Large extra dimensions

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Abstract

In this master of science thesis the concerns of large extra dimensions are discussed. The thesis begins with an introduction to string theory followed by a review of large extra dimensional topics. A motivation for large extra dimensions is found within the heterotic string theory. By imposing a compactification volume of the order of TeV it is possible to break the supersymmetry and obtain the sought after theory with one supersymmetry. The relations between the different string theories are investigated. A theory with two extra dimensions can possibly have extra dimensional radii of $\sim$mm size. In the end I discuss experimental verification in the context of accelerators and gravitational experiments. It is argued that a change of the gravitational force from $\sim 1/r^2$ to $\sim 1/r^{2+n}$ has very little effect on macroscopic systems.

Contents

Introduction

One of the main goals in the history of science has been unification. In the 19th century Maxwell understood that the electric and magnetic forces could be united into one -electromagnetism. Einstein formed the theory of general relativity from special relativity and gravity. In the 1940s it was clear that quantum field theory was the correct framework for the unification of quantum mechanics and special relativity.

Today we stand before yet another challenge of the same character, the unification of quantum mechanics and general relativity. Any "straightforward" attempt to quantize general relativity leads to a non-renormalizable theory. They way string theory takes care of this is by assuming that elementary particles, instead of being point like, are one-dimensional extended objects -strings. Even though string theory is arising within the framework of a consistent quantum field theory, which does not allow gravity, it necessarily incorporates gravity in the form proposed by Einstein, as desired.

String theory divides into several different theories. First we have the Bosonic string theory which is a good starting point for understanding the string theory basics. It has a spacetime dimensionality of 26, contains a tachyon and lacks fermions and is therefore not thought to be the theory we are looking for. To introduce fermions we make the theory supersymmetric. This takes us from plain string theory to superstring theory. Supersymmetry says, among other things, that there must be one fermion for each type of boson and vice versa. It also reduces the number of spacetime dimensions to 10. The superstring theory contains open and closed strings. The type I super string theory contains open strings while type II theory contains closed strings. We will later see that type I theory separates into type I and type I' and type II into IIA and IIB. Finally we have the heterotic string which is a hybrid of a bosonic and superstring.

Very important discoveries done recently are the two duality transformations. They make it possible to transform one theory into another. The T-duality inverts the radius of a compact dimension and S-duality inverts the coupling constant. This might seem trivial, but the impact and consequences of the dualities are of great importance. Instead of dealing with five different theories they allow us to focus on one underlaying theory - the M-theory.

One of the most stunning predictions made by string theory is a spacetime dimensionality of 10, or 11 in M-theory. One immediately wonders how this is compatible with our four dimensional world. The answer to this is extra dimensional compactification. The extra dimensions are made periodic, one can think of them as curled. This gives rise to a compactfication volume and a compactification manifold which surrounds the volume. But how large is the compactification volume, is it infinitely small or maybe large enough to be seen somehow? Before trying to answer this question or at least discussing it, we need to understand a couple of things concerning the extra dimensional volume. First of all, in the heterotic theory, which provides a motivation for a large compactification volume through supersymmetry braeking and gauge coupling unification, the volume it self is proportional to the string coupling constant squared. In other words, strong coupling is necessary for a large compactification volume. The second thing is that we need a weak coupling to be able to speak of the smallest constituents of matter, strings. This simply comes from the fact that strings and other particles in general tend to interact and form more massive objects under strong coupling. Hence, if we want large extra dimensions we must find a way to get rid of the strong coupling but keeping the spatial properties strong coupling implies. The string dualities are the answer to this problem. T-duality turns a small dimension into a large and vice versa while S-duality inverts the coupling, making strong coupling weak and weak coupling strong. It is therefore possible to transform the above mentioned heterotic theory to a theory with a weak coupling and a large compactification volume. The question is if the dual theory is a candidate for describing our universe.

Despite all its success string theory is far from being complete. It might even turn out to be inconsistent or wrong in certain aspects which may lead scientists on to other paths looking for a theory of unification. We shall see.

The first chapter of this thesis is devoted to the basics of string theory. Chapter two deals with large extra dimensions and their experimental implications. The last chapter is rather short and handles the aftermath and references.

Chapter 1

Basic string theory

In this chapter the basics of string theory will be discussed briefly. It starts with bosonic string theory, which is not satisfactory in many ways. The bosonic theory lacks fermions and contains a tachyon. Despite this the bosnic string is never the less important. It is a good starting point and a crucial ingredient in the heterotic string theory which seems to be one of the more promising theories. The chapter continues with superstrings where open and closed strings are discussed. The features of a ten dimensional action are mentioned very briefly. We move on to the heterotic string which is a hybrid of a bosonic and a super string. Thereafter comes the important dualities. In the end M-theory is discussed and as an example a sequence of dualities is performed on a compactified M-theory.

1.1 The bosonic string

We will start by considering the simplest string possible, a free string propagating in a D-dimensional spacetime. The string action corresponding to these circumstances is the Nambu-Goto action

$$S_{NG} = T \int dA.$$

This action makes perfect sense since it is the analog to the particle action $S = m \int ds$. But instead of minimizing the length of a world line we minimize the area of a world sheet. T denotes the tension of the string and is given by $T = \frac{1}{2\pi\alpha'}$ where α is proportional to the string length squared, l_s^2. Distances are measured on the world sheet according to $ds = \frac{\partial X^\mu}{\partial \sigma^\alpha} d\sigma^\alpha$ this implies that $ds^2 = g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta} d\sigma^\alpha d\sigma^\beta = h_{\alpha\beta} d\sigma^\alpha d\sigma^\beta$ where $h_{\alpha\beta}$ is the metric induced on the world sheet. The area element is given by

$$dA = |e_1||e_2| sin\theta$$

where $e_1 = (d\sigma_1, 0)$ and $e_2 = (0, d\sigma_2)$ implying $|e_1| = h_{11}^{1/2} d\sigma_1$ and $|e_2| = h_{22}^{1/2} d\sigma_2$. We also know that

$$\bar{e}_1 \cdot \bar{e}_2 = |e_1||e_2| cos\theta = h_{12} d\sigma_1 d\sigma_2$$

Using this and the above we see that $cos\theta = \frac{h_{12}}{h_{11}^{1/2} h_{22}^{1/2}}$ which gives us that $sin\theta = \sqrt{1 - \frac{h_{12}^2}{h_{11}h_{22}}} = \sqrt{\frac{h_{11}h_{22} - h_{12}^2}{h_{11}h_{22}}}$. Hence we have

$$dA = |e_1||e_2| sin\theta = h_{11}^{1/2} d\sigma_1 h_{22}^{1/2} d\sigma_2 \sqrt{\frac{h_{11}h_{22} - h_{12}^2}{h_{11}h_{22}}}$$

The area element will thus be

$$dA = \sqrt{h} d\sigma_1 d\sigma_2$$

where h is the determinant of the metric. Hence the Nambu-Goto action will now take the form

$$S_{NG} = -T \int \sqrt{-h} d\sigma_1 d\sigma_2$$

The square root of makes this action a bit tricky to handle why we turn our attention elsewhere. Now consider the Polyakov action

$$S_p = -\frac{T}{2} \int d\sigma_1 d\sigma_2 \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu$$

where γ^{ab} is a metric. This action can be shown to be same as the Nambu-Goto action if we in addition the Polyakov action also take into account the constraint $\frac{\delta S_p}{\delta \gamma^{ab}} = 0$. This is by definition equal to $T_{ab} = 0$. Varying the Polyakov action with respect to the metric gives

$$\delta_{\gamma^{ab}} S_P = -\frac{T}{2} \int d\sigma_1 d\sigma_2 \sqrt{-\gamma} \delta\gamma^{ab} (h_{ab} - \frac{1}{2} \gamma_{ab} \gamma^{cd} h_{cd})$$

where we have used that the variation of the determinant is $\delta\gamma = \gamma\gamma^{ab}\delta\gamma_{ab} = -\gamma\gamma_{ab}\delta\gamma^{ab}$ which gives $\delta(-\gamma)^{1/2} = \frac{1}{2}(-\gamma)^{-1/2}\delta\gamma = \frac{1}{2}(-\gamma)^{1/2}\gamma_{ab}\delta\gamma^{ab}$. Requiring that $T_{ab} = \frac{\delta S_p}{\delta \gamma^{ab}} = 0$ implies

$$h_{ab} = \frac{1}{2} \gamma_{ab} \gamma^{cd} h_{cd}.$$

Dividing this expression by the square root of its determinant gives the relation

$$h_{ab}(-h)^{-1/2} = \gamma_{ab}(-\gamma)^{-1/2}$$

which is equal to

$$(-h)^{1/2} = \gamma^{ab}(-\gamma)^{1/2} h_{ab}.$$

This can be used to eliminate γ^{ab} in the Polyakov action. When this is done we see that $S_P \to S_{NG}$. Hence the Polyakov action can be used instead of the more complicated Nambu-Goto action if we in addition take into consideration the condition $T_{ab} = 0$.

The equation of motion obtained form S_p is a wave equation. This is not all though, depending on whether the string is closed or open different boundary conditions are imposed. Closed strings have ends which are connected to form closed loops and are hence subject to the constraints

$$X^\mu(\tau, l) = X^\mu(\tau, 0) \tag{1.1}$$

$$\partial_\sigma X^\mu(\tau, 0) = \partial_\sigma X^\mu(\tau, l). \tag{1.2}$$

The solution to the equations of motion with the above conditions imposed is

$$X^\mu(\sigma, \tau) = x^\mu + p^\mu \tau + \sum_{n \neq 0} \left(\frac{\tilde{\alpha}_n^\mu}{n} e^{-2in(\tau+\sigma)} + \frac{\alpha_n^\mu}{n} e^{-2in(\tau+\sigma)} \right). \tag{1.3}$$

This expression can be split into left and right moving parts. The two terms in the sum does in fact represent the left and right moving parts.

