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# Gauge Unification In Six Dimensions

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## Abstract

We study the breaking of a supersymmetric  $SO(10)$  GUT in 6 dimensions by orbifold compactification. In 4 dimensions we obtain a  $N=1$  supersymmetric theory with the standard model gauge group enlarged by an additional  $U(1)$  symmetry. The 4-dimensional gauge symmetry is obtained as intersection of the Pati-Salam and the Georgi-Glashow subgroups of  $SO(10)$ , which appear as unbroken subgroups in the two 5 dimensional subspaces, respectively. The doublet-triplet splitting arises as in the recently discussed  $SU(5)$  GUTs in 5 dimensions.

The simplest grand unified theory (GUT) which unifies one generation of quarks and leptons, including the right-handed neutrino, in a single irreducible representation is based on the gauge group  $SO(10)$  [1]. Important subgroups, whose phenomenology has been studied in great detail, are the Pati-Salam group  $SU(4) \times SU(2) \times SU(2)$  [2] and the GUT group  $SU(5)$  of Georgi and Glashow [3].

The breaking of these GUT groups down to the standard model gauge group is in general rather involved and requires often large Higgs representations. In particular, the mass splitting between the weak doublet and the colour triplet Higgs fields requires either a fine-tuning of parameters or additional, sophisticated mechanisms. An attractive new possibility has been suggested by Kawamura [4]: compactifying a  $SU(5)$  GUT in 5 dimensions (5d) on an orbifold, which breaks  $SU(5)$  to the standard model group  $SU(3) \times SU(2) \times U(1)$  only on one of the boundary branes, one achieves in 4d the correct gauge symmetry breaking and in addition the wanted doublet-triplet splitting. Various aspects of such 5d  $SU(5)$  GUTs, including fermion masses, have recently been studied by several groups [5, 6]. The goal of the present paper is to extend the orbifold breaking to the GUT group  $SO(10)$ .

The breaking of  $G=SU(5)$  is achieved by means of a ‘parity’  $P$ , under which the generators of  $G$  are either even ( $S$ ) or odd ( $A$ ),

$$PSP^{-1} = S, \quad PAP^{-1} = -A. \quad (1)$$

Clearly, the set of generators  $S$  and  $A$  satisfy the relations

$$[S, S] \subseteq S, \quad [S, A] \subseteq A, \quad [A, A] \subseteq S. \quad (2)$$

Hence, the group  $G_S$  generated by  $S$  is a symmetric subgroup of  $G$ . Together with the identity  $P$  forms the discrete group  $Z_2$ . In orbifold compactifications of 5d  $SU(5)$  GUTs the theory is assumed to be invariant under the parity transformation of gauge fields  $V^M(x, y)$ ,  $M = (\mu, 5)$ ,  $\mu = 0 \dots 3$ ,  $x^5 = y$ , and matter fields  $H_{\pm}(x, y)$ ,

$$PV_{\mu}(x, -y + a)P^{-1} = +V_{\mu}(x, y + a), \quad PV_5(x, -y + a)P^{-1} = -V_5(x, y + a), \quad (3)$$

$$PH_{\pm}(x, -y + a) = \pm H_{\pm}(x, y + a), \quad (4)$$

where  $a = 0$  or  $a = \pi R/2$ , and  $R$  is the radius of the compact dimension. On the orbifold,  $M = \mathcal{R}^4 \times S^1/Z_2$ , the GUT symmetry is then broken on branes which are fixed points of the transformation. Note, that this symmetry breaking preserves the rank of the group<sup>1</sup>.

The extension of this procedure to the GUT group  $SO(10)$  is not straightforward since the standard model group  $G_{SM}$  is not a symmetric subgroup of  $SO(10)$  [7, 6]. It is

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<sup>1</sup>For a detailed discussion of GUT breaking by orbifolding, see e.g. [6].

