Closed timelike curves in asymmetrically warped brane universes

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Abstract

We discuss causality properties of asymmetrically warped space-times and argue that such scenarios may allow for timelike curves which can be closed via paths in the extra-dimensional bulk. We find a metric where the null, weak and dominant energy conditions are violated in the bulk, but satisfied on the brane. Such scenarios are interesting, since in principle gravitons or gauge-singlet ("sterile") fermions propagating in the extra dimension may be manipulated in a way to test the chronology protection conjecture experimentally.

1 Introduction

The physics of time travel has fascinated science fiction aficionados and scientists alike. In particular, the seminal papers of Morris, Thorne and Yurtserver [1] on traversable wormholes initiated a considerable research library of serious attempts to transmit information to the past, i.e. to generate closed timelike curves (CTCs). Several space-time settings, mostly contrived or oversimplified in some way, have been discussed in the literature. These include Gödel's rotating universe [2], van Stockum's and Tipler's rotating cylinders [3, 4], Gott's pair of moving cosmic strings [5], Wheeler's space-time foam [6], regions inside the horizon of Kerr- and Kerr-Newman geometries [7], Alcubierre's warp drive [8], and Ori's vacuum torus [9]. Typically these space-times suffer from obstacles of either unphysically fast rotation to tip the Lorentz cones, or the requirement of exotic matter with negative energy density which violates the so-named null, weak, strong and dominant energy conditions. Several analyses indicate possible instabilities of such space-times to classical perturbations and/or quantum fluctuations [10]. This situation has inspired Stephen Hawking's "chronology

protection conjecture" [11], which states that the ultimate laws of physics prevent the appearance of CTCs. Hawking's "chronology horizon" [11] is a stable, special type of Cauchy horizon, which separates space-time regions where CTCs occur from space-time regions where chronology is protected. Although apparently plausible situations seem naively to violate causality, quantum corrections to the stress-energy tensor diverge at the Cauchy horizon. It has been argued that the backreaction to the metric would destroy the potential time-machine on the horizon, leaving a protected region.

Whether Hawking's chronology protection conjecture holds beyond the semi-classical treatment, so that chronology is truly protected, is still not known. Very probably a better understanding of quantum gravity will be necessary to resolve this issue in the future. In the meantime, a study of physics under the unusual conditions surrounding the chronology horizon may provide more insight into chronology protection. One might glimpse some fascinating new physics proposed to avoid the obvious paradoxes associated with time travel. These paradoxes include the Grandfather and Bootstrap paradoxes. In the Grandfather paradox, one modifies the initial conditions that lead to one's own existence; in the Bootstrap pardox, an effect is its own cause. If the chronology protection conjecture is false, even more wonderful discoveries may await the serious researcher. Proposals include non-Hausdorff manifold geometry [12], where the same event has multiple futures or pasts, and the many-world interpretation of quantum mechanics, with switching between parallel histories [13].

The advent of theories with large extra dimensions in string theory has provided yet new room for chronology violations (see e.g. [14]). Extra dimensions were originally motivated by the consistency of string theory. More recently, large (or even infinite) extra dimensions have been discussed as a possible new way to understand the hierarchy problem ($M_{\rm weak} \ll M_P$) [15, 16] and to keep neutrino masses small [17]. In many extra-dimension models, ordinary Standard Model (SM) fields are confined on a brane (our three-surface), while gravitons and SM singlets are allowed to propagate also in the extra-dimensional bulk. A generic feature of such space-times seems to be the existence of signals, mediated by the graviton or SM singlets, taking "shortcuts" through the extra dimension. As viewed from our brane world, these shortcuts appear as superluminal communication [18]. Such apparent superluminal communication, via graviton shortcuts in the bulk [19], or earlier, via wormholes [20], has been proposed as a possible solution to the cosmological horizon problem (obviating one of the needs for an inflationary epoch in the early universe). While there seems to be agreement in the literature that extra dimensional space-times admit bulk shortcuts under

rather generic conditions, whether these shortcuts solve the horizon problem depends on the details of the specific extra-dimensional model [21, 22, 23, 24].