Open strings, on the other hand, have ends moving freely in spacetime independent of each other. This leads to slightly different constraint. Either Neumann or Dirichlet boundary conditions can be imposed,

$$\partial_\sigma X^\mu|_{\sigma=0,l} = 0 \tag{1.4}$$

$$\partial_\tau X^\mu|_{\tau=0,l} = 0. \tag{1.5}$$

The Dirichlet condition can be integrated which gives a specific location in space time on which the string ends. This only makes sense if the open string ends on some sort of physical object, a D-brane, D for Dirichlet. The Neumann conditions are usually imposed and and we shall assume so from now on unless else is stated. The solution for the open string looks like

$$X^\mu(\sigma, \tau) = x^\mu + \pi l p^\mu \tau + il \sum_{n \neq 0} \frac{1}{n} \alpha_n^\mu e^{-i\frac{\pi n \tau}{l}} cos \left(\frac{\pi n \sigma}{l} \right) \tag{1.6}$$

We also have to take into consideration the energy-momentum constraints, $T^{ab} = 0$. This can be done by introducing the light cone coordinates

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1) \tag{1.7}$$

$$x^i, i = 2, ..., D-1. \tag{1.8}$$

The energy momentum-constraints imposes a spacetime dimensionality of 26 for the bosonic theory. This is shown in great detail in [3]. A dimensionality of 26, a tachyon and no fermions make the bosonic string unsuitable for our purposes. However, it is good as a starting point since the Nambu-Goto action is intuitively clear and, as pointed out before, the bosonic string is a necessary ingredient in the heterotic string theory, which is one of the most interesting theories.

1.2 The superstring

Since the real world contains fermions, we would like to somehow add fermions to our theory. This can be done by introducing a supersymmetric world-sheet and hence creating superstrings [1]. Super symmetry is implemented by requiring the action to be invariant under the supersymmetry transformations

$$\delta X^\mu = \bar{\epsilon}\psi^\mu \tag{1.9}$$

$$\delta\psi^\mu = -\rho^\alpha \partial_\alpha X^\mu \epsilon. \tag{1.10}$$

This is fulfilled by the Polyakov action with added world-sheet fermions

$$S = -T \int d^2\sigma \sqrt{-\gamma}(\partial_a X^\mu \partial^a X_\mu + i\bar{\psi^\mu}\rho^a \partial_a \psi_\mu) \tag{1.11}$$

where ψ are the fermionic fields, $\bar{\psi^\mu} = \psi^{\mu\dagger}\rho^0$, $\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ and $\{\rho^a, \rho^b\} = 2\eta^{ab}I$. The equations of motion will, for the bosons, be an ordinary wave equation and for the fermions the dirac equation

$$i\rho^a \partial_a \psi^\mu = 0 \tag{1.12}$$

Supersymmetry makes sure that for each type of boson there is a fermion and vice versa. Due to the added fermions a quantum field theoretical calculation of the central charge, [3], shows that the number of spacetime dimensions decreases from 26 to 10.

The solutions of the above dirac equation can be split into left- and right-parts.

$$\psi^\mu = \psi^\mu_+ + \psi^\mu_- \tag{1.13}$$

where ψ^μ_+ describes the right movers and ψ^μ_- the left movers. The solutions can represent either open or closed strings and we must treat them separately.

In case of open strings it turns out that we need $\psi^\mu_+ = \pm\psi^\mu_-$ at the two ends ($\sigma = 0, \pi$) in order to have a well defined action [4]. We can choose one without loss of generality, say

$$\psi^\mu_+ = \psi_- \tag{1.14}$$

This leaves two possibilities for the other string end which can be either periodic or anti periodic.

$$\psi_+^\mu(\pi,\tau) = \psi_-^\mu(\pi,\tau) \tag{1.15}$$

$$\psi_+^\mu(\pi,\tau) = -\psi_-^\mu(\pi,\tau). \tag{1.16}$$

The periodic condition is called Ramond, R, and the anti-periodic Neveu-Schwarz, NS. The solutions are

$$\psi_{NS,\pm}^\mu = \sum_{r\in Z+1/2} b_r^\mu e^{-ir(\tau\pm\sigma)} \tag{1.17}$$

$$\psi_{R,\pm}^\mu = \sum_{r\in Z} d_r^\mu e^{-ir(\tau\pm\sigma)}. \tag{1.18}$$

Open strings are called type I strings and are divided into two sectors NS and R.

In the closed string case the situation is slightly different. The left and right movers of the fermionic field can either be periodic or anti-periodic

$$\psi_-^\mu(\tau,\sigma+2\pi) = \pm\psi_-^\mu(\tau,\sigma) \tag{1.19}$$

$$\psi_+^\mu(\tau,\sigma+2\pi) = \pm\psi_+^\mu(\tau,\sigma). \tag{1.20}$$

The periodic conditions are still denoted R, and the antioperiodic NS.

In order to get the complete closed string states we must combine left movers with right movers. This gives us four sectors of interest

$$(NS,NS),(R,R),(R,NS),(NS,R).$$

It is also appropriate to count the number of worldsheet fermions one needs for a certain state and make a difference between odd and even. In the NS case this means that N^{NS} is half integer or integer. NS_+ will denote integer and NS_- will denote half integer values of N^{NS}. R_+ and R_- simply denote different chiralities. It also turns out that some sectors contain a tachyon. The tachyon free options are

$$(NS_+,NS_+),(R_+,NS_+),(NS_+,R_-),(R_+,R_-)$$

which is called type IIA string theory and

$$(NS_+,NS_+),(R_+,NS_+),(NS_+,R_+),(R_+,R_+)$$

which is called type IIB. The (NS,NS) states give 64 bosons, the (NS,R) and (R,NS) states give us 128 fermions and finally (R,R) give 64 bosons. Totally we hence have 128 bosons and 128 fermions. Equally many bosons and fermions, as required by supersymmetry.

Unfortunately supersymmetry does not imply four spacetime dimensions but ten. There are however ways of taking care of the extra dimenions.

This is done by a process called compactification. One identifies points in spacetime and thereby creates a compactification manifold. This must be done with great care since the wrong manifold increases the number of supersymmtries and the correct number of supersymmetries is thought to be one.

The world sheet action is not the only way to represent strings. We can instead choose to look at them from a ten-dimensional point of view. This leads to a ten dimensional action containing different terms representing different parts of physics. It is not necessary to include all these terms when looking at a certain phenomena. If for example, we are interested in gravity only, it suffices to consider

$$\int d^{10}x e^{-2\phi}\mathcal{R}. \tag{1.21}$$

Where $\mathcal{R}$ is the Ricci scalar and ϕ represents something called the dilaton field which is related to the coupling, $e^{-2\phi}$ is in fact nothing but the coupling constant, in this case the Newton's constant in ten dimensions. As we see the coupling is not fixed but rather the expectation value of the dilaton field. The equation of motion for this action is nothing but Einstein's field equations in ten dimensions. The action can contain many other kinds of terms representing for example p-dimensional objects called branes, which simply are necessary in string theory, the earlier mentioned D-branes for example.

1.2.1 The heterotic string

Nothing keeps us from combining a bosonic string with a super string. We can do this by choosing a bosonic string as left mover and a superstring as right mover. We need $X_L^\mu, X_L^m, X_R^\mu, \psi_R^\mu$ where $\mu = 1, ..., 10$ and $m = 11, ..., 26$. To get rid of the extra 16 dimensions we compactify with the help of a 16 dimensional lattice. Identifying points in this lattice leaves us with a ten dimensional bosonic string part. There are two lattices suitable for this, the root lattices of the gauge groups $SO(32)$ and $E_8 \times E_8$ [1].

The heterotic string theory seems to be one of the more promising theories in the context of large extra dimensions.

1.3 String dualities

String theories compactified on certain manifolds can be transformed to other theories under certain mappings called string dualities. The different string theories are said to be dual to each other under the particular transformation. In this section we will look at two types of dualities, T-duality which relates a theory compactified on a small circle with another theory compactified on a large circle and S-duality which transforms a theory with strong coupling into a theory with weak coupling.

1.3.1 T-duality

T-duality, a very interesting result obtained in late 1980s, relates one string theory with a circular dimension of radius R with another theory with a circular dimension of radius 1/R. This can be seen by studying a closed string with one circular dimension. The wave function in the circular dimension must periodic and hence we must have

$$e^{ipx} = e^{ip(x+2\pi R)} \tag{1.22}$$

this implies $p2\pi R = n2\pi$ which equals

$$p = \frac{n}{R}. \tag{1.23}$$

This gives rise to one class of excitations called Kaluza-Klein excitations. There is another kind of excitation which is relevant in the closed string case, namely the winding. A closed string can wind ω times around the circular dimension we must have $X^\mu(\sigma + 2\pi) = X^\mu(\sigma) + 2\pi R\omega$. Where ω is the so called winding number. In a non compactified 10-dimensional spacetime a graviton is massless and

$$m^2 = 0 = E^2 - \sum_{i=1}^{9} p_i^2 \tag{1.24}$$

is true. If we compactify this theory on a circle as discussed above and hence consider it from a 1-dimensional lower point of view the situation changes radically. The momentum in the compact dimension will now look like a mass from our point of view. We have

$$m^2 = p_9^2 = \left(\frac{n}{R}\right)^2 = E^2 - \sum_{i=1}^{8} p_i^2. \tag{1.25}$$

The string winding also gives a contribution to the mass equal to ωR [1]. The total mass shift will hence be

$$m^2 = \left(\frac{n}{R}\right)^2 + (\omega R)^2. \tag{1.26}$$

We see that this quantity is unchanged if we invert the radius, R. In other words, from this perspective the closed string theories IIA and IIB are T-dual.

If we perform a T-duality on a theory containing open strings, with the Neumann boundary condition imposed the Neumann condition is turned

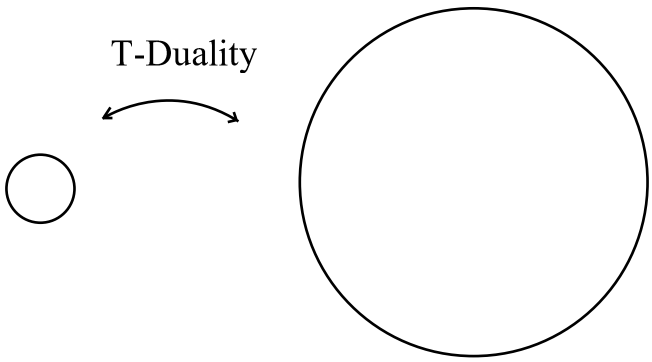

Figure 1.1: T-duality inverts the compactification radius.

into a Dirichlet boundary condition. The string now lives in a the presence of two big walls. This is called a type I' theory. The type I' theory turns out to be nothing else but a type IIA theory living between these two walls.

