Improved bounds on universal extra dimensions and consequences for Kaluza-Klein dark matter

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(Received 28 October 2005; published 4 May 2006)

We study constraints on models with a flat universal extra dimension in which all standard model fields propagate in the bulk. A significantly improved constraint on the compactification scale is obtained from the extended set of electroweak precision observables accurately measured at LEP1 and LEP2. We find a lower bound of $M_c \equiv R^{-1} > 700(800)$ GeV at the 99% (95%) confidence level. We also discuss the implications of this constraint on the prospects for the direct and indirect detection of Kaluza-Klein dark matter in this model.

DOI: 10.1103/PhysRevD.73.095002 PACS numbers: 12.60.−i, 11.25.Mj, 12.15.Lk, 95.35.+d

I. INTRODUCTION

Models with flat "universal" extra dimensions (UED) in which all fields propagate in the extra-dimensional bulk have received a much recent attention [1] (for predating ideas closely related to UED models see Refs. [2]). They are of phenomenological interest for two primary reasons. First, among all extra-dimensional models with standard model (SM) charged fields propagating in the bulk, the mass scale of the compactification is most weakly constrained [1,3–10], this mass scale being well within the reach of future collider experiments. Moreover the collider signatures of Kaluza-Klein (KK) particle production in UED models are easily confused with those of superpartner production in some supersymmetric models [11]. Second, UED models provide a viable dark matter candidate—the lightest Kaluza-Klein particle (LKP)—which is stable by virtue of a conserved discrete quantum number intrinsic to the model. Electroweak radiative corrections imply that the LKP is neutral [12] with a thermal relic density consistent with observation for the mass range allowed by collider bounds [13,14].

Both of these features result from the existence of a conserved Z_2 KK-parity. Models in which all fields propagate in the extra-dimensional bulk allow conservation of a discrete (due to the finite volume) subgroup of translation invariance, implying that the momentum in the extra dimension(s), p_i, remains a conserved discrete quantity. In terms of the 4D effective theory, this translates into the conservation of KK-mode number. However, in order to obtain chiral fermions in the 4D effective theory, the extra dimension(s) have to be compactified on an orbifold, which inevitably breaks translation invariance and hence induces KK-number violation, but still preserves KK-parity as a conserved Z_2 quantum number. This KK-parity implies processes with a single first KK-excitation and standard model particles only are forbidden, and the lightest KK-particle is stable.

Specializing to the case of one extra dimension, the bound on the compactification scale $M_c \equiv 1/R$ from direct nondetection is $M_c \gtrsim 300$ GeV [1,5] being a factor of 2 lower than the naïve estimate for models without KK-parity conservation. Comparable bounds have been derived from the analysis of electroweak precision tests (EWPT) [1,3], the improvement of which is a primary focus of this paper. The bound from the $Z \to b\bar{b}$ branching ratio is of order $M_c \gtrsim 200$ GeV [1] (the bounds from the $Z \to b\bar{b}$ left-right asymmetry, and from the muon anomalous magenetic moment, are significantly weaker), while $b \to s\gamma$ leads to $M_c \gtrsim 280$ GeV [5] and constraints from FCNC processes [10,9] are of the order of $M_c \gtrsim 250$ GeV. Concerning current and future experiments, Run II at the Tevatron will be able to detect KK-particles if $M_c \lesssim 600$ GeV while the LHC will reach $M_c \sim 3$ TeV [6,7], well beyond the currently excluded compactification scale.

In this paper, we will extend the analysis of constraints from EWPT to arrive at a significantly more stringent bound on the compactification scale. Within the remaining allowed parameter space, we then investigate the direct and indirect detection prospects of the LKP dark matter candidate.

Specifically, in Sec. II we briefly review UED models, following Refs. [1,3]. In Secs. III and IV, the constraints from EWPT are investigated including the full LEP2 data set following the general analysis of [15]. As has been shown in [15], this *a priori* requires an extended set of EWPT parameters, and we calculate the contributions from universal extra dimension models to this extended set. A fit to the LEP1 and LEP2 data set leads to significantly improved bounds on M_c as a function of the unknown Higgs mass m_H. We emphasize that the two-loop standard model Higgs contributions to the EWPT parameters are included in this fit following the simple accurate numerical interpolation of [15]. In Sec. V we discuss the implications of this improved bound for KK dark matter, particularly the prospects of direct and indirect detection. Finally Sec. VI contains our conclusions while an appendix examines, in an improved version of naïve dimensional analysis, the maximum scale of applicability of the 5-dimensional UED theory with which we calculate.