In this paper we discuss causality violations arising in asymmetrically warped brane-bulk models. As a rule of thumb, once a space-time model admits effective faster-than-light travel on a background having global Lorentz invariance, then the twin pseudo-paradox of special relativity is elevated to the time travel paradox [12] whereby a signal may arrive at the spatial point from which it was emitted *at a time before it was emitted!*[1]. This occurs whether the source of the superluminal signal is wormholes, warp drives, geodesics in extra dimensions, etc. Thus, we seek a metric (i) describing a globally Lorentz invariant brane, and (ii) admitting shortcut geodesics through the bulk. We show by construction that it is not difficult to find such a metric.

2 Causality of asymmetrically warped space-times

In space-times allowing for shortcuts in the extra dimensions, the brane defines a hypersurface which in general is not "totally geodesic", meaning that an on-brane geodesic is not in general a geodesic in the brane+bulk space-time (and vice versa) [21]. It is thus crucial to discuss Lorentz invariance on the brane separately from invariance in the brane+bulk, i.e. to analyze which line element remains invariant under Lorentz transformations. The possibility that the complete extra-dimensional line element ds^2 is invariant can be discarded, as Lorentz invariance will be broken by compactifying and orbifolding or warping the bulk (compare the discussion in [26]). Thus Lorentz invariance holds on the brane but not in the bulk, or it is broken both in the bulk and on brane. In the latter case, Lorentz violation would be observable experimentally, for example, as bremsstrahlung due to inertial motion on the brane. The latter case includes space-times where the extrinsic brane curvature generates the effective superluminality as observed on the brane, somewhat analogous to light propagating in a curved fiber cable. Examples in the literature are described by the line elements (17) in [19] and (8) in [27]. Here, we credit the apparently good Lorentz symmetry of our world, and pursue the former case where Lorentz invariance is intact on our brane. Interestingly, bulk shortcuts in this scenario resemble simple examples of wormhole space-times.

[1]For a careful discussion of why superluminal travel in a space-time without global Lorentz invariance does not necessarily lead to CTCs, see, e.g. [25].

Consider the asymmetrically warped line element with extra dimension "u"

$$ds^2 = dt^2 - \sum_i \alpha^2(u)\,(dx^i)^2 - du^2, \tag{1}$$

$i = 1, 2, 3$, with our brane located at the $u = 0$ submanifold. The warped space-time of (1) allows shortcut geodesics connecting spacelike-separated events on the brane, if $|\alpha(u)| < |\alpha(0)|$ for any $u \neq 0$.

Variants of this warped space-time (1) can be generated by AdS-Schwarzschild or AdS-Reissner-Nordström black holes in the bulk [28], and have been discussed as solutions to the cosmological horizon problem [19], and as a possible way around Weinberg's no-go theorem for the adjustment of the cosmological constant [28]. Very recently it has been shown that sterile neutrinos propagating in such a space-time can account for the LSND neutrino oscillation evidence, without the problems faced by conventional four-dimensional four-neutrino scenarios [27].

The metric in (1) is in Gaussian normal form with respect to u (i.e., $g_{tu} = g_{x_i u} = 0$), so the induced metric on each hypersurface with constant u is simply given by the extra-dimensional metric evaluated on the hypersurface. These induced metrics are purely Minkowskian, albeit with a different constant limiting velocity $c(u) = \alpha^{-1}(u)$ on each hypersurface [2]. This means that a Lorentz symmetry can be defined for each hypersurface, but each hypersurface's Lorentz symmetry will not hold on any other hypersurface, as we now discuss.

It is natural to choose $c(u = 0) = 1$ such that the induced metric on the brane is given by $ds^2_{\text{brane}} = dt^2 - dx^2$ (we set $x^1 \equiv x$ and will omit the transverse coordinates $x^{2,3}$ for brevity). There then follows the usual Lorentz symmetry under the familiar transformations on our brane:

$$x' = \gamma\,(x - \beta t)\,, \qquad t' = \gamma\,(t - \beta x)\,, \qquad u' = u = 0, \tag{2}$$

or equivalently, the inverse transformation

$$x = \gamma\,(x' + \beta t')\,, \qquad t = \gamma\,(t' + \beta x')\,, \tag{3}$$

with the usual definition $\gamma = (1 - \beta^2)^{-1/2}$. However, physics at $u \neq 0$ (in the "bulk") is not invariant under this transformation.

[2] Note that in the limiting case of vanishing α the space-time (1) becomes equivalent to Minkowski space-time on the brane with all space-time points being identified through the bulk, i.e. an "ubiquitous wormhole". As it is well known, that wormhole space-times allow for CTCs, it is not too surprising that CTCs can be generated also in the case of finite α presented in this paper.

