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Chronology protection conjecture

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It has been suggested that an advanced civilization might have the technology to warp spacetime so that closed timelike curves would appear, allowing travel into the past. This paper examines this possibility in the case that the causality violations appear in a finite region of spacetime without curvature singularities. There will be a Cauchy horizon that is compactly generated and that in general contains one or more closed null geodesics which will be incomplete. One can define geometrical quantities that measure the Lorentz boost and area increase on going round these closed null geodesics. If the causality violation developed from a noncompact initial surface, the averaged weak energy condition must be violated on the Cauchy horizon. This shows that one cannot create closed timelike curves with finite lengths of cosmic string. Even if violations of the weak energy condition are allowed by quantum theory, the expectation value of the energy-momentum tensor would get very large if timelike curves become almost closed. It seems the back reaction would prevent closed timelike curves from appearing. These results strongly support the chronology protection conjecture: *The laws of physics do not allow the appearance of closed timelike curves.*

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I. INTRODUCTION

There have been a number of suggestions that we might be able to warp spacetime in such a way as to allow rapid intergalactic space travel or travel back in time. Of course, in the theory of relativity, time travel and faster-than-light space travel are closely connected. If you can do one, you can do the other. You just have to travel from A to B faster than light would normally take. You then travel back, again faster than light, but in a different Lorentz frame. You can arrive back before you left.

One might think that rapid space travel might be possible using the wormholes that appear in the Euclidean approach to quantum gravity. However, one would have to be able to move in the imaginary direction of time to use these wormholes. Further, it seems that Euclidean wormholes do not introduce any nonlocal effects. So they are no good for space or time travel.

Instead, I shall consider real-time, Lorentzian metrics. In these, the light-cone structure forces one to travel at less than the speed of light and forward in time in a local region. However, the global structure of spacetime may allow one to take a shortcut from one region to another or may let one travel into the past. Indeed, it has been suggested by Morris and Thorne and others [1–3] that in the future, with improved technology, we might be able to create traversable wormholes connecting distant regions of spacetime. These wormholes would allow rapid space travel and, thus, travel back in time. However, one does not need anything as exotic as wormholes. Gott [4] has pointed out that an infinite cosmic string warps spacetime in such a way that one can get ahead of a beam of light. If one has two infinite cosmic strings, moving at high velocity relative to each other, one can get from A to B and back again before one sets out. This example is worrying, because unlike wormholes, it does not involve negative-energy densities. However, I will show that one cannot create a spacetime in which one can travel into the past if one only uses finite lengths of cosmic string.

The aim of this paper is to show that even if it is possible to produce negative-energy densities, quantum effects are likely to prevent time travel. If one tries to warp spacetime to allow travel into the past, vacuum polarization effects will cause the expectation value of the energy-momentum tensor to be large. If one fed this energy-momentum tensor back into the Einstein equations, it appears to prevent one from creating a time machine. It seems there is a chronology protection agency, which prevents the appearance of closed timelike curves and so makes the universe safe for historians.

Kim and Thorne [5] have considered the expectation value of the energy-momentum tensor in a particular model of a time machine. They find that it diverges, but argue that it might be cut off by quantum-gravitational effects. They claim that the perturbation that it would produce in the metric would be so small that it could not be measured, even with the most sensitive modern technology. Because we do not have a well-defined theory of quantum gravity, it is difficult to decide whether there will be a cutoff to quantum effects calculated on a background spacetime. However, I shall argue that even if there is a cutoff, one would not expect it to come into effect until one was a Planck distance from the region of closed timelike curves. This Planck distance should be measured in an invariant way, not the frame-dependent way that Kim and Thorne adopt. This cutoff would lead to an energy density of the Planck value, 10^{94} g/cc, and a perturbation in the metric of order 1. Even if "order 1" meant 10^{-2} in practice, such a perturbation would create a disturbance that was enormous compared with chemical binding energies of order 10^{-9} or 10^{-10}. So one could not hope to travel through such a region and back into the past. Furthermore, the sign of the energy-momentum tensor of the vacuum polarization seems to be such as to resist the warping of the light cones to produce closed timelike curves.