T duality must also leave Newton's constant invariant. We study this by considering the term in the ten dimensional action that contains the Ricci scalar $\mathcal{R}$. The following must hold

$$\int d^{10} e^{-2\phi}\mathcal{R} = \int d^9 R e^{-2\phi}\mathcal{R} = \int d^9 \frac{R}{g^2}\mathcal{R} = \int d^9 \frac{R'}{g'^2}\mathcal{R} \tag{1.27}$$

where the primed parameters are for the dual theory. In the first step one dimension has been compactified on a circle . This makes the R pop out. In the second we have taken into account that $e^{-2\phi}$ is in fact the squared coupling constant inverted. Since the Newton's constant of gravity must be unchanged under T-duality the last step must hold and hence we have that $\frac{R}{g^2} = \frac{R'}{g'^2}$. Using that $R' = \frac{l_s^2}{R}$ one sees that

$$g' = \frac{l_s}{R} g \tag{1.28}$$

which means that the string coupling is rescaled.

Type II theory does not only contain strings, it also contains p dimensional objects called p-branes. Branes have been mentioned earlier in the context of Dirichlet boundary conditions imposed on open strings. Type IIA theory contains only even branes while IIB contains only odd branes. The consistency of T-duality requires that these branes are interchanged somehow. This is indeed the case. Performing a T-duality on a p-brane in a IIA theory turns it into a p-1 brane in the IIB theory if the duality is performed in a direction parallel to the brane. Performing the duality in a tranverse direction results in a p+1 brane.

The two heterotic theories $SO(32)$ and $E_8 \times E_8$ are also T-dual to each other if compactified on a circle.

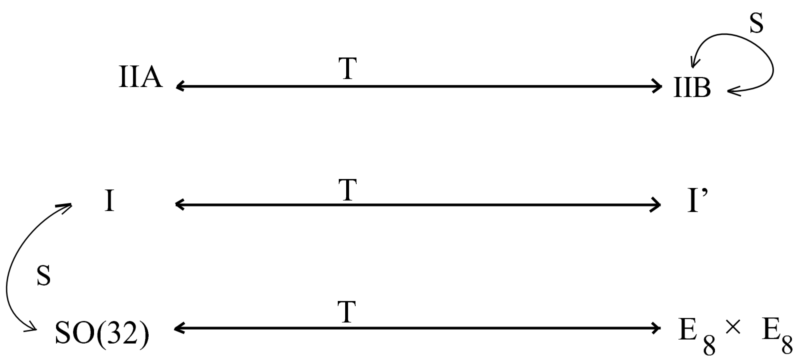

Figure 1.2: String dualities after compactification on a circle.

1.3.2 S-duality

S-duality works in the same as T-duality but instead of inverting the radius the coupling is inverted. The transformation is

$$g' = \frac{1}{g}. \tag{1.29}$$

Demanding that Newton's constant is unchanged and by inspecting the gravitational term in the ten dimensional action we see that the following must hold $\frac{1}{g'^2 l_s'^8} = \frac{1}{g^2 l_s^8}$. This gives an expression for the string scale rescaling under S-duality

$$l_s' = g^{1/2} l_s. \tag{1.30}$$

The coupling constant and the string scale are the two parameters which are altered during S-duality. Under T-duality the parameters were the coupling and the compactification radius.

S-duality can be investigated more carefully by studying the low-energy action of the IIB and type I theories. From this one sees that the type IIB theory is in fact self-dual. By this is meant that we get back a type IIB theory after having performed an S-duality on a IIB theory. The only difference is that certain terms have switched places in the action. In other words constituent A in IIB theory under strong coupling look just like constituent B under weak coupling.

A more careful study also reveals that type I theory is S-dual to the SO(32) heterotic theory when compactified on a circle.

These are just some examples of theories which are dual to each other. Using other higher dimensional compactification manifolds leads to entirely different duality relations among the different theories. This will be mentioned later on. Dualities are discussed in greater detail in [1] and especially [3].

1.4 M-Theory

1.4.1 Eleven dimensional supergravity

Let us for a while consider a massless graviton moving in an eleven dimensional space with one dimension compactified on a circle of radius R_{11}. The momentum of the graviton must be quantized in this direction giving a mass constribution proportional to $\frac{1}{R_{11}}$, the inverted radius. We have $E^2 - p^2 - q^2 = 0$ for the massless graviton and $q = \frac{n}{R_{11}}$. On the other hand, in type IIA theory in ten dimensions, we have the D-particles which have a discrete mass spectrum of order $\frac{1}{gl_s}$. A discrete mass spectrum is indeed what we get if we compactify a eleven dimensional super gravity. If we assume that the IIA D-particles are in fact gravitons in a higher dimensional space we get

$$R_{11} = gl_s \tag{1.31}$$

We now turn our attention to five branes in ten and eleven dimensions. In string theory such a brane has a tension of $T_{10} = \frac{1}{g^2 l_s^6}$, while the eleven dimensional five brane has tension $T_{11} = \frac{1}{l_p^6}$. The need for $T_{11} = T_{10}$ gives

$$l_p = g^{1/3} l_s. \tag{1.32}$$

Eleven dimensional supergravity does only permit membranes as solutions. This means that strings in ten dimensions are obtained by compactifying the eleventh dimension and wrapping one dimension of the brane in that direction. In eleven dimesions an M2-brane has tension $T_{11} = \frac{1}{l_p^3}$ after compactifying one dimension, the ten dimensional tension will be $T_{10} = \frac{R}{l_p^3} = \frac{g_s l_s}{g_s l_s^3} = \frac{1}{l_s^2}$. Which gives the relation $l_p^3 = g_s l_s^3$. Which is consistent with what we have derived above. This in turn will lead to

$$R_{11} = g_s^{2/3} l_P \tag{1.33}$$

in the Planck scale. This means that we have related the eleven dimensional Planck scale, l_p, and the compactification radius to the ten dimensional string scale, l_s, and the string coupling, g_s.

1.4.2 The $E_8 \times E_8$ heterotic story

The type IIA string theory can be seen as a eleven dimensional supergravity theory compactified on a circle. What happens if we compactify the eleven dimensional theory on an intervall instead of a circle?

We now consider the $E_8 \times E_8$ heterotic string with coupling g_H and a compact dimension of radius R_H. We want to see what happens when the heterotic coupling becomes large and the compact dimension arbitrarily large, i.e. a strongly coupled heterotic string in ten dimensions. This is done by performing a series of dualities on the $E_8 \times E_8$ heterotic string. We would like the final theory to have a weak coupling an a large compact radius.

We start by letting a T-duality transform the $E_8 \times E_8$ string to a $SO(32)$ string. The compact radius and the coupling rescales as

$$R'_H = \frac{1}{R_H} \tag{1.34}$$

$$g'_H = \frac{g_H}{R_H}. \tag{1.35}$$

Now we perform an S-duality to take us to a type I theory. The parameters become

$$g_I = \frac{1}{g'_H} = \frac{R_H}{g_H} \tag{1.36}$$

and the string scale is rescaled

$$l_I = g'^{1/2}_H l'_H. \tag{1.37}$$

In the new string scale the compact radius becomes

$$R_I = \frac{R'_H}{g'^{1/2}_H} = \frac{1}{g^{1/2}_H R^{1/2}_H}. \tag{1.38}$$

As we let R_H be large but fixed and g_H arbitrarily large we see that g_I becomes small as we wanted but R_I becomes small as well. We can not stop here! We proceed by performing another T-duality which takes us to the type I' theory. After the duality the radius and coupling constant will become

$$R_{I'} = \frac{1}{R_I} = g^{1/2}_H R^{1/2}_H \tag{1.39}$$

$$g_{I'} = \frac{g_I}{R_I} = \frac{R^{3/2}_H}{g^{1/2}_H}. \tag{1.40}$$

Letting g_H assume a large, but fixed value, and R_H an arbitrary large value, $R_{I'}$ indeed becomes large but so does $g_{I'}$. Fortunately there is a solution to this problem.

Type I' string theory is nothing but a type IIA theory between two planes. We can think of it as type IIA on the segment $S_1/\mathbf{Z_2}$. Type IIA theory immediately brings the M-theory to mind. And indeed, type IIA theory on $S_1/\mathbf{Z_2}$ is related to M-theory on $S_1 \times S_1/\mathbf{Z_2}$. The eleventh direction must then behave as

$$R_{11} = g^{2/3}_{I'} = g^{-1/3}_H R_H. \tag{1.41}$$

The distance between the planes become

$$R = \frac{R_{I'}}{g^{1/3}_{I'}} = g^{2/3}_H \tag{1.42}$$

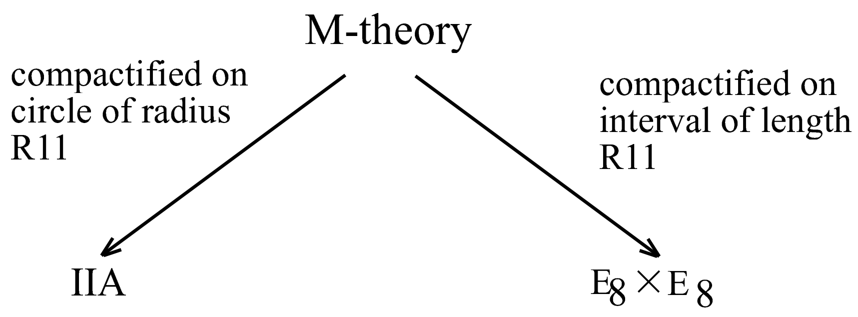

Figure 1.3: Type IIA or $E_8 \times E_8$ theories can be achieved depending on compactification.

in the Planck scale. And hence, while type IIA theory is M-theory compactfied on a circle, the $E_8 \times E_8$ heterotic theory is M-theory compactified on an intervall. This is discussed in detail in [3]. In M-theory we gave a good candidate for one underlying theory.

1.4.3 TST-duality on M-theory compactified on $S_1 \times S_1$

It is interesting to see what happens to an M-theory compactified on $S_1 \times S_1$, this is a type IIA theory with one compact dimension, when performing a series of S- and T-dualties, TST. One could expect to end up with the same thoery as we started with. We will see that this is not the case. Recall that under a T-duality $R' = \frac{l_s^2}{R}$ and $g' = \frac{gl_s}{R}$ while an S-duality has the effects $g' = \frac{1}{g}$ and $l_s' = g^{1/2} l_s$. In the first T-duality the type IIA theory is turned into a type IIB theory. The compact radius and the coupling constant become

$$R_B = \frac{l^2}{R_A} \tag{1.43}$$

$$g_B = \frac{g_A l_s}{R_A} \tag{1.44}$$

where R_B and g_B are the new radius and coupling constant respectively. We now perform an S-duality. The parameters become

$$g_{B'} = \frac{1}{g_B} = \frac{R_A}{l_s g_A} \tag{1.45}$$

$$l_{s'} = g_B^{1/2} l_s = (\frac{g_A}{R_A})^{1/2} l_s^{3/2} \tag{1.46}$$

where l_s' is the new string scale. The compactification radius is constant. Finally we do a T-duality which results in

$$R_{A'} = \frac{l_s'^2}{R_B} = \frac{g_A l_s^3 R_A}{R_A l_s^2} = g_A l_s \tag{1.47}$$

$$g_{A'} = \frac{g_{B'} l_{s'}}{R_B} = \left(\frac{R_A}{l_s}\right)^{3/2} \frac{1}{g_A^{1/2}} \tag{1.48}$$

From this we clearly see that the compactification radius and the coupling constant are far from having the same values we started with. There is another very important observation to make. After having performed the TST-duality sequence we see that

$$R_{A'} = g_A l_s = R_{11} \tag{1.49}$$

keeping (1.31) in mind. We see that $R_{A'}$ now has the same meaning as R_{11} had before the TST-duality. It is also possible to use (1.32) to rewrite (1.48) as $g_{A'} = \left(\frac{R_A}{l_P}\right)^{3/2}$ which is equal to

$$R_A = g_{A'}^{2/3} l_P. \tag{1.50}$$

We can now conclude that under the TST-duality transformation the dependence of the extra dimensional radii R_{10} and R_{11} have been interchanged. In other words the compact eleventh dimension has been turned into the tenth compact dimension and vice versa, a very interesting result.

Chapter 2

Large extra dimensions in experiments

2.1 The size of the extra dimensions

In this chapter the size of the extra dimensions will be studied. We will also discuss some of the effects large extra dimensions will have on experiments of various kinds.

The extra dimensions are in general thought to be to small to be detected. A more careful study reveals that this is not the case or at least not the whole story.

What we seek is a theory which has a large compactification volume and a weak coupling. The compactification volume, V, must be much larger than the corresponding volume in the string scale, this is what is meant by large. The string scale needs to be larger than the Planck scale. To achieve $V > l_s^6$ T-dualities are performed in the dimensions demanded. Strong coupling is dealt with similarly, by an S-duality transformation.

A great deal of the material presented in section 2.1 and the beginning of 2.2 is discussed in greater detail in [2] why references to this article will be given sparsely.

2.1.1 Planck scales in different dimensions

In this section we will develop relations between the Planck scales in different dimensions. As we know the Planck scale and Newton's constant are related. In for instance four dimensions the relation is $G_{N(4)} = \frac{1}{M_{Pl}^2}$. We start by stating the laws of gravitational force in 4 and 4+n dimension

$$F_{(4+n)}(r) = G_{N(4+n)} \frac{m_1 m_2}{r^{2+n}} \tag{2.1}$$

$$F_{(4)}(r) = G_{N(4)} \frac{m_1 m_2}{r^2}. \tag{2.2}$$

We now compactify the 4+n spacetime by making the identification $x_\mu = x_\mu + L$ for the n extra dimensions x_μ. Suppose that a mass m is located at the origin in the 4 dimensional space. The 4+n dimensional analog is produced by placing mirror masses periodically in the extra dimensions. At a distance r from the origin and $r \ll L$ the mirror masses make a very small contribution to the force why we have the 4+n dimensional force law (2.1). If instead $r \gg L$ the mass distribution looks like an n-dimensional line with uniform mass density. The gravitational field generated by the n dimensional line mass can be calculated by applying the 4+n dimensional gauss law. From this we find the relation $G_{N(4)} = \frac{S_{(3+n)}}{4\pi}\frac{G_{N(4+n)}}{V_n}$ where $S_{3+n} = 2\pi^{3+n}/\Gamma(3+n)$ is the area of the unit sphere in 3+n spatial dimensions. Leaving out the numerical factors we get

$$G_{N(4)} \sim \frac{G_{N(4+n)}}{V_n} \tag{2.3}$$

where $V_n \sim L^n \sim r_n^n$ is the compactification volume. By compactifying the 10 dimensional Lagrangian and using the above relation we get the following

$$M_{Pl(4+n)}^{n+2} = G_{N(4+n)}^{-1} \tag{2.4}$$
$$M_{Pl(4)}^2 = M_{Pl(4+n)}^{2+n} V_n \tag{2.5}$$

where we have left out factors such as for instance powers of 2π. This whole procedure is carried out in detail in [5]. For $n = 2$ and $M_{Pl(6)} = 1TeV$ we have that $r_{1,2} \sim \frac{M_{Pl(4)}}{M_{Pl(6)}^2}$ where $r_{1,2}$ is expressed in units of inverted energy, eV^{-1}. To get an explicit value of $r_{1,2}$ in meters we note that $M_{Pl} l_{Pl} = M_s l_s$ since both $M_{Pl} = l_{Pl}^{-1}$ and $M_s = l_s^{-1}$ holds. Subscript s simply denotes any arbitrary scale. Since $r_{1,2}$ is expressed in eV^{-1} we simply multiply by $M_{Pl} l_{Pl}$ to change units to meters. This gives extra dimensional radii of $r_{1,2} \sim 1mm$.

The above relations can also be intuitively understood from (2.1) and (2.2). The 4+n dimensional force law must of course reduce to the 4 dimensional one if we let the distance in the extra dimensions be maximal. In 4+n dimensions we have $F_{(4+n)} = G_{N(4+n)}\frac{m_1 m_2}{r_n^n r^2} = \frac{G_{N(4+n)}}{V}\frac{m_1 m_2}{r^2}$ where r_n is the size of the extra dimension n. Comparing to $F_{(4)} = G_{(4)}\frac{m_1 m_2}{r^2}$ we see that (2.3) must hold and since $G_{N(4)} \sim \frac{1}{M_{Pl(4)}^2}$ and its 4+n dimensional analog is true, the above follows.

2.1.2 Motivations for large extra dimensions

A motivation for large volume compactifications is found within the heterotic string theory. Let us start with a heterotic string theory in ten dimension and compactify it to four dimensions. The action becomes

$$S = \int d^4x \frac{V}{g_H^2}\left(\frac{1}{l_H^8}\mathcal{R} + \frac{1}{l_H^6}F^2\right) \tag{2.6}$$

where V is the volume of the compactification manifold. For simplicity only the gravitation and the kinetic gauge terms are kept [2]. It is worth pointing out that the factor in front of the second term completely determines which theory we are discussing. In the next two sections the corresponding actions of both type I and II strings are stated. As we see the factor in front of the kinetic gauge term differs from case to case while the factor in front of the Ricci scalar always has the same power dependence of the string scale and coupling.

We can now identify the Planck mass and the gauge coupling

$$M_{Pl}^2 = \frac{V}{g_H^2 l_H^8} \tag{2.7}$$

$$\lambda^2 = \frac{g_H^2 l_H^6}{V} \tag{2.8}$$

The heterotic mass scale (or length) M_H ($M_H = l_H^{-1}$) and the string coupling g_H can now be expressed in terms of the Planck mass and gauge coupling.

$$M_H = \frac{1}{l_H} = \frac{\sqrt{V}}{g_H l_H^4} \frac{g_H l_H^3}{\sqrt{V}} = \lambda M_{Pl} \tag{2.9}$$

$$g_H = \frac{\lambda\sqrt{V}}{l_H^3} \tag{2.10}$$

Assuming that the gauge coupling is of order unity ($\sim \frac{1}{5}$) we see that the string scale is of order 10^{18} GeV. If we further let $g_H \leq 1$ (weak coupling) we see that $V \sim l_H^6$. A result which does not point to large extra dimensions. Despite this fact there are arguments for large (larger) extra dimensions. These come from gauge coupling unification and supersymmetry breaking by compactification.

If extrapolating the three gauge coupling constants of the standard model at high energies they meet at an energy scale of $M_{GUT} \simeq 2 \times 10^{16} GeV$. M_{GUT} is very near the heterotic string scale but differs by roughly a factor of 100. This difference can in a way be explained by introducing a large compactification volume. We can for instance consider a compactification manifold with one large dimension with radius R, we have $V \sim R l_H^5$. Identifying M_{GUT} with the compactification scale R^{-1}, implies $R \sim 100 l_H$ since we the M_{GUT} differs from the string scale l_H by a factor 100. Using (2.10) we see that this gives a string coupling in the strong regime, $g_H \sim 2$.

Another argument for large extra dimensions is supersymmetry. In ten dimensions the type II strings have two supersymmetries while the type I and heterotic string have one supersymmetry. When compactfying to a less number of dimensions the number of supersymmetries increase depending on the compactification manifold. A theory with with few supersymmetries is desirable. By introducing a compactification volume of the order of a few TeV the right number of supersymmetries can be broken [2].

2.1.3 M-theory on $\mathbf{S^1/Z_2}\times$ Calabi-Yau

M-theory compactified on an interval of length πR_{11} and a Calabi-Yau manifold is the same as the $E_8\times E_8$ strongly coupled heterotic theory compactified on the same Calabi-Yau manifold. In this theory gravity acts on the whole eleven dimensional bulk while the gauge interactions take place on two ten dimensional boundaries (9-branes) localized at each endpoint of the interval. The 9-branes contain one E_8 factor each. The corresponding action is

$$S_H = \int d^4xV(\frac{R_{11}}{l_M^9}\mathcal{R} + \frac{1}{l_M^6}F^2) \tag{2.11}$$

Identifying the gauge coupling λ we see that $\frac{1}{\lambda^2} = \frac{V}{l_M^6}$ and the Planck length l_P is given by $\frac{1}{l_P^2} = \frac{VR_{11}}{l_M^6}$. These relations can be written as

$$l_M = (\lambda^2 V)^{1/6} \tag{2.12}$$

$$R_{11} = \lambda^2 \frac{l_M^3}{l_P^2} \tag{2.13}$$

To have a valid eleven dimensional supergravity we must have $R_{11} > l_M$ and $V > l_M^6$. Looking at the above equations we see that this implies $g < 1$, weak coupling. By using the relations (2.9) and (2.10) this yields:

$$l_M = l_H g_H^{1/3} \tag{2.14}$$

$$R_{11} = l_H g_H. \tag{2.15}$$

In other words in heterotic units, R_{11} is the heterotic string coupling. As a result at strong coupling, ($g_H > 1$), we get the relation $R_{11} > l_M > l_H$.

Imposing the M-theory scale $M_M \sim 1$ TeV and a Planck scale $M_{Pl} \sim 10^{19}$ GeV and multiplying the rhs of the relation (2.13) by $M_{Pl}l_{Pl}$ gives us R_{11} in units of meters. The size of the eleventh dimension turns out to be of order $\sim 10^8$ km. If we on the other hand impose a value of 1 mm for R_{11} which is the shortest length scale over which gravity has been tested we find a lower bound of l_M^{-1} of 10^7 GeV.

2.1.4 Type I/I' strings

Type I/I' theory consists of closed and open strings. Closed strings describes gravity while open strings describe gauge interactions. The ends of the open strings propagate on D-branes. As a result of that gravitation and gauge interactions appear at different orders in perturbation theory. The effective action is

$$S_I = \int d^{10}x \frac{1}{g_I^2 l_I^8}\mathcal{R} + \int d^{p+1}x \frac{1}{g_I l_I^{p-3}}F^2. \tag{2.16}$$

Here we have assumed that the standard model is located on a p-brane, $p \geq 3$. Hence there are $p-3$ parallel and $9-p$ transverse compact dimensions.

When compactifying to four dimensions the Planck length (mass) and gauge coupling constant can be identified

$$\frac{1}{l_P^2} = \frac{V_\parallel V_\perp}{g_I^2 l_I^8}$$
$$\frac{1}{\lambda^2} = \frac{V_\parallel}{g_I l_I^{p-3}}$$

$V_\parallel$ is the compactification volume parallel to the p-brane while $V_\perp$ is the tranverse volume. For weak coupling $g_I < 1$ it follows from the above that the parallel volume must be of order $V_\parallel \sim l_I^{p-3}$. There is however no restriction on the transverse volume. We can choose to express the parallel volume in string units as $v_\parallel \sim 1$. It also appropriate to identify $V_\perp \sim R_\perp^n$. Where $n = 9-p$ is number of tranverse dimensions and $R_\perp$ is the transverse radius. Remebering the relation $M_s = \frac{1}{l_s}$ leads us to the relations

$$M_P^2 = \frac{1}{l_P^2} = \frac{1}{\lambda^4 v_\parallel} M_I^{2+n} R_\perp^n \tag{2.17}$$
$$g_I = \lambda^2 v_\parallel \tag{2.18}$$

Solving (2.17) for $R_\perp$ gives an answer in eV^{-1} to convert meters we simply multiply by $M_{Pl} l_{Pl}$. Using $v_\parallel \sim 1$ we have the following

$$R_\perp = \left(\frac{M_{Pl}^2 \lambda^4}{M_I^{2+n}} \right)^{1/n} M_{Pl} l_{Pl}. \tag{2.19}$$

Using this relation and letting the type I string scale take a value of ~ 1 TeV we see that the size of the transverse dimensions vary from 10^8 km for n=1, .1 mm for n=2 ($10^{-3} eV$) down to 10^{-14} m for n=6 (10 MeV).

Relations between heterotic and type I/I' strings

The above type I/I' theories do in fact describe strongly coupled heterotic strings with large dimensions. Let us consider the heterotic string compactified on a 6 dimensional manifold with k large dimensions of radius $R \gg l_H$ and 6-k string size dimensions ($\sim l_H$).

In ten dimensions with one compact circular dimension the heterotic and type I string are S-dual to each other. The new type I parameters expressed in the old heterotic are

$$g_I = \frac{1}{g_H} \tag{2.20}$$
$$l_I = g_H^{1/2} l_H \tag{2.21}$$

From the relation (2.10) we can under the assumption that $\lambda \sim 1$ and the observation that $V = R^k l_H^{6-k}$ obtain $g_H \sim \left(\frac{R}{l_H}\right)^{k/2}$. Using this we can write the (2.20) and (2.21) as

$$g_I \sim \left(\frac{R}{l_H}\right)^{-k/2} \tag{2.22}$$

$$l_I \sim \left(\frac{R}{l_H}\right)^{k/4} l_H \tag{2.23}$$

We see that for $k < 4$, $R^{-1} < M_I < M_H$ which means that $R > l_I > l_H$. For $k = 4$ it follows that $R^{-1} \sim M_I$ and for $k > 4$ we have $M_I < R^{-1}$ (i.e. $l_I > R$). In order to get a compactification volume larger than the string scale ($V > l_I^6$) we need to do a number of T-dualities. In the $k < 4$ case we need to do 6-k T-dualities on the heterotic size dimensions. Using the above expression for l_I and g_I the new coupling constant and string scale will be

$$g_{I'} = g_I \left(\frac{l_I}{l_H}\right)^{6-k} = \left(\frac{R}{l_H}\right)^{k(4-k)/4}$$

$$l'_H = \frac{l_I^2}{l_H} = \left(\frac{R}{l_H}\right)^{k/2} l_H$$

Unfortunately the string coupling is strong ($g_{I'} \gg 1$), making this case uninteresting. If we instead consider the case $k \geq 4$ T-dualities have to be performed in all six directions. The coupling constant becomes

$$g_{I'} = g_I \left(\frac{l_I}{R}\right)^k \left(\frac{l_I}{l_H}\right)^{6-k} \sim 1 \tag{2.24}$$

hence we have weak coupling (of order unity).The different compactification radii will be, using above relations

$$R'_1 = \frac{l_I^2}{R} = \left(\frac{R}{l_H}\right)^{k/2-1} l_H \tag{2.25}$$

$$l'_H = \frac{l_I^2}{l_H} = \left(\frac{R}{l_H}\right)^{k/2} l_H \tag{2.26}$$

As we see $l'_H > R'_1$ for k=4,5,6. For the k=4 case we have 2 dimensions of size $\sim l_I$ and 4 dimensions of size $\sim R$ and we see from (2.23) that $R \sim l_I$. This means that a weakly coupled theory with 2 large dimensions offers a weak description to a strongly coupled theory with 4 large dimensions. If we have k=5 we still have one dimension $\sim l'_H$ but also 5 dimensions of size $\sim R'$. In all we have a weak type I' theory with 5 large dimensions and one extra large. In the k=6 case we have no dimension $\sim l_H$, all 6 dimensions are of size $\sim R'$, in other words, we have 6 large extra dimensions.

2.1.5 Type II string theories

When compactifying to six or fewer dimensions a duality arises between the heterotic and type II strings. Type II strings have N=2 supersymmtries in ten dimensions while heterotic strings have N=1. This is obviously a problem. One way of dealing with this problem is to compactify the different theories on different manifolds. For example, in six dimensions the $E_8 \times E_8$ compactified on T^4 is S-dual to type IIA compactified on a K3 manifold [2]. A K3 manifold is a manifold suitable for string compactification since the number of supersymmtries is less than it would be if compactifying on T^6 instead. This leads to the demand that the map between different manifolds has to be taken into consideration when performing the duality.

In type II non-commutative gauge symmtries arise non-perturbatively in singular compactifications. Massless gauge bosons are provided by D2-branes in IIA theory (D3-branes in IIB) wrapped in 2 (3) compact directions. From this it follows that gauge kinetic terms are independent of the string coupling λ_{IIA}. The corresponding effective action is

$$S_{IIA} = \int d^{10}x \frac{1}{g_{IIA}^2 l_{IIA}^8} \mathcal{R} + \int d^6x \frac{1}{l_{IIA}^2} F^2 \tag{2.27}$$

This is the analog to (2.6) in the heterotic case and (2.16) in the type I case. We would now like to compactify this theory to four dimensions. Since the action contains both a six and ten dimensional intergral the compactification manifold(s) have to be chosen with care. In this case we can for example use the product of a K3 manifold and a two dimensional torus, $K3 \times T^2$, for the gravitation term and just T^2 for the gauge term [2]. Noting that $g_{6IIA} = \frac{g_{IIA}}{\sqrt{V_{K3}}} l_{IIA}^2$ this gives

$$\frac{1}{\lambda^2} = \frac{V_{T^2}}{l_{IIA}^2} \tag{2.28}$$

$$\frac{1}{l_P^2} = \frac{V_{T^2}}{g_{6IIA}^2 l_{IIA}^4} = \frac{1}{g_{6IIA}^2} \frac{1}{\lambda^2 l_{IIA}^2} \tag{2.29}$$

The string scale can thus be expressed as

$$M_{IIA} = \lambda g_{IIA} M_P \frac{l_{IIA}^2}{\sqrt{V_{K3}}} \tag{2.30}$$

Comparing to the type I case (2.17) where only the volume appears, we can now use both the compactification volume of the K3 manifold and the string coupling constant to make the Planck mass and string scale have the desired relation. In particular, having a string scale of order $\sim$ TeV, we can choose a compactification volume of string size and to keep λ of order $\sim$unity (to account for the hierarchy between the electroweak and and the Planck scale) this implies $g_{6IIA} = 10^{-14}$, indeed a weak coupling. As a result gravity remains weak up to the Planck scale and string interactions are suppressed by the weak string coupling. If we study this in a particle accelerator with the energies available today we will see nothing but Kaluza-Klein excitations along the T^2 directions [2].