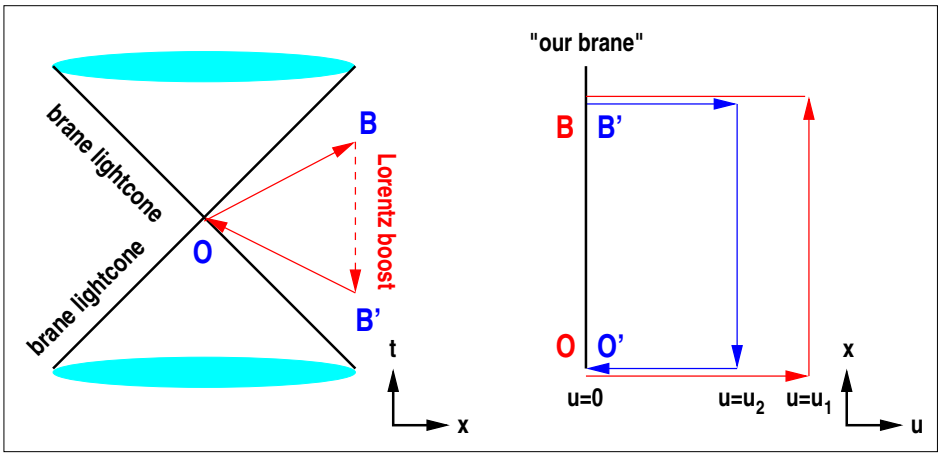

Figure 1: Closed timelike curve in an asymmetrically warped universe: (i) A signal takes a spacelike shortcut via a path of constant $u = u_1$ from point O to point B. (ii) A Lorentz boost transforms B into B' with negative time coordinate. (iii) A return shortcut at constant $u = u_2$ closes the timelike curve.

In the following we consider a signal following a particular path as given in Fig. 1. The signal leaves our brane at the space-time point $O = (t = 0, x = 0, u = 0)$, and propagates on the hypersurface at u_1 for a travel time t with the limiting velocity $(\alpha(u_1))^{-1} \equiv \alpha_1^{-1}$. We will assume that $0 < \alpha_1 < 1$, so that the travel speeds in the bulk is superluminal relative to travel speed on our metric. At time t, the signal may reenter our brane. In the limit $u_1 \ll \alpha_1^{-1} t$, which is always fulfilled for sufficiently large t, the reentry point on our brane is $B^\mu \approx (t, x = \alpha_1^{-1} t, u = 0)$. Since the distance to the reentry point B^μ is spacelike (i.e. outside the brane's lightcone), it may be transformed to negative time by a boost on our brane. The boosted point B'^μ is obtained by using the transformation (2). The point B'^μ has coordinates

$$x' = \gamma\, t \left(\alpha_1^{-1} - \beta\right), \qquad t' = \gamma\, t \left(1 - \beta \alpha_1^{-1}\right). \tag{4}$$

It is clear that for

$$0 < \alpha_1 < \beta < 1 \tag{5}$$

an observer in the boosted frame on our brane sees the signal arrive in time with $t' < 0$, i.e., before it was emitted. This result alone does not imply any conflict with causality. In particular, it does not necessarily imply that space-time is blessed with CTCs. To close the timelike curve, one has to show that the time t' during which the signal traveled backwards in time, is sufficiently large to allow a return from the space-time point $B'^\mu = (t', (x = \alpha_1^{-1} t)', 0)$ to the space-time point of origin, $O = O' = (0, 0, 0)$. The speed required to close the lightlike

curve of the signal, as seen by the boosted observer on the brane, is

$$c'_{\rm req} = \frac{(x = \alpha_1^{-1} t)'}{|t'|} = \frac{1 - \beta\alpha_1}{\beta - \alpha_1}\,, \tag{6}$$

where the latter expression results from inputting Eq. (2). It is easy to show that the condition $0 < \alpha_1 < \beta < 1$ implies that $c'_{\rm req}$ itself is superluminal. Thus there is no return path on our brane which leads to a CTC. To generate a CTC the signal has to traverse another path (say, at constant u_2) which has a limiting velocity satisfying $c'_{\rm bulk} \geq c'_{\rm req}$ in the boosted frame.