Morris and Thorne build their time machine out of traversable Lorentzian wormholes, that is, Lorentzian spacetimes of the form $\Sigma \times R$. Here R is the time direction and Σ is a three-dimensional surface, that is, asymptotically flat, and has a handle or wormhole connecting two mouths. Such a wormhole would tend to collapse with time, unless it were held up by the repulsive gravity of a negative-energy density. Classically, energy densities are always positive, but quantum field theory allows the energy density to be negative locally. An example is the Casimir effect. Morris and Thorne speculate that with future technology it might be possible to create such wormholes and to prevent them from collapsing.

Although the length of the throat connecting the two mouths of the wormhole will be fairly short, the two mouths can be arbitrarily far apart in the asymptotically flat space. Thus going through a wormhole would be a way of traveling large distances in a short time. As remarked above, this would lead to the possibility of travel into the past, because one could travel back to one's starting point using another wormhole whose mouths were moving with respect to the first wormhole. In fact, it would not be necessary to use two wormholes. It would be sufficient just for one mouth of a single wormhole to be moving with respect to the other mouth. Then there would be the usual special-relativistic time-dilation factor between the times as measured at the two mouths. This would mean that at some point in the wormhole's history it would be possible to go down one mouth and come out of the other mouth in the past of when you went down. In other words, closed timelike curves would appear. By traveling in a space ship on one of these closed timelike curves, one could travel into one's past. This would seem to give rise to all sorts of logical problems, if you were able to change history. For example, what would happen if you killed your parents before you were born. It might be that one could avoid such paradoxes by some modification of the concept of free will. But this will not be necessary if what I call the *chronology protection conjecture* is correct: *The laws of physics prevent closed timelike curves from appearing.*

Kim and Thorne [5,6] suggest that they do not. I will present evidence that they do.

II. CAUCHY HORIZONS

The particular time machine that Kim and Thorne [5] consider involves wormholes with nontrivial topology. But as I will show, to create a wormhole, one has to distort the spacetime metric so much that closed timelike curves appear. I shall therefore consider the appearance of closed timelike curves in general, without reference to any particular model.

I shall assume that our region of spacetime develops from a spacelike surface S without boundary. By going to a covering space if necessary [7], one can assume that spacetime is time orientable and that no timelike curve intersects S more than once. Let us suppose that the initial surface S did not contain any wormholes: Say it was simply connected, like R^3 or S^3. But let us suppose we had the technology to warp the spacetime that developed from S, so that a later spacelike surface S' had a different topology, say, with a wormhole or handle. It seems reasonable to suppose that we would be able to warp spacetime only in a bounded region. In other words, one could find a timelike cylinder T which intersected the spacelike surfaces S and S' in compact regions S_T and S'_T of different topology. In that case the topology change would take place in the region of spacetime M_T bounded by T, S, and S'. The region M_T would not be compact if it contained a curvature singularity or if it went off to infinity. But in that case, extra unpredictable information would enter the spacetime from the singularity or from infinity. Thus one could not be sure that one's warping of spacetime would achieve the result desired if the region M_T were noncompact. It therefore seems reasonable to suppose that M_T is compact. In Sec. V, I show that this implies that M_T contains closed timelike curves. So if you try to create a wormhole to use as a time machine, you have to warp the light-cone structure of spacetime so much that closed timelike curves appear anyway. Furthermore, one can show the requirement that M_T have a Lorentz metric and a spin structure imply that wormholes cannot be created singly, but only in multiples of 2 [8]. I shall therefore just consider the appearance of closed timelike curves without there necessarily being any change in the topology of the spatial sections.

