Shortcuts in extra dimensions and neutrino physics

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Abstract. We discuss the possibility of sterile neutrinos taking shortcuts in extra dimensions, and the consequences for neutrino oscillations. This effect influences the active-sterile neutrino mixing and provides a possibility to accommodate the LSND evidence for neutrino oscillations together with bounds from accelerator and reactor experiments. We briefly comment on causality in such schemes.

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A typical feature of theories with large extra dimensions is the idea to avoid the large gauge hierarchy by lowering the Planck scale. Such absence of any large energy scale in the theory invalidates the most prominent seesaw suppression of neutrino masses and requires an alternative mechanism for neutrino mass generation. However, string theories usually predict singlet fermions in the bulk (e.g. superpartners of moduli fields) which have the right quantum numbers to act as right-handed neutrinos. This picture leads to a suppressed wave function overlap between the left-handed neutrino localized on a 3+1 dimensional brane and its right-handed partner, and small neutrino masses can be obtained from volume-suppressed couplings to right-handed neutrinos in the bulk [1].

Another interesting feature of extra-dimensional theories has been first proposed as an alternative to inflationary cosmology. Such an era of exponential expansion in the early universe explains why the universe seems to be homogenous over distances without causal contact, and is basic to the concordance model. The alternative idea put forward in [2] is that bulk gravitons may take shortcuts in the extra dimensions and this way establish causal contact between space-time regions apparently causally separated for paths traversing the brane.

One can easily sketch three different mechanisms for such bulk shortcuts:

- Self-gravity effects in the presence of matter localized on the brane imply extrinsic brane curvature.
- Thermal or quantum fluctuations lead to brane bending.
- The extra dimension can be warped asymmetrically, i.e. warp factors can shrink the space dimensions x parallel to the brane but leave the time and bulk dimension t and u unaffected, as in the following example [2, 3]:

$$ds^2 = dt^2 - [e^{-2ku}a^2(t)dx^2 + du^2].\tag{1}$$

In a recent paper we have analyzed the effect of such shortcuts on neutrino oscillations in the presence of sterile bulk neutrinos, and its consequences for the LSND dilemma [4]. The LSND experiment has reported evidence for $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations with $\delta m^2_{\text{LSND}} \sim 1 \text{ eV}^2$. In order to accommodate the LSND result with the confirmed solar ($\delta m^2_{\text{sun}} \sim 10^{-4} \text{ eV}^2$) and atmospheric ($\delta m^2_{\text{atm}} \sim (\text{few})10^{-3} \text{ eV}^2$) neutrino results, a 4-th neutrino is required. Moreover, since the width of the Z-boson as measured by LEP allows only for three neutrinos coupling to the Z, this 4-th neutrino has to be sterile. Thus the four states can be arranged as either the two pairs of solar and atmospheric neutrinos, separated by the large LSND mass gap (the 2+2 spectrum) or as a triplet of active neutrinos complemented by a single, predominantly sterile state (the 3+1 spectrum). Hereby the first option is excluded, since neither solar nor atmospheric neutrinos have been observed to oscillate into steriles. On the other hand, the second option is strongly constrained by the negative results of reactor (BUGEY) and accelerator (CDHS) experiments searching for electron or muon neutrino disappearance.

The oscillation amplitudes for BUGEY, CDHS and LSND are approximately given by $\sin^2 2\theta_{ee'} \approx 4U^2_{e4}$, $\sin^2 2\theta_{\mu\mu} \approx 4U^2_{\mu4}$, and $\sin^2 2\theta_{\text{LSND}} = 4U^2_{e4}U^2_{\mu4}$, where the mixing matrix elements $U_{e/\mu4}$ parametrize the admixture of ν_e, ν_μ in the 4-th state. Consequently the LSND amplitude is doubly suppressed by the bounds obtained from BUGEY and CDHS, invalidating the 3+1 spectrum. While one solution to the LSND dilemma is that LSND might be wrong (results of the ongoing test at MinibooNE are due!), it also may hint towards deviations from the usual oscillation mechanism and may be a messenger of the mechanism of neutrino mass generation. A particular attractive example for the latter case are extra dimensions and bulk shortcuts.

If we consider neutrino oscillations in the presence of bulk shortcuts the evolution factor $\int Hdt$ in the path integral is altered, as the bulk signal gains a time shift Δt, implying

$$\Delta \int Hdt = H\Delta t \to \Delta H_{\text{eff}}T. \tag{2}$$

By introducing the shortcut parameter $\varepsilon \equiv (t_{\text{brane}} - t_{\text{bulk}})/t_{\text{brane}} = \Delta t/T$ the effective Hamiltonian in flavor space is given by

$$\Delta H_{\text{eff}} = +\frac{\delta m^2}{4E}\left(\begin{array}{cc} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{array}\right) + E\frac{\varepsilon}{2}\left(\begin{array}{cc} -1 & 0 \\ 0 & 1 \end{array}\right). \tag{3}$$