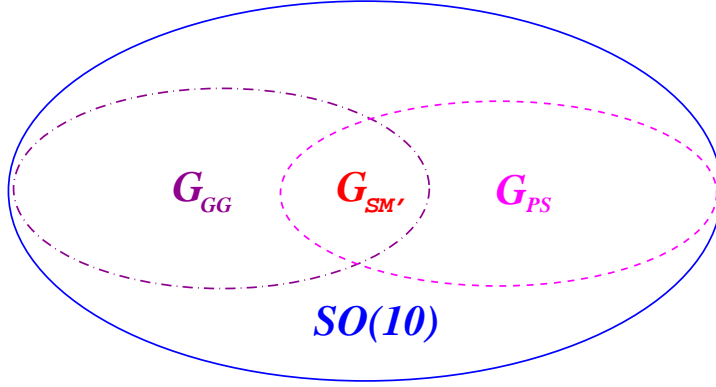


Figure 1: The extended standard model gauge group  $G_{SM'} = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)^2$  as intersection of the two symmetric subgroups of  $\text{SO}(10)$ ,  $G_{GG} = \text{SU}(5) \times \text{U}(1)$  and  $G_{PS} = \text{SU}(4) \times \text{SU}(2) \times \text{SU}(2)$ .

remarkable, however, that the extended standard model group  $G_{SM'} = G_{SM} \times \text{U}(1)$  is the maximal common subgroup of two symmetric subgroups of  $\text{SO}(10)$ ,  $\text{SU}(5) \times \text{U}(1)$  and  $\text{SU}(4) \times \text{SU}(2) \times \text{SU}(2)$  (cf. fig. 1). This suggests to realize the wanted  $\text{SO}(10)$  breaking by starting in 6 dimensions and orbifolding to the two different subgroups in the two orthogonal compact dimensions.

Such an extension to 6 dimensions is also welcome with respect to supersymmetry. It is well known that the extended supersymmetry in 4d is easily obtained by dimensional reduction from 6d [8]. The corresponding unwanted  $\text{N}=2$  supersymmetry in 4d can also be broken by appropriate orbifolding [9]. The irreducible  $\text{N}=1$  multiplets in 6d then split into reducible  $\text{N}=1$  multiplets in 4d.

In the following we shall start from an  $\text{SO}(10)$  GUT in 6 dimensions. First, we consider the compactification on a torus, i.e.,  $M = \mathcal{R}^4 \times T^2$ . The goal is the construction of a  $\text{N}=1$  supersymmetric Yang-Mills theory with extended standard model gauge symmetry. The necessary breaking of the extended supersymmetry in 4d and the breaking of the  $\text{SO}(10)$  GUT group are then achieved in three steps of orbifolding leading finally to the manifold  $M = \mathcal{R}^4 \times T^2 / (Z_2 \times Z_2^{GG} \times Z_2^{PS})$ .

Let us then consider the  $\text{N}=1$  supersymmetric Yang-Mills theory in 6 dimensions. The corresponding lagrangian reads [8],

$$\mathcal{L}_{6d}^{YM} = \text{tr} \left( -\frac{1}{2} V_{MN} V^{MN} + i \bar{\Lambda} \Gamma^M D_M \Lambda \right). \quad (5)$$

Here  $V_M = T^A V_M^A$  and  $\Lambda = T^A \Lambda^A$ , where  $T^A$  are the  $\text{SO}(10)$  generators;  $D_M \Lambda = \partial_M \Lambda - ig[V_M, \Lambda]$  and  $V_{MN} = [D_M, D_N]/(ig)$ ; the  $\Gamma$ -matrices are

$$\Gamma^\mu = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 0 & i\gamma_5 \\ i\gamma_5 & 0 \end{pmatrix}, \quad \Gamma^6 = \begin{pmatrix} 0 & \gamma_5 \\ -\gamma_5 & 0 \end{pmatrix}, \quad (6)$$

with  $\gamma_5^2 = I$  and  $\{\Gamma_M, \Gamma_N\} = 2\eta_{MN} = \text{diag}(1, -1, -1, -1, -1, -1)$ . The gaugino is composed of two Weyl fermions with opposite 4d chirality,  $\Lambda = (\lambda_1, -i\lambda_2)$ , with  $\gamma_5\lambda_1 = -\lambda_1$  and  $\gamma_5\lambda_2 = \lambda_2$ .  $\Lambda$  has negative 6d chirality,  $\Gamma_7\Lambda = -\Lambda$ , with  $\Gamma_7 = \text{diag}(\gamma_5, -\gamma_5)$ .

