# Time machines constructed from anti−de Sitter space

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In this paper time machines are constructed from anti−de Sitter space. One is constructed by identifying points related via boost transformations in the covering space of anti−de Sitter space and it is shown that this Misner-like anti−de Sitter space is just the Lorentzian section of the complex space constructed by Li, Xu, and Liu [Phys. Rev. D **48**, 4735 (1993)]. The others are constructed by gluing an anti−de Sitter space to a de Sitter space, which could describe an anti−de Sitter phase bubble living in a de Sitter phase universe. Self-consistent vacua for a massless conformally coupled scalar field are found for these time machines, whose renormalized stress-energy tensors are finite and solve the semiclassical Einstein equations. The extensions to electromagnetic fields and massless neutrinos are discussed. It is argued that, in order to make the results consistent with Euclidean quantization, a new renormalization procedure for quantum fields in Misner-type spaces (Misner space, Misner-like de Sitter space, and Misner-like anti−de Sitter space) is required. Such a ''self-consistent'' renormalization procedure is proposed. With this renormalization procedure, self-consistent vacua exist for massless conformally coupled scalar fields, electromagnetic fields, and massless neutrinos in these Misner-type spaces. [S0556-2821(99)02108-6]

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## I. INTRODUCTION

In classical general relativity there are many solutions of Einstein equations with closed timelike curves (CTC's) [1–6]. However, some early calculations of vacuum polarization in spacetimes with CTC's indicated that the renormalized stress-energy tensor diverged at the Cauchy horizon or the polarized hypersurfaces [7–10]. Hawking thus proposed the chronology protection conjecture which stated that physical laws do not allow the appearance of CTC's [11]. But many counterexamples to the chronology protection conjecture have been found [12–22]. In particular, Li and Gott [21] have found a self-consistent vacuum for a massless conformally coupled scalar field in Misner space (see, also, [20]), which gives an example of a time machine (i.e., a spacetime with CTC's) at the semiclassical level (i.e., the background spacetime is classical but the matter fields are quantized).

Of more interest, recently Gott and Li have discovered that CTC's could play an important role in the early universe: if we trace backward the history of time, we may enter an early epoch of CTC's, which means that there is no earliest event in time [22]. According to the theory of quantum foam [23], in the early universe (at the Planck epoch), quantum fluctuations of spacetime should be very important and the spacetime might have a very complicated topology. Very strong fluctuations in the metric of spacetime could cause the light cones to distribute randomly, which could give rise to a sea of CTC's in the early universe. Therefore, we might expect that at very early epochs the universe could have a tangled network of CTC's.

One model of the creation of the universe is the model of ''tunneling from nothing'' [24,25]. In this model the universe is supposed to be a Lorentzian spacetime [with signature $(-,+,+,+)$] glued to an early Euclidean space [with signature $(+,+,+,+)$]; thus the universe has a beginning in time (i.e., the beginning of the Lorentzian section). This model of ''tunneling from nothing'' has some shortcomings (see [22], and Penrose in [26]). Contrasting with the model of ''tunneling from nothing,'' in the model of Gott and Li [22], the universe does not need a signature change and has no beginning in time. The Gott-Li universe is always a Lorentzian spacetime but at a very early epoch there is a loop of time. The universe could thus be its own mother and create itself. The model of Gott and Li has some additional interesting features: the present epoch of the universe is separated from the early CTC's epoch by a past chronology horizon. The only self-consistent solution with this geometry has pure retarded potentials, creating naturally an arrow of time in our current universe, which is consistent with our experience [22]. Thus, CTC's have potentially important applications in the early universe.

Anti−de Sitter space is a spacetime which has CTC's everywhere. It is a solution of the vacuum Einstein equation with a negative cosmological constant and has maximum symmetry [27]. Anti−de Sitter space plays a very important role in theories of supergravity and superstrings [28,29]. If we ''unfold'' anti−de Sitter space and go to its covering space, the CTC's disappear. However, if we identify the events related by boost transformations in the covering space of anti−de Sitter space, we will get a spacetime with an infinite number of regions with CTC's and an infinite number of regions without CTC's, where the regions with CTC's and the regions without CTC's are separated by chronology horizons. The causal structure is similar to that of Misner space, except that Misner space has only two regions with CTC's and two regions without CTC's. For Misner space, Li and Gott have found a self-consistent vacuum for a massless conformally coupled scalar field, which is an ''adapted'' Rindler vacuum (i.e., a Rindler vacuum with multiple images) [21]. In this paper we will show that a self-consistent vacuum also exists for a massless conformally coupled scalar field in the Misner-like anti−de Sitter space constructed above, which is simply the conformal transformation of the

 

Li-Gott adapted Rindler vacuum.

In 1993, Li, Xu, and Liu constructed a time machine in a space with a complex metric [13]. In this paper we will show that the Lorentzian section of that solution is just the Misner-like anti–de Sitter space described above.

Inflation theory proposes that during an early epoch the universe was in a state with an effective positive cosmological constant at the grand unified theory (GUT) (or Planck) scale [30–32], which is well described as a de Sitter phase. By transition to a zero cosmological constant (through either quantum tunneling or classical evolution), the universe then enters a Friedmann Big Bang stage. But, if there is a transition to a negative cosmological constant (it does not seem physical theories exclude a negative cosmological constant—especially since in supergravity and superstring theories anti–de Sitter space is the only known self-consistent solution besides Minkowski space [28]), the universe could enter an anti–de Sitter phase where CTC's exist. In this paper we will consider some models describing the transition between a de Sitter space and an anti–de Sitter space, which are obtained by gluing a de Sitter space to an anti–de Sitter space along a bubble wall, and we will show that self-consistent vacua for these solutions also exist.

The generalization to the case of electromagnetic fields and massless neutrinos will also be considered. It is argued that, in order to be consistent with Euclidean quantization, a new renormalization procedure for quantum fields in Misner-type spaces is required. A ''self-consistent'' renormalization procedure is then proposed. With this renormalization procedure, self-consistent vacua exist for massless conformally coupled scalar fields, electromagnetic fields, and massless neutrinos in the Misner-type spaces.

## II. MISNER-LIKE ANTI–de SITTER SPACE

Anti–de Sitter space $AdS^4$ is a hyperbola

$$V^2+W^2-X^2-Y^2-Z^2=\alpha^2 \tag{1}$$

embedded in a five-dimensional space $R^5$ with metric

$$ds^2=-dV^2-dW^2+dX^2+dY^2+dZ^2. \tag{2}$$

Anti–de Sitter space has topology $S^1\times R^3$ and is a solution of the vacuum Einstein equations with a negative cosmological constant $\Lambda=-3/\alpha^2$, which has maximum symmetry (i.e., it has ten Killing vectors).

Several coordinate systems can be defined in anti–de Sitter space: (1) *Global static coordinates*. Define

$$\begin{aligned} V&=-\alpha\cosh\chi\cos\frac{t}{\alpha},\\ W&=\alpha\cosh\chi\sin\frac{t}{\alpha},\\ X&=\alpha\sinh\chi\cos\theta,\\ Y&=\alpha\sinh\chi\sin\theta\cos\phi,\\ Z&=\alpha\sinh\chi\sin\theta\sin\phi, \end{aligned} \tag{3}$$

where $0\leqslant t/\alpha<2\pi$, $0<\chi<\infty$, $0<\theta<\pi$, and $0\leqslant\phi<2\pi$. Then the anti–de Sitter metric can be written as

$$\begin{aligned} ds^2=&-\cosh^2\chi dt^2+\alpha^2 d\chi^2\\ &+\alpha^2\sinh^2\chi(d\theta^2+\sin^2\theta\, d\phi^2). \end{aligned} \tag{4}$$

The global static coordinates (3) cover the whole anti–de Sitter space (except the coordinate singularities at $\chi=0$ and $\theta=0,\pi$), the time coordinate $t$ has a period of $2\pi\alpha$. (2) *Local static coordinates*. Define

$$\begin{aligned} V&=(r^2-\alpha^2)^{1/2}\sinh\frac{t}{\alpha},\\ W&=r\cosh\theta,\\ X&=(r^2-\alpha^2)^{1/2}\cosh\frac{t}{\alpha},\\ Y&=r\sinh\theta\cos\phi,\\ Z&=r\sinh\theta\sin\phi, \end{aligned} \tag{5}$$

where $-\infty<t<\infty$, $r>\alpha$, $0<\theta<\infty$, and $0\leqslant\phi<2\pi$. Then the anti–de Sitter metric can be written (in a Schwarzschild-like form) as

$$\begin{aligned} ds^2=&-\left(\frac{r^2}{\alpha^2}-1\right)dt^2+\left(\frac{r^2}{\alpha^2}-1\right)^{-1}dr^2\\ &+r^2(d\theta^2+\sinh^2\theta d\phi^2). \end{aligned} \tag{6}$$

The local static coordinates (5) cover only the region with $|V|<X$ and $W>0$ in anti–de Sitter space. (3) *Nonstationary coordinates*. Define

$$\begin{aligned} V&=\alpha\cos t\cosh\chi,\\ W&=\alpha\sin t,\\ X&=\alpha\cos t\sinh\chi\cos\theta,\\ Y&=\alpha\cos t\sinh\chi\sin\theta\cos\phi,\\ Z&=\alpha\cos t\sinh\chi\sin\theta\sin\phi, \end{aligned} \tag{7}$$

where $-\pi/2<t<\pi/2$, $0<\chi<\infty$, $0<\theta<\pi$, and $0\leqslant\phi<2\pi$. Then the anti–de Sitter metric can be written (in an open cosmological form) as

$$ds^2=\alpha^2\{-dt^2+\cos^2 t[d\chi^2+\sinh^2\chi(d\theta^2+\sin^2\theta d\phi^2)]\}. \tag{8}$$

The nonstationary coordinates (7) cover the region with $V>0$ and $|W|<\alpha$, but they can by extended to the region with $V<0$ and $|W|<\alpha$ by the transformations $t\rightarrow\pi+t$, $\chi\rightarrow-\chi$, $\theta\rightarrow\pi-\theta$, and $\phi\rightarrow\pi+\phi$. (4) The nonstationary coordinates can also be extended to the regions with $|W|>\alpha$. Define

$$V=\alpha\sinh t\sinh\chi,$$

$$W=-\alpha\cosh\chi,$$

$$X=\alpha\cosh t\sinh\chi\cos\theta, \tag{9}$$

$$Y=\alpha\cosh t\sinh\chi\sin\theta\cos\phi,$$

$$Z=\alpha\cosh t\sinh\chi\sin\theta\sin\phi,$$

where $-\infty<t<\infty$, $0<\chi<\infty$, $0<\theta<\pi$, and $0\leqslant\phi<2\pi$. The coordinates $(t,\chi,\theta,\phi)$ cover the region with $W<-\alpha$, where the anti–de Sitter metric can be written as

$$ds^2=\alpha^2[-\sinh^2\chi dt^2+d\chi^2 + \cosh^2 t\sinh^2\chi(d\theta^2+\sin^2\theta d\phi^2)]. \tag{10}$$

