quantum theory can be derived from topological assumptions in extra dimensions. In the eyes of modern physicists, the meaning of Klein's deduction is flawed: the *classical* theory of fields, even in 5-D, is not able to provide a description of quantum phenomena.

Usually, the major criticism raised against Kaluza-Klein theory consists in its lack of predictions. For many physicists, a unification is successful only when making new predictions that are confirmed by experiment.[81] The charge quantization, the external symmetry of **EM** and the existence of some particles were brute facts for Kaluza as well as for **EM** or **GR**, whereas in Klein's theory they become *explananda*. Once one has accepted `COMP`, one hits the ground of explanation and no explanation is needed any longer. The "unexplained explainer" is that the fifth dimension is curled and this for Klein the finale, no other *explanans* is necessary for this brute fact. As part of its unificatory virtues, `COMP`, a *topological* brute fact, explains and predicts *physical* facts. Klein aimed higher when he envisaged to explain particles. However, can a vacuum theory predict the existence of particles? Another unexpected explanation of Klein's theory: the photon and, albeit Klein was not aware of it, the graviton and the "dilaton" can be deduced from `COMP` as expectation values of $\langle A_\mu \rangle, \langle g_{\mu\nu} \rangle, \langle \phi \rangle$ by assuming a first-order approximation in which massive states are disregarded.[82] Even if the interpretation of zero modes as masses was too bold for the 1920s, Klein correctly inferred the photon from $\langle A_{\mu\nu} \rangle$. For him, as for de Broglie, material particles are solutions to fields and their motion reflects the propagation of waves: "the observed motion as a kind of projection onto space-time of a wave propagation taking place in a space of five dimensions."[83] The scalar field ϕ as well as g itself signal the presence of an *unobservable* particle. However, the 5-D wavefunction comes with its own troubles. A tower of massive, charged and spin particles with mode $n > 1$ having the mass $m_n = |n|m$ pops into existence. It is easy to see whyt in its original formulation Klein's theory was not renormalizable.[84]

[80] The question whether a 5-D theory can capture the description of other interpretations of quantum mechanics (Bohmian mechanics, for example) is way beyond the scope of the present paper.

[81] For example, see (Smolin, 2006, 47, 125) for quotes from Richard Feynman and Sheldon Glashow on superstring theory. Smolin rightly emphasizes that what used to be critiques against Kaluza-Klein is in the present times directed against string theory.

[82] This is similar to the "dimensional reduction" used in modern Kaluza-Klein theories with D=11 by Scherk, Julia and Cremmer (1978), see Appelquist et al. (1987).

[83] (Klein, 1926a, 905).

[84] One can associate these massive multiplets with the symmetry group of the theory.

Klein's world with a curled x^4 is operationally indistinguishable from a 4-D world with an infinite mass spectrum. The "dimension reduction" is necessary precisely to avoid embarrassing predictions. But in order to explain *massive* particles one needs non-geometrical fields "coupled" with the metric which indicates that the geometrical reduction is not fundamental. Despite Klein's attempts, "matter fields" must remain on the brute facts side and cannot be explicated away.

Notwithstanding these shortcomings, when Klein modified the original formulation following Kaluza, he was clearly motivated to develop a theory with explanations, with fewer types of brute facts and more capable of solving problems. Klein's case study comes to odds with Morisson's decoupling general claim: while the wavefunction plays the role of the "theoretical element of unification", Klein's `COMP` is a mechanism crucial for unification with novel and unexpected explanations, beyond the scope of the original approach (otherwise similar enough to Kaluza). But some unintended results came out from his very theory. [85]

Last but not least, Klein is a contrast case to Morrison's analysis in another respect, too. Morrison tried to show that in Maxwell's unification of **EM**, the theory's commitment to the existence of ether gradually lessened. [86] Kaluza-Klein illustrates the opposite trend of an increasingly realist commitments to the existence of an extra dimension and to its topology as the theory boosted its explanatory store. In Einstein's and Pauli's approaches to extra dimensions, but especially in the later stage of the theory, the realism commitment became more transparent. At the renaissance of the extradimension theory, Cremmer & Scherk (1976) and Witten (1981) have approached Kaluza-Klein with an explicit realist stance in which the "mechanism" of compactification was based on spontaneous symmetry breaking [87]. From an unexplained explainer, `COMP` became an *explanandum* of the Kaluza-Klein type of cosmology.

Klein's place in Maudlin's ranking. In Klein, gravity and electromagnetism are coupled. In Klein's "line element" (42) there is no longer a pure gravitational "piece of metric". The interaction term $A_\mu A_\nu$ represents the coupling between gravitation and electromagnetism (on which Kaluza remained silent) which affects the 4-D gravitational metric. Unlike Kaluza, Klein's theory meets Maudlin's three conditions a)-c). Klein qualifies as a non-trivial unification which is not a mere conjunction of **GR** and **EM**. Klein's theory is not decom-

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According to Salam (1982), the non-compact symmetries are spontaneously broken and are nothing more than spectrum generating terms.

[85] Similarly, Maxwell had intended to unify electric and magnetic fields, but what he accomplished at the end of the day was the unification of light with electromagnetic waves as well.