Instead of assuming that both directions of T^2 are of the same size, as we did above, we can let one dimension be much larger and the other much smaller. We will now study the case when we use a rectangle of radii r and R, with $V_{T^2} = rR \sim l_{IIA}^2$ and $r \gg l_{IIA} \gg R$. As we see R is smaller than the string scale so we perform a T-duality in the R direction which results in a IIB theory with parameters

$$R' = \frac{l_{IIA}^2}{R} \tag{2.31}$$

$$g_{IIB} = g_{IIA}\frac{l_{IIA}}{R} \tag{2.32}$$

and $l_{IIA} = l_{IIB}$. We now have

$$\frac{1}{\lambda^2} = \frac{r}{R'} \tag{2.33}$$

$$\frac{1}{l_P^2} = \frac{V_{T^2}}{g_{6IIB}^2 l_{IIB}^4} = \frac{R'^2}{g_{6IIB}^2}\frac{1}{\lambda^2 l_{IIB}^4} \tag{2.34}$$

Hence the gauge coupling is now determined by the shape of T^2, while the Planck length is determined by the size of the torus and the six dimensional type IIB coupling constant. The IIB string scale can be expressed as

$$\frac{1}{l_{IIB}^2} = \lambda g_{6IIB}\frac{M_P}{R'} \tag{2.35}$$

Comparing these relations to (2.28) and (2.29) we see that the situation in IIA theory is the same as in IIB theory, unless the size of T^2 is much larger than the string length, $R \gg l_{IIB}$.

Relations between type II and heterotic strings

We will now show that relations between the above type II theories and heterotic strings with strong coupling exist. As described above in 6 dimensions the $E_8 \times E_8$ heterotic string compactified on T^4 is S-dual to type IIA theory compactified on K3. The string coupling relation is

$$g_{6IIA} = \frac{1}{g_{6H}} \tag{2.36}$$

Where g_6 simply denotes the string coupling in 6 dimensions, in this case we have

$$g_{6H} = \frac{g_H l_H^2}{\sqrt{V_{T^4}}} \tag{2.37}$$

$$g_{6IIA} = \frac{g_{IIA} l_{IIA}^2}{\sqrt{V_{K^3}}}. \tag{2.38}$$

If we now use (2.9) and (2.36) in (2.29) we get $\frac{1}{l_P^2\lambda^2} = \frac{g_{6H}^2}{\lambda^2 l_{IIA}^2}$. This can be written as

$$l_{IIA} = g_{6H} l_H \tag{2.39}$$

This is not all however. We also have to bare in mind that the compactfication manifolds are not the same for the type IIA and heterotic theory and hence this relation has to be studied more carefully.

Let us consider M-theory compactified on the cartesian product space of an interval of length πR_I and four circles of radii $R_1, ..., R_4$, in other words $S^1/Z_2(R_I) \times S^1(R_1) \times T^3(R_2, R_3, R_4)$. By identifying the eleventh dimension with one of the compactification radii the above compactified M-theory can be interpreted as various theories. For example letting $R_I = R_{11}$ is corresponding to the heterotic string compactified on $T^4(R_1, ..., R_4)$, while choosing $R_1 = R_{11}$ corrsponds to a IIA theory compactified on K3 of "squashed" shape, $S^1/Z_2(R_I) \times T^3(\tilde{R}_2, \tilde{R}_3, \tilde{R}_4)$. Using (2.15) and (2.37) we can write R_I as

$$R_I = g_H l_H = g_{6H} \frac{V_{T^4}^{1/2}}{l_H} \tag{2.40}$$

and for R_1 we use (1.31) and (2.38) and can hence write

$$R_1 = g_{IIA} l_{IIA} = g_{6IIA} \frac{V_{K3}^{1/2}}{l_{IIA}}. \tag{2.41}$$

using (2.36) and (2.39) the above equations can be written as

$$\frac{R_I}{l_{IIA}} = \frac{V_{T^4}^{1/2}}{l_H^2} \tag{2.42}$$

$$\frac{R_1}{l_H} = \frac{V_{K3}^1/2}{l_{IIA}^2} \tag{2.43}$$

When performing the S-duality between the two compactified theories the shape of T^3 does in fact remain invariant. This can be expressed by the following relations

$$\frac{R_i}{R_j} = \frac{\tilde{R}_i}{\tilde{R}_j} \tag{2.44}$$

where $i, j = 2, 3, 4$. Which yields $\tilde{R}_i = l_M^3/(R_j R_k)$ with $i \neq j \neq k \neq i$ and $l_M^3 = g_H l_H^3$. This relation together with (2.42) and (2.43) completely determines the mapping for the compactification manifolds under S-duality.

2.2 Experimental verification of string theory

Hopefully string theory will be experimentally verified one day. A possible way do this is in the sense of large extra dimensions. Large extra dimensions have at least in theory an effect on gravity as well as events and energy experiments performed in particle accelerators. In for instance particle experiments excited Kaluza-klein states is one of the expected most common effects while in gravity a shift in the force law from $\frac{1}{r^2}$ to $\frac{1}{r^{2+n}}$ dependence in sub $\sim$ mm measurements is expected. Also, in gravitational accelerator experiments gravitons might escape into the extra dimension leading to missing expected events and interactions. A Kaluza-Klein state is a contribution to the mass energy and can be written as $\frac{n^2}{R_i^2}$ where n is the KK quantum number and R_i is the compact radius of the ith dimension. We see from this that if the radius R_i is large, the energy contribution will be relatively small and hence less energy is needed for detection. The main experimental verification of string theory is due to three different phenomena large longitudinal dimensions felt by gauge interactions, extra large transverse dimensions felt only by gravity and strings with low tension.

2.2.1 Longitudinal dimensions

The main experimental prediction in the large longitudinal case is the direct discovery of KK excitations for all standard model gauge bosons in particle colliders such as the LHC. They couple to quarks and leptons in the compact space. The coupling of a bulk to two boundary fields contains a form factor which exponentially suppresses the heavy KK modes while in the large radius limit it reduces to the 4d gauge coupling. This form factor regulates the otherwise divergent sum of KK excitations [2].

2.2.2 Transverse dimensions

Large transverse dimensions exist in type I/I' theory and can be up to millimeter size. In experiments these would show as graviton emissions into the higher dimensions which would lead to for instance missing of certain energy events. An example of such a process is gluon annihilation into a graviton which goes into the extra dimensions. The corrsponding cross section is, [2],

$$\sigma(E) \sim \frac{E^n}{M_I^{n+2}} \frac{\Gamma(1-\frac{2E^2}{M_I^2})^2}{\Gamma(1-\frac{E^2}{M_I^2})^4} \tag{2.45}$$

This process exhibits three different kinematic regimes with different behaviours. At high energies $E \gg M_I$, is the centre of mass energy, the cross section falls of exponentially. At energies of the same order as the string scale $E \sim M_I$, there is a sequence of poles. At low energies $E \ll M_I$, the cross section falls of as $\sim \frac{E^n}{M_I^{n+2}}$.

2.2.3 Low-scale strings

In type II theory low-scale strings with extremely weak coupling $\sim 10^{-14}$, string interactions are suppressed and the only observable effects are KK excitations.

2.2.4 Gravity

Since it seems like gravity goes like $\frac{1}{r^{2+n}}$ instead of $\frac{1}{r^2}$ it is not irrelevant to ask one self how this effects systems where gravity is important and the particle separation is smaller than the size of the extra dimensions $\sim 1mm$, in for instance stars. Let us for example consider a large body and split it in spherical balls of density ρ and radius R. We now compute the gravitational energy of such a ball. We have

$$E_{grav} \sim \int_0^{r_n} d^3r \frac{G_{N(4+n)}\rho}{r^{n+1}} + \int_{r_n}^{R} d^3r \frac{G_{N(4)}\rho}{r} \sim \int_0^{r_n} dr \frac{G_{N(4+n)}\rho}{r^{n-1}} + \int_{r_n}^{R} dr G_{N(4)}\rho r.$$

The first term uses the 4+n dimensional gravitational potential and the second term uses the usual one. The second term is dominated by large distances and gives a contribution of $\sim G_{N(4+n)}\rho R^2$. We note that for n=2 the new contribution is logarithmic divergent but is in reality at short distances cutoff by the inter-particle separation r_{min} and at long distances by $r_n \leq R$. For $n > 2$ it is again cutoff by r_{min}. The change in gravitational energy is for $n > 2$ $\Delta E_{grav} = G_{N(4+n)}\rho / r_{min}^{n-2}$ and for n=2 we have $\Delta E_{grav} = G_{N(4+n)} log(r_{min})$. The fractional change in gravitational energy for $n > 2$ is then

$$\frac{\Delta E_{grav}}{E_{grav}} \sim \frac{G_{N(4+n)}}{G_{N(4)} r_{min}^{n-2} R^2} \sim \frac{r_n^n}{r_{min}^{n-2} R^2} \tag{2.46}$$

In the last step we have used (2.3). If $n = 2$ we would instead have

$$\frac{\Delta E_{grav}}{E_{grav}} \sim log(r_{min}) \frac{r_n^n}{R^2} \tag{2.47}$$

If r_{min} is larger than $\sim (TeV)^{-1}$ and n=2 the fraction goes like

$$\frac{\Delta E_{grav}}{E_{grav}} < \left(\frac{1mm}{R}\right)^2 \tag{2.48}$$

from which we see that the contribution to the gravitational energy is completely irrelevant even for the smallest object of interest, neutron stars. Neutron stars have radius of about 10 km yielding a fractional change in gravitational energy of $\sim 10^{-12}$.