Anti–de Sitter space is multiply connected and has CTC's everywhere. For example, in the global static coordinates (3), the world line with $\chi=$const, $\theta=$const, and $\phi=$const (which is the intersection of the hyperbola given by Eq. (1) with the surface with $X=$const, $Y=$const, and $Z=$const) is a CTC with the proper period $2\pi\alpha\cosh\chi$. If we unfold the anti–de Sitter space along the time coordinate $t$ in the global static coordinates (then $t$ goes from $-\infty$ to $\infty$), we obtain the covering space of the anti–de Sitter space, which is simply connected with the topology $R^4$ and does not contain CTC's anymore. (However, there is no Cauchy surface in this covering space [27].) The Penrose diagram of the covering space of anti–de Sitter space is shown in Fig. 1. Anti–de Sitter space has maximum symmetry, which has one time translation Killing vector, three space rotation Killing vectors, and six boost Killing vectors. In the local static coordinates in (5), $\partial/\partial t$ is a boost Killing vector. By the continuation

$$t\to l-i\frac{\pi}{2}\alpha,\quad r\to\tilde{t}, \tag{11}$$

where $-\infty<l<\infty$ and $-\alpha<\tilde{t}<\alpha$, the local static coordinates can be extended to the region with $V>|X|$ on the hyperbola defined by Eq. (1), where the boost Killing vector becomes $\partial/\partial l$ and the anti–de Sitter metric can be written as

$$ds^2=-\left(1-\frac{\tilde{t}^2}{\alpha^2}\right)^{-1}d\tilde{t}^2+\left(1-\frac{\tilde{t}^2}{\alpha^2}\right)dl^2 + \tilde{t}^2(d\theta^2+\sin^2\theta d\phi^2). \tag{12}$$

In the covering space of the anti–de Sitter space, if we identify all points related by boost transformations, then we obtain a *Misner-like anti–de Sitter space*. With this identification, there are CTC's in the region with $|V|<|X|$ but no CTC's in the region with $|V|>|X|$. On the boundary $|V|=|X|$, there are closed null curves. $|V|=|X|$ is the chronology horizon. (See Fig. 2 for the causal structure of Misner-like anti–de Sitter space.)

A coordinate system $(t,x,y,z)$ covering the region $V+X>0$ of anti–de Sitter space can also be found, which are given by

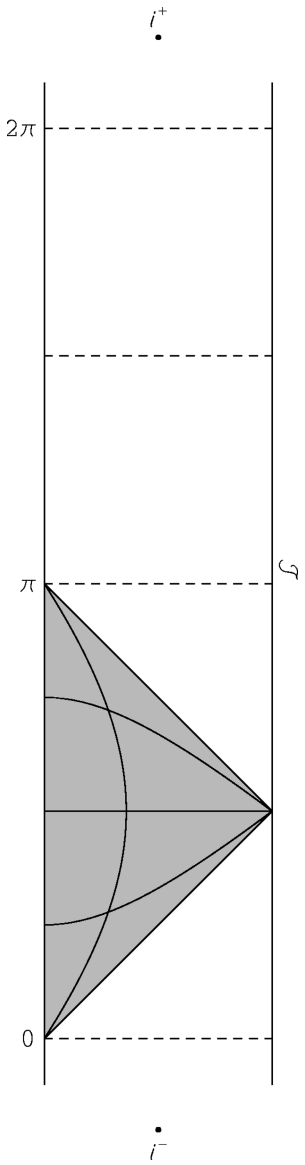


FIG. 1. The Penrose diagram of the covering space of anti–de Sitter space. The left vertical line represents the hypersurface in anti–de Sitter space with $X=0$, where the global static coordinates $(t,\chi,\theta,\phi)$ defined by Eq. (3) are singular ($\chi=0$). The right vertical line labeled with $\mathcal{J}$ represents null infinity ($\chi=\infty$). The horizontal dashed lines represent hypersurfaces with $t=$const, the labels 0, $\pi$, and $2\pi$ refer $t/\alpha=0$, $\pi$, and $2\pi$, respectively. The grey triangle represents the region covered by the nonstationary (open cosmological) coordinates $(t,\chi,\theta,\phi)$ defined by Eqs. (7) and (8). The two isolated points labeled with $i^+$ and $i^-$ represent future timelike infinity and past timelike infinity, respectively. If the hypersurfaces with the global time $t/\alpha=0$ and $t/\alpha=2\pi$ are identified, we obtain the usual anti–de Sitter space with CTC's everywhere.

$$V=\alpha\cosh\psi+\frac{1}{2\alpha}e^{-\psi}(y^2+z^2-t^2),$$

$$W=t,$$

$$X=\alpha\sinh\psi-\frac{1}{2\alpha}e^{-\psi}(y^2+z^2-t^2), \tag{13}$$

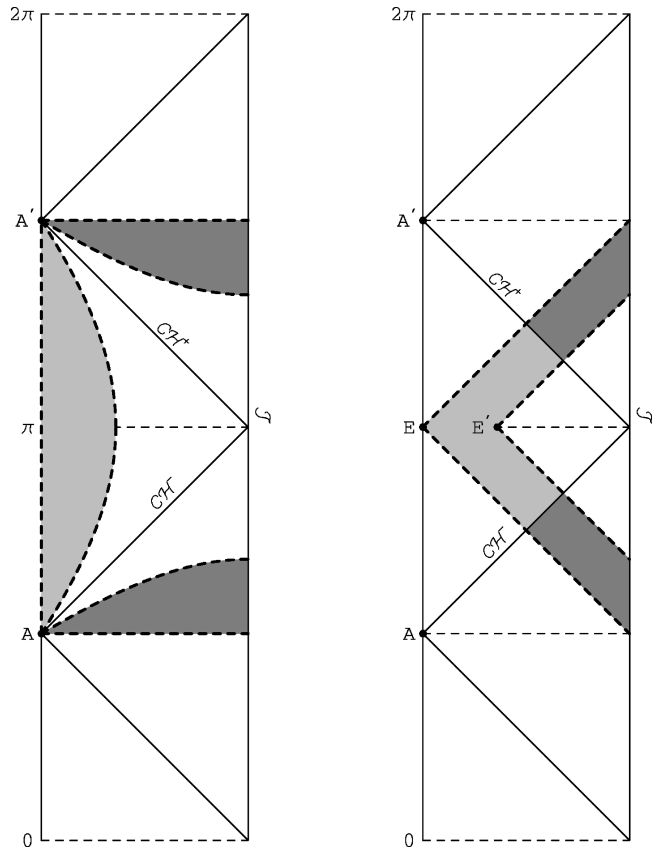


FIG. 2. The Penrose diagrams of the Misner-like anti–de Sitter space constructed by identifying points related by boost transformations in the covering space of anti–de Sitter space. With these boost transformations, $A$ and $A'$ are unchanged. (a) In the left diagram, the light and dark grey regions represent unit cells in the Misner-like anti–de Sitter space, whose opposite boundaries (heavy dashed lines) being identified. The chronology horizons $\mathcal{CH}^+$ and $\mathcal{CH}^-$ separate the regions with CTC's (the dark grey regions) from that without CTC's (the light grey region). (b) The right diagram is equivalent to the left one, except that the fundamental cell is chosen to be one bounded with null hypersurfaces. $E'$ is the image of $E$ under the boost transformation.

$$Y = y,$$

$$Z = z,$$

where $-\infty < t,\psi,y,z < \infty$. With these coordinates, the anti–de Sitter metric can be written as

$$ds^2 = -(dt - t\,d\psi)^2 + \alpha^2 d\psi^2 + (dy - y\,d\psi)^2 + (dz - z\,d\psi)^2. \tag{14}$$

$\partial/\partial\psi$ is a boost Killing vector. In the covering space of anti–de Sitter space, if the points $(t,\psi,y,z)$ are identified with $(t,\psi+2n\pi,y,z)$ $(n=\pm1,\pm2,\ldots)$, we obtain a Misner-like anti–de Sitter space. In this space there are CTC's in the regions with $t^2 > \alpha^2 + y^2 + z^2$, but no CTC's in the region with $t^2 < \alpha^2 + y^2 + z^2$.

In 1993, Li, Xu, and Liu [13] constructed a complex space $S^1 \times R^3$ with the metric

$$ds^2 = (dw - w\,d\psi)^2 + d\psi^2 + (y - y\,d\psi)^2 + (z - z\,d\psi)^2, \tag{15}$$

where $\psi$, $y$, and $z$ are real but $w$ is complex, and $\psi$ has a period $2\pi$ [i.e., $(w,\psi,y,z)$ are identified with $(w,\psi+2n\pi,y,z)$ where $n=\pm1\pm2,\ldots$]. They showed that this space is a solution of the vacuum complex Einstein equation with a negative cosmological constant $\Lambda = -3$. Here we find that, the Lorentzian section of the metric in Eq. (15) (i.e., let $w = it$) is just the anti–de Sitter metric in Eq. (14) with $\alpha = 1$ (i.e., $\Lambda = -3$). Thus the Lorentzian section of the complex space of Li, Xu, and Liu is just the Misner-like anti–de Sitter space obtained from the covering space of the anti–de Sitter space by identifying points related by boost transformations.

## III. SELF-CONSISTENT VACUUM IN MISNER-LIKE ANTI–de SITTER SPACE

Usually there is no well-defined quantum field theory in a spacetime with CTC's. However the problem can be worked using Hawking's Euclidean quantization procedure [33,34]. Alternatively, in the case where a covering space exists, we can do it in the covering space with the method of images. In fact, in most cases where the renormalized energy-momentum tensor in spacetimes with CTC's has been calculated, this method has been used (for the theoretical basis for the method of images see Ref. [8], and references cited therein). Thus we will begin by using this method to deal with quantum field theory in anti–de Sitter space and Misner-like anti–de Sitter space.