II. THE UED MODEL

We consider the 5-dimensional extension of the single Higgs doublet standard model with all fields propagating in the extra dimension. The $5D$ Lagrangian is

$$\begin{aligned}\mathcal{L}_{5D} = &-\frac{1}{4}G^A_{MN}G^{AMN} - \frac{1}{4}W^I_{MN}W^{IMN} - \frac{1}{4}B_{MN}B^{MN} \\ &+ (D_M H)^\dagger (D^M H) + \mu^2 H^\dagger H - \frac{1}{2}\lambda (H^\dagger H)^2 \\ &+ i\bar{\psi}\gamma^M D_M \psi + (\hat{\lambda}_E \bar{\mathcal{L}}\mathcal{E}H + \hat{\lambda}_U \bar{\mathcal{Q}}\mathcal{U}\tilde{H} \\ &+ \hat{\lambda}_D \bar{\mathcal{Q}}\mathcal{D}H + \text{h.c.}) + \ldots \end{aligned} \quad (1)$$

where G_{MN}, W_{MN}, B_{MN} are the $5D$ $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge field strengths, the covariant derivatives are defined as $D_M = \partial_M + i\hat{g}_3 G^A_M T^A + i\hat{g}_2 W^I_M T^I + i\hat{g}_1 Y B_M$, where $\hat{g}_i$ are the $5D$ gauge couplings, with engineering dimension $m^{-1/2}$. The ellipses in Eq. (1) denote higher-dimension operators whose effect we discuss below and in the appendix. For compactification on S^1, the $5D$ matter fermions $\psi = (\mathcal{Q}, \mathcal{U}, \mathcal{D}, \mathcal{L}, \mathcal{E})$ contain in $4D$ language both left-handed and right-handed chirality zero modes, e.g., $\mathcal{Q} = (Q_L, Q_R)$. The $5D$ Higgs scalar H is in the representation $(1, 2)_{1/2}$ and $\tilde{H} = i\sigma_2 H^*$, and for simplicity the family indices on the fields and $5D$ Yukawa couplings, $\hat{\lambda}$, are suppressed.

Chiral fermions are obtained at the KK-zero mode level by compactifying the extra dimension on the orbifold S^1/Z_2. The length of the orbifold is πR and the associated KK mass scale $M_c \equiv 1/R$. By integrating out the extra dimension, every $5D$ field yields an infinite tower of effective $4D$ Kaluza-Klein modes. For compactification on S^1, the theory would be translation invariant in the extra dimension, implying conservation of 5-momentum and therefore KK-mode number k ($\sum_i k_i = 0$) in every vertex, however, for compactification on an orbifold, the orbifold boundaries break translation invariance and therefore 5-momentum conservation. In the KK picture, this corresponds to the existence of KK-number violating operators (which are induced at loop level even if not included in the bare $5D$ Lagrangian). By definition of the S^1/Z_2 orbifold, the $5D$ theory however still has a symmetry under reflection in the extra dimension $y \to -y$. In terms of KK-modes this translates into the conservation of KK-parity:

$$\sum_i k_i = 0 \text{ mod } 2. \quad (2)$$

Choosing even boundary conditions for all gauge fields and H as well as the chiral fermion components Q_L, U_R, D_R, L_L, and E_R (but not their would-be mirror partners), the resulting KK-zero-modes are identical to the SM fields. At tree-level, the parameters in Eq. (1) are defined in terms of the SM couplings and R via $\hat{g}_i^2 \equiv \pi R g_i^2$ and $\hat{\lambda}_i^2 \equiv \pi R \lambda_i^2$. At tree-level the mass of the j-th KK-mode is given by $m_i^{(j)} = \sqrt{(M_c)^2 + m_i^2}$ for fermions and gauge bosons with the first KK-excitation of the photon being the lightest KK-particle. Because of KK-parity conservation, the LKP is stable, providing a dark matter candidate. The KK-interaction vertices for the UED model are given in Refs. [1,8,9].

III. CONSTRAINTS FROM PRECISION MEASUREMENTS AT LEP1 AND LEP2

Even below the KK-particle production threshold $\sim 2M_c$, KK-particles enter experimental constraints via virtual effects in radiative corrections. At one-loop level and at LEP energies, vertex corrections and box diagrams are suppressed compared to self-energy contributions [3]. Higgs-KK-mode contributions to vertices and box diagrams are suppressed by small Yukawa couplings because no top pair can be produced at LEP energies, while KK-gauge-boson contributions are parametrically suppressed by a factor of $(m_W/M_c)^2$. Thus the UED radiative corrections are approximately oblique, i.e. flavor independent. Oblique corrections are traditionally parameterized by the Peskin-Takeuchi $\hat{S}$, $\hat{T}$, $\hat{U}$ parameters [16] or equivalently by the EWPT parameters $\epsilon_{1,2,3}$ [17], both of which are defined in terms of the gauge-boson self-energies.

Using an effective field theory approach, it has been shown in [15] that $\hat{S}$, $\hat{T}$, and $\hat{U}$ do not form a complete parameterization of relevant corrections to the standard model, where the only significant deviations from the SM reside in the self-energies of the vector bosons. (We will call these corrections *oblique* as opposed to *universal* as is done in [15] to avoid confusion). If the scale of new physics is above LEP2 energies then the new-physics contributions to the transverse gauge-boson self-energies are analytic in q^2 and can be power series expanded. The full independent set of electroweak precision observables (EWPO) that are well-determined by the LEP1 and LEP2 data sets can be defined by[1]

$$\begin{aligned} \hat{T} &\equiv \frac{1}{m_W^2}(\Pi_{W_3W_3}(0) - \Pi_{W^+W^-}(0)) & \hat{S} &\equiv \frac{g}{g'}\Pi'_{W_3B}(0) \\ \hat{U} &\equiv \Pi'_{W^+W^-}(0) - \Pi'_{W_3W_3}(0) & X &\equiv \frac{m_W^2}{2}\Pi''_{W_3B}(0) \\ Y &\equiv \frac{m_W^2}{2}\Pi''_{BB}(0) & W &\equiv \frac{m_W^2}{2}\Pi''_{W_3W_3}(0) \end{aligned} \quad (3)$$

where Π denote the new-physics contributions to the

[1]The observables defined in (3) differ by factors of g and g' compared to [15] as we employ canonical normalizations for the gauge bosons.