If there were a closed timelike curve through a point p to the future of S, then p would not lie in the future Cauchy development [7] $D^+(S)$. This is the set of points q such that every past-directed curve through q intersects S if continued far enough. So there would have to be a future Cauchy horizon $H^+(S)$ which is the future boundary of $D^+(S)$. I wish to study the creation of closed timelike curves from the warping of the spacetime metric in a bounded region. I shall therefore consider Cauchy horizons $H^+(S)$ that are what I shall call "compactly generated." That is, all the past-directed null geodesic generators of $H^+(S)$ enter and remain within a compact set C. One could generalize this definition to a situation in which there were a countable number of disjoint compact sets C, but for simplicity I shall consider only a single compact set.

What this condition means is that the generators of the Cauchy horizon do not come in from infinity or a singularity. Of course, in the presence of closed timelike curves, the Cauchy problem is not well posed in the strict mathematical sense. But one might hope to predict events beyond the Cauchy horizon if it is compactly generated, because extra information will not come in from infinity or singularities. This idea is supported by some calculations that show there is a unique solution to the wave equation on certain wormhole spacetimes that contain closed timelike curves [15]. But even if there is not a unique solution beyond the Cauchy horizon, it will not affect the conclusions of this paper because the quantum effects that I shall describe occur in the future Cauchy development $D^+(S)$, where the Cauchy problem is well posed and where there is a unique solution, given the initial data and quantum state on S.

The inner horizons of the Reissner-Norström and Kerr solutions are examples of Cauchy horizons that are not compactly generated. Beyond the Cauchy horizon, new information can come in from singularities or infinity, and so one could not predict what will happen. In this paper I will restrict my attention to compactly generated Cauchy horizons. It is, however, worth remarking that the inner horizons of black holes suffer similar quantum-mechanical divergences of the energy-momentum tensor. The quantum radiation from the outer black-hole horizon will pile up on the inner horizon, which will be at a different temperature.

By contrast, the Taub-Newman-Unti-Tamburino (NUT) universe is an example of a spacetime with a compactly generated Cauchy horizon. It is a homogeneous anisotropic closed universe, where the surfaces of homogeneity go from being spacelike to null and then timelike. The null surface is a Cauchy horizon for the spacelike surfaces of homogeneity. This Cauchy horizon will be compact and therefore will automatically be compactly generated. However, I have deliberately chosen the definition of compactly generated, so that it can apply also to Cauchy horizons that are noncompact. Indeed, if the initial surface S is noncompact, the Cauchy horizon $H^+(S)$ will be either noncompact or empty. To show this one uses the standard result, derived in Sec. V, that a manifold with a Lorentz metric admits a timelike vector field V^a. (Strictly, a Lorentz metric implies the existence of a vector field up to a sign. But one can choose a consistent sign for the vector field if the spacetime is time orientable, which I shall assume.) Then the integral curves of the vector field give a mapping of the future Cauchy horizon $H^+(S)$ into S. This mapping will be continuous and one to one onto the image of $H^+(S)$ in S. But the future Cauchy horizon $H^+(S)$ is a three-manifold without boundary. So, if S is noncompact, $H^+(S)$ must be noncompact as well. However, that need not prevent it from being compactly generated.

An example will illustrate how closed timelike curves can appear without there being any topology change. Take the spacetime manifold to be R^4 with coordinates t,r,θ,ϕ. Let the initial surface S be $t=0$ and let the spacetime metric g_{ab} be the flat Minkowski metric η_{ab} for t negative. For positive t let the metric still be the flat Minkowski metric outside a timelike cylinder, consisting of a two-sphere of radius L times the positive-time axis. Inside the cylinder let the light cones gradually tip in the ϕ direction, until the equator of the two-sphere, $r=\frac{1}{2}L$, becomes first a closed null curve γ and then a closed timelike curve. For example, the metric could be

$$ds^2=-dt^2+2f\,dt\,d\phi-f\,d\phi^2+dr^2+r^2(d\theta^2+\sin^2\theta\,d\phi^2)\ ,$$

$$f=r^2t^2\sin^4\theta\sin^2\left[\frac{\pi r}{L}\right]\ .$$

The Cauchy horizon will be generated by null geodesics that in the past direction spiral toward the closed null geodesic γ. They will all enter and remain within any compact neighborhood C of γ. Thus the Cauchy horizon will be compactly generated.