Anti–de Sitter space is conformally flat. With the transformation $\chi' = 2\arctan e^{\chi} - \frac{1}{2}\pi$, the anti–de Sitter metric in Eq. (4) can be written as

$$ds^2 = \alpha^2 \cosh^2\chi\, d\tilde{s}^2 = \alpha^2 \cosh^2\chi[-dt^2 + d\chi'^2 + \sin^2\chi'(d\theta^2 + \sin^2\theta d\phi^2)], \tag{16}$$

where $d\tilde{s}^2 = -dt^2 + d\chi'^2 + \sin^2\chi'(d\theta^2 + \sin^2\theta d\phi^2)$ is the metric of the Einstein static universe. With more transformation $r = \sin\chi'/[2(\cos t + \cos\chi')]$ and $t' = \sin t/[(\cos t + \cos\chi')]$, the metric of anti–de Sitter space can be written as

$$ds^2 = \Omega^2 d\bar{s}^2 = \Omega^2[-dt'^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \tag{17}$$

where $d\bar{s}^2 = -dt'^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$ is just the Minkowski metric, and $\Omega^2$ is given by

$$\Omega^2 = 4\alpha^2 \cosh^2\chi(\cos t + \cosh\chi)^2. \tag{18}$$

Equation (17) demonstrates that anti–de Sitter space is conformally flat.

For a massless conformally coupled scalar field in a conformally flat spacetime, there exists a conformal vacuum whose Hadamard function $G^{(1)}(X,X')$ is related to the Had-

amard function $\overline{G}^{(1)}(X,X')$ of the corresponding vacuum of the massless conformally coupled scalar field in flat Minkowski space via [35]

$$G^{(1)}(X,X')=\Omega^{-1}(X)\overline{G}^{(1)}(X,X')\Omega^{-1}(X'). \tag{19}$$

The corresponding renormalized stress-energy tensors are related via

$$\langle T_a{}^b\rangle_{\rm ren}=\Omega^{-4}\langle \overline{T}_a{}^b\rangle_{\rm ren}+\frac{1}{16\pi^2}\left[\frac{1}{9}a_1\,{}^{(1)}H_a{}^b+2a_3\,{}^{(3)}H_a{}^b\right], \tag{20}$$

where

$${}^{(1)}H_{ab}=2\nabla_a\nabla_b R-2g_{ab}\nabla^c\nabla_c R-\frac{1}{2}R^2 g_{ab}+2RR_{ab}, \tag{21}$$

$${}^{(3)}H_{ab}=R_a{}^c R_{cb}-\frac{2}{3}RR_{ab}-\frac{1}{2}R_{cd}R^{cd}g_{ab}+\frac{1}{4}R^2 g_{ab}, \tag{22}$$

and for scalar field we have $a_1=\frac{1}{120}$ and $a_3=-\frac{1}{360}$ [35]. [The sign before $1/16\pi^2$ is positive here because we are using signature $(-,+,+,+)$.] For anti–de Sitter space we have $R_{ab}=\Lambda g_{ab}$, $R=4\Lambda$, and thus ${}^{(1)}H_{ab}=0$, ${}^{(3)}H_{ab}=\frac{1}{3}\Lambda^2 g_{ab}=(3/\alpha^4)g_{ab}$. Inserting these into Eq. (20), we have

$$\langle T_a{}^b\rangle_{\rm ren}=\Omega^{-4}\langle \overline{T}_a{}^b\rangle_{\rm ren}-\frac{1}{960\pi^2\alpha^4}\,\delta_a{}^b. \tag{23}$$

In Minkowski spacetime, for the massless conformally coupled scalar field with the Minkowski vacuum, the Hadamard function is

$$\overline{G}^{(1)}_{\rm M}(X,X')=\frac{1}{2\pi^2}\,\frac{1}{-(t'-t'')^2+r^2+r'^2-2rr'\cos\Theta_2}, \tag{24}$$

where $\cos\Theta_2=\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\phi-\phi')$. The corresponding renormalized stress-energy tensor of the Minkowski vacuum is

$$\langle \overline{T}_{ab}\rangle_{\rm ren}=0. \tag{25}$$

Inserting Eq. (24) into Eq. (19), we get the Hadamard function for the massless conformally coupled scalar field in the conformal Minkowski vacuum in anti–de Sitter space

$$G^{(1)}_{\rm CM}(X,X')=\frac{1}{4\pi^2\alpha^2}\,\frac{1}{\cos[(t-t')/\alpha]\cosh\chi\cosh\chi'-1-\sinh\chi\sinh\chi'\cos\Theta_2}, \tag{26}$$

where $(t,\chi,\theta,\phi)$ are the global static coordinates of anti–de Sitter space. Clearly $G^{(1)}_{\rm CM}$ satisfies the periodic boundary condition

$$G^{(1)}_{\rm CM}(t,\chi,\theta,\phi;t',\chi',\theta',\phi')=G^{(1)}_{\rm CM}(t+2n\pi\alpha,\chi,\theta,\phi;t',\chi',\theta',\phi'), \tag{27}$$

thus it is a suitable Hadamard function in anti–de Sitter space which has CTC's everywhere. Inserting Eq. (25) into Eq. (23), we get the renormalized stress-energy tensor for the massless and conformally coupled scalar field in the conformal Minkowski vacuum in anti–de Sitter space

$$\langle T_{ab}\rangle_{\rm ren}=-\frac{1}{960\pi^2\alpha^4}g_{ab}, \tag{28}$$

which is the same as that for de Sitter space with radius $\alpha$.

If we insert the energy-momentum tensor in Eq. (28) into the semiclassical Einstein equations

$$G_{ab}+\Lambda g_{ab}=8\pi\langle T_{ab}\rangle_{\rm ren}, \tag{29}$$

and recall that for anti–de Sitter space we have $G_{ab}=R_{ab}-\frac{1}{2}Rg_{ab}=(3/\alpha^2)g_{ab}$, we find that the semiclassical Einstein equations are satisfied if and only if

$$\Lambda+\frac{3}{\alpha^2}+\frac{1}{120\pi\alpha^4}=0. \tag{30}$$

If $\Lambda=0$, the two solutions to Eq. (30) are $\alpha=\infty$, which corresponds to Minkowski space; and $\alpha=i(360\pi)^{-1/2}$, which corresponds to a de Sitter space with radius $|\alpha|=(360\pi)^{-1/2}$. Thus, if the bare cosmological constant $\Lambda=0$, there is no self-consistent anti–de Sitter space (though there are a self-consistent de Sitter space with radius $|\alpha|=(360\pi)^{-1/2}$ [36,22] and a self-consistent Minkowski space). If $\Lambda<0$, the two solutions to Eq. (30) are

$$\alpha^2=-\frac{3}{2\Lambda}\left(1+\sqrt{1-\frac{\Lambda}{270\pi}}\right)>0, \tag{31}$$

which corresponds to an anti–de Sitter space with radius $\alpha$, and

$$\alpha^2=-\frac{3}{2\Lambda}\left(1-\sqrt{1-\frac{\Lambda}{270\pi}}\right)<0, \tag{32}$$

which corresponds to a de Sitter space with radius $|\alpha|$. (If $\Lambda>0$, the two solutions both correspond to de Sitter spaces [22].) It is interesting to note that if $\Lambda<0$ there are two

self-consistent spaces, one of which is an anti–de Sitter space, and the other is a de Sitter space. Equation (30) tells us that, if $\Lambda<0$ and $\alpha^2<0$, we have $|\alpha|<(360\pi)^{-1/2}$, which implies that the self-consistent de Sitter space supported by a negative cosmological constant has a sub-Planckian radius. (See Sec. V for further discussion.)

For the Misner-like anti–de Sitter space which is obtained from the covering space of anti–de Sitter space by identifying points related by boost transformations, as in the case of Misner-like de Sitter space [22], it is easy to show that the adapted conformal Minkowski vacuum is not a self-consistent quantum state for the massless conformally coupled scalar field. The renormalized stress-energy tensor of the adapted conformal Minkowski vacuum diverges as the chronology horizon is approached. But, as in the cases of Misner space [21] and Misner-like de Sitter space [22], we can show that an adapted conformal Rindler vacuum is a self-consistent vacuum state for a massless conformally coupled scalar field in Misner-like anti–de Sitter space. To do so, it will be more convenient to write the anti–de Sitter metric in the local static coordinates [Eqs. (5) and (6)] and the Minkowski metric in Rindler coordinates

$$d\bar{s}^2=-\xi^2 d\eta^2+d\xi^2+dy^2+dz^2, \tag{33}$$

where the Rindler coordinates $(\eta,\xi,y,z)$ are defined by

$$\begin{aligned} t&=\xi\sinh\eta,\\ x&=\xi\cosh\eta,\\ y&=y,\\ z&=z, \end{aligned} \tag{34}$$

where $(t,x,y,z)$ are the Cartesian coordinates in Minkowski spacetime. With the transformation

$$\begin{aligned} \eta&=\frac{t}{\alpha},\\ \xi&=\frac{\sqrt{r^2/\alpha^2-1}}{(r/\alpha)\cosh\theta-1},\\ y&=\frac{(r/\alpha)\sinh\theta\cos\phi}{(r/\alpha)\cosh\theta-1},\\ z&=\frac{(r/\alpha)\sinh\theta\sin\phi}{(r/\alpha)\cosh\theta-1}, \end{aligned} \tag{35}$$

where $(t,r,\theta,\phi)$ are the local static coordinates in Eq. (5), the anti–de Sitter metric (6) can be written as

$$ds^2=\Omega^2 d\bar{s}^2=\Omega^2(-\xi^2 d\eta^2+d\xi^2+dy^2+dz^2), \tag{36}$$

here $\Omega^2$ is

$$\Omega^2=\alpha^2[(r/\alpha)\cosh\theta-1]^2. \tag{37}$$

With the conformal relation between anti–de Sitter space and Rindler space given by Eqs. (35) and (36), the time coordinate $t$ of the anti–de Sitter space in local static coordinates is mapped to the Rindler time coordinate $\eta$. Construct a Misner space with all $(\eta+n\eta_0,\xi,y,z)$ $(n=0,\pm1,\dots)$ identified, then the map given by Eq. (35) gives rise to a Misner-like anti–de Sitter space with all $(t+nt_0,r,\theta,\phi)$ $(n=0,\pm1,\dots)$ identified, where

$$t_0=\alpha\eta_0. \tag{38}$$

Equations (35)–(38) give a natural conformal map between Misner space and Misner-like anti–de Sitter space.