transverse gauge-boson vacuum polarization amplitudes, with $\Pi'(0) = d\Pi(q^2)/dq^2|_{q^2=0}$, etc. *A priori*, four more parameters are needed in addition to S, T, U in order to parameterize the full freedom in the electroweak gauge-boson self-energy corrections up to order $(q^2)^2$, but only X, Y and W are well-determined by the LEP data sets.

A convenient way of expressing the Z-pole LEP1 experimental constraints on the electroweak precision observables is in terms of the ϵ_1, ϵ_2, ϵ_3 parameters whose determination are independent of the unknown mass of the Higgs. The experimental constraints from LEP1 determine:

$$\begin{aligned} \epsilon_1 &= +(5.0 \pm 1.1)10^{-3} \\ \epsilon_2 &= -(8.8 \pm 1.2)10^{-3} \\ \epsilon_3 &= +(4.8 \pm 1.0)10^{-3} \end{aligned}$$

with correlation matrix (4)

$$\rho = \begin{pmatrix} 1 & 0.66 & 0.88 \\ 0.66 & 1 & 0.46 \\ 0.88 & 0.46 & 1 \end{pmatrix}$$

These observables are related to the $\hat{S}$, $\hat{T}$, $\hat{U}$, X, Y and W parameters by [15]

$$\begin{aligned} \epsilon_1 &= \epsilon_{1,SM} + \hat{T} - W + 2X\frac{\sin\theta_W}{\cos\theta_W} - Y\frac{\sin^2\theta_W}{\cos^2\theta_W} \\ \epsilon_2 &= \epsilon_{2,SM} + \hat{U} - W + 2X\frac{\sin\theta_W}{\cos\theta_W} \\ \epsilon_3 &= \epsilon_{3,SM} + \hat{S} - W + \frac{X}{\sin\theta_W \cos\theta_W} - Y. \end{aligned} \tag{5}$$

where $\epsilon_{i,SM}$ denote the Higgs-dependent contributions to the electroweak gauge-boson radiative corrections, which by definition are not included in $\hat{S}$, $\hat{T}$, $\hat{U}$, X, Y and W. The full Higgs-dependent corrections $\epsilon_{i,SM}$ have been calculated to 2-loop order [18,19] and implemented in precision electroweak codes such as as TopaZ0 [20,21]. The authors of Ref. [15] give a simple but accurate numerical interpolation to these full 2-loop order results, as shown in Fig. 1.[2]

$$\begin{aligned} \epsilon_{1,SM} &= \left(+6.0 - 0.86\ln\frac{m_H}{m_Z}\right)10^{-3} \\ \epsilon_{2,SM} &= \left(-7.5 + 0.17\ln\frac{m_H}{m_Z}\right)10^{-3} \\ \epsilon_{3,SM} &= \left(+5.2 + 0.54\ln\frac{m_H}{m_Z}\right)10^{-3}. \end{aligned} \tag{6}$$

Similarly the full LEP2 data set at center-of-mass energies varying from 189 GeV to 207 GeV lead to the

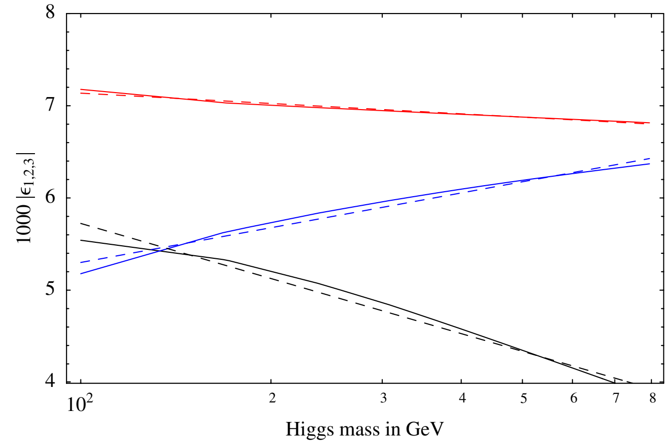

FIG. 1 (color online). Comparison of the 2-loop order Higgs-dependent contributions to the electroweak gauge-boson radiative corrections as implemented in the TopaZ0 code (solid lines) with the simple numerical interpolations (dashed lines) given in Eq. (6).

following determination of the X, Y, W parameters [15]

$$\begin{aligned} X &= (-2.3 \pm 3.5)10^{-3} \\ Y &= (+4.2 \pm 4.9)10^{-3} \\ W &= (-2.7 \pm 2.0)10^{-3} \end{aligned}$$

with correlations (7)

$$\rho = \begin{pmatrix} 1 & -0.96 & +0.84 \\ -0.96 & 1 & -0.92 \\ +0.84 & -0.92 & 1 \end{pmatrix}.$$