[86] (Morrison, 2000, 84)

[87] (Appelquist et al., 1987, 278sqq)

posable in a simple group, so it does not constitute a *Level III* unification à la Maudlin. I situate Klein's unification as an intermediate position between *Level I* and textitLevel II in Maudlin's schema. Although there are observable particles generated by the unification procedure: the photon and, although not directly, the graviton and dilation, there are no particles generated by the mixing angle similar to those in the electroweak unification.

Both the Kaluza and Klein cases reveal two amendments to Maudlin. Although **GR** is not, strictly speaking, a gauge theory (it can be interpreted as a gauge theory, but this was not available to Klein), its very presence complicates Maudlin's hierarchy of gauge unifications.[88] Secondly, Kaluza's case in which condition c) is not met, albeit it has the symmetry of a simple group ($GL(5)$), reveals that there are missing pieces in the puzzle of unification of gauge theories at least at *Level III*, where further conditions are needed. The same can be said about the condition of the mixing angle at *Level II*. Therefore, it would not be fair to downgrade Klein's unification well below the electroweak one. Klein's theory is not worse (or better) than electroweak theory in another respect: they both postulate a scalar field in order to achieve unification. I conclude that Klein's unification is at least as powerful as the electroweak one, even if one must admit plenty of structural differences between them. This suggested that for gauge theories unified with gravity Mauldin's hierarchy should be based on more than one criterion or at least to be more precise in the conditions impose at *Level III*.

Klein's unification: a problem solver and a problem maker. I want to finally address question V) and VI). Besides the aforementioned explanatory and unificatory boost, `COMP` acted like a "problem solver" for Kaluza's theory: Klein substantially relaxed the approximation of weak fields, took out the slow motion constraint and showed that electrons move on geodesics. This takes his theory to a higher level of unification. Klein also assumed that only the *first term* (n=0) in the Fourier expansion on x^4 counts in our everyday observations. Given the smallness of λ_4, the modes $n > 0$ are taken to be large enough to not be visible from our 4-D world and consequently the field values in 5-D do not depend on x^4.

What are the major limitations of the Kaluza-Klein theory? In the fourth decade of the last century, physicists were preoccupied with the new nuclear forces discovered. Quantum physics swamped the research in Kaluza-Klein which seemed unable to render a description of these new, quantized interactions.[89] Because of these historical reasons, the Kaluza-Klein program has

[88] The author acknowledges this (Maudlin, 1996, 143)

[89] In the meantime, speculations about curled-up extra dimensions seemed "as crazy and unproductive as studying UFOs. There were no implications for experiment, no new predictions, so, in a period when theory developed hand in hand with experiment, no reason to pay attention."(Smolin, 2006, 52)

been stalemated for about half of a century.[90] Notwithstanding the history of classical field theories, one should always recall that the original Kaluza-Klein theory is strictly speaking a false theory.[91] Besides the tower of massive particles mentioned above, there are two difficulties generated by `COMP`. Firstly, the λ_4 as a parameter is instable, almost chaotical: when perturbed, either it collapses to a singularity or it becomes visible.[92] Secondly, as directly linked to the elementary charge of the electron, λ_4 is not a dynamic parameter, but a "frozen" parameter of the manifold. This undermines the essence of Einstein's **GR** for which geometry is dynamical.[93] Klein's theory is dependent then on a background manifold with a fixed topology. Last but not least, from a methodological point of view, the generalization of the Kaluza-Klein theory shows that there are always too many ways to achieve unification. Whenever there are more than one hidden dimensions, there are infinite ways to curl them up, so there are an infinite number of possible versions of the theory.[94]

6 Conclusion

The evolution of this theory from Kaluza to Klein brought about an increased unificatory and explanatory power, a reduction of types of brute facts while solving previous problems and removing triviality and *ad-hocness*. Although the commitment to realism is not transparent in either of these stages, one can see how Klein opened the road to a more realistic interpretation of the higher dimensions. Its potential to be generalized as well as the paradigmatic mechanism of unification in which internal symmetries are reduces is worthy of further philosophical analysis. The main conclusion to be drawn from my analysis is that in this case unification was not possible from within two theories. The external factor exploited by Klein, the wavefunction behavior on x^4, reveals that **GR** and **EM** do not have enough internal resources to be unified

[90] The theory eventually resurfaced due to its generalization to Yang-Mills fields by adding extra dimensions with more and more sophisticated topologies and by including quantum effects.
[91] (Wesson, 2006, 5).
[92] (Smolin, 2006, 48).
[93] (Smolin, 2006, 48).
[94] "The more dimensions, the more degrees of freedom — and the more freedom is accorded to the geometry of the extra dimensions to wander away from the rigid geometry needed to reproduce the forces known in our three-dimensional world."(Smolin, 2006, 51). This inflation of models chases nowadays' string theory, too. The supersymmetric theories are so rich that they can explain almost any imaginable universe. And this affects Kaluza-Klein generalizations which seem to be nothing more than a mathematical tool of representation and not a physical theory that reflects reality.

in 5-D. Wave mechanics, or at least a primitive notion of quantum mechanics, was the external element of unification also endowed with explanatory power.

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