For a massless conformally coupled scalar field in Misner space, the Hadamard function for the adapted Rindler vacuum is [21]

$$G^{(1)}(X,X')=\frac{1}{2\pi^2}\sum_{n=-\infty}^{\infty}\times\frac{\gamma}{\xi\xi'\sinh\gamma[-(\eta-\eta'+n\eta_0)^2+\gamma^2]}, \tag{39}$$

where $\gamma$ is defined by

$$\cosh\gamma=\frac{\xi^2+\xi'^2+(y-y')^2+(z-z')^2}{2\xi\xi'}. \tag{40}$$

The corresponding renormalized stress-energy tensor of the adapted Rindler vacuum is [21]

$$\langle T_\mu{}^\nu\rangle_{\rm R,ren}=\frac{1}{1440\pi^2\xi^4}\left[\left(\frac{2\pi}{\eta_0}\right)^4-1\right]\begin{pmatrix}-3&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}, \tag{41}$$

which is expressed in the Rindler coordinates. We see that $\langle T_\mu{}^\nu\rangle_{\rm R,ren}$ (also $\langle T_{\mu\nu}\rangle_{\rm R,ren}\langle T^{\mu\nu}\rangle_{\rm R,ren}$) diverges at the chronology horizon where $\xi=0$ unless $\eta_0=2\pi$. If $\eta_0=2\pi$, however, we have $\langle T_\mu{}^\nu\rangle_{\rm R,ren}=0$ throughout the Misner space (though Rindler coordinates cover only a quadrant in Misner space, the results can be analytically extended to the whole Misner space [21,22] where $\langle T_\mu{}^\nu\rangle_{\rm R,ren}$ is also zero, see [22] for further discussion).

Inserting Eq. (39) into Eq. (19), we obtain the Hadamard function for the adapted conformal Rindler vacuum of the massless and conformally coupled scalar field in Misner-like anti–de Sitter space

$$G^{(1)}_{\rm CR}(X,X')=\frac{1}{2\pi^2}\sum_{n=-\infty}^{\infty}\frac{\gamma}{\sinh\gamma\sqrt{(r^2/\alpha^2-1)(r'^2/\alpha^2-1)}[-(t-t'+nt_0)^2+\alpha^2\gamma^2]}, \tag{42}$$

where $\gamma$ is written in $(t,r,\theta,\phi)$ as

$$\cosh\gamma=\frac{1}{\sqrt{(r^2/\alpha^2-1)(r'^2/\alpha^2-1)}}\left\{1-\frac{rr'}{\alpha^2}[\cosh\theta\cosh\theta'-\sinh\theta\sinh\theta'\cos(\phi-\phi')]\right\}. \tag{43}$$

Clearly the Hadamard function in Eq. (42) satisfies the periodic boundary condition

$$G^{(1)}(t,r,\theta,\phi)=G^{(1)}(t+nt_0,r,\theta,\phi), \tag{44}$$

where $n=0,\pm1,\ldots,$ thus it defines a reasonable quantum state in the Misner-like anti–de Sitter space. Inserting Eq. (41) into Eq. (23), we obtain the renormalized stress-energy tensor of the adapted conformal Rindler vacuum of the conformally coupled scalar field in Misner-like anti–de Sitter space

$$\langle T_\mu{}^\nu\rangle_{\rm CR,ren}=\frac{1}{1440\pi^2\alpha^4(r^2/\alpha^2-1)^2}\left[\left(\frac{2\pi\alpha}{t_0}\right)^4-1\right]\times\begin{pmatrix}-3&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}-\frac{1}{960\pi^2\alpha^4}\delta_\mu{}^\nu, \tag{45}$$

where the coordinate system is the local static coordinate system $(t,r,\theta,\phi)$. Again, $\langle T_\mu{}^\nu\rangle_{\rm CR,ren}$ (also $\langle T_{\mu\nu}\rangle_{\rm CR,ren}\langle T^{\mu\nu}\rangle_{\rm CR,ren}$) diverges at the chronology horizon $r=\alpha$, unless $t_0=2\pi\alpha$. But, if

$$t_0=2\pi\alpha, \tag{46}$$

we get a renormalized stress-energy tensor which is regular throughout the Misner-like anti–de Sitter space

$$\langle T_{ab}\rangle_{\rm CR,ren}=-\frac{1}{960\pi^2\alpha^4}g_{ab}, \tag{47}$$

which is exactly the same as that of de Sitter space and anti–de Sitter space. [Though the local coordinates $(t,r,\theta,\phi)$ cover only the region with $|V|<X$ in anti–de Sitter space, the results can be easily extended to the whole Misner-like anti–de Sitter space (where $\langle T_{ab}\rangle_{\rm CR,ren}$ is finite and given by Eq. (47)), as in the cases of Misner space and Misner-like de Sitter space [21,22].]

Similarly, the Misner-like anti–de Sitter space solves the semiclassical Einstein equation with a negative cosmological constant $\Lambda$ and the energy-momentum tensor in Eq. (47) (and thus is self-consistent) if $\alpha^2=-(3/2\Lambda)(1+\sqrt{1-\Lambda/270\pi})$.

## IV. TRANSITION BETWEEN de SITTER AND ANTI–de SITTER SPACES

Phase transitions play important roles in the evolution of the early universe. With phase transitions various bubbles could form; inside and outside the bubbles the spacetimes have different stress-energy tensors and thus are described by different spacetime metrics [38,39]. The inside and outside of a bubble are separated by a wall—a spacetime structure which can be approximately treated as a three-dimensional hypersurface. Usually it is assumed that the outside of the bubble is in a state dominated by a positive cosmological constant at GUT (or Planckian) scale. Thus the spacetime metric outside the bubble is well approximated with a de Sitter metric. Inside the bubble, the cosmological constant could be zero and the stress-energy tensor could be zero thus the spacetime inside would be Minkowskian—which is a model of inflation decaying through a first-order phase transition in the old inflation theory [30]; or, inside the bubble the cosmological constant could also be positive and at GUT (or Planckian) scale so the spacetime inside the bubble is still inflating, but after a while the cosmological constant falls off a plateau and evolves classically to zero and the universe inside the bubble enters a hot Big Bang phase—which is a model of transition from inflation to an open Big Bang cosmology through a second-order phase transition [36,37,31,32]. But, either via the first-order phase transition or the second-order phase transition, as another alternative, the inside of the bubble could become dominated by a negative cosmological constant instead and thus the spacetime inside the bubble would be described with an anti–de Sitter metric. In this paper we are interested in this case since anti–de Sitter space has CTC's.

In this paper we will discuss bubbles that are pre-existing rather than ones that form by quantum tunneling. We thus only consider how to glue a de Sitter space and an anti–de Sitter space together at a boundary (i.e., at a wall), and we investigate the causal structure of the spacetime so obtained.

The conditions for two spacetimes to be glued together along a wall (so that the Einstein equations are satisfied at the wall) are [40]: (1) the metrics on the wall induced from the spacetimes at the two sides agree; (2) the surface stress-energy tensor of the wall defined by $S_\alpha{}^\beta=\lim_{\epsilon\to0}\int_{-\epsilon}^{\epsilon}T_\alpha{}^\beta dn$ should satisfy (in the Gaussian normal coordinates near the wall)

$$S_n{}^n = S_n{}^i = 0, \quad S_i{}^j = -\frac{n^a n_a}{8\pi}(\gamma_i{}^j - \gamma\delta_i{}^j), \tag{48}$$

where $n^a$ is the normal vector of the hypersurface $\Sigma$ of the wall ($n^a n_a = 1$ if the $\Sigma$ is timelike; $n^a n_a = -1$ if the wall is spacelike. Regarding $\Sigma$ as a hypersurface embedded in either the spacetime inside it, or the spacetime outside it, by the first condition, $\Sigma$ should be either timelike or spacelike in both. Here we do not consider the case of a null wall), and

$$\gamma_{ab} \equiv [K_{ab}] \equiv K^+_{ab} - K^-_{ab} \tag{49}$$

is the difference of the extrinsic curvature of $\Sigma$ embedded in the spacetimes at the two sides of the wall [40], and $\gamma \equiv \gamma_a{}^a$. The definition for the extrinsic curvature $K_{ab}$ is $K_{ab} = \nabla_a n_b$ where we have treated $n^a$ as an vector field in the neighborhood of $\Sigma$ extended from the normal vector defined only on $\Sigma$. In the Gaussian normal coordinates of $\Sigma$, the components of $K_{ab}$ can be written as

$$K_{\mu\nu} = \frac{1}{2}\frac{\partial h_{\mu\nu}}{\partial n}. \tag{50}$$

(Here we use the definition of the extrinsic curvature $K_{ab}$ with an opposite sign as that used in [40].) The evolution of the wall is governed by

$$S^{ij}{}_{|j} + [T^{in}] = 0, \tag{51}$$

where ''$|j$'' denotes the covariant derivative associated with the three-metric $h_{ij}$ on $\Sigma$. (Here the Greek letters $\mu, \nu, \ldots$ label vectors and tensors in the four-dimensional spacetime, the Latin letters $i, j, \ldots$ label vectors and tensors in the three-dimensional space $\Sigma$.)

de Sitter space is a hyperbola

$$-V^2 + W^2 + X^2 + Y^2 + Z^2 = \beta^2, \tag{52}$$

embedded in a five-dimensional space $R^5$ with the metric

$$ds^2 = -dV^2 + dW^2 + dX^2 + dY^2 + dZ^2. \tag{53}$$

de Sitter space is a solution of the vacuum Einstein equations with a positive cosmological constant $\Lambda = 3/\beta^2$, which has maximal symmetry (it has six space rotation Killing vectors and four boost Killing vectors). There are various coordinate systems for de Sitter space [41,27,35]. For our purpose here, the following two coordinate systems are convenient: (1) Define

$$\begin{aligned} V &= \beta \sinh t \cosh\chi, \\ W &= \beta \cosh t, \\ X &= \beta \sinh t \sinh\chi \cos\theta, \\ Y &= \beta \sinh t \sinh\chi \sin\theta \cos\phi, \\ Z &= \beta \sinh t \sinh\chi \sin\theta \sin\phi, \end{aligned} \tag{54}$$

where $0 < t < \infty$, $0 < \chi < \infty$, $0 < \theta < \pi$, and $0 \leqslant \phi < 2\pi$. The coordinates $(t, \chi, \theta, \phi)$ in Eq. (54) cover the region on the hyperbola (52) with $V > 0$ and $W > \beta$, where the de Sitter metric can be written as

$$ds^2 = \beta^2\{-dt^2 + \sinh^2 t[d\chi^2 + \sinh^2\chi(d\theta^2 + \sin^2\theta d\phi^2)]\}. \tag{55}$$

The section $t = \text{const}$ in de Sitter space is an open (i.e., negatively curved), homogeneous, and isotropic space. The metric in Eq. (55) could describe an open inflation [36,37]. The coordinates $(t, \chi, \theta, \phi)$ in Eq. (54) can be extended to the region with $V < 0$ and $W > \beta$ by the reflection $t \to -t$ and $\chi \to -\chi$, $\theta \to \pi - \theta$, and $\phi \to \pi + \phi$. (2) The coordinates in Eq. (54) could be extended to the region with $|W| < \beta$. Define

$$\begin{aligned} V &= \beta \sinh t \cos\chi, \\ W &= \beta \sin\chi, \\ X &= \beta \cosh t \cos\chi \cos\theta, \\ Y &= \beta \cosh t \cos\chi \sin\theta \cos\phi, \\ Z &= \beta \cosh t \cos\chi \sin\theta \sin\phi, \end{aligned} \tag{56}$$

where $-\infty < t < \infty$, $-\pi/2 < \chi < \pi/2$, $0 < \theta < \pi$, and $0 \leqslant \phi < 2\pi$, then the de Sitter metric can be written as

$$ds^2 = \beta^2[-\cos^2\chi dt^2 + d\chi^2 + \cosh^2 t \cos^2\chi(d\theta^2 + \sin^2\theta d\phi^2)]. \tag{57}$$

The coordinates $(t, \chi, \theta, \phi)$ cover the region with $|W| < \beta$ in the de Sitter space as a hyperbola given by Eq. (52). The Penrose diagram of de Sitter space is shown in Fig. 3.