It is obvious, that for a certain resonance energy the shortcut term cancels the diagonal piece of the brane Hamiltonian, implying a resonance at $E_{\text{res}} = \sqrt{\frac{\delta m^2 \cos 2\theta}{2\varepsilon}}$. Then the oscillation amplitude becomes

$$\sin^2 2\tilde{\theta} = \left[\frac{\sin^2 2\theta}{\sin^2 2\theta + \cos^2 2\theta(1-A)^2}\right] \tag{4}$$

with $A = (E/E_{\text{res}})^2$. This implies that oscillations at energies $E \gg E_{\text{res}}$ are suppressed, and that the CDHS bound with $E_{\text{CDHS}} \gg E_{\text{LSND}}$ may not apply to oscillations as observed at LSND. Thus, in the presence of bulk shortcuts, the 3+1 spectrum is allowed again.
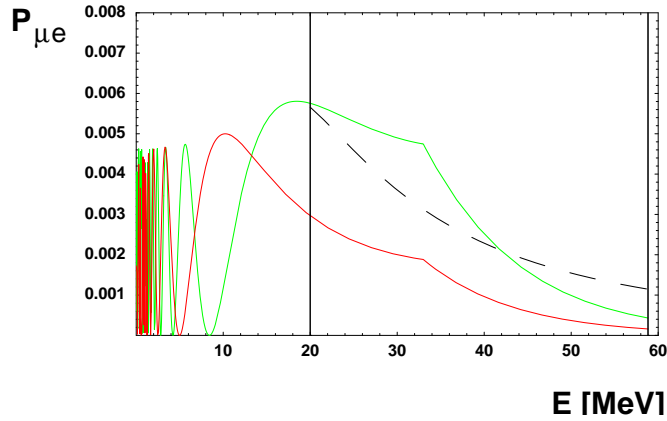
FIGURE 1. Bulk shortcut oscillation probabilities for LSND (light/green) and KARMEN (dark/red, reporting a null result) as a function of the neutrino energy, for $E_{res} = 33$ MeV; For comparison, a standard oscillation probability for LSND is displayed (dashed).
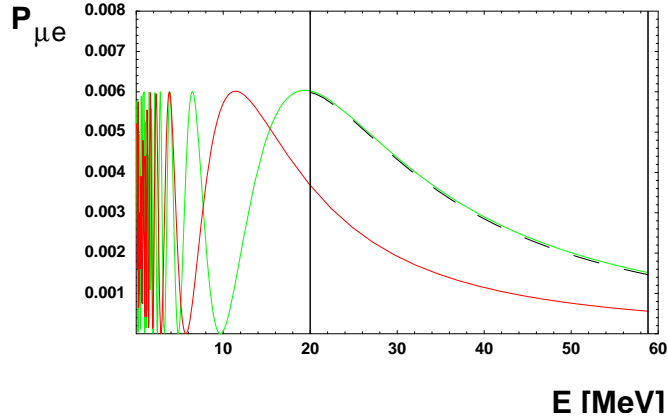

FIGURE 2. As Fig. 1, but for $E_{res} = 400$ MeV.

Results for the LSND oscillation probability for two limiting cases ($E_{res} = 33$ MeV and $E_{res} = 400$ MeV) are shown in Figs. 1 and 2. The LSND energy spectrum in both cases resembles the case of standard oscillations, and the constraint from the KARMEN experiment reporting a null result, is satisfied. The large resonance energy of the latter case leads to strongly enhanced muon neutrino oscillations at MiniBooNE as shown in Fig. 3. The small resonance of the first case predicts no neutrino oscillations at MiniBooNE, but does predict a distorted LSND spectrum and enhanced oscillations at a proposed experiment at the SNS [5].

In conclusion, we have discussed the effect of bulk shortcuts, which arise quite naturally in extra-dimensional theories, on neutrino oscillations. In this framework bulk shortcuts affect neutrino mixing and imply a new resonance, which implies a suppression of the oscillation amplitude at energies $E \gg E_{res}$. This effect accommodates the LSND result in a consistent framework with results from other neutrino oscillation experiments, and predicts strongly enhanced oscillations at MiniBooNE for $E_{res} \gg 100$ MeV. More-
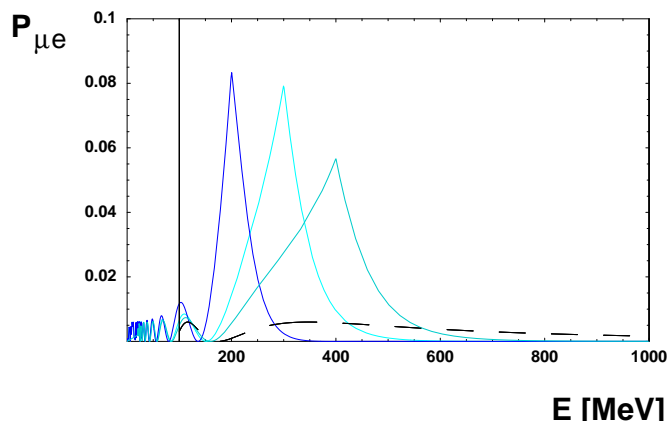
FIGURE 3. Bulk shortcut oscillation probabilities for MiniBooNE as a function of the neutrino energy, for $E_{\text{res}} = 200, 300, 400$ MeV, from left to right (dark to light). For comparison, the expectation for a standard oscillation solution for LSND is displayed (dashed).

over the BBN bound on a sterile neutrino abundance in the early universe may be evaded, if bulk shortcut effects like gravitational brane bending, brane fluctuations, or scattering off our brane into an asymmetrically warped bulk are amplified by higher temperatures and densities in the early universe in a manner which lowers the resonance energy below the MeV scale. In this case, oscillations into sterile neutrinos would be suppressed at BBN, the sterile state would not be populated, and fatal cosmological consequences would be evaded.

Open questions concern the details of BBN, supernova neutrinos, effects on atmospheric neutrino oscillations, realistic 3+1 neutrino fits, the relation of the LSND neutrino to cosmological dark matter, and influences on the horizon problem. We have started to analyze causality properties of shortcut spacetimes, and have pointed out that certain examples of asymmetrically warped spacetimes may allow for closed timelike curves [6]. It is an attractive thought that sterile neutrinos, if their existence is confirmed in future experiments, may help to experimentally probe Hawking's chronology protection conjecture, or even to realize neutrino time travel.

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