We now perform a torus compactification, i.e., we choose the manifold  $M = \mathcal{R}^4 \times T^2$ . For the fields  $\Phi = (V_M, \Lambda)$  one then has the mode expansion ( $x^6 = z$ ),

$$\Phi(x, y, z) = \frac{1}{2\pi\sqrt{R_1 R_2}} \sum_{m,n} \Phi^{(m,n)}(x) \exp \left\{ i \left( \frac{my}{R_1} + \frac{nz}{R_2} \right) \right\}, \quad (7)$$

where  $R_1$  and  $R_2$  are the two radii of the torus. Since the vector field is hermitian the corresponding coefficients satisfy the relation  $V_M^{(-m,-n)} = V_M^{(m,n)\dagger}$ .

It is straightforward to work out the 4d lagrangian obtained by integrating over the 5th and the 6th dimensions. For the 4d scalars a convenient choice of variables is

$$\Pi_1^{(m,n)}(x) = \frac{i}{M(m,n)} \left( \frac{m}{R_1} V_5^{(m,n)}(x) + \frac{n}{R_2} V_6^{(m,n)}(x) \right), \quad (8)$$

$$\Pi_2^{(m,n)}(x) = \frac{i}{M(m,n)} \left( -\frac{n}{R_2} V_5^{(m,n)}(x) + \frac{m}{R_1} V_6^{(m,n)}(x) \right), \quad (9)$$

where  $M(m,n) = \sqrt{\left(\frac{m}{R_1}\right)^2 + \left(\frac{n}{R_2}\right)^2}$ . The kinetic term for gauge fields and scalar fields is then given by

$$\begin{aligned} \mathcal{L}_4^{(1)} = & \sum_{m,n} \text{tr} \left( -\frac{1}{2} \tilde{V}_{\mu\nu}^{(m,n)\dagger} \tilde{V}^{(m,n)\mu\nu} + M(m,n)^2 V_\mu^{(m,n)\dagger} V^{(m,n)\mu} \right. \\ & + \partial_\mu \Pi_2^{(m,n)\dagger} \partial^\mu \Pi_2^{(m,n)} + M(m,n)^2 \Pi_2^{(m,n)\dagger} \Pi_2^{(m,n)} \\ & + \partial_\mu \Pi_1^{(m,n)\dagger} \partial^\mu \Pi_1^{(m,n)} \\ & \left. - M(m,n) \left( V_\mu^{(m,n)\dagger} \partial^\mu \Pi_1^{(m,n)} + \partial^\mu \Pi_1^{(m,n)\dagger} V_\mu^{(m,n)} \right) \right), \quad (10) \end{aligned}$$

where  $\tilde{V}_{\mu\nu}^{(m,n)} = \partial_\mu V_\nu^{(m,n)} - \partial_\nu V_\mu^{(m,n)}$ . Massless states are obtained for  $m = n = 0$  (zero modes). The mass generation for the massive Kaluza-Klein (KK) states is analogous to the Higgs mechanism. Here  $\Pi_1^{(m,n)}$  play the role of the Nambu-Goldstone bosons and  $M(m,n)$  correspond to the Higgs vacuum expectation values.

Similarly, one obtains for the gauginos,

$$\begin{aligned} \mathcal{L}_4^{(2)} = & \sum_{m,n} \text{tr} \left( i \bar{\lambda}_1^{(m,n)} \gamma^\mu \partial_\mu \lambda_1^{(m,n)} + i \bar{\lambda}_2^{(m,n)} \gamma^\mu \partial_\mu \lambda_2^{(m,n)} \right. \\ & \left. - \left( \frac{m}{R_1} - i \frac{n}{R_2} \right) \bar{\lambda}_1^{(m,n)} \lambda_2^{(m,n)} + c.c. \right). \quad (11) \end{aligned}$$

This is the kinetic term for the Dirac fermion  $\lambda_D = (\lambda_1, \lambda_2)$  with mass  $M(m,n)$ . Together with the vector  $V_\mu^{(m,n)}$  and the scalars  $\Pi_{1,2}^{(m,n)}$ ,  $\lambda_D$  forms a massive  $N = 1$  vector multiplet in 4d.