The EWPO $\hat{S}$, $\hat{T}$, Y and W break different parts of $SU(2)_L \times SU(2)_{\text{custodial}}$. Without knowing the specific high-energy completion there is no physical reason for a hierarchy between them whereas $\hat{U}$ and X correspond to higher derivative operators with the same symmetry properties as $\hat{T}$ and $\hat{S}$ respectively. Thus if M is the scale of new physics, the expectation is that new-physics contributions to $\hat{U}$ and X will be suppressed by powers of $(m_W/M)^2$ compared to the new-physics contributions to $\hat{S}$, $\hat{T}$, Y and W. However for the purpose of investigating the bounds on new physics, such as the KK mass scale M_c, it is not correct to restrict the analysis to $\hat{S}$, $\hat{T}$, Y, W, and exclude $\hat{U}$ and X. Even though the new-physics contributions to $\hat{U}$ and X are expected to be suppressed from an effective theory point of view, this is not reflected in the experimental constraints on these EWPO. For example the LEP2 data determine $X = (-2.3 \pm 3.5)10^{-3}$ with a similar accuracy as the "dominant" parameters. The fact that the experimentally preferred value is *not* suppressed ought to be included in the analysis. We therefore keep the full parameter set.

IV. CONSTRAINTS ON UED MODELS FROM EWPO

In this section we calculate the contributions to the extended set of EWPO from UED models and obtain an

[2]We thank the authors of Ref. [15] for communications regarding this interpolation, and especially Alessandro Strumia for generating Fig. 1 for us.

improved constraint on $1/R$ by performing a χ^2-fit to the experimental values given in Eqs. (4)–(7). We note in passing that the consequences of the combined LEP1 and LEP2 constraints have so far been explored in $5D$ models with gauge bosons in the extra dimension and Higgsless models [15], for supersymmetry [22] and for Little Higgs Models [15,23].

Concerning UED models, for low Higgs mass, the dominant constraint on $1/R$ is expected from the measurement of T rather than S while U is further suppressed. For large m_H however, the one-loop standard model contribution to T can compensate for the KK-contribution such that a combined S, T analysis is necessary. In [3], one-loop KK-contributions to the S and T parameters are fitted to the experimentally determined values of S, T from LEP1, yielding a constraint on the $(1/R, m_H)$ parameter space (see Fig. 3 of Ref. [3]). The lowest allowed compactification scale is $1/R \sim 300$ GeV at high $m_H \sim 800$ GeV as a result of this cancellation in the T parameter. Because of the significant dependence of the current limit on the cancellation in contributions to T at large m_H where higher terms in the loop expansion for the Higgs-dependent contributions are becoming large, it is important to include the two-loop SM contributions to test if this cancellation is stable. We show below that the 2-loop Higgs contributions destroy the cancellation resulting in an improved constraint on $M_c = 1/R$. A further improvement results from the inclusion of LEP2 data, specifically the measurements of the X, Y, W parameters.

We have calculated the one-loop KK-contributions to the full set of electro-weak precision observables $\hat{S}$, $\hat{T}$, $\hat{U}$, X, Y and W. These contributions, which for one extra dimension $\delta = 1$ are functions of $1/R$ and m_H are in general extremely complicated and rather unilluminating in their explicit form.

As an example of the KK-contributions, Fig. 2 shows the standard model contribution to ϵ_1 and the first three KK-modes of $\hat{T}$ as well as the sum over the first 10 KK-modes at $1/R = 400$ GeV. Similar behavior occurs for the other EWPO. In all cases the sum over KK-modes converges sufficiently fast such that in our further analysis we approximate the UED contributions to the EWPO by the sum over the first 10 modes.

Using the expressions for $\hat{S}$, $\hat{T}$, $\hat{U}$, X, Y W we have derived from the UED model we have performed a χ^2 fit to the LEP1 and LEP2 experimental data encapsulated in Eqs. (4)–(7). Figure 3 shows the resulting constraints on the $1/R$, m_H parameter space.

We find that the lower bound on the mass of the first KK level is improved to $M_c \equiv R^{-1} > 700(800)$ GeV at the 99% (95%) confidence level. There are two origins for the improvement of the bound compared to [3]. First, when taking two-loop standard model contributions to the electroweak precision parameters into account, the KK-contributions to the $\hat{T}$ parameter no longer cancel against Higgs-dependent contributions in the heavy-Higgs-mass limit. This differs from the situation of Ref. [3] where only one-loop Higgs-dependent contributions were taken in to account. This lack of cancellation is the reason for elimination of the heavy-Higgs-mass and low M_c region. Second, the inclusion of LEP2 data into the analysis necessitated the use of an extended set of well-determined electroweak precision observables as shown in Ref. [15]. These new EWPO provide additional constraints, further lifting the bound on M_c.