One could calculate the Einstein tensor of this metric. As I will show, it will necessarily violate the weak energy condition. But one could take the attitude that quantum field theory in curved space allows violations of the weak energy condition, as in the Casimir effect. One might hope, therefore, that in the future we might have the technology to produce an energy-momentum tensor equal to the Einstein tensor of such a spacetime. It is worth remarking that, even if we could distort the light cones in the manner of this example, it would not enable us to travel back in time to before the initial surface S. That part of the history of the universe is already fixed. Any time travel would have to be confined to the future of S.

I shall mainly be interested in the case where the initial surface S is noncompact, because that corresponds to building a time machine in a local region. However, most of the results in this paper will also apply to the cosmological case, in which S can be compact.

The Cauchy horizon is generated by null geodesic segments [7]. These may have future end points, where they intersect another generator. The future end points will form a closed set B of measure zero. On the other hand, the generators will not have past end points. If the horizon is compactly generated, the generators will enter and remain within a compact set C. One can introduce a null tetrad $l^a,n^a,m^a,\overline{m}^a$ in a neighborhood of $(H^+(S)-B)\cap C$. The vector l^a is chosen to be the future-directed tangent to the generators of the Cauchy horizon. The vector n^a is another future-directed null vector. Because I am using the signature $-+++$, rather than the $+---$ signature of Newman and Penrose, I normalize them by $l^a n_a=-1$. The complex-conjugate null vectors m^a and $\overline{m}^a$ are orthogonal to l^a and n^a and are normalized by $m^a\overline{m}_a=1$. One can then define the Newman-Penrose quantities [9,10]

$$\epsilon=-\tfrac{1}{2}(n^a l_{a;c}l^c-\overline{m}^a m_{a;c}l^c)\ ,$$

$$\kappa=-m^a l_{a;c}l^c\ ,$$

$$\rho=-m^a l_{a;c}\overline{m}^c\ ,$$

$$\sigma=-m^a l_{a;c}m^c\ .$$

Note that these definitions have the opposite sign to those of Newman and Penrose. This is because of the different signature of the metric.

Because the generators are null geodesics and lie in a null hypersurface, $\kappa=0$ and $\rho=\overline{\rho}$. The convergence ρ and shear σ obey the Newman-Penrose equations along γ:

$$\frac{d\rho}{dt}=\rho^2+\sigma\overline{\sigma}+(\epsilon+\overline{\epsilon})\rho+\tfrac{1}{2}R_{ab}l^a l^b\ ,$$

$$\frac{d\sigma}{dt}=2\rho\sigma+(3\epsilon-\overline{\epsilon})\sigma+C_{abcd}l^a m^b l^c\overline{m}^d\ ,$$

where t is the parameter along the generators such that

$l^a = dx^a/dt$.

The real and imaginary parts of ϵ, respectively, measure how the vectors l^a and m^a change compared to a parallel-propagated basis. By choosing an affine parameter $\tilde{t}$ on the generators, one can rescale the tangent vector l^a so that $\epsilon + \bar{\epsilon} = 0$. The generators may be geodesically incomplete in the future direction; i.e., the affine parameter may have an upper bound. But one can adapt the lemma in Ref. [7], p. 295, to show that the generators of the horizon are complete in the past direction.

Now suppose the weak energy condition holds:

$$T_{ab} l^a l^b \geq 0 ,$$

for any null vector l^a. Then the Einstein equations (with or without cosmological constant) imply

$$R_{ab} l^a l^b \geq 0 .$$

It then follows that the convergence ρ of the generators must be non-negative everywhere on the Cauchy horizon. For suppose $\rho = \rho_1 < 0$ at a point p on a generator γ. Then one could integrate the Newman-Penrose equation for ρ in the negative $\tilde{t}$ direction along γ to show that ρ diverged at some point q within an affine distance ρ_1^{-1} to the past of p. Such a point q would be a past end point of the null geodesic segment γ in the Cauchy horizon. But this is impossible because the generators of the Cauchy horizon have no past end points. This shows that ρ must be everywhere non-negative on a compactly generated Cauchy horizon if the weak energy condition holds.