So far the massless sector of the theory has an unwanted N=2 supersymmetry. This can be reduced by considering instead of the torus  $T^2$  the orbifold  $T^2/Z_2$ . Under the corresponding reflection  $(y, z) \rightarrow (-y, -z)$  vectors and scalars are even and odd, respectively,

$$PV_\mu(x, -y, -z)P^{-1} = +V_\mu(x, y, z) , \quad PV_{5,6}(x, -y, -z)P^{-1} = -V_{5,6}(x, y, z) , \quad (12)$$

where we choose  $P = I$ . This implies for the Kaluza-Klein modes,

$$V_\mu^{(-m, -n)} = +V_\mu^{(m, n)} = +V_\mu^{(m, n)\dagger} , \quad (13)$$

$$V_{5,6}^{(-m, -n)} = -V_{5,6}^{(m, n)} = +V_{5,6}^{(m, n)\dagger} . \quad (14)$$

In this way scalar zero modes are obviously eliminated. Further, the number of massive KK modes is halved. Since the derivatives  $\partial_{5,6}$  are odd under reflection the two Weyl fermions  $\lambda_1$  and  $\lambda_2$  must have opposite parities,

$$P\lambda_1(x, -y, -z)P^{-1} = +\lambda_1(x, y, z) , \quad P\lambda_2(x, -y, -z)P^{-1} = -\lambda_2(x, y, z) . \quad (15)$$

Comparison of eqs. (12) and (15) shows that  $(V_\mu, \lambda_1)$  and  $(V_{5,6}, \lambda_2)$  form vector and chiral multiplets, respectively. Only vector multiplets have zero modes. The orbifold compactification breaks the extended supersymmetry which one obtains from the 6d theory by dimensional reduction. This is completely analogous to the previously discussed compactification of 5d theories on  $S^1/Z_2$  [9].

The zero modes obtained by compactification on the orbifold  $T^2/Z_2$  form a  $N = 1$  supersymmetric SO(10) theory in 4d. A breaking of the full SO(10) gauge group can be achieved by using the two parities  $P_{GG}$  and  $P_{PS}$  which define the symmetric subgroups  $G_{GG} = \text{SU}(5) \times \text{U}(1)$  and  $G_{PS} = \text{SU}(4) \times \text{SU}(2) \times \text{SU}(2)$ , respectively. In the vector representation the parities can be taken as ( $\sigma_0 = I$ ),

$$P_{GG} = \begin{pmatrix} \sigma_2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_2 \end{pmatrix} , \quad P_{PS} = \begin{pmatrix} -\sigma_0 & 0 & 0 & 0 & 0 \\ 0 & -\sigma_0 & 0 & 0 & 0 \\ 0 & 0 & -\sigma_0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_0 \end{pmatrix} . \quad (16)$$

For the vector fields and the gauginos  $\lambda_1$  one demands

$$\begin{aligned} P_{GG}V_\mu(x, -y, -z + \pi R_2/2)P_{GG}^{-1} &= +V_\mu(x, y, z + \pi R_2/2) , \\ P_{PS}V_\mu(x, -y + \pi R_1/2, -z)P_{PS}^{-1} &= +V_\mu(x, y + \pi R_1/2, z) . \end{aligned} \quad (17)$$

Component fields belonging to the symmetric subgroup  $G_s$  then have positive parity, those of the coset space  $\text{SO}(10)/G_s$  have negative parity. The restrictions of the discrete

$G_{SM'}$	$G_{GG}$	$G_{PS}$	$(V_\mu, \lambda_1)$			$(V_{5,6}, \lambda_2)$		
			$Z_2$	$Z_2^{GG}$	$Z_2^{PS}$	$Z_2$	$Z_2^{GG}$	$Z_2^{PS}$
$(\mathbf{8}, \mathbf{1}, \mathbf{0}, \mathbf{0})$	(24, 0)	(15,1,1)	+	+	+	−	−	−
$(\mathbf{3}, \mathbf{2}, -5, \mathbf{0})$	(24, 0)	(6,2,2)	+	+	−	−	−	+
$(\bar{\mathbf{3}}, \mathbf{2}, 5, \mathbf{0})$	(24,0)	(6,2,2)	+	+	−	−	−	+
$(\mathbf{1}, \mathbf{3}, \mathbf{0}, \mathbf{0})$	(24,0)	(1,3,1)	+	+	+	−	−	−
$(\mathbf{1}, \mathbf{1}, \mathbf{0}, \mathbf{0})$	(24,0)	(1,1,3)	+	+	+	−	−	−
$(\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{4})$	(10,4)	(6,2,2)	+	−	−	−	+	+
$(\bar{\mathbf{3}}, \mathbf{1}, -4, \mathbf{4})$	(10,4)	(15,1,1)	+	−	+	−	+	−
$(\mathbf{1}, \mathbf{1}, \mathbf{6}, \mathbf{4})$	(10,4)	(1,1,3)	+	−	+	−	+	−
$(\bar{\mathbf{3}}, \mathbf{2}, -1, -4)$	$(\overline{10}, -4)$	(6,2,2)	+	−	−	−	+	+
$(\mathbf{3}, \mathbf{1}, \mathbf{4}, -4)$	$(\overline{10}, -4)$	(15,1,1)	+	−	+	−	+	−
$(\mathbf{1}, \mathbf{1}, -6, -4)$	$(\overline{10}, -4)$	(1,1,3)	+	−	+	−	+	−
$(\mathbf{1}, \mathbf{1}, \mathbf{0}, \mathbf{0})$	(1,0)	(15, 1, 1)	+	+	+	−	−	−

Table 1: Parity assignment for the components  $V_M^A = \frac{1}{2}\text{tr}(T^A V_M)$  of the **45**-plet of SO(10).  $G_{SM'} = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)^2$ ,  $G_{GG} = \text{SU}(5) \times \text{U}(1)$  and  $G_{PS} = \text{SU}(4) \times \text{SU}(2) \times \text{SU}(2)$ .

symmetry  $Z_2$  require the opposite parities for the scalars and the gauginos  $\lambda_2$ ,

$$\begin{aligned}
P_{GG} V_{5,6}(x, -y, -z + \pi R_2/2) P_{GG}^{-1} &= -V_{5,6}(x, y, z + \pi R_2/2), \\
P_{PS} V_{5,6}(x, -y + \pi R_1/2, -z) P_{PS}^{-1} &= -V_{5,6}(x, y + \pi R_1/2, z).
\end{aligned} \tag{18}$$

The component fields are again split. Their parities are given in table 1 for the different  $G_{SM'}$  representations contained in the **45**-plet of SO(10).

The mode expansion for the fields  $\Phi(x, y, z)$  with any combination of parities reads explicitly,

$$\Phi_{+++}(x, y, z) = \frac{1}{\pi\sqrt{R_1 R_2}} \sum_{m \geq 0} \frac{1}{2^{\delta_{m,0} \delta_{n,0}}} \phi_{+++}^{(2m, 2n)}(x) \cos\left(\frac{2my}{R_1} + \frac{2nz}{R_2}\right), \tag{19}$$

$$\Phi_{++-}(x, y, z) = \frac{1}{\pi\sqrt{R_1 R_2}} \sum_{m \geq 0} \phi_{++-}^{(2m, 2n+1)}(x) \cos\left(\frac{2my}{R_1} + \frac{(2n+1)z}{R_2}\right), \tag{20}$$

$$\Phi_{++-}(x, y, z) = \frac{1}{\pi\sqrt{R_1 R_2}} \sum_{m \geq 0} \phi_{++-}^{(2m+1, 2n)}(x) \cos\left(\frac{(2m+1)y}{R_1} + \frac{2nz}{R_2}\right), \quad (21)$$

$$\Phi_{+--}(x, y, z) = \frac{1}{\pi\sqrt{R_1 R_2}} \sum_{m \geq 0} \phi_{+--}^{(2m+1, 2n+1)}(x) \cos\left(\frac{(2m+1)y}{R_1} + \frac{(2n+1)z}{R_2}\right), \quad (22)$$

$$\Phi_{-++}(x, y, z) = \frac{1}{\pi\sqrt{R_1 R_2}} \sum_{m \geq 0} \phi_{-++}^{(2m+1, 2n+1)}(x) \sin\left(\frac{(2m+1)y}{R_1} + \frac{(2n+1)z}{R_2}\right), \quad (23)$$

$$\Phi_{-+-}(x, y, z) = \frac{1}{\pi\sqrt{R_1 R_2}} \sum_{m \geq 0} \phi_{-+-}^{(2m+1, 2n)}(x) \sin\left(\frac{(2m+1)y}{R_1} + \frac{2nz}{R_2}\right), \quad (24)$$

$$\Phi_{--+}(x, y, z) = \frac{1}{\pi\sqrt{R_1 R_2}} \sum_{m \geq 0} \phi_{--+}^{(2m, 2n+1)}(x) \sin\left(\frac{2my}{R_1} + \frac{(2n+1)z}{R_2}\right), \quad (25)$$

$$\Phi_{---}(x, y, z) = \frac{1}{\pi\sqrt{R_1 R_2}} \sum_{m \geq 0} \phi_{---}^{(2m, 2n)}(x) \sin\left(\frac{2my}{R_1} + \frac{2nz}{R_2}\right). \quad (26)$$

Only fields for which all parities are positive have zero modes; they form an  $N = 1$  massless vector multiplet in the adjoint representation of the unbroken group  $G_{SM'}$ . All other fields with one or more negative parity combine to massive vector multiplets with some  $G_{SM'}$  quantum numbers.

It is interesting to consider the two limiting cases  $R_1 \rightarrow 0$  with  $R_2$  fixed, and  $R_1$  fixed with  $R_2 \rightarrow 0$ . The dependence on one of the compact dimensions then disappears and one considers effectively a 5 dimensional subspace. In the first case  $SO(10)$  is broken to the Pati-Salam group  $G_{PS} = SU(4) \times SU(2) \times SU(2)$ , in the second case it is broken to the extended Georgi-Glashow group  $G_{GG} = SU(5) \times U(1)$ . Only for finite  $R_1$  and  $R_2$  one obtains the extended standard model group  $G_{SM'} = SU(3) \times SU(2) \times U(1)^2$ .

Matter fields are easily added to the 6d supersymmetric Yang-Mills theory. Of particular interest is the **10**-plet of ‘Higgs’ fields. It contains two complex scalars  $H$  and  $H'$ , and a fermion  $h = (h, h')$  where  $h$  and  $h'$  have 4d chiralities  $\gamma_5 h = h$ ,  $\gamma_5 h' = -h'$ , and positive 6d chirality  $\Gamma_7 h = h$ . The corresponding lagrangian reads [8]

$$\begin{aligned} \mathcal{L}_{6d}^M = & |D_M H|^2 + |D_M H'|^2 - \frac{1}{2} g^2 (H^\dagger T^A H + H'^\dagger T^A H')^2 \\ & + i \bar{h} \Gamma^M D_M h - i \sqrt{2} g (\bar{h} \Lambda H + \bar{h} \Lambda^c H' + c.c.) . \end{aligned} \quad (27)$$

After integrating over the compact dimensions one obtains for the kinetic terms of the 4d fields,

$$\begin{aligned} \mathcal{L}_4^{(3)} = & \sum_{m, n} \left( i \bar{h}^{(m, n)} \gamma^\mu \partial_\mu h^{(m, n)} + i \bar{h}'^{(m, n)} \gamma^\mu \partial_\mu h'^{(m, n)} \right. \\ & + \left( \frac{m}{R_1} - i \frac{n}{R_2} \right) \bar{h}^{(m, n)} h'^{(m, n)} + c.c. \\ & + \partial_\mu H^{(m, n) \dagger} \partial^\mu H^{(m, n)} + M(m, n)^2 H^{(m, n) \dagger} H^{(m, n)} \\ & \left. + \partial_\mu H'^{(m, n) \dagger} \partial^\mu H'^{(m, n)} + M(m, n)^2 H'^{(m, n) \dagger} H'^{(m, n)} \right) . \end{aligned} \quad (28)$$