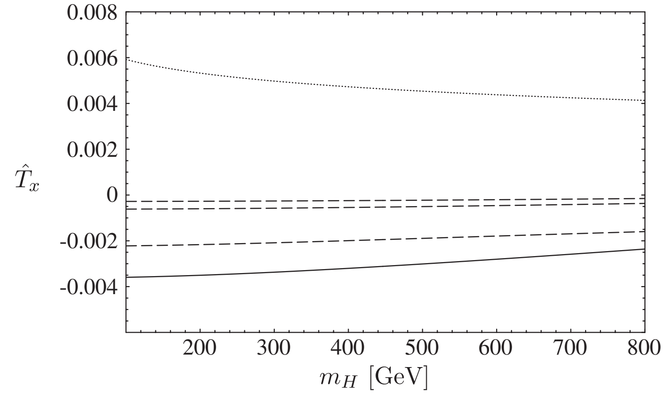

FIG. 2. The contribution to $\hat{T}$ from the first three KK levels (dashed lines) for $M_c = 400$ GeV as a function of Higgs mass in the range 100 to 800 GeV, as well as the sum over the first 10 KK-modes (solid line) and the numerically-interpolated Higgs-dependent correction (dotted line) arising from $\epsilon_{1,SM}$.

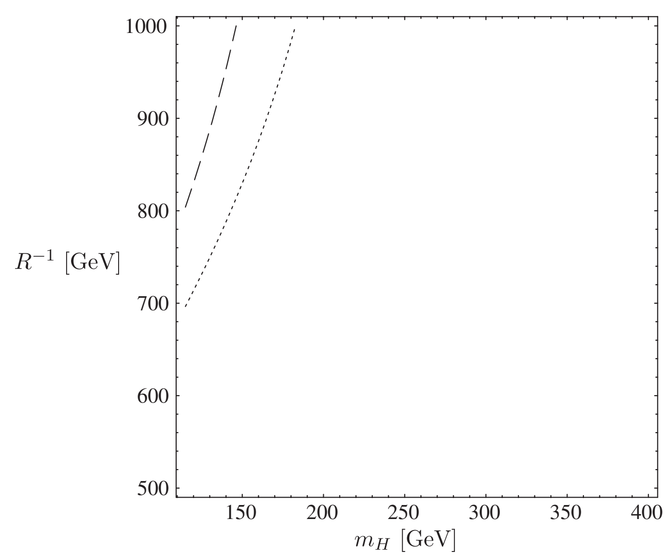

FIG. 3. The 95% (dashed line) and 99% (dotted line) confidence limit exclusion zones for the UED model, as a function of Higgs mass in the range 115 GeV to 400 GeV, and mass $M_1 = 1/R$ of the lightest KK-excitation in the range 500 GeV to 1 TeV. The excluded regions are towards the bottom right of the figure.

V. KK DARK MATTER

In recent years, the Lightest KK-Particle (LKP) in UED models has become a rather popular candidate for the dark matter of our Universe [13,14]. In this section, we will review the phenomenology of KK dark matter in this scenario, and discuss the detection prospects for such a particle in light of the new electroweak precision constraints presented in this article.

As stated earlier, the most natural choice for the LKP in UED models is the first KK-excitation of the hypercharge gauge boson, $B^{(1)}$. Such a state, being electrically neutral and colorless, can serve as a viable candidate for dark matter. One attractive feature of this candidate is that its thermal relic abundance is naturally near the measured quantity of cold dark matter for $M_c \equiv R^{-1} \sim$ TeV.

The number density of the LKP evolves according to the Boltzman equation

$$\frac{dn_{B^{(1)}}}{dt} + 3Hn_{B^{(1)}} = -\langle\sigma v\rangle[(n_{B^{(1)}})^2 - (n^{\rm eq}_{B^{(1)}})^2], \quad (8)$$

where H is the Hubble rate, $n^{\rm eq}_{B^{(1)}}$ denotes the equlibrium number density of the LKP, and $\langle\sigma v\rangle$ is the LKP's self-annihilation cross section, given by[3]

$$\langle\sigma v\rangle \simeq \frac{95 g_1^4}{324\pi m^2_{B^{(1)}}}. \quad (9)$$

Numerical solutions of the Boltzman equation yield a relic density of

$$\Omega_{B^{(1)}} h^2 \approx \frac{1.04 \times 10^9 x_F}{M_{\rm Pl}\sqrt{g^*}(a + 3b/x_F)}, \quad (10)$$

where $x_F = m_{B^{(1)}}/T_F$, T_F is the relic freeze-out temperature, g^* is the number of relativistic degrees of freedom available at freeze-out ($g^* \simeq 92$ for the case at hand), and a and b are terms in the partial wave expansion of the annihilation cross section, $\sigma v = a + bv^2 + \vartheta(v^4)$. Note that in this simple case with only the LKP participating in the freeze-out process, b can safely be neglected. Evaluation of x_F leads to

$$x_F = \ln\left[c(c+2)\sqrt{\frac{45}{8}}\frac{g m_{B^{(1)}} M_{\rm Pl}(a + 6b/x_F)}{2\pi^3\sqrt{g^* x_F}}\right], \quad (11)$$

where c is an order 1 parameter determined numerically and g is the number of degrees of freedom of the LKP. Note that since x_F appears in the logarithm as well as on the left hand side of the equation, this expression must be solved by iteration. WIMPs generically freeze-out at temperatures in the range of approximately $x_F \approx 20$ to 30.