I shall now establish a contradiction in the case that the initial surface S is noncompact. The argument is similar to that in Ref. [7], p. 297. On C one can introduce a unit timelike vector field V^a. One can then define a positive definite metric by

$$\hat{g}_{ab} = g_{ab} + 2V_a V_b .$$

In other words, $\hat{g}$ is the spacetime g with the sign of the metric in the timelike V^a direction reversed.

One can normalize the tangent vector to the generators by $g_{ab} l^a V^b = 1/\sqrt{2}$. The parameter t on the generators then measures the proper distance in the metric $\hat{g}_{ab}$. One can define a map

$$\mu_t : (H^+(S) - B) \cap C \rightarrow (H^+(S) - B) \cap C ,$$

by moving each point of the Cauchy horizon a parameter distance t to the past along the generators. The three-volume (measured with respect to the metric $\hat{g}_{ab}$) of the image of the Cauchy horizon under this map will change according to

$$\frac{d}{dt} \int_{\mu_t(H^+(S) \cap C)} dA = 2 \int_{\mu_t(H^+(S) \cap C)} \rho \, dA .$$

The change in volume cannot be positive because the Cauchy horizon is mapped into itself. If the initial surface S is noncompact, the change in volume will be strictly negative, because the Cauchy horizon will be noncompact and will not lie completely in the compact set C. This would establish a contradiction with the requirement that $\rho \geq 0$ if the weak energy condition is satisfied. Thus a compactly generated Cauchy horizon cannot form if the weak energy condition holds and S is noncompact.

On the other hand, the example of the Taub-NUT universe shows that it is possible to have a compactly generated Cauchy horizon if S is compact. However, in that case the weak energy condition would imply that ρ and σ would have to be zero everywhere on the Cauchy horizon. This would mean that no matter or information, and in particular no observers, could cross the Cauchy horizon into the region of closed timelike curves. Moreover, as will be shown in the next section, the solution will be classically unstable in that a small matter-field perturbation would pile up on the horizon. Thus the chronology protection conjecture will hold if the weak energy condition is satisfied whether or not S is compact. In particular, this implies that if no closed timelike curves are present initially, one cannot create them by warping the metric in a local region with finite loops of cosmic string. If the weak energy condition is satisfied, closed timelike curves require either singularities (as in the Kerr solution) or a pathological behavior at infinity (as in the Godel and Gott spacetimes).

The weak energy condition is satisfied by the classical energy-momentum tensors of all physically reasonable fields. However, it is not satisfied locally by the quantum expectation value of the energy-momentum tensor in certain quantum states in flat space. In Minkowski space the weak energy condition is still satisfied if the expectation value is averaged along a null geodesic [11], but there are curved-space backgrounds where even the averaged expectation values do not satisfy the weak energy condition. The philosophy of this paper is therefore not to rely on the weak energy condition, but to look for vacuum polarization effects to enforce the chronology protection conjecture.

III. CLOSED NULL GEODESICS

The past-directioned generators of the Cauchy horizon will have no past end points. If the horizon is compactly generated, they will enter and remain within a compact set C. This means they will wind round and round inside C. In Sec. V it is shown that there is a nonempty set E of generators, each of which remains in a compact set C in the future direction, as well as in the past direction.

The generators in E will be *almost* closed. That is there will be points q such that a generator in E will return infinitely often to any small neighborhood of q. But they need not actually close up. For example, if the initial surface is a three-torus, the Cauchy horizon will also be a three-torus, and the generators can be nonrational curves that do not close up on themselves. However, this kind of behavior is unstable. The least perturbation of the metric will cause the horizon to contain closed null geodesics. More precisely, the space of all metrics on the spacetime manifold M can be given a C^∞ topology. Then, if g is a metric that has a compactly generated horizon which does not contain closed null geodesics, any neighborhood of g will contain a metric g' whose Cauchy horizon *does* contain closed null geodesