Universal extra dimensions and the muon magnetic moment

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Abstract

We analyze the muon anomalous magnetic moment in the context of universal extra dimensions. Our computation shows that the bound from electroweak data on the size of these dimensions allows only a small shift in the muon magnetic moment given by Kaluza–Klein modes of standard model fields. In the well-motivated case of two universal extra dimensions, additional contributions arising from physics at scales where the effective 6-dimensional standard model breaks down, given by dimension-ten operators, have a natural size comparable to the sensitivity of the muon $(g - 2)$ experiment at BNL. © 2001 Published by Elsevier Science B.V.

1. Introduction

There are good reasons to imagine that *all* the standard model fields propagate in a larger number of spatial dimensions compactified at a scale $1/R \lesssim 1$ TeV. This framework could provide a mechanism for electroweak symmetry breaking [1] and supersymmetry breaking [2], and it relates the number of fermion generations to the requirement of anomaly cancellation [3]. Recently, it was pointed out that the compactification radius R of these universal extra dimensions, can be surprisingly large, as large as $1/(300$ GeV$)$. The reason is that momentum conservation in extra dimensions leads to Kaluza–Klein (KK) number conservation and therefore to the absence of vertices with a single nonzero KK mode. There are thus no tree-level contributions to the electroweak observables, and no single KK mode production at colliders. Interestingly, the tightest bounds on R, derived from the ex-

perimental constraints on the ρ parameter [4] and on the $b \to s\gamma$ process [5], leave room for a discovery of KK modes in Run II at the Tevatron.

The higher-dimensional standard model is an effective field theory, valid below some scale M_s in the multi-TeV range. Its lowest dimension operators correspond directly to the familiar terms of the 4-dimensional standard model. Corrections to the leading low energy theory are encoded in a tower of operators of increasing dimension allowed by the field content and the symmetries. This effective higher-dimensional theory, after compactification to four dimensions, leads to the standard model together with two classes of corrections. The first arises from physics above $1/R$ but below the cutoff M_s, and corresponds to virtual KK modes of the standard model particles. The second arises from the unknown physics at scales M_s and above, and is parametrized by the coefficients of the tower of higher dimension operators.

In this Letter, we extend the considerations of Ref. [4] by discussing the muon anomalous magnetic moment in the context of universal extra dimensions. We compute the one-loop contribution from standard

model KK modes in universal extra dimensions, and find that it is too small to be seen by the Muon $g - 2$ Collaboration [6]. This computation confirms the estimate of Agashe, Deshpande and Wu [7].

We then consider the effects of the higher dimension operators. Concentrating on the well motivated case of the (chiral) 6-dimensional standard model, we find that the contributions to the muon magnetic moment from dimension-ten operators could naturally be large enough to be experimentally measurable for typical values of M_s, of a few TeV.

In Section 2, we discuss universal extra dimensions in the framework of effective field theory, and display the leptonic sector of this theory, appropriate for the computation of the muon anomalous magnetic moment. In Section 3, we compute at one-loop level the muon magnetic moment induced by the KK modes of standard model fields, and in Section 4 we analyze higher dimension operators within the chiral 6-dimensional standard model. Conclusions and a comparison with the ρ parameter computation are contained in Section 5.

2. Universal extra dimensions

The idea of universal extra dimensions is very simple: *all* standard model fields propagate in some extra spatial dimensions. These universal dimensions are taken to be flat, and must be compactified on an orbifold such that the zero-mode fermions are endowed with 4-dimensional chirality. The simplest δ-dimensional orbifold of this kind is obtained by compactifying each pair of extra dimensions on T^2/Z_2 and, for odd δ, the remaining extra dimension on S^1/Z_2, as explicitly shown in [4]. The $(4 + \delta)$-dimensional quarks, $\mathcal{Q}_i, \mathcal{U}_i, \mathcal{D}_i$, and leptons, $\mathcal{L}_i, \mathcal{E}_i$, ($i = 1, 2, 3$ is a generational index) are decomposed in KK modes such that only one left- (right-)handed component of each weak doublet (singlet) is even under the orbifold projection. For example, the second generation lepton fields have zero-modes $\mathcal{L}_2^{(0)} = (\nu_{\mu L}, \mu_L)$ and $\mathcal{E}_2^{(0)} = \mu_R$.

The $(4 + \delta)$-dimensional Lagrangian looks very similar to that of the 4-dimensional standard model. There are kinetic terms for the $(4 + \delta)$-dimensional $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge fields, a kinetic term and potential for the $(4 + \delta)$-dimensional Higgs doublet, $\mathcal{H}$, as well as the following terms involving quarks and leptons:

$$(\overline{\mathcal{Q}}, \overline{\mathcal{U}}, \overline{\mathcal{D}}, \overline{\mathcal{L}}, \overline{\mathcal{E}}) i \Gamma^\alpha D_\alpha (\mathcal{Q}, \mathcal{U}, \mathcal{D}, \mathcal{L}, \mathcal{E})$$
$$- \left[\overline{\mathcal{Q}}(\hat{\lambda}_{\mathcal{U}} \mathcal{U} i \sigma_2 \mathcal{H}^* + \hat{\lambda}_{\mathcal{D}} \mathcal{D} \mathcal{H}) + \overline{\mathcal{L}} \hat{\lambda}_{\mathcal{E}} \mathcal{E} \mathcal{H} + \text{h.c.} \right]. \tag{2.1}$$

We use the notation x^α or x^β, for all space–time coordinates ($\alpha, \beta = 0, 1, \ldots, 4 + \delta$), and x^ρ or x^τ for the noncompact coordinates ($\rho, \tau = 0, 1, 2, 3$). D_α are the covariant derivatives associated with the $SU(3)_C \times SU(2)_W \times U(1)_Y$ group. Γ^α are anticommuting $2^{k+2} \times 2^{k+2}$ matrices, where k is an integer such that $\delta = 2k$ or $\delta = 2k + 1$. When δ is even, the quark and lepton fields may have $4 + \delta$ chirality, defined by the eigenvalues ± 1 of $\Gamma_{4+\delta}$, the analogue of γ_5 in four dimensions. Anomaly considerations are discussed in Refs. [3,4]. A summation over a generational index is implicit in Eq. (2.1). The $(4 + \delta)$-dimensional Yukawa couplings, $\hat{\lambda}_{\mathcal{U}}, \hat{\lambda}_{\mathcal{D}}, \hat{\lambda}_{\mathcal{E}}$, are 3×3 matrices and have mass dimension $-\delta/2$.

The standard model operators listed above have mass dimension ranging up to $4 + 2\delta$. They are the lowest dimension operators allowed by gauge symmetry. Corrections are described by a tower of higher dimension operators, each suppressed by inverse powers of the multi-TeV scale M_s at which the effective theory breaks down.

In Ref. [4], this effective theory was used to show that, because the KK number is conserved and thus the contributions to experimental observables arise only from loops, the bound from the electroweak data on the size of universal extra dimensions is rather loose. The main constraint comes from weak-isospin violating effects, encoded in the $\Delta\rho = \alpha T$ parameter. In the case of a single extra dimension the electroweak parameters may be computed reliably at one-loop level using the Lagrangian (2.1), revealing that the compactification scale $1/R$ could be as low as 300 GeV. Higher order corrections, suppressed by inverse powers of RM_s, are small. In the case of two universal extra dimensions, the contributions of the KK modes to electroweak observables become logarithmically sensitive to M_s, meaning that they are not reliably computable by relying only on physics below M_s. A rough estimate can be made, however, indicating that in this case $1/R$ could be as low as

roughly 500 GeV. We return to this discussion in Section 5, after describing our results for the muon $g-2$.

The 4-dimensional Lagrangian is obtained by dimensional reduction from the $(4+\delta)$-dimensional theory. The decomposition of the standard model fields in KK modes leads to a variety of trilinear and quartic interactions. Contributions to the one-loop muon anomalous magnetic moment arise from the leptonic part of Eq. (2.1). The vector-like KK modes associated with the weak-doublet, $\mathcal{L}_2^j = (\mathcal{L}_{\nu_\mu}^j, \mathcal{L}_\mu^j)$, and -singlet, $\mathcal{E}_2^j$, muon fields have electroweak symmetric masses M_j, with $j \geqslant 1$. In the case of one universal extra dimension $M_j = j/R$, while for more dimensions the KK spectrum is denser. The zero-mode Higgs doublet acquires a VEV, breaking the electroweak symmetry, and leading to mass mixing between the $\mathcal{L}_\mu^j$ and $\mathcal{E}_2^j$ KK modes, level by level. The mixing angle is suppressed by the ratio of the muon mass to KK mass,

$$\sin\alpha_j \approx \frac{m_\mu}{2M_j} + \mathcal{O}\big(m_\mu^3/M_j^3\big). \tag{2.2}$$

The trilinear interaction of the zero-mode muon mass eigenstate, μ', with the higher KK modes of the Z boson, Z_α^j, and the muon KK mass eigenstates, $\mathcal{L}_\mu^{\prime j}$ and $\mathcal{E}_2^{\prime j}$, is described by the following terms in the 4-dimensional Lagrangian:

$$\frac{g}{\cos\theta_W}\Big\{ Z_\rho^j\big[\overline{\mathcal{L}}_\mu^{\prime j}\gamma^\rho(g_{\mu_L}\cos\alpha_j P_L + g_{\mu_R}\sin\alpha_j P_R)$$
$$- \overline{\mathcal{E}}_2^{\prime j}\gamma^\rho(g_{\mu_R}\cos\alpha_j P_R + g_{\mu_L}\sin\alpha_j P_L)\big]\mu'$$
$$- i Z_4^j\big[\overline{\mathcal{L}}_\mu^{\prime j}(g_{\mu_L}\cos\alpha_j P_L - g_{\mu_R}\sin\alpha_j P_R)$$
$$- \overline{\mathcal{E}}_2^{\prime j}(g_{\mu_R}\cos\alpha_j P_R - g_{\mu_L}\sin\alpha_j P_L)\big]\mu'\Big\}$$
$$+ \text{h.c.} \tag{2.3}$$

As usual, g is the $SU(2)_W$ gauge coupling, $P_{L,R} = (1 \mp \gamma_5)/2$ and

$$g_{\mu_L} = -\frac{1}{2} + \sin^2\theta_W, \qquad g_{\mu_R} = \sin^2\theta_W. \tag{2.4}$$

In Eq. (2.3) we have displayed the interactions involving the KK modes of only one scalar component of the Z. For δ extra dimensions there are δ scalars associated with a gauge boson, at each KK level. For the one-loop computations that we perform in the next section each scalar KK mode contributes by the same

amount, so it is sufficient to consider the exact form of the interactions involving Z_4^j.

Another interaction entering the muon $g-2$ computation is that of a photon KK mode, A_ρ^j and A_4^j, with a muon zero-mode and a muon j-mode. It may be obtained from Eq. (2.3) by substituting $Z_{\rho,4}^j$ with $-A_{\rho,4}^j \sin\theta_W \cos\theta_W$ and setting $g_{\mu_L} = g_{\mu_R} = 1$. It is also straightforward to write the interactions of the W scalar KK modes with a muon zero-mode and a muon–neutrino KK mode in the weak eigenbasis:

$$-\frac{ig}{\sqrt{2}} W_4^{+j}\,\bar{\nu}_{\mu_R}^j \mu_L + \text{h.c.} \tag{2.5}$$

The $W_\rho^{+j}\bar{\nu}_{\mu_L}^j\gamma^\rho\mu_L$ vertex, involving the W boson KK modes, as well as the $A_\rho W_\rho^{+j} G^{-j}$ and $G^{+j}\bar{\nu}_{\mu_L}^j\mu_R$ vertices, involving the KK modes of the charged Goldstone boson eaten by the W, are identical with the standard model ones for the corresponding zero-modes. Finally, the interactions of the photon zero-mode with the muon or W boson KK modes are diagonal and determined by the corresponding electric charge.

3. $g_\mu - 2$ from KK modes of standard model fields

The anomalous magnetic moment of the muon is the coefficient $a_\mu \equiv g_\mu - 2$ in the 4-dimensional momentum space operator

$$-a_\mu \frac{e}{2m_\mu} A_\rho\big(p^{\text{out}} - p^{\text{in}}\big)\bar{\mu}'\big(p^{\text{out}}\big)i\sigma^{\rho\tau}\big(p_\tau^{\text{out}} - p_\tau^{\text{in}}\big)$$
$$\times \mu'\big(p^{\text{in}}\big). \tag{3.1}$$

In this section, we compute the contribution a_μ^{KK} arising from KK modes associated with universal extra dimensions.

The standard model in universal extra dimensions leads to the one-loop corrections to a_μ shown in Figs. 1 and 2. Each diagram gives a contribution of order $(\alpha/\pi)m_\mu^2/M_j^2$, reflecting the decoupling of the KK modes. [1] An important feature of these diagrams is that the contributions from individual KK levels are

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[1] The result is proportional to $(m_\mu R)^2$, unlike the linear dependence on $m_\mu R$ in Ref. [8], due to the chiral couplings of the muon to the gauge boson KK modes.
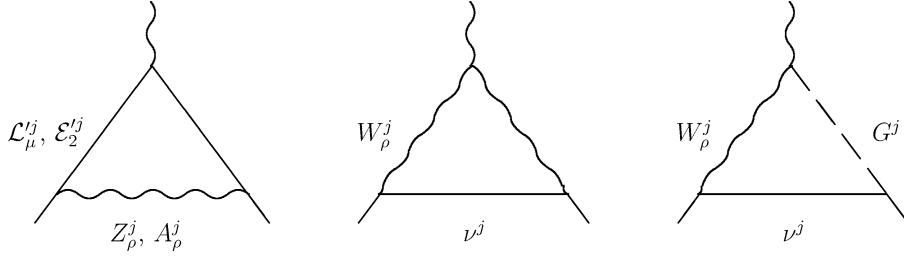
Fig. 1. Contributions to $g_\mu - 2$ from KK modes of the $SU(2)_W \times U(1)_Y$ gauge fields, labeled by $c(Z^\rho)$, $c(A^\rho)$, $c(W^\rho)$, $c(G^\pm)$ in Eq. (3.2).
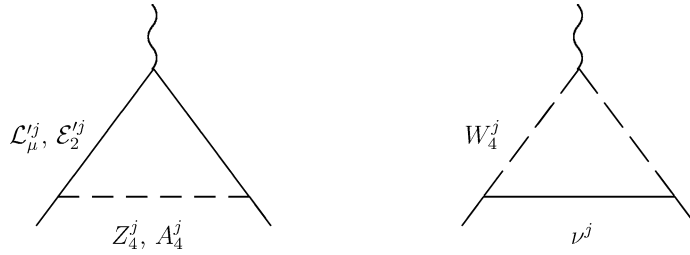

Fig. 2. Contributions to $g_\mu - 2$ from the gauge fields polarized in extra dimensions, labeled by $c(A^4)$, $c(Z^4)$, $c(W^4)$ in Eq. (3.2).

independent, so that the result is simply a sum over KK levels:

$$a_\mu^{\text{KK}} = \frac{\alpha}{8\pi} \sum_j D_j \frac{m_\mu^2}{M_j^2} \Big\{ c(Z^\rho) + c(A^\rho) + c(W^\rho)$$
$$+ c(G^\pm) + \big[c(A^4) + c(Z^4) + c(W^4) \big] \delta \Big\},$$
$$(3.2)$$

where D_j is the degeneracy of the jth KK level. The coefficients $c(Z^\rho)$, $c(A^\rho)$, $c(W^\rho)$ and $c(G^\pm)$ correspond to the diagrams shown in Fig. 1. The coefficients $c(A^4)$, $c(Z^4)$ and $c(W^4)$, given by diagrams which involve scalar KK modes of the photon, Z and W (see Fig. 2), are multiplied by the number of extra dimensions because the higher-dimensional gauge fields have $4 + \delta$ components. Using the interactions displayed in Section 2, we compute the diagrams of Figs. 1 and 2 in the Feynman gauge, and obtain the following values for each of the coefficients:

$$c(A^\rho) = -\frac{2}{3} c(A^4) = \frac{2}{3}, \tag{3.3}$$

$$c(Z^\rho) = -\frac{3 + 4\sin^2\theta_W \cos 2\theta_W}{3\sin^2 2\theta_W}, \tag{3.4}$$

$$c(Z^4) = \frac{1 + 12\sin^2\theta_W \cos 2\theta_W}{6\sin^2 2\theta_W}, \tag{3.5}$$

$$c(W^\rho) = 2c(W^4) = -c(G^\pm) = -\frac{1}{3\sin^2\theta_W}. \tag{3.6}$$

The sum of all the diagrams takes the form

$$a_\mu^{\text{KK}} = \frac{\alpha}{24\pi \sin^2 2\theta_W}$$
$$\times \left[-3 + 4\sin^2\theta_W - \frac{\delta}{2}\big(3 + 8\sin^2\theta_W\big) \right]$$
$$\times \sum_j D_j \frac{m_\mu^2}{M_j^2}, \tag{3.7}$$

and using $\sin^2\theta_W \approx 0.231$, we find

$$a_\mu^{\text{KK}} \approx -5.8 \times 10^{-11}(1 + 1.2\delta)S_{\text{KK}}, \tag{3.8}$$

where we defined

$$S_{\text{KK}} \equiv \sum_j \frac{6D_j}{\pi^2} \left[\frac{300\,\text{GeV}}{M_j} \right]^2. \tag{3.9}$$

With a single universal extra dimension, the degeneracy factor D_j is unity and the sum is convergent

as in the case of precision electroweak observables. The smallest value for $1/R$ allowed by the electroweak data is approximately 300 GeV, giving $S_{KK} \approx 1$, and thus leading to a negative value for a_μ^{KK} of order 10^{-10}. This is smaller than the final expected 1σ sensitivity of the muon $g-2$ experiment at BNL [6]. The negative sign of this small contribution increases slightly the discrepancy between the standard model prediction [9] and experiment.

For the more interesting case of two universal extra dimensions, the KK sum diverges logarithmically, indicating that as in the case of the electroweak observables, important contributions can arise from physics at scales above M_s as well as below. The contribution from physics below M_s can be estimated by cutting off the KK mode sum at an M_j of order M_s. As noted in Ref. [4], this procedure for the electroweak observables leads to $1/R \gtrsim 500$ GeV and $M_s R \lesssim 5$. This leads to $S_{KK} \lesssim 1$, and therefore a_μ^{KK} is of order 10^{-10} or smaller in the 6-dimensional standard model.

4. The 6-dimensional standard model and short distance effects on $g_\mu - 2$

In the previous section we showed that, with one or two universal extra dimensions, the value of $g_\mu - 2$ $\equiv a_\mu$ induced by loops with standard model KK fields below the effective theory cutoff M_s is smaller than the final expected 1σ sensitivity of the muon $g-2$ experiment at BNL [6]. However, physics above M_s also contributes to a_μ, and these effects could be large given that the $SU(3)_C \times SU(2)_W \times U(1)_Y$ interactions are strongly coupled at these scales. From a low-energy effective theory point of view, the effect of physics above M_s is parametrized by higher dimension operators suppressed by powers of M_s. In the case of one universal extra dimension, the effective 5-dimensional theory breaks down at $M_s \gtrsim 10$ TeV, so the operators suppressed by powers of M_s are not likely to induce a large a_μ. With more dimensions the cut-off M_s is lower. We concentrate in what follows on the case of two universal extra dimensions.

In six dimensions, the standard model is chiral as in four dimensions and is highly constrained by anomaly cancellation and Lorentz invariance. The quarks and leptons are 4-component Weyl fermions of defi-

nite chirality which we label by '+' and '−'. The cancellation of local anomalies imposes one of the following two chirality assignments: $\mathcal{Q}_+, \mathcal{U}_-, \mathcal{D}_-, \mathcal{L}_\mp, \mathcal{E}_\pm$. Each of these 6-dimensional chiral fermions leads in the effective 4-dimensional theory to either a left- or right-handed zero-mode fermion depending on the orbifold boundary conditions. The 6-dimensional standard model is the only known theory that constrains the number of fermion generations to be $n_g = 3 \bmod 3$, based on the global anomaly cancellation condition [3].

The gravitational anomaly cancels only if within each generation there is a gauge singlet fermion with 6-dimensional chirality opposite to that of the lepton doublet [1,3]. These gauge singlet fermions can have Yukawa couplings to the Higgs and lepton doublet fields, which at one loop give rise to a negative shift in a_μ. However, the Yukawa couplings of the zero-modes have to be smaller than $\sim 10^{-10}$ in order to avoid too large Dirac neutrino masses. There are mechanisms to explain this small parameter, involving additional dimensions accessible only to gravity and the singlet fermions [10]. The large number of singlet fermion KK modes associated with these additional dimensions enhance the contribution to the muon anomalous magnetic moment, but even then there is no reason to expect a sizable a_μ from the neutrino sector.

Here we point out that the chiral 6-dimensional standard model includes higher dimension operators suppressed by powers of M_s that can naturally have a substantial contribution to a_μ. In the 6-dimensional Lagrangian these appear as dimension-ten operators:

$$\overline{\mathcal{L}}_+ \frac{i}{2}[\Gamma^\alpha, \Gamma^\beta] \frac{\hat{\lambda}_\mathcal{E}}{M_s^2} \left(C_B \frac{\hat{g}'}{2} \mathcal{B}_{\alpha\beta} - C_W \frac{\vec{\sigma}}{2} \hat{g} \vec{\mathcal{W}}_{\alpha\beta} \right) \mathcal{E}_- \mathcal{H}$$
$$+ \text{h.c.}, \qquad\qquad\qquad (4.1)$$

where $\mathcal{W}_{\alpha\beta}$, $\mathcal{B}_{\alpha\beta}$ are the 6-dimensional $SU(2)_W \times U(1)_Y$ field strengths, and C_W, C_B are dimensionless parameters determined by the unknown physics above M_s. We have defined them by extracting the 6-dimensional $SU(2)_W \times U(1)_Y$ gauge couplings $\hat{g}, \hat{g}'$, and charged lepton Yukawa coupling matrix, $\hat{\lambda}_\mathcal{E}$. These have inverse mass dimension, and are related to the corresponding 4-dimensional couplings by

$$\{\hat{g}, \hat{g}', \hat{\lambda}_\mathcal{E}\} = \sqrt{2}\pi R \{g, g', \lambda_\mathcal{E}\}. \qquad (4.2)$$

After the two extra dimensions are integrated out, the operator (4.1) gives rise to a number of terms in

the 4-dimensional Lagrangian. Only the Higgs doublet zero-mode acquires a VEV, leading to interactions of the leptons with gauge bosons described by dimension-five operators. Among those that involve only zero-modes, the following operator contributes to a_μ:

$$\frac{em_\mu}{2M_s^2}\bar{\mu}'\big[U^\dagger(C_B+C_W)U\big]_{22}\frac{i}{2}[\gamma^\rho,\gamma^\tau]\mu' F_{\rho\tau}, \quad (4.3)$$

where $F_{\rho\tau}$ is the electromagnetic field strength, μ' is the muon mass eigenstate, and U is the unitary matrix that relates the mass eigenstate charged leptons to the weak eigenstates. In general, C_W and C_B are 3×3 matrices in flavor space. However, the gauge fields have generational-independent couplings, so we expect that the flavor-dependence of the operator (4.1) is due only to the presence of the Higgs field and shows up predominantly through the Yukawa coupling matrix $\lambda_{\mathcal{E}}$. In other words, we expect C_W and C_B to be approximately flavor independent:

$$C_{W,B}^{ii'} = c_{W,B}\big(\delta_{ii'}+\epsilon_{W,B}^{ii'}\big), \quad (4.4)$$

with $\epsilon_{W,B}^{ii'} \ll 1$ ($i, i' = 1, 2, 3$), on the order of the squared lepton Yukawa couplings. Therefore, the muon anomalous magnetic moment induced by the operator (4.3) is given by

$$a_\mu^{\mathrm{op}} \approx \frac{2m_\mu^2}{M_s^2}(c_B+c_W). \quad (4.5)$$

If M_s is taken to be the scale where the standard model gauge interactions become nonperturbative, then $RM_s \approx 5$. The bound on the size of two universal extra dimensions imposed by the electroweak data is $1/R \gtrsim 500$ GeV [4]. Since anomaly cancellation in six dimensions does not allow a straightforward supersymmetric extension of the standard model [3], the scale where the 6-dimensional standard model breaks down should be not much higher than a few TeV in order to avoid fine-tuning in the Higgs sector. The result for a_μ^{op} can be written as

$$a_\mu^{\mathrm{op}} \approx 3.6 \times 10^{-9}(c_B+c_W)\left(\frac{2.5\,\mathrm{TeV}}{M_s}\right)^2. \quad (4.6)$$

The operators (4.1) arise at scales of order M_s, and because they involve gauge fields, their coefficients are expected to be proportional to the 6-dimensional gauge couplings. At the same time, these operators

break the chiral symmetry of the leptons, and from the Yukawa terms in the 6-dimensional Lagrangian we know that such breaking is accompanied by Yukawa couplings. Hence, it is natural to expect the coefficients C_B and C_W, defined in Eq. (4.1) by extracting the 6-dimensional gauge and Yukawa couplings, to be of order unity at the scale M_s. Furthermore, upon dimensional reduction the volume suppression is entirely absorbed in the gauge and Yukawa couplings.[2] As a result, the values of c_B and c_W at scales comparable to the muon mass differ from those at the scale M_s by factors of order one, mostly due to the one-loop running. Notice that the theory is perturbative at scales below M_s.

We conclude that physics from above the cutoff M_s of the effective, 6-dimensional standard model naturally gives a contribution to a_μ comparable to the current sensitivity of the muon $g - 2$ experiment at BNL. The sign of this contribution cannot be determined within the effective 6-dimensional standard model. Future reductions in the experimental uncertainty and improvements in the estimate of hadronic contributions to a_μ would allow a measurement of $(c_B + c_W)/M_s^2$ in the context of universal extra dimensions.

Although the operators (4.1) are expected to be approximately flavor diagonal, the constraints on flavor-changing neutral currents are severe enough to warrant attention. The process $\mu^- \to e^-\gamma$ is the most constraining in this context. The tree level decay width for this process is

$$\Gamma(\mu^- \to e^-\gamma) \approx \frac{\alpha m_\mu^5}{2M_s^4}\big\{\big[U^\dagger(C_B+C_W)U\big]_{12}\big\}^2. \quad (4.7)$$

The experimental limit of $\Gamma(\mu^- \to e^-\gamma) < 3.6 \times 10^{-27}$ MeV [12] imposes a bound on the off-diagonal ($i \neq i'$) elements of $C_{W,B}$,

$$\epsilon_{W,B}^{ii'} \lesssim \frac{10^{-4}}{c_{W,B}}\left(\frac{2.5\,\mathrm{TeV}}{M_s}\right)^2. \quad (4.8)$$

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[2] Consequently, the estimate (4.5) is similar to that arising from higher dimension operators associated with muon substructure in a 4-dimensional context [11].

5. Conclusions

In Ref. [4], it was pointed out that all the standard model fields could propagate in a larger number of spatial dimensions, compactified at a scale $1/R$ as small as 300 GeV. In this Letter, we addressed the implications of this idea of "universal extra dimensions" for the muon anomalous magnetic moment. For one or two extra dimensions, we computed the one-loop contribution of the KK modes of the standard model fields and found that it is too small to be detected by the muon $(g - 2)$ experiment at BNL [6]. We then analyzed higher dimension operators in the context of the 6-dimensional standard model. For the cut-off M_s in a range such that fine-tuning in the Higgs sector is eschewed (a few TeV), the contribution to the muon anomalous magnetic moment is naturally as large as the currently quoted discrepancy [6]. The sign of this contribution, however, is determined by the unknown physics above M_s.

It is interesting that the natural expectation of the contribution to $g_\mu - 2$ from physics above M_s is an order of magnitude larger than the contribution from physics below M_s, arising from the KK modes of standard model fields. This is in contrast to the case of the weak-isospin violating ρ parameter discussed in [4]. The dimension-ten weak-isospin violating operator $(c_T \hat{\lambda}_\mathcal{H}/2M_s^2)(\mathcal{H}^\dagger D_\alpha \mathcal{H})^2$ has a coefficient c_T (defined by extracting the 6-dimensional quartic Higgs coupling $\hat{\lambda}_\mathcal{H}$) of order unity if the weak-isospin is maximally violated by physics above M_s. The volume suppression resulting from integration over the two extra dimensions is absorbed in the Higgs and gauge couplings, so this operator gives $\Delta\rho \sim M_h^2/M_s^2$ (M_h is the Higgs boson mass), comparable to the one-loop KK contribution. The reason for this difference is partly that the one-loop contributions to $g_\mu - 2$ involve only lepton KK modes, while $\Delta\rho$ is enhanced by a color factor and the largeness of the top Yukawa coupling.

We emphasize that the low scale of new physics, of a few TeV, where the 6-dimensional standard model is expected to break down, is an opportunity, allowing phenomenologically interesting higher dimension operators, but also a challenge requiring further study of mechanisms that suppress dangerous operators [13].

Acknowledgements

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