[3]The LKP self-annihilation cross section may be enhanced by processes involving the resonant s-channel exchange of second level KK-modes, particularly if the mass of the higgs boson is somewhat large [24].

When the cross section of Eq. (9) is inserted into Eqs. (10) and (11), a relic abundance within the range of the cold dark matter density measured by WMAP ($0.095 < \Omega h^2 < 0.129$) [25] can be attained for $m_{B^{(1)}}$ approximately in the range of 850 to 950 GeV. This conclusion can be substantially modified if other KK-modes contribute to the freeze-out process, however.

To include the effects of other KK-modes in the freeze-out process, we adopt the following formalism. In Eqs. (10) and (11), we replace the cross section (σ, denoting the appropriate combinations of a and b) with an effective quantity which accounts for all particle species involved

$$\sigma_{\rm eff} = \sum_{i,j}\sigma_{i,j}\frac{g_i g_j}{g^2_{\rm eff}}(1+\Delta_i)^{3/2}(1+\Delta_j)^{3/2}e^{-x(\Delta_i+\Delta_j)}. \quad (12)$$

Similarly, we replace the number of degrees of freedom, g, with the effective quantity

$$g_{\rm eff} = \sum_i g_i(1+\Delta_i)^{3/2}e^{-x\Delta_i}. \quad (13)$$

In these expressions, the sums are over KK species, $\sigma_{i,j}$ denotes the coannihilation cross section between species i and j and the Δ's denote the fractional mass splitting between that state and the LKP.

To illustrate how the presence of multiple KK species can affect the freeze-out process, we will describe two example cases. First, consider a case in which the coannihilation cross section between the two species, $\sigma_{1,2}$, is large compared to the LKP's self-annihilation cross section, $\sigma_{1,1}$. If the second state is not much heavier than the LKP (Δ_2 is small), then $\sigma_{\rm eff}$ may be considerably larger than $\sigma_{1,1}$, and thus the residual relic density of the LKP will be reduced. Physically, this case represents a second particle species depleting the WIMP's density through coannihilations. This effect is often found in the case of supersymmetry models in which coannihilations between the lightest neutralino and another superpartner, such as a chargino, stau, stop, gluino or heavier neutralino, can substantially reduce the abundance of neutralino dark matter.

The second illustrative case is quite different. If $\sigma_{1,2}$ is comparatively small, then the effective cross section tends toward $\sigma_{\rm eff} \approx \sigma_{1,1}g_1^2/(g_1+g_2)^2 + \sigma_{2,2}g_2^2/(g_1+g_2)^2$. If $\sigma_{2,2}$ is not too large, $\sigma_{\rm eff}$ may be smaller than the LKP's self-annihilation cross section alone. Physically, this scenario corresponds to two species freezing out quasi-independently, followed by the heavier species decaying into the LKP, thus enhancing its relic density. Although this second case does not often apply to neutralinos, KK dark matter particles may behave in this way for some possible arrangements of the KK spectra.

Very recently, the LKP freeze-out calculation has been performed, including all coannihilation channels, by two independent groups [26,27]. We will summarize their conclusions briefly here.

As expected, the effects of coannihilation on the LKP relic abundance depend critically on the KK spectrum considered. If strongly interacting KK states are less than roughly $\sim 10\%$ more heavy than the LKP mass, the effective LKP annihilation cross section can be considerably enhanced, thus reducing the relic abundance. KK quarks which are between 5% and 1% more massive than the LKP lead to an LKP with the measured dark matter abundance over the range of masses, $m_{B^{(1)}} \approx 1500$ to 2000 GeV. If KK gluons are also present with similar masses, $m_{B^{(1)}}$ as heavy as 2100 to 2700 GeV is required to generate the observed relic abundance. We thus conclude that if KK quarks or KK gluons are not much more massive than the LKP, the new constraints presented in this article do not reach the mass range consistent with the observed abundance of cold dark matter.

On the other hand, it is possible that all of the strongly interacting KK-modes may be considerably more heavy than the LKP. In this circumstance, other KK states may still affect the LKP's relic abundance. If, for example, all three families of KK leptons are each 1% more massive than the LKP, the observed relic abundance is generated only for $m_{B^{(1)}}$ between approximately 550 and 650 GeV. This range is excluded by the constraints presented in this article. If the KK leptons are instead 5% more massive than the LKP, the observed abundance is found for $m_{B^{(1)}} \approx 670$ to 730 GeV, which is excluded at around the 99% confidence level. We thus conclude that electroweak precision measurements are not consistent with dark matter in this model if KK leptons are within approximately 5% of the LKP mass, unless other KK states are also quasidegenerate.