$G_{SM'}$	$G_{GG}$	$G_{PS}$	$(H_1, h_1)$			$(H_2, h_2)$		
			$Z_2$	$Z_2^{GG}$	$Z_2^{PS}$	$Z_2$	$Z_2^{GG}$	$Z_2^{PS}$
$(3, 1, -2, 2)$	$(5, 2)$	$(6, 1, 1)$	+	+	-	+	-	-
$(\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{2})$	$(5, 2)$	$(1, 2, 2)$	+	+	+	+	-	+
$(\bar{3}, 1, 2, -2)$	$(\bar{5}, -2)$	$(6, 1, 1)$	+	-	-	+	+	-
$(\mathbf{1}, \mathbf{2}, -\mathbf{3}, -\mathbf{2})$	$(\bar{5}, -2)$	$(1, 2, 2)$	+	-	+	+	+	+

Table 2: Parity assignment for the components of the Higgs **10**-plets of SO(10). The parities of  $H'_{1,2}$  and  $h'_{1,2}$  are opposite to the listed parities of  $H_{1,2}$  and  $h_{1,2}$ .  $G_{SM'} = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)^2$ ,  $G_{GG} = \text{SU}(5) \times \text{U}(1)$  and  $G_{PS} = \text{SU}(4) \times \text{SU}(2) \times \text{SU}(2)$ .

We now have to define the action of the parities on the Higgs fields. For the N=1 supermultiplets  $H = (H, h)$  and  $H' = (H', h')$  we have opposite parities with respect to every  $Z_2$ . Note, however, that there is an ambiguity in the global sign of each parity, as long as we do not consider a superpotential for the matter fields.

For the  $Z_2$  parity we choose

$$PH(x, -y, -z) = +H(x, y, z), \quad (29)$$

$$PH'(x, -y, -z) = -H'(x, y, z), \quad (30)$$

with  $P = I$ . This breaks the extended supersymmetry as in the case of the **45**-plet. For  $Z_2^{GG}$  we can take

$$P_{GG}H(x, -y, -z + \pi R_2/2) = +H(x, y, z + \pi R_2/2), \quad (31)$$

$$P_{GG}H'(x, -y, -z + \pi R_2/2) = -H'(x, y, z + \pi R_2/2), \quad (32)$$

where the matrix representations of  $P_{GG}$  is defined in eq. (16). Note that the SU(5) **5**- and **5\***-plets contained in the **10**-plet have opposite parities with respect to  $Z_2^{GG}$  (cf. table 2). The parity  $P_{PS}$  yields the desired doublet-triplet splitting. As mentioned above, it is ambiguous up to a sign, like Kawamura's parity for the SU(5) GUT [4]. With

$$P_{PS}H(x, -y + \pi R_1/2, -z) = +H(x, y + \pi R_1/2, z), \quad (33)$$

$$P_{PS}H'(x, -y + \pi R_1/2, -z) = -H'(x, y + \pi R_1/2, z), \quad (34)$$

one obtains one SU(2) doublet N=1 supermultiplet as zero modes, as given in table 2. The related SU(3) triplet is heavy. The opposite choice of sign leads to a massless colour triplet and a heavy weak doublet.

In order to obtain the wanted two Higgs doublets as zero modes one has to introduce two **10**-plets  $H_1$  and  $H_2$ . Their parities must be different with respect to  $Z_2^{GG}$ , as listed in table 2. Note, that the irreducible 6d gauge anomalies of the two **10**-plets cancel the one of the **45**-plet [6].

So far we have only considered the breaking of the SO(10) GUT symmetry. Quarks and leptons can be included along the lines discussed for the 5d SU(5) GUTs [5, 6], e.g. as **16**-plets on a brane. The electroweak gauge group  $SU(2) \times U(1)^2$ , which contains  $U(1)_{B-L}$ , is the minimal extension of the chiral standard model electroweak symmetry which can keep all fermions, including the right-handed neutrino, massless. Its breaking is related to the generation of fermion masses which will be discussed elsewhere.

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