By gluing anti–de Sitter space onto de Sitter space we can obtain various spacetimes with CTC's. These spacetimes with CTC's differ from the anti–de Sitter space with CTC's by the fact that in these glued spacetimes there are regions without CTC's which are separated from the regions with CTC's by chronology horizons, while in the usual anti–de Sitter space CTC's exist everywhere. Here we only show one typical example, which is obtained by gluing an anti–de Sitter space onto a de Sitter along a timelike hypersurface ($n^a n_a = 1$). This spacetime could describe a bubble of anti–de Sitter space existing for all time in an eternal de Sitter space.

Consider a de Sitter space as a hyperbola described by Eq. (52) in the embedding space with the metric in Eq. (53). Cut this de Sitter space along the hypersurface $\Sigma_1$ with $W = w_1 > 0$ and throw away the part with $W > w_1$. Denote the part of the de Sitter space with $W < w_1$ as $dS^-$. Then we have a de Sitter space with a boundary $\Sigma_1$ at $W = w_1$. Suppose $w_1 < \beta$, then $\Sigma_1$ is timelike. In such a case, the hypersurface $\Sigma_1$ is a three-dimensional timelike hyperbola with $-V^2 + X^2 + Y^2 + Z^2 = \beta^2 - w_1^2 > 0$. With the coordinates in Eq. (56), $\Sigma_1$ is at $\chi = \arcsin(w_1/\beta)$. The metric $h_{ab}$ on $\Sigma_1$ induced from the de Sitter metric is

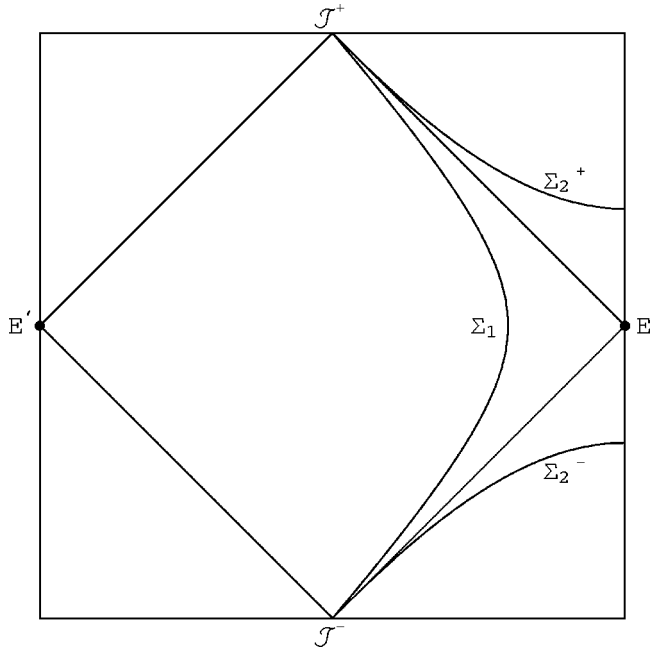


FIG. 3. The Penrose diagram of de Sitter space. The event $E'$ is the antipodal point of the event $E$. The upper horizontal line labeled with $\mathcal{J}^+$ represents future null infinity, the lower horizontal line labeled with $\mathcal{J}^-$ represents past null infinity. Two (future and past) light cones from $E$ and $E'$ are shown. The curve labeled with $\Sigma_1$ represents a timelike hypersurface with $W=\text{const}<\beta$. The curves labeled with $\Sigma_2{}^{\pm}$ represent the spacelike hypersurfaces with $W=\text{const}>\beta$ (two leaves).

$$ds_1^2=(\beta^2-w_1^2)[-dt^2+\cosh^2 t(d\theta^2+\sin^2\theta d\phi^2)]. \tag{58}$$

The normal vector of $\Sigma$ is $n^a=\beta^{-1}(\partial/\partial\chi)^a(n^a n_a=1)$. The extrinsic curvature $K^-_{ab}$ of $\Sigma_1$ is

$$K^-_{ab}=-\frac{w_1}{\beta\sqrt{\beta^2-w_1^2}}h_{ab}. \tag{59}$$

Consider an anti–de Sitter space as the hyperbola given by Eq. (1) in the embedding space with the metric in Eq. (2). Cut the anti–de Sitter space along the hypersurface $\Sigma_2$ with $W=-w_2<0$, throw away the part with $W<-w_2$. We denote the anti–de Sitter space with $W>-w_2$ as $\text{AdS}^+$. Then we have an anti–de Sitter space with $W>-w_2$ and a boundary $\Sigma_2$ at $W=-w_2$. Suppose $w_2>\alpha$, then $\Sigma_2$ is timelike. In such a case, $\Sigma_2$ is a three-dimensional timelike hyperbola with $-V^2+X^2+Y^2+Z^2=w_2^2-\alpha^2>0$. With the coordinates $(t,\chi,\theta,\phi)$ in Eq. (9), the hypersurface $\Sigma_2$ is at $\chi=\text{arccosh}(w_2/\alpha)$. The metric $h_{ab}$ on $\Sigma_2$ induced from the anti–de Sitter metric is

$$ds_3^2=(w_2^2-\alpha^2)[-dt^2+\cosh^2 t(d\theta^2+\sin^2\theta d\phi^2)]. \tag{60}$$

The normal vector of $\Sigma_2$ is $n^a=-\alpha^{-1}(\partial/\partial\chi)^a(n^a n_a=1)$. The extrinsic curvature $K^+_{ab}$ of $\Sigma_2$ is

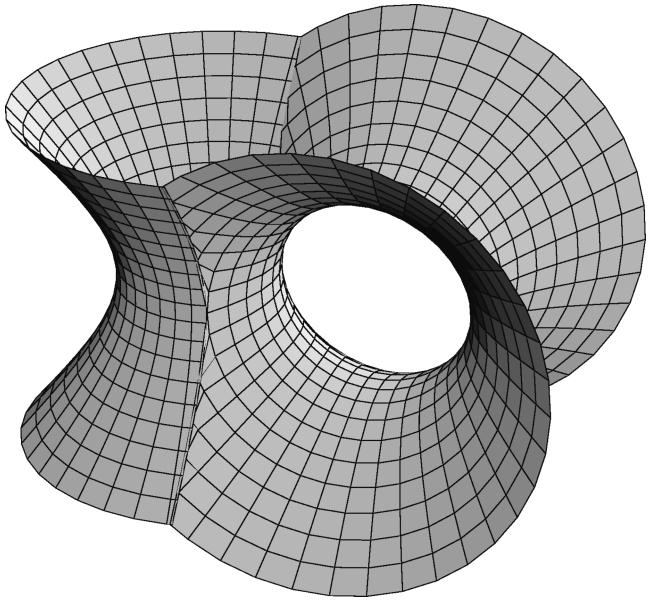

FIG. 4. A schematic diagram for the spacetime obtained by gluing a de Sitter space to an anti–de Sitter space along a timelike wall. The vertical hyperbola of one sheet (to the left) represents a de Sitter space in the embedding space of Eq. (53), the horizontal hyperbola of one sheet (to the right) represents an anti–de Sitter space in the embedding space of Eq. (2). They are glued along a timelike hypersurface (a bubble wall) on which a surface stress-energy tensor is induced so that Einstein equations are satisfied there. The two embedding spaces match at the bubble wall ($W=\text{const}$). The Penrose diagram of this spacetime is shown in Fig. 5.

$$K^+_{ab}=-\frac{w_2}{\alpha\sqrt{w_2^2-\alpha^2}}h_{ab}. \tag{61}$$

To glue the anti–de Sitter space and the de Sitter space together, let us identify $\Sigma_1$ with $\Sigma_2$ by identifying their coordinates $(t,\theta,\phi)$. The spacetime so obtained is schematically shown in Fig. 4. In the sections with $V=\text{const}$ in de Sitter space and anti–de Sitter space, an $S^3$ is glued together with an $H^3$ at a cross section $S^2$. From Eqs. (58) and (60), the metric $h_{ab}$ on $\Sigma_1=\Sigma_2\equiv\Sigma$ induced from de Sitter space and anti–de Sitter space agree if and only if

$$w_1^2+w_2^2=\alpha^2+\beta^2. \tag{62}$$

By gluing two spacetimes along a hypersurface, usually a surface stress-energy tensor is induced. The surface stress-energy tensor induced on the hypersurface $\Sigma$ is determined by the difference in the extrinsic curvature of $\Sigma_1$ which is embedded in the de Sitter space and the extrinsic curvature of $\Sigma_2$ which is embedded in the anti–de Sitter space through Eq. (48) with $n^a n_a=1$. By inserting the extrinsic curvatures derived above into Eqs. (49) and (48), we obtain the surface stress-energy tensor for $\Sigma$:

$$S_{ab}=-\frac{1}{4\pi\mu}\left(\sqrt{1+\frac{\mu^2}{\alpha^2}}-\sqrt{1-\frac{\mu^2}{\beta^2}}\right)h_{ab}, \tag{63}$$

where $\mu=\sqrt{w_2^2-\alpha^2}=\sqrt{\beta^2-w_1^2}$, the metric $h_{ab}$ on the timelike $\Sigma$ is given by

$$ds^2=\mu^2[-dt^2+\cosh^2 t(d\theta^2+\sin^2\theta d\phi^2)]. \quad (64)$$

The surface stress-energy tensor given by Eq. (63) is like a positive three-dimensional cosmological constant. The timelike hypersurface $\Sigma$ with the metric (64) is a three-dimensional de Sitter space. The Penrose diagram of the spacetime obtained by gluing $dS^-$ with $AdS^+$ along the timelike $\Sigma$ is shown in Fig. 5. There are CTC's in the region of $dS^-$ with $W>-\beta$ and the whole $AdS^+$. But there are no CTC's in the region of $dS^-$ with $W<-\beta$. The null hypersurface $W=-\beta$ is the chronology horizon which separates the region with CTC's from that without CTC's. (See Fig. 5.)

Mathematically, a de Sitter space can also be glued to an anti–de Sitter space along a spacelike hypersurface, the resultant spacetime has similar properties as the example described above. The surface stress-energy tensor induced on the spacelike hypersurface is like a negative cosmological constant in a three-dimensional Euclidean space. The spacelike hypersurface is a three-dimensional hyperbola $H^3$. The causal structure of the spacetime obtained by gluing a de Sitter space to an anti–de Sitter space along a spacelike hypersurface is the same as that of the spacetime obtained by gluing a de Sitter space to an anti–de Sitter space along a timelike hypersurface. There are CTC's in the region of $dS^-$ with $W>-\beta$ and the whole $AdS^+$. But there are no CTC's in the region of $dS^-$ with $W<-\beta$. The null hypersurface $W=-\beta$ is the chronology horizon which separates the region with CTC's from that without CTC's. For both cases, we have $T^{in}=0$ and $h^{ij}{}_{|j}=0$, thus the evolution equation (51) is satisfied automatically.

From Eqs. (63) we see that if $\mu\to 0$, i.e., if $\Sigma$ becomes null, we have $S_{ab}\to 0$. Though the coordinates $(t,\chi,\theta,\phi)$ are singular at the null $\Sigma$ with $w_1=\beta$ and $w_2=\alpha$, we can show that as $\mu\to 0$, the scalars $S\equiv h^{ab}S_{ab}$ and $S^{ab}S_{ab}$ also vanish as $\mu\to 0$. (These conclusions also hold if $\Sigma$ is spacelike.) However, since the metric $h_{ab}$ on a null hypersurface is degenerate, the formalism discussed above cannot be used to the junction at null hypersurfaces. Thus the case of null $\Sigma$ requires more discussion. Here we do not discuss this complicated but interesting issue. (For detailed discussions on the junction conditions at null hypersurfaces, see Refs. [42,43].)

## V. SELF-CONSISTENT VACUA FOR SPACETIMES WITH CTC'S

As discussed in Sec. III, an anti–de Sitter space with radius $\alpha$ is self-consistent (i.e., the semiclassical Einstein equations (29) are satisfied with $\langle T_{ab}\rangle_{\rm ren}$ being the renormalized stress-energy tensor of vacuum polarization) if the negative cosmological constant is

$$\Lambda_1=-\frac{3}{\alpha^2}\left(1+\frac{1}{360\pi\alpha^2}\right), \quad (65)$$

while a de Sitter space with radius $\beta$ is self-consistent if the cosmological constant is [22]

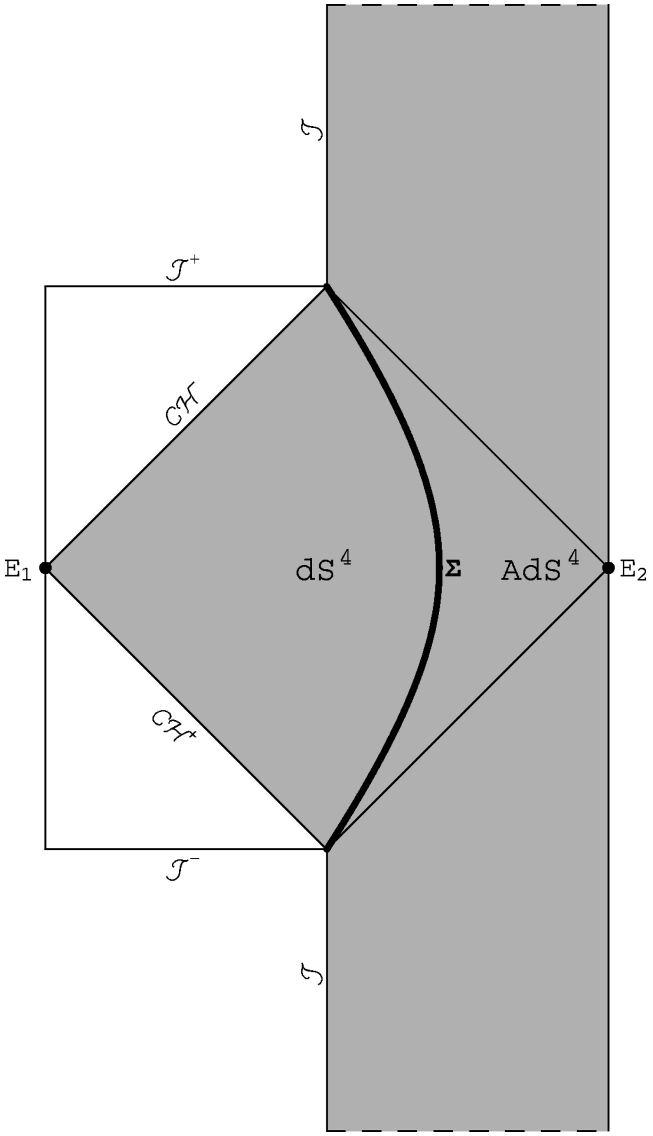


FIG. 5. The Penrose diagram of the spacetime obtained by gluing anti–de Sitter space to de Sitter space along a timelike hypersurface. The heavy curve labeled with $\Sigma$ represents a timelike hypersurface (the bubble wall) on the left side of which there is a de Sitter space ($dS^4$), and on the right side of which there is an anti–de Sitter space ($AdS^4$). On the anti–de Sitter side, the spacelike hypersurfaces denoted with two dashed lines are identified. This spacetime contains CTC's in the grey region, but no CTC's in the blank region. The region with CTC's is separated from that without CTC's by the chronology horizons $\mathcal{CH}^{\pm}$. $\mathcal{J}^+$, $\mathcal{J}^-$, and $\mathcal{J}$ represent future null infinity, past null infinity, and null infinity, respectively. (Compare with Fig. 4.)

$$\Lambda_2=\frac{3}{\beta^2}\left(1-\frac{1}{360\pi\beta^2}\right). \quad (66)$$

$\Lambda_1$ is always negative. $\Lambda_2$ could be either positive or negative, depending on the value of $\beta$. From Eq. (66), if $\beta>(360\pi)^{-1/2}$, $\Lambda_2$ is positive; if $\beta<(360\pi)^{-1/2}$, $\Lambda_2$ is negative. Thus, interestingly, *a de Sitter space with sub-Planckian radius could be self-consistent only if the bare cosmological constant is negative.*

From Eqs. (65) and (66), we see that if $\beta < (360\pi)^{-1/2}$, $\Lambda_1 = \Lambda_2$ if and only if

$$\frac{1}{\beta^2} - \frac{1}{\alpha^2} = 360\pi. \tag{67}$$

This together with Eq. (62) gives a self-consistent spacetime which is obtained by gluing a de Sitter space to an anti–de Sitter space with *a unique negative bare cosmological constant throughout*. This could be realized since for a negative cosmological constant there are two self-consistent solutions of the semiclassical Einstein equations, one is anti–de Sitter space, the other is de Sitter space, and these two could be glued together—as discussed in Sec. III.

The spacetime obtained by gluing a de Sitter space with an anti–de Sitter space as discussed in the last section, is a self-consistent solution of the semiclassical Einstein equations if Eqs. (65) and (66) are satisfied and on the wall separating the de Sitter region and the anti–de Sitter region there is a surface stress-energy tensor given by Eqs. (63). Since the Hadamard function is continuous across the wall (the Hadamard function does not contain any derivatives of the metric), the wall does not introduce any additional vacuum polarization effects.

The above discussions of self-consistent solutions are for a massless conformally coupled scalar field in de Sitter/anti–de Sitter spaces. The results can be easily extended to other matter fields. If their vacua are invariant under de Sitter/anti–de Sitter transformations, it could be expected that their renormalized stress-energy tensor should have the form of a constant$\times g_{ab}$. If there are many matter fields with their vacua being invariant under de Sitter/anti–de Sitter transformations, the renormalized stress-energy tensor could be written as

$$\langle T_{\rm ab} \rangle_{\rm ren} = -\frac{g_*}{960\pi^2 r_0^4} g_{ab}, \tag{68}$$

where $g_*$ is a dimensionless number determined by the number and spins of matter fields existing in the de Sitter/anti–de Sitter space with radius $r_0$ ($r_0 = \alpha$ for anti–de Sitter space, $r_0 = \beta$ for de Sitter space, in practice $g_* \sim 100$). Correspondingly, with the appearance of many matter fields with de-Sitter/anti–de Sitter invariant vacua, Eqs. (65) and (66) should be replaced by

$$\Lambda_1 = -\frac{3}{\alpha^2}\left(1 + \frac{g_*}{360\pi\alpha^2}\right), \tag{69}$$

and

$$\Lambda_2 = \frac{3}{\beta^2}\left(1 - \frac{g_*}{360\pi\beta^2}\right). \tag{70}$$

$\Lambda_1$ is always negative. $\Lambda_2$ is negative if $\beta < (g_*/360\pi)^{1/2}$, positive if $\beta > (g_*/360\pi)^{1/2}$. And, $\Lambda_1 = \Lambda_2$ if

$$\frac{1}{\beta^2} - \frac{1}{\alpha^2} = 360\pi g_*^{-1}. \tag{71}$$

If Eqs. (62) and (71) are satisfied simultaneously, the spacetime obtained by gluing the de Sitter space to the anti–de Sitter space is self-consistent and has a unique negative bare cosmological constant through both de Sitter and anti–de Sitter regions. Such a solution is very interesting. The semiclassical Einstein equations with a negative bare cosmological constant can have two self-consistent solutions, one being an anti–de Sitter space, the other being a de Sitter space. These two spacetimes could transit from one to the other across a bubble wall (through whatever quantum processes) without changing the cosmological constant.

But, for Misner-type spaces (Misner space, Misner-like de Sitter space, or Misner-like anti–de Sitter space), the situation for matter fields other than the massless and conformally coupled scalar field is more complicated since the self-consistent vacua are not Lorentzian, de Sitter, or anti–de Sitter invariant. To see this, let us consider the electromagnetic field in Misner space. As an alternative to the method of images, Euclidean quantization is another more powerful tool for dealing with quantum field theory in an acausal space [33,34]. The Euclidean method provides a convenient bridge between the conical space around a cosmic string [44,45] and Misner space, which could conveniently translate the results of quantum field theory in a conical space to that in Misner space (since a conical space and Misner space have the same Euclidean section—the Euclidean conical space). In [20], from the well-known renormalized stress-energy tensor of the conformally coupled scalar field in the conical space around a cosmic string, using the Euclidean method (first translate the results in the conical space around the string to that in the Euclidean conical space, then translate these results to that in Misner space), Cassidy has successfully predicted that there should be a quantum state with vanishing renormalized stress-energy tensor in Misner space when the boost parameter $\eta_0$ is $2\pi$, which corresponds to no cosmic string. With Li and Gott's independent discovery of the self-consistent vacuum state (an adapted Rindler vacuum) for a conformally coupled scalar field in Misner space [21], Cassidy's prediction has been confirmed. And, the Euclidean quantization procedure gives a beautiful geometrical explanation for the self-consistent vacua in Misner-type spaces namely that when $\eta_0 = 2\pi$ the corresponding Euclidean section is flat with no conical singularity and thus has $\langle T_\mu{}^\nu \rangle_{\rm ren} = 0$ throughout [21,22]. Recently, with the method of Euclidean quantization, Li and Gott have found a self-consistent vacuum for a model of inflation in the Kaluza-Klein theory, and found a relation between the fine-structure constant and the inflationary energy scale which is consistent with the energy scale usually talked about in inflation and GUT theory [46]. Thus, we adopt the method of Euclidean quantization.

The Euclidean section of Misner space is a Euclidean conical space with metric $ds^2 = d\xi^2 + \xi^2 d\phi^2 + dy^2 + dz^2$ where $(\xi, \phi, y, z)$ is identified with $(\xi, \phi + n\phi_0, y, z)$ ($n = \pm 1, \pm 2, \ldots$). If we make the continuation $\phi \to i\eta$ and $\phi_0 \to i\eta_0$, we obtain Misner space [21]. (On the other hand, if we make the continuation $y \to it$, we obtain the spacetime of a cosmic string.) The quantum field effects in the conical space have been investigated by many people (see [47,48]

and references therein). Thus, for the electromagnetic field in the Euclidean conical space, the renormalized stress-energy tensor of vacuum polarization is

$$\langle T_\mu{}^\nu\rangle_{\rm ren}=\frac{1}{720\pi^2\xi^4}\left[\left(\frac{2\pi}{\phi_0}\right)^2-1\right]\left[\left(\frac{2\pi}{\phi_0}\right)^2+11\right]\times\begin{pmatrix}1&0&0&0\\0&-3&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix},\tag{72}$$

where the cylindrical coordinates $(\xi,\phi,y,z)$ are used. Clearly, if $\phi_0=2\pi$, the renormalized stress-energy tensor of the electromagnetic field is zero and thus the semiclassical vacuum Einstein equations (i.e., the Einstein equations with the renormalized stress-energy tensor being that of vacuum polarization) are satisfied. This is what we expect since for $\phi_0=2\pi$ the Euclidean conical space becomes the regular flat Euclidean space $R^4$. But, when the Euclidean conical space is continued to Misner space by $\phi\to i\eta$ and $\phi_0\to i\eta_0$ in Eq. (72), the renormalized stress-energy tensor becomes

$$\langle T_\mu{}^\nu\rangle_{\rm ren}=\frac{1}{720\pi^2\xi^4}\left[-\left(\frac{2\pi}{\eta_0}\right)^2-1\right]\left[-\left(\frac{2\pi}{\eta_0}\right)^2+11\right]\times\begin{pmatrix}-3&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix},\tag{73}$$

where the Rindler coordinates $(\eta,\xi,y,z)$ are used. We see that, unlike the case for a massless conformally coupled scalar field, even if $\eta_0=2\pi$, the renormalized stress-energy tensor given by Eq. (73) is nonzero. In fact, $\langle T_\mu{}^\nu\rangle_{\rm ren}$ in Eq. (73) diverges at $\xi=0$ unless $\eta_0=2\pi/\sqrt{11}$. If $\eta_0=2\pi/\sqrt{11}$, the $\langle T_\mu{}^\nu\rangle_{\rm ren}$ given by Eq. (73) is zero and thus it would solve the semiclassical Einstein equations for this locally flat space. But, if this were a self-consistent solution, it would be surprising that the self-consistent vacua of different matter fields have different values of $\eta_0$. Recall that, for the massless conformally coupled scalar field the self-consistent vacuum has $\eta_0=2\pi$ ([21,22], and Sec. III of this paper). And, for the case with $\eta_0=2\pi$, there is an excellent geometric explanation: the corresponding Euclidean section with $\phi_0=2\pi$ is the regular Euclidean space $R^4$ without conical singularity [21,22]; while for $\eta_0=2\pi/\sqrt{11}$, we cannot find any simple geometric explanation. Another surprise would be that, for the case of the electromagnetic field, the Euclidean section with $\phi_0=2\pi$ is a self-consistent solution of the semiclassical Euclidean Einstein equations; but, by adopting the continuation $\phi\to i\eta$ and $\phi_0\to i\eta_0$, the resultant Misner space with $\eta_0=2\pi$ is *not* a self-consistent solution of the semiclassical Einstein equations. This implies that the semiclassical Einstein equations are broken during this continuation: that is, with this particular continuation the solutions of the semiclassical Einstein equations cannot be translated between the Lorentzian and Euclidean sections. This raises a question as to the procedure of Euclidean quantization: is the Euclidean quantization still valid for the electromagnetic field in Misner space? A fundamental requirement for Euclidean quantization should be that *during the continuation between the Lorentzian section and the Euclidean section, the Einstein equations should be preserved* (otherwise the Euclidean quantization loses its significance). Note that if we first take $\phi_0=2\pi$ in the Euclidean section of Misner space, $\langle T_\mu{}^\nu\rangle_{\rm ren}$ will be zero in the Euclidean section according to Eq. (72). Then, if we use the continuation $\phi\to i\eta$ and $\phi_0$ (in this case $=2\pi$)$\to i\eta_0$ (in this case $=i2\pi$) when going from the Euclidean conical space to Misner space, naturally we expect ''zero'' should be continued to ''zero'' in the renormalized stress-energy tensor. Then we should expect that the $\langle T_\mu{}^\nu\rangle_{\rm ren}$ in the Misner space should also be zero, which would conflict with the results obtained above by going to the Lorentzian section first and then setting $\eta_0=2\pi$. What causes this non-self-consistency?

If we check the continuation procedure carefully, we find that the problem is at renormalization. In the Euclidean section, the renormalization is equivalent to subtracting from the original nonrenormalized Hadamard function $G^{(1)}(X,X')$ a reference Hadamard function $G^{(1)}_{\rm ref}(X,X')$ with $\phi_0=2\pi$, i.e., the regularized Hadamard function is

$$G^{(1)}_{\rm reg}(X,X';\phi_0)=G^{(1)}(X,X';\phi_0)-G^{(1)}_{\rm ref}(X,X';2\pi),\tag{74}$$

where

$$G^{(1)}_{\rm ref}(X,X';2\pi)\equiv G^{(1)}(X,X';\phi_0=2\pi).\tag{75}$$

If we make the continuation with $\phi\to i\eta$ and $\phi_0\to i\eta_0$ but keep $\phi_0=2\pi$ unchanged in $G^{(1)}_{\rm ref}$, $G^{(1)}_{\rm ref}$ would be continued as

$$G^{(1)}_{\rm ref}(\xi,\phi,y,z;\xi',\phi',y',z';\phi_0=2\pi)\to G^{(1)}_{\rm ref}(\xi,i\eta,y,z;\xi',i\eta',y',z';\phi_0=2\pi=i\eta_0),\tag{76}$$

then the obtained $G^{(1)}_{\rm ref}$ in the Lorentzian section is the usual Hadamard function for the Minkowski vacuum (see [35]), and the corresponding renormalized stress-energy tensor (which is obtained by operating on $G^{(1)}_{\rm reg}$ with a differential operator [35]) is just given by Eq. (73). But, in this continuation procedure, it would be surprising why in both $G^{(1)}_{\rm reg}$ and $G^{(1)}$ the parameter $\phi_0$ was changed to $i\eta_0$ but in $G^{(1)}_{\rm ref}$ the parameter $\phi_0=2\pi$ was unchanged, thus this procedure is not self-consistent. The result of this non-self-consistent procedure is that the semiclassical Einstein equations are broken, as mentioned above.

For a flat Euclidean space $R^4$ with Cartesian coordinates $(\tau,x,y,z)$ and metric $ds^2=d\tau^2+dx^2+dy^2+dz^2$, if we go to the Lorentzian section by the continuation $\tau\to it$, the Euclidean space $R^4$ is naturally continued to the usual simply connected Minkowski space with Cartesian coordinates $(t,x,y,z)$ and metric $ds^2=-dt^2+dx^2+dy^2+dz^2$. Since in the Euclidean section $\tau$ goes from $-\infty$ to $\infty$, naturally in the

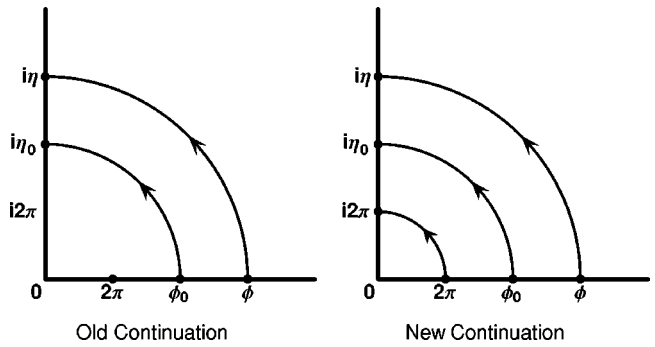


FIG. 6. A schematic diagram of the old and the new renormalization procedures. The horizontal lines are real axes, the vertical lines are imaginary axes. The arcs with arrows represent the continuation from the Euclidean section to the Lorentzian section. (a) The left diagram describes the old renormalization procedure in Euclidean quantization. As one goes from the Euclidean section of a conical space to its Lorentzian section (Misner space), $\phi$ is changed to $i\eta$, $\phi_0$ is changed to $i\eta_0$, while the parameter $2\pi$ in the reference Hadamard function is unchanged. With this old renormalization procedure, the semiclassical Einstein equations are broken during the continuation, as discussed in the text. (b) The right diagram describes the new self-consistent renormalization procedure in Euclidean quantization. With this new procedure, as one goes from the Euclidean section to the Lorentzian section, $\phi$ is changed to $i\eta$, $\phi_0$ is changed to $i\eta_0$, and $2\pi$ in the reference Hadamard function is changed to $i2\pi$. The semiclassical Einstein equations are preserved during this new continuation.

Lorentzian section $t$ also goes from $-\infty$ to $\infty$. With this continuation, the Hadamard function for the Euclidean vacuum in the Euclidean $R^4$ space—which is the reference Hadamard function for renormalization in the Euclidean section—is continued to the Hadamard function for the Lorentzian vacuum in Minkowski space—which is the reference Hadamard function for renormalization in Minkowski space. But in our case, we start with a flat Euclidean space $R^4$ with cylindrical coordinates $(\xi,\phi,y,z)$ and metric $ds^2=d\xi^2+\xi^2d\phi^2+dy^2+dz^2$ where $\phi$ has a period of $2\pi$. When we go to the Lorentzian section by the continuation $\phi\rightarrow i\eta$, very naturally $\eta$ has also a period of $2\pi$. Then the obtained Lorentzian section is a Misner space with the boost parameter $\eta_0=2\pi$. With this continuation, the Hadamard function for the Euclidean vacuum in the Euclidean section—which is the reference Hadamard function for renormalization in the Euclidean section—is continued to the Hadamard function for Li and Gott's adapted Rindler vacuum with $\eta_0=2\pi$—very naturally which should also be the reference Hadamard function for renormalization in Misner space. We start with the reference space for a Euclidean conical space—which is the flat Euclidean space without conical singularity, and end with the corresponding reference space for Misner space—which is the Misner space with $\eta_0=2\pi$. This procedure is very natural. Therefore, here we propose the following *self-consistent renormalization procedure* for quantum fields in Misner-type spaces: When we make the continuation from the Euclidean section to the Lorentzian section by $\phi\rightarrow i\eta$ and $\phi_0\rightarrow i\eta_0$, we should also make the continuation $2\pi\rightarrow i2\pi$ (see Fig. 6), then the reference Hadamard function is continued to be the Hadamard function in the Lorentzian section (Misner space) with $\eta_0=2\pi$. With this self-consistent renormalization procedure, instead of Eq. (76), the reference Hadamard function is continued to be

$$G^{(1)}_{\rm ref}(\xi,\phi,y,z;\xi',\phi',y',z';\phi_0=2\pi)\rightarrow G^{(1)}_{\rm ref}(\xi,i\eta,y,z;\xi',i\eta',y',z';\eta_0=2\pi), \tag{77}$$

making all the problems mentioned above go away. With this self-consistent renormalization procedure, instead of Eq. (73), the renormalized stress-energy tensor of electromagnetic field in Misner space becomes [substituting $i\eta_0$ for $\phi_0$ and $i2\pi$ for $2\pi$ in Eq. (72)]

$$\langle T_\mu{}^\nu\rangle_{\rm ren}=\frac{1}{720\pi^2\xi^4}\left[\left(\frac{2\pi}{\eta_0}\right)^2-1\right]\left[\left(\frac{2\pi}{\eta_0}\right)^2+11\right]\times\begin{pmatrix}-3&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}, \tag{78}$$

in the Rindler coordinates $(\eta,\xi,y,z)$. Clearly, if $\eta_0=2\pi$, we have $\langle T_\mu{}^\nu\rangle_{\rm ren}=0$ and thus the semiclassical Einstein equations are satisfied. The situation is similar for massless neutrinos in Misner space. If we make the continuation $\phi\rightarrow i\eta$, $\phi_0\rightarrow i\eta_0$, $2\pi\rightarrow i2\pi$, using the self-consistent renormalization procedure, we obtain the renormalized stress-energy tensor of vacuum polarization for massless neutrinos in Misner space

$$\langle T_\mu{}^\nu\rangle_{\rm ren}=\frac{1}{5760\pi^2\xi^4}\left[\left(\frac{2\pi}{\eta_0}\right)^2-1\right]\left[\left(\frac{2\pi}{\eta_0}\right)^2+17\right]\times\begin{pmatrix}-3&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}, \tag{79}$$

which is also zero for $\eta_0=2\pi$. Thus, with the new renormalization procedure, we obtain self-consistent vacua for electromagnetic fields and massless neutrinos in Misner space. It is very easy to check that this new renormalization procedure does not change the results for a massless conformally coupled scalar field in Misner space. The Euclidean result for a conical space is

$$\langle T_\mu{}^\nu\rangle_{\rm R,ren}=\frac{1}{1440\pi^2\xi^4}\left[\left(\frac{2\pi}{\phi_0}\right)^4-1\right]\begin{pmatrix}-3&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}. \tag{80}$$

Substituting $i\eta_0$ for $\phi_0$ and $i2\pi$ for $2\pi$ into Eq. (80) gives Eq. (41) just as before. Thus, Li and Gott's adapted Rindler vacuum [21] is still a self-consistent vacuum for the massless conformally coupled scalar field in Misner space with $\eta_0 = 2\pi$. These results can be easily transplanted to Misner-like de Sitter space and Misner-like anti–de Sitter space, since these fields (massless conformally coupled scalar fields, electromagnetic fields, and massless neutrinos) are conformally invariant and de Sitter space and anti–de Sitter space are conformally flat. The results are: if the boost period is $2\pi r_0$ (where $r_0$ is the radius of the de Sitter space or anti–de Sitter space), the renormalized stress-energy tensors are finite and given by Eq. (68) with $g_*=11/2$ for one neutrino field and $g_*=62$ for the electromagnetic field. With Eqs. (69) and (70), self-consistent solutions of the semiclassical Einstein equations with cosmological constant can be found. Thus, with our new renormalization method, various self-consistent vacuum states in Misner-type spaces with CTC's can be found.

The chronology protection conjecture [11] was originally based on the fact that for a massless conformally coupled scalar field with the adapted Minkowski vacuum in the Misner space the renormalized stress-energy tensor of vacuum polarization diverges at the chronology horizon [7]. After the appearance of many counterexamples [12–20], Cassidy and Hawking [49] demonstrated that the backreaction of vacuum polarization does not enforce chronology protection. The results in this paper (and [21,22]) support this demonstration. However, Cassidy and Hawking [49] turned to the proposition that the effective action of matter fields in spacetimes with CTC's always diverges at the chronology horizon and that it is this that enforces chronology protection. The effective action plays an important role in Euclidean quantum gravity, which gives the probability for the creation of spacetime through quantum tunneling. Consider a simple example of a massless conformally coupled scalar field in Misner space. The Euclidean effective Lagrangian density is [48]

$$\mathcal{L}=\frac{1}{1440\pi^2\xi^4}\left[\left(\frac{2\pi}{\phi_0}\right)^2-1\right]\left[\left(\frac{2\pi}{\phi_0}\right)^2+11\right]. \tag{81}$$

The Euclidean effective action is obtained by integrating the Euclidean effective Lagrangian density over a suitable volume of the Euclidean section. Clearly, if $\phi_0\neq 2\pi$, the effective Euclidean Lagrangian diverges at $\xi=0$ (thus the Euclidean effective action diverges if the domain for integration includes the conical singularity at $\xi=0$). But, if $\phi_0=2\pi$, we have $\mathcal{L}=0$, which is not surprising since when $\phi_0=2\pi$ the conical singularity at $\xi=0$ disappears and the Euclidean section becomes the usual regular flat $R^4$ space. The Euclidean effective action is also zero since the Euclidean effective Lagrangian is zero everywhere. By the continuation $\phi \rightarrow i\eta$, $\phi_0\rightarrow i\eta_0$, and with the self-consistent renormalization procedure outlined above, the reference Hadamard function is continued to be that with $\eta_0=2\pi$ in Misner space (which means that the ratio $2\pi/\phi_0$ in Eq. (81) is changed to $2\pi/\eta_0$ since $\phi_0\rightarrow i\eta_0$ and $2\pi\rightarrow i2\pi$), then the effective Lagrangian for a massless conformally coupled scalar field in Misner space is

$$\mathcal{L}=\frac{1}{1440\pi^2\xi^4}\left[\left(\frac{2\pi}{\eta_0}\right)^2-1\right]\left[\left(\frac{2\pi}{\eta_0}\right)^2+11\right], \tag{82}$$

which is zero everywhere if $\eta_0=2\pi$. Thus the Lorentzian effective action is also zero for $\eta_0=2\pi$. This shows that, for the self-consistent vacuum in Misner space, the effective action is zero. It can be expected that this result can be extended to other quantum fields or other spacetimes with CTC's, where the (Euclidean or Lorentzian) effective Lagrangian would be finite throughout the space [and thus the (Euclidean or Lorentzian) effective action would also be finite] for the self-consistent vacua. Thus Cassidy and Hawking's argument that the effective action (or equivalently the entropy) enforce chronology protection is questionable.

## VI. CONCLUSIONS

From the covering space of anti–de Sitter space, a Misner-like anti–de Sitter space can be constructed. This Misner-like anti–de Sitter space has CTC's but the regions with CTC's are separated from the regions without CTC's by chronology horizons. In the appropriate coordinates, this Misner-like anti–de Sitter space is just the Lorentzian section of the complex space with CTC's constructed by Li, Xu, and Liu in 1993 [13]. For a massless conformally coupled scalar field in this space, a self-consistent vacuum is found, whose renormalized stress-energy tensor is like that of a positive cosmological constant—which when added to an appropriate negative bare cosmological constant can self-consistently solve the semiclassical Einstein equations.

By gluing a de Sitter space to an anti–de Sitter space along a bubble wall, another new spacetime with CTC's is obtained. This spacetime could describe the transition between de Sitter space and anti–de Sitter space. In this spacetime, the region with CTC's and the region without CTC's are separated via chronology horizons. For the de Sitter or anti–de Sitter invariant vacua in these spacetimes, the renormalized stress-energy tensors are like positive cosmological constants. A self-consistent solution can be obtained if there is a single negative bare cosmological constant in the two regions with the renormalized stress-energy tensor of vacuum polarization adding different positive cosmological constants to the two sides of the bubble wall so that the effective cosmological constant (bare + renormalized) is positive on one side and negative on the other. On the hypersurface separating de Sitter space from anti–de Sitter space, in order that the Einstein equations are satisfied, a surface stress-energy tensor should be induced. If the hypersurface is timelike, the surface stress-energy tensor is like that of a three-dimensional positive cosmological constant.

The self-consistent solutions of the semiclassical Einstein equations with cosmological constant and the renormalized stress-energy tensor of vacuum polarization in de Sitter/anti–de Sitter space are investigated. If the bare cosmological constant is positive, there are two self-consistent solutions, both of them are de Sitter spaces. If the bare cosmological constant is zero then there are two self-consistent solutions—one is Minkowski space and the other is a Planckian scale de Sitter space. If the bare cosmological

constant is negative, there are also two self-consistent solutions, one of them is an anti–de Sitter space, but the other is a sub-Planckian scale de Sitter space. And, at the sub-Planckian scale, self-consistent solutions (either de Sitter space or anti–de Sitter space) exist only for a bare negative cosmological constant.

The generalization to electromagnetic fields and massless neutrinos in spacetimes with CTC's are discussed. It is argued that, for Misner-type spacetimes, in order that the semiclassical Einstein equations are preserved under continuation between the Euclidean and Lorentzian sections, a new renormalization procedure should be introduced. We have proposed such a self-consistent renormalization procedure, with which self-consistent vacua for electromagnetic fields and neutrinos are found.

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