The constraints put forth here can have a substantial impact on the prospects for the direct and indirect detection of KK dark matter. Firstly, direct detection experiments benefit from the larger elastic scattering cross sections found for smaller values of $m_{B^{(1)}}$. Spin-dependent scattering of the LKP scales with $1/m^4_{B^{(1)}}$, while the spin-independent cross section goes like $1/(m^2_{B^{(1)}} - m^2_{q^{(1)}})^2$ [28]. In either case, the largest scattering rates are expected for the lightest LKPs. Furthermore, heavier WIMPs have a smaller local number density, and thus a smaller scattering rate in direct detection experiments.

Even if the new constraints presented here are not taken into account, the prospects for the direct detection of KK dark matter is somewhat poor. Cross sections are expected to be smaller than roughly 10^{-9} pb and 10^{-4} pb for spin-independent and dependent scattering, respectively [28], both of which are well beyond the reach of existing experiments. Next generation experiments may be able to reach this level of sensitivity, however.

The situation is rather different for the case of indirect detection. The annihilation cross section of LKPs is proportional to $1/m^2_{B^{(1)}}$, and thus a heavier LKP corresponds to a lower rate of annihilation products being generated in regions such as the galactic center, the local halo, external galaxies and in local substructure. For dark matter searches using gamma-rays from regions such as the galactic center [29], this is probably of marginal consequence, considering the very large astrophysical uncertainties involved. The annihilation rate of LKPs in the local halo, however, has less associated astrophysical uncertainty. LKP annihilations in the galactic halo to antimatter particles [30], positrons in particular, may be potentially observable if the LKP annihilation cross section is large enough, i.e.. if the LKP is sufficiently light. It has been shown [31] that the cosmic positron excess observed by the HEAT experiment [32] could have been generated through the annihilations of LKPs in the surrounding few kiloparsecs of our galaxy. This, however, requires $m_{B^{(1)}}$ to be in the range of approximately 300 to 400 GeV,[4] which is strongly excluded by the results presented in this article. Even if the HEAT excess is not a product of dark matter annihilations, the presence of KK dark matter in the local halo will possibly be within the reach of future cosmic positron measurements, particularly those of the AMS-02 experiment [33]. Assuming a modest degree of local inhomogenity, LKP masses up to $m_{B^{(1)}} \approx$ 900 GeV should be within the reach of AMS-02 [34].

Indirect detection of dark matter using neutrino telescopes, on the other hand, relies on WIMPs being efficiently captured in the Sun, where they then annihilate and generate high-energy neutrinos. KK dark matter becomes captured in the Sun most efficiently through its spin-dependent scattering off of protons. Since this cross section scales with $1/m^4_{B^{(1)}}$, the constraints presented in this article somewhat limit the rates which might be observed by next generation high-energy neutrino telescopes, such as IceCube [35]. In particular, a 800 GeV LKP could generate approximately 20 or 3 events per year at IceCube for KK quark masses 10% or 20% larger than the LKP mass, respectively [36]. Over several years of observation, a rate in this range could potentially be distinguished from the atmospheric neutrino background. Larger volume experiments (multicubic kilometer) would be needed to detect an LKP which was significantly heavier than $\sim$TeV.

VI. CONCLUSIONS

In this paper we have reinvestigated the bounds on the compactification scale of Universal Extra Dimension extension arising from electroweak precision observables measured at LEP1 and LEP2. The lower bound is improved to be $M_c \equiv R^{-1} > 700(800)$ GeV at the 99% (95%) confidence level. There are two origins for the improvement of the bound compared to [3]. First, when taking two-loop standard model contributions to the electroweak precision parameters into account, the KK-contributions to the $\hat{T}$

[4]A large degree of local dark matter substructure, if present, may potentially enable this mass range to be extended to considerably higher values.

parameter no longer cancel against Higgs-dependent contributions in the heavy-Higgs-mass limit. This differs from the situation of Ref. [3] where only one-loop Higgs-dependent contributions were taken in to account. This lack of cancellation is the reason for elimination of the heavy-Higgs-mass and low M_c region. Second, the inclusion of LEP2 data into the analysis necessitated the use of an extended set of well-determined electroweak precision observables as shown in Ref. [15]. These new EWPO provide additional constraints, further lifting the bound on M_c.

The new constraint presented in this article can have a significant impact on the phenomenology of Kaluza-Klein dark matter. Indirect detection techniques often rely on the efficient annihilation of dark matter particles in the galactic center, galactic halo, or in dark substructure. The models with the highest annihilation rates of Kaluza-Klein dark matter are those with a low compactification scale, and are thus excluded by the results of this study. The prospects for direct detection are also somewhat reduced by this constraint.

ACKNOWLEDGMENTS

We are grateful to Riccardo Rattazzi and Alessandro Strumia for helpful communications. J. M. R. and T. F. would especially like to thank the Department of Physics of the University of California, Davis, where much of this work was done, for their kind hospitality and for support from the HEFTI visitors program. The work of T. F. was supported by "Evangelisches Studienwerk Villigst e.V." and PPARC Grant No. PPA/S/S/2002/03540A. D. H. is supported by the US Department of Energy and by NASA grant No. NAG5-10842. This work was also supported by the "Quest for Unification" network, MRTN 2004-503369.