The complete metric in the boosted system is given by the tensor transformation law

$$g'_{\alpha\beta} = \frac{\partial x^\mu}{\partial x'^\alpha}\frac{\partial x^\nu}{\partial x'^\beta}\, g_{\mu\nu}, \tag{7}$$

where $g_{\mu\nu} = {\rm diag}(1, -\alpha^2, -1)$ is the Gauss-normal metric of Eq. (1). Using Eq. (3), the resulting boosted metric is

$$g'_{\mu\nu} = \begin{pmatrix} \gamma^2(1-\beta^2\alpha^2) & \frac{1}{2}\gamma^2\beta(1-\alpha^2) & 0 \\ \frac{1}{2}\gamma^2\beta(1-\alpha^2) & -\gamma^2(\alpha^2-\beta^2) & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{8}$$

Notice that only for $\alpha^2 = 1$ is the metric Lorentz invariant. Such is the case on our brane, but generally not the case on other hypersurfaces, where the limiting velocity as seen by local inhabitants in the rest frame is $\alpha^{-1}(u_j) \equiv \alpha_j^{-1}$. However, these limiting velocities are not invariant under Lorentz boosts defined on our brane. Using the general expression for the metric in Eq. (8), the null line element for this hypersurface is

$$0 = ds'^2 = \gamma^2 \left\{ \left(1 - \beta^2\alpha_2^2\right) dt'^2 - \beta\left(\alpha_2^2 - 1\right) dx'dt' - \left(\alpha_2^2 - \beta^2\right) dx'^2 - du'^2 \right\}. \tag{9}$$

There results a quadratic equation for $c'_{\rm bulk}$, with solutions

$$c'_{\rm bulk} \equiv \frac{dx'}{dt'} = \frac{\beta(1-\alpha_2^2)}{2(\alpha_2^2 - \beta^2)} \pm \sqrt{\frac{\beta^2(1-\alpha_2^2)^2}{4(\alpha_2^2-\beta^2)^2} + \frac{1-\beta^2\alpha_2^2}{\alpha_2^2-\beta^2}}\,. \tag{10}$$

We choose $\alpha_2 > \beta$ to maintain the negative sign for the $x' - x'$ metric component [3]. Including the ordering $0 < \alpha_1 < \beta < 1$ noted previously, we arrive at $0 < \alpha_1 < \beta < \alpha_2 < 1$. The CTC is then established if we can find $(\alpha_1, \beta, \alpha_2)$ satisfying the condition and satisfying

[3] To ensure that the signature of the metric $g_{(2)}$ for the $t' - x'$ subspace is maintained, it is enough to require ${\rm Det}(g_{(2)}) < 0$. It is easily verified that the choice $\alpha_2 > \beta$ is sufficient to ensure this.

$c'_{\rm bulk} > c'_{\rm req}$. This is demontrated as follows. We substitute into (10) the relation $\alpha_2^2 = \beta^2 + \epsilon$, with $\epsilon \ll 1$. The resulting solutions to (10) are

$$c'_{\rm bulk+} = \frac{\beta}{\gamma^2 \epsilon} + \frac{1}{\beta}(1+\beta^2)\left(1 + \gamma^2\epsilon + \mathcal{O}(\epsilon^2)\right), \tag{11}$$

$$c'_{\rm bulk-} = -\frac{1}{\beta}(1+\beta^2)\left(1 + \gamma^2\epsilon + \mathcal{O}(\epsilon^2)\right). \tag{12}$$

To lowest order in ϵ, the solution $c'_{\rm bulk-}$ moving along negative x' exceeds $c_{\rm req}$ if $\frac{1}{\beta}(1+\beta^2) > \frac{1-\beta\alpha_1}{\beta-\alpha_1}$, or equivalently, if $\alpha_1 < \beta^3$; An acceptable set of parameters to close the CTC is thus $0 < \alpha_1^{\frac{1}{3}} < \beta < \alpha_2 < 1$. It is almost always possible to find such a combination of parameters for a given warp factor $\alpha(u)$.

It should be stressed that realistic graviton or bulk fermion signals, rather than following restricted bulk trajectories with constant u as constructed here, will instead propagate on the path of least action to minimize the travel time. Since the effectively superluminal velocities in our constructed example produced a CTC, we expect that a truly geodesic signal will also generate a CTC.

To summarize this section, we have identified a CTC beginning and ending on our brane and superluminally transiting two parallel paths in an asymmetrically warped bulk.