When measuring gravity at small distances it is important to know how its strength is related to other forces. At large distances the usual Newtonian gravity is valid while at small distances the force law goes as $\sim 1/r^{2+n}$ instead of $1/r^2$. This has to do with that the new higher dimensional gravity catches up at 1 TeV rather than at 10^{19} GeV. Even though gravity is much stronger than before it is still much weaker than other forces at distances larger than the weak scale. This is obvious if we study the relation between the electromagnetic force and gravity between a proton and an electron a distance r apart. For the electromagnetic force $F_{em} = \frac{Q^2}{4\pi\varepsilon_0 r^2}$ and for gravity $F_{grav} = G_{N(4+n)} \frac{m_p m_e}{r^{2+n}}$ yielding the fraction

$$\frac{F_{grav}}{F_{em}} \sim \frac{G_{N(4+n)} m_e m_p}{4\pi\varepsilon_0 r^n} \sim 10^{-7} \left(\frac{10^{-17} cm}{r} \right)^n . \tag{2.49}$$

The smallest value of r for which the electromagnetic effects are dominant is $r \sim 10^{-8} cm$ and even then the fraction is as small as $\sim 10^{-25}$. At larger distances the electromagnetic forces are of course screened, gravity is not. But still, other forces dominate over the 4+n dimensional gravity. Consider for example the Van der Waals force between two hydrogen atoms. The VdW force comes from the dipole-dipole interaction between the atoms. The dipole moment of the first atom in the electromagnetic field of the second one (and vice versa) gives rise to a force, the VdW force. The interacting energy for the dipole of one hydrogen atom in the field of the other behaves like

$$V_{dipint} \sim \frac{d_1 d_2}{R^3} \tag{2.50}$$

where d_1 and d_2 are the dipole moments of atom 1 and 2 respectively anf R is the distance. The energy splitting is calculated using perturbation theory. The first order contribution vanishes. The second order contribution gives the usual VdW $1/r^6$ potential

$$F_{VdW} \sim \frac{C_{VdW}}{r^7} \tag{2.51}$$

where C_{VdW} is the VdW constant which of course varies depending on the system [6]. In this case, considering two hydrogen atoms, $C_{VdW} \sim 6\alpha r_{bohr}^5$ [5], where α is the fine structure constant and r_{bohr} is the bohr radius. As we know $F_{grav} = G_{N(4)} \frac{m_H^2}{r^2}$ this gives us the ratio of the VdW and 4 dimensional gravity

$$\frac{F_{VdW}}{F_{grav}} \sim \frac{7\alpha r_{Bohr}^5}{G m_H^2 r^5} . \tag{2.52}$$

Dimensional analysis tells us that this has to be multiplied with the Planck energy times the Planck length. Doing so and using numerical values of the fine structure constant, the hydrogen mass etc. gives an expression for the relation between the VdW force and gravity

$$\frac{F_{VdW}}{F_{grav}} \sim \left(\frac{1mm}{r}\right)^5 \tag{2.53}$$

from which we see that the VdW force dominate over gravity up to $\sim$ mm distances while gravity takes over at larger distances. This is in fact one of the main obstacles to sub-millimeter measurements of gravitational strength forces. Even if we instead have higher dimensional gravity this problem is not solved. For n=2 large extra dimensions, the extra dimensions are of millimeter size and the force behaves like $\sim \frac{1}{r^4}$ which is still inferior to the gravitational force. For n=3 the large extra dimesions open at 10^{-7} cm and the VdW force dominates at larger distances than that.

In the n=2 scenario, we expect gravity to switch from $1/r^2$ to $1/r^4$ some where around $\sim$ 1 mm. The only experiments conducted on $1/r^4$ potentials under the millimeter distance that have been conducted are measuring of the Casimir forces at distances of $\sim 5mm$. The Casimir force can be seen for instance between two parallel conducting plates placed in vacuum. It arises due to the fact that the electromagnetic field fluctuates rather than being constantly zero which gives rise to a higher energy density between the plates than outside which in turn implies an attractive force between the conductors. Considering two objects composed of N_1 and N_2 nucleons separated by a distance r the Casimir potential behaves as

$$V_{Csr}(r) = CN_1N_2\frac{(10^{-15}m)^2}{r^3}. \tag{2.54}$$

The best measurements made proposes $C <\sim 7 \times 10^{-17}$. If we just for a glimpse of a second would equate this with 6 dimensional gravity we would have the relation

$$C = \frac{G_{N(6)}(m_N \times 10^{15}m^{-1})^2}{3} = \frac{1GeV^4}{50M^4_{Pl(6)}} \tag{2.55}$$

implying $M_{Pl(6)} \geq 4.5TeV$. This makes r_2 shrink to a size of $\sim 30\mu m$.

2.2.5 Decay to higher dimension gravitons

As mentioned in section 2.2.2 in a theory with large transverse extra dimensions graviton emission into the extra dimensions may occur. In this section we will study the probability and boundaries of such processes. Emission of a single graviton into the extra dimension does not conserve extra dimensional momentum. This is not a problem since the wall on which the standard model is localized breaks translational invariance in the extra dimension. Time translation is still valid though, which implies conservation of energy.

The graviton is a massless particle which couples to the standard model particles with a coupling suppressed by $\frac{1}{TeV}$ $(\frac{1}{M_{Pl}})$. It is similar to other light particles which are known to be in disastrous conflict with decay experiments with decay constants of $\sim TeV$. It is important the check that the graviton is not far off in this context.

Consider for instance a Kaon decaying to a pion and a graviton $K \rightarrow \pi + graviton$. This is schematically depicted in the figure below.

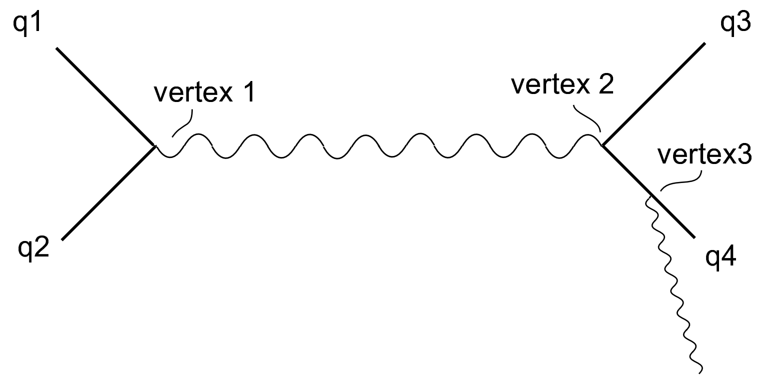

Figure 2.1: A schematic feynmann diagram for a two-quark particle decay into another two-quark particle and a graviton emitted at vertex 3.

The order of the decay width for the above process can be determined in a qualitative hand waving way. The process is a weak decay process why the decay amplitude will be proportional to $\frac{1}{M_W^2}$ where M_W is the mass of the W-boson which is the exchange quanta of the weak force. Since we are looking for a decay width rather than an amplitude we must square this and then multiply with enough powers of m_K (the mass of the decaying kaon) to make it an energy. We thus have

$$\Gamma \propto \frac{m_K^5}{M_W^4}. \tag{2.56}$$

If we now add the emission of a graviton in any KK mode to the decay, the width becomes

$$\Gamma \sim \frac{m_K^5}{M_W^4} \times \frac{m_K^2}{M_{(4)}^2}. \tag{2.57}$$

The amplitude for decay into any KK mode is $\sim \frac{m_K}{M_{(4)}}$ since the graviton couples as $\frac{1}{M_{(4)}}$ in 4 dimensions and its rate of interaction is proportional to the mass m_K. There are also a large number of KK modes, differing with $\frac{1}{r_n}$, with mass $\leq m_K$ which are energetically allowed. In each dimension there are $\sim m_K r_n$ such modes but since we have n dimensions the total contribution is $\sim (m_K r_n)^n$. It is worth noting that the number of such KK modes vastly succeeds the suppression of their coupling which makes them responsible for the conversion from $\frac{1}{r^2}$ to $\frac{1}{r^{2+n}}$ gravity as discussed above. The total decay width to gravitons is given by

$$\Gamma_{K\to\pi+gravitons} \sim \frac{m_K^5}{M_W^4} \times \left(\frac{m_K}{M_{4+n}}\right)^{n+2}. \tag{2.58}$$

Hence probability for extra-dimensional graviton production (branching ratio) is

$$B_{K\to\pi+graviton} \sim \left(\frac{m_K}{M_{(4+n)}}\right)^{n+2} \tag{2.59}$$

In the n=2 case we, $M_{(6)} \sim 1$ TeV, the branching ratio is $\sim 10^{-12}$. The same ratio for the familon process, which provides the strongest bound, is $\sim 10^{12}$ GeV. And we see that the graviton process is safely smaller than this bound. Astrophysics and cosmology seem to require $M_{(6)} \geq 10$ TeV for n=2, in which case the above ratio will be $\sim 10^{-16}$ instead of $\sim 10^{-12}$.

2.3 Cosmology

In this section we shall discuss for how long the extra dimensions have had their current size. We are all familiar with the theory of big bang which says that our universe has been expanding ever since the very first moment. This must of course also be true for the extra dimensions, or at least they must have expanded until a certain stage and then been frozen. Very little is known about the mechanism controlling the extra dimensional radii. The only thing we know with certainty is that the Big Bang Nucleosynthesis (BBN) started when the universe had a temperature of ~ 1 MeV. The cosmological observations of vast amounts of light nuclei tells us that the expansion rate of the universe during BBN can not be modified by more than 10%. And since the extra dimensions determine $G_{N(4)}$ and hence the expansions rate of the universe, we know that they must have settled to their current size before the onset of BBN. When extrapolating back in time we assume that the universe normal up to some temperature, T*. By normal is meant that the extra dimensions are frozen and nearly empty of energy.

Below we shall discuss boundaries of such a temperature T*. The boundaries implied by string theory mainly comes from gravitons escaping into the bulk, escaping from the bulk and decaying to photons

2.3.1 Expansion dominated cooling

The cooling of the radiation energy density in the universe on the wall is due to two phenomena. The first is the normal expansion of the universe. This can be seen by studying the Friedmann equations from which a third equation can be derived

$$\frac{d\rho}{dt} = -\frac{3}{a}\frac{da}{dt}(\rho + p). \tag{2.60}$$

Where p is the pressure and can be expressed as $p = \sigma\rho$ and σ is the parameter describing the matter. In this case we have relativistic matter and radiation which implies of value of $\sigma = \frac{1}{3}$ [7]. Solving (2.60) for ρ as a function of a gives $\rho \sim a^{-3(1+\sigma)}$ where a is the time dependent parameter in the Robertson-Walker metric describing the expansion of the universe. Using $\sigma = \frac{1}{3}$ gives us the solution $\rho \sim a^{-4}$. Differentiating this with respect to time leads to

$$\frac{d\rho}{dt} = -4\frac{1}{a^5}\frac{da}{dt} = -4H\rho \tag{2.61}$$

since $\dot{a}/a = H$ is the Hubble constant. We would like to have an expression relating $\frac{d\rho}{dt}$ with the temperature T. In this way we would be able to make a comparison to T*. We therefore need to express H in terms of T. By using relations from [7] we see that $H \sim t^{-1}$, $a \sim t$ and $T \sim \frac{1}{a}$ where t is the time. We hence have the relation $H \sim \frac{1}{t} \sim \frac{1}{a^2} \sim T^2$. This allows us to write the above expression as

$$\frac{d\rho}{dt}|_{expansion} \sim -4H\rho \sim -4\frac{T^2}{M_{Pl}}\rho \tag{2.62}$$

The second contribution to the decrease of energy density is evaporative cooling due to production of gravitons which escape into the extra-dimensional bulk. Graviton production is proportional to $\frac{1}{M_{(4+n)}^{n+2}}$. The rate for evaporative cooling will be

$$\frac{d\rho}{dt}|evap. \sim -\frac{T^{n+3}}{M_{4+n}^{n+2}} \tag{2.63}$$

The requirement of a normal universe requires that the cooling due to normal expansion is larger than the cooling due to graviton evaporation. This put an upper bound to T*,

$$\frac{T*^2}{M_{Pl}}\rho \geq \frac{T*^{n+3}}{M_{4+n}^{n+2}} \Rightarrow$$

$$T* \leq \left(\frac{M_{(4+n)}^{n+2}}{M_{Pl}}\right)^{1/(n+1)} \sim 10^{\frac{6n-9}{n+1}} MeV \times \left(\frac{M_{(4+n)}}{1TeV}\right)^{(n+2)/(n+1)} \tag{2.64}$$

For the case n=2 and $M_{(4+n)} \sim 1$ TeV we have $T* \leq 10$ MeV. Astrophysics however, seem to require $M_{(4+n)} \sim 10$ TeV which moves the boundary of T* to ~ 100 MeV. For n=6 we have $T* \leq 10$ GeV. In all cases we can have $T* \geq 1$ MeV, so BBN will not be interfered with.

2.3.2 BBN constraints

It is important to see that the produced gavitons does not affect the expansion rate of the universe during BBN, which gives the demand $\frac{\rho_{grav.}}{\rho_\gamma} \leq 1$. The ratio of the energy density in gravitons versus photons by the time of BBN is then

$$\frac{\rho_{grav.}}{\rho_\gamma}|_{BBN} \sim \frac{T*}{1Mev} \times \frac{T*^{n+1} M_{Pl}}{M_{(4+n)}^{n+2}} \tag{2.65}$$

To have a normal expansion rate during BBN the bound on T* is slightly stronger now

$$T* \leq 10^{\frac{6n-9}{n+2}} \times \frac{M_{4+n}}{1TeV} \tag{2.66}$$

2.3.3 Over-closure by gravitons

Produced gravitons which escapes into the bulk contributes with yet a constraint on T*. Gravitons propagating in the bulk tend to have a rather long lifetime. We must check that the graviton energy density is not higher than the critical density of the universe. Consider the width for a graviton propagating in the bulk to decay into two photons on the wall. This can only happen if the graviton is within its compton wavelength $\sim E^{-1}$ from the wall and the probability that this happens in a n-dimensional bulk of volume r_n^n is

$$P_{grav.nearwall} \sim (Er_n)^{-n}. \tag{2.67}$$

The graviton decays into photons with a coupling suppressed by $\sim \frac{1}{M_{(4+n)}^{(n+2)/2}}$ (in n+4 dimensions). Therefore width of the decay from graviton to two photons is

$$\Gamma_{grav.to\gamma} \sim \frac{E^{n+3}}{M_{(4+n)}^{n+2}}. \tag{2.68}$$

Where multiplication with E^{n+3} is done on basis of analyzing the dimension. Hence the total width becomes

$$\Gamma = P_{grav.nearwall} \times \Gamma_{grav.to\gamma} \sim \frac{E^3}{r_n^n M_{4+n}^{n+2}} \sim \frac{E^3}{M_{(4)}^2}. \tag{2.69}$$

This implies that gravitons in the bulk can be very long lived since as long as the momenta in the extra dimensions is conserved, the graviton can not decay into two other massless particles. Interaction with the wall allows extra dimensional momentum non-conservation but this requires the decay to take place on the wall. The lifetime of a graviton is

$$\tau(E) = \frac{1}{\Gamma} \sim \frac{M_{(4)}^2}{E^3} \sim 10^{10} yr \times \left(\frac{100MeV}{E}\right)^3. \tag{2.70}$$

This formula tells us that gravitons produced beneath a temperature of ~ 100 MeV have a lifetime at least the age of the present universe. This will put an even harder constraint on the ratio n_{grav}/n_γ which has to be much smaller than 1 in order to not have a universe over closed by gravitons. The energy density stored in gravitons produced at temperature T* is

$$\rho_{grav} \sim T* \times n_{grav} \sim \frac{T*^{n+5} M_{Pl}}{M_{(4+n)}^{n+2}}. \tag{2.71}$$

which is then red-shifted due to the expansion of the universe. The ratio ρ_{grav}/T^3 is however invariant. The critical density of the universe today corresponds to $\rho_{crit}/T^3 \sim 3 \times 10^{-9}$ GeV. To avoid graviton over closure we therefore require

$$3 \times 10^{-9} GeV \geq \rho_{grav}/T^3 \sim \frac{T*^{n+2} M_{Pl}}{M_{(4+n)}^{n+2}}$$

which gives

$$T* \leq 10^{\frac{6n-15}{n+2}} MeV \times \frac{M_{(4+n)}}{TeV}. \tag{2.72}$$

We would prefer to get $T* \geq 1$ MeV, but it turns out that in order to even get $T* \sim 1$ MeV for n=2, we need to push $M_{(4+n)}$ to ~ 10 TeV, which is astrophysically preferred though. This is a very serious boundary and it must be studied in detail since it is on the borderline of not being compatible with what we know about BBN today.

2.3.4 Late decays to photons

We now turn to the bounds coming from decay of gravitons into photons showing up today as distortions of the diffuse photon spectrum. For $T* \leq 100$ MeV the graviton has a lifetime longer than the age of the universe, but a fraction $\sim \left(\frac{T*}{100MeV}\right)^3$ of them have already decayed producing photons of energy $\sim T*$. The flux of these photons is

$$\frac{d\mathcal{F}}{d\Omega} \sim n_{grav} H_0^{-1} \times \left(\frac{T*}{100MeV}\right)^3. \tag{2.73}$$

This is to be compared with the observational bound on the diffuse background radiation at photon energy E

$$\frac{d\mathcal{F}}{d\Omega} \leq \frac{1MeV}{E} cm^{-2} sr^{-1} s^{-1}. \tag{2.74}$$

This gives a bound on T*

$$T* \leq 10^{\frac{6n-15}{n+5}} MeV \times \left(\frac{M_{(4+n)}}{TeV}\right)^{\frac{n+2}{n+5}}. \tag{2.75}$$

For n=2 and putting $M_{(4+n)} \sim 10TeV$ we have $T* \leq 1$ MeV which is also very serious. In the n=6 case we have for $M_{(4+n)} \sim 1$ TeV, $T* \leq 100$ MeV.

2.3.5 Cosmological thoughts

It is interesting that the cosmological bounds on T* are compatible with the Big Bang Nucleosynthesis, even though a more careful analysis has to be done in the n=2 case. This might of course suggest that n=2 is not enough, perhaps we need n=3 to have a consistent theory. There is however a way around this. The n=2 problem arose because we assumed that all gravitons going into the bulk eventually have to return to our 4-dimensional wall. If we for a moment assume the existence of another brane somewhere in the bulk, and especially if the new brane is of higher dimension than ours, a so called fat-brane, the probability of extra-dimensional graviton decay on our wall would be radically reduced [5]. It follows that (2.73) would have a much lower value and hence (2.75) would imply a larger bound on T*. The same argument holds for graviton over-closure. Since a fat-brane would decrease the lifetime of gravitons in the bulk and hence also the energy density stored in extra-dimensional gravitons, (2.71) would have a much lower value. Which in turn would leads to a higher bound for T* in (2.72). The fat-branes might also contribute to the understanding of the dark matter issue. In [5] it is argued that bulk-gravitons decaying on a fat-brane might provide a dark matter candidate.

Chapter 3

Conclusions

So what does all this tell us? It is clear string theory changes the way we view upon the universe, everything from subatom level to stars and beyond. Just the fact that particles, which earlier were considered as just particles, now turn out to be strings or even higher dimensional branes, changes our whole intuitive view of the fundamentals of nature. One of the most spectacular predictions made by string theory is the possibility of large extra dimensions. These are not only interesting in themselves but they might also provide possibilities for a verifcation of the previously unverified string theory.

The most problematic feature of string theory is without doubt the lack of contact with experiments. No matter how beautiful a theory of physics is, if it is impossible to confirm experimentally it is practically worthless in the end. The possibility of large extra dimensions drastically changes that. The perhaps most important features of large extra dimensions are the "low" energetic Kaluza- Klein states which, if small enough, possibly allow us to get an experimental verification of string theory in particle accelerator experiments. Large extra dimensions also imply changes to the normal Newtonian law of gravitation. Instead of having the normal $\frac{1}{r^2}$ the gravitational force behaves like $\frac{1}{r^{n+2}}$. Other forces do however provide difficulties when performing sub mm gravitational experiments. Despite this sub mm gravitational experiments are planned. Higher dimensional gravity does not change the physics of macroscopic systems, such as stars etc, considerably.

Extra dimensions as large as TeV^{-1} arise naturally when breaking the supersymmetry. Large extra dimensions are also motivated in the context of gauge coupling unification. This is indeed interesting and our ability to handle strong coupling makes it even more interesting. The string dualities provide powerful tools for transforming a strongly coupled into a theory with weak coupling and one or more large extra dimensions. A theory with one large dimension implies an extra dimensional radius of order $\sim 10^9$ km, which of course is excluded experimentally. With two large dimensions the compact radii shrinks to around mm size. Constraints from cosmology might not be compatible with this though. Late decaying photons discussed earlier do in fact require a string scale of 10 TeV instead of 1 TeV to make the Big Bang Nucleosynthesis possible when the universe had a temperature of 1 MeV, which is experimentally confirmed.

Whether or not this is true or not remains to be seen. It might turn out that a theory suitable for our universe does not possess large extra dimensions which forces us to find other ways of experimentally verifying string theory.

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