APPENDIX A: NDA FOR UNIVERSAL EXTRA DIMENSIONS

Here we outline the naïve dimensional analysis (NDA) of the UED contributions to the EWPO. The $4D$ effective action of a universal EW theory is [15]

$$\begin{aligned}\mathcal{L}_{4\,\mathrm{eff}} = &-\frac{1}{4}b_{\mu\nu}b^{\mu\nu} - \frac{1}{4}w^I_{\mu\nu}w^{I\mu\nu} + (D_\mu h)^\dagger D^\mu h \\ &+ \frac{1}{v^2}(c_{wb}O_{wb} + c_h O_h + c_{ww}O_{ww} + c_{bb}O_{bb}) \\ &+ \ldots,\end{aligned} \tag{A1}$$

where the lower case fields denote the $4D$ effective fields, $v = 174$ GeV is the Higgs VEV, the operators O are defined below and the parentheses contain Higgs potential and Yukawa terms. O_{wb}, O_h, O_{ww}, O_{bb} form a basis of the universal dimension 6 operators [15,37]. All other universal dimension 6 operators are equivalent to the ones given via the equations of motion.

Starting from the $5D$ theory and KK-expanding the fields, the only operators which in the 4D effective theory lead to dimension 6 operators which solely depend on light fields (zero-modes) are the $5D$ analogues of the operators O_{wb}, O_h, O_{ww}, O_{bb}

$$\begin{aligned} O_{WB} &\equiv H^\dagger T^I H W^I_{MN} B^{MN} \qquad & O_H &\equiv |H^\dagger D_M H|^2 \\ O_{BB} &\equiv \frac{1}{2}(\partial_R B_{MN})^2 & O_{WW} &\equiv \frac{1}{2}(D_R W_{MN})^2 \end{aligned} \tag{A2}$$

and operators which are equivalent to them via the $5D$ equations of motion. The parameters $\hat{S}$, $\hat{T}$, Y, W are related to the operator coefficients by [15]

$$\begin{aligned} \hat{S} &= \frac{2}{\tan\theta_w}c_{wb} \qquad & \hat{T} &= -c_h \\ W &= -g^2 c_{ww} & Y &= -g^2 c_{bb}. \end{aligned} \tag{A3}$$

Naïve dimensional analysis of the $5D$ action yields[5]

$$\begin{aligned}\mathcal{L}_{5D} = \frac{\Lambda^5 \pi R}{24\pi^2}\bigg[&-\frac{1}{4\Lambda^2}B_{MN}B^{MN} - \frac{1}{4\Lambda^2}W^I_{MN}W^{IMN} \\ &+ \frac{1}{\Lambda^2}(D_M H)^\dagger D^M H + \frac{1}{\Lambda^2}O_{WB} + \frac{1}{\Lambda^2}O_H \\ &+ \frac{1}{\Lambda^4}O_{WW} + \frac{1}{\Lambda^4}O_{BB}\bigg] + \ldots,\end{aligned} \tag{A4}$$

where here Λ is the strong coupling scale of the $5D$ theory. After evaluating this action on the zero modes, integrating over the extra dimension, and then canonically normalizing the fields, the $4D$ effective action for the light fields reads

$$\begin{aligned}\mathcal{L}_{4\,\mathrm{eff}} = &-\frac{1}{4}b_{\mu\nu}b^{\mu\nu} + (D_\mu h)^\dagger D^\mu h - \frac{1}{4}w^I_{\mu\nu}w^{I\mu\nu} \\ &+ \frac{24\pi^2}{\pi R \Lambda^3}O_{wb} + \frac{24\pi^2}{\pi R\Lambda^3}O_h + \frac{1}{\Lambda^2}O_{ww} \\ &+ \frac{1}{\Lambda^2}O_{bb} + \ldots,\end{aligned} \tag{A5}$$

together with the NDA expression for the largest gauge coupling of the theory—the QCD coupling g_3 (a related expression holds for the largest Yukawa coupling if it becomes strong at the same scale Λ)

$$g_3^2 \sim \frac{24\pi^2}{\pi R \Lambda}. \tag{A6}$$

Further, matching the NDA action Eq. (A1) and (A5) and using Eq. (A3) leads to the NDA estimates for the

[5] Note that NDA numerical factor differs [38] from the usually quoted $24\pi^3$ of Ref. [39] for a $5D$ theory. We thank Riccardo Rattazzi for discussions on this point.

contributions to the leading EWPO

$$\hat{S} \sim \frac{48\pi v^2}{\tan\theta_w R\Lambda^3} \qquad \hat{T} \sim \frac{24\pi v^2}{R\Lambda^3}$$
$$W \sim \frac{g^2 v^2}{\Lambda^2} \qquad Y \sim \frac{g^2 v^2}{\Lambda^2}. \tag{A7}$$

Given the measured size of the QCD coupling at m_Z, the NDA expression for the largest gauge coupling Eq. (A6) leads to a limit on the ratio of the cutoff Λ to the KK mass scale $M_c = 1/R$ given by

$$R\Lambda \lesssim 48 \tag{A8}$$

If we assume this estimate of the bound on the cutoff scale is saturated then from the NDA estimates of the EWPO, we expect $W \sim Y \sim 0.1\hat{T} \sim 0.03\hat{S} \sim 10^{-4}(vR)^2$

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