3 Stress-energy tensor and energy conditions

As a check on the consistency of the picture, we should diagnose the stress-energy tensor $T^{\mu\nu}$ which sources the extra-dimensional metric, for any pathologies. Thus, our task is to construct the stress-energy tensor

$$T_{\mu\nu} = \frac{1}{8\,\pi\,{\rm G_N}}\,G_{\mu\nu} \tag{13}$$

for the space-time (1). It is straightforward to show that the only nonzero Christoffel symbols

$$\Gamma^{\kappa}_{\lambda\mu} = \frac{g^{\kappa\nu}}{2}\left(\frac{\partial g_{\mu\nu}}{\partial x^\lambda} + \frac{\partial g_{\lambda\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\lambda}}{\partial x^\nu}\right) \tag{14}$$

for the metric (1) are

$$\Gamma^j_{5j} = \Gamma^j_{j5} = \frac{\alpha'}{\alpha}, \tag{15}$$

$$\Gamma^5_{jj} = -\alpha\alpha', \tag{16}$$

with $\alpha' = \partial_u \alpha$. Thus the non-vanishing terms in the Ricci tensor

$$R_{\mu\nu} = -\frac{\partial \Gamma^\rho_{\mu\rho}}{\partial x^\nu} + \frac{\partial \Gamma^\rho_{\mu\nu}}{\partial x^\rho} - \Gamma^\sigma_{\mu\rho}\Gamma^\rho_{\sigma\nu} + \Gamma^\sigma_{\mu\nu}\Gamma^\rho_{\sigma\rho}, \tag{17}$$

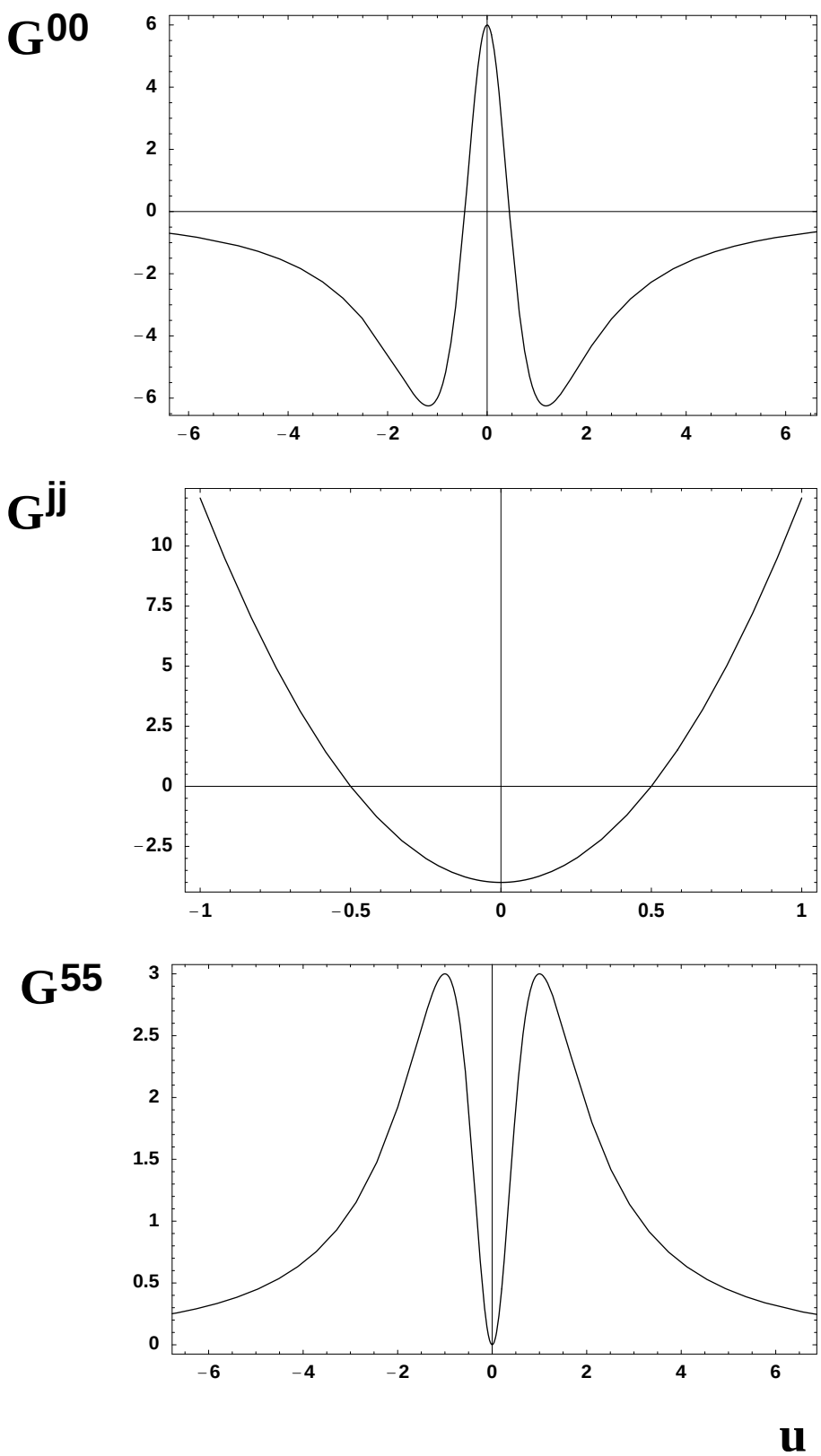

Figure 2: Elements of the Einstein tensor $G^{\mu\nu}$, as a function of u, for the warp factor $\alpha(u) = 1/(u^2 + c^2)$, with $c = 1$. While energy conditions are violated in the bulk, they are conserved on the brane.

are

$$R_{jj} = -(2\alpha'^2 + \alpha\alpha'') \tag{18}$$

$$R_{55} = -3\frac{\alpha''}{\alpha}. \tag{19}$$

Conseqently, the resulting curvature scalar is

$$R = g^{\mu\nu} R_{\mu\nu} = 6\left(\frac{\alpha''}{\alpha} + \left(\frac{\alpha'}{\alpha}\right)^2\right) \tag{20}$$

and finally, the Einstein tensor is given by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{21}$$

i.e.

$$G^{00} = 8\,\pi\,\mathrm{G_N}\,\rho = -3\left(\frac{\alpha''}{\alpha} + \left(\frac{\alpha'}{\alpha}\right)^2\right), \tag{22}$$

$$G^{jj} = 8\,\pi\,\mathrm{G_N}\,p_j = \frac{1}{\alpha^4}(\alpha'^2 + 2\alpha\alpha''), \tag{23}$$

$$G^{55} = 8\,\pi\,\mathrm{G_N}\,p_5 = 3\left(\frac{\alpha'}{\alpha}\right)^2. \tag{24}$$

The null, weak, strong and dominant energy conditions are defined by

$$\text{NEC} \;:\; \rho + p_j \geq 0 \quad \forall j; \tag{25}$$

$$\text{WEC} \;:\; \rho \geq 0 \quad \text{and} \quad \forall j, \quad \rho + p_j \geq 0; \tag{26}$$

$$\text{SEC} \;:\; \forall j, \quad \rho + p_j \geq 0 \quad \text{and} \quad \rho + \sum_j p_j \geq 0; \tag{27}$$

$$\text{DEC} \;:\; \rho \geq 0 \quad \text{and} \quad \forall j, \quad p_j \in [\rho, -\rho]. \tag{28}$$

It is not difficult to find a functional form for the warp factor α, which conserves some of the energy conditions, at least on the brane. One such example is $\alpha(u) = 1/(u^2 + c^2)$. For this case the elements of the Einstein tensor are shown as a function of u in Fig. 2. The null, weak and dominant energy conditions are conserved on the brane, while the strong energy condition is violated on the brane.

4 Discussion and Conclusion

We have demonstrated the existence of closed timelike curves (CTC) for a rather generic asymmetrically warped metric. In addition, we have found a particular metric yielding positive energy density on the brane.

A thorough discussion of whether CTCs in the observable universe are prevented by stable chronology horizons where the stress-energy tensor diverges (consult the discussion in [29, 30]), is beyond the scope of this work. We have confined ourselves to the pragmatic attitude that even if chronology is protected by some mechanism operative near the chronology horizon, it remains a highly rewarding effort to study the physics near this horizon. The CTC we have constructed is particularly interesting in this respect, since it is available to particles which have previously been hypothesized to propagate in the extra-dimensional bulk [4]. Such particles include the graviton and sterile (gauge singlet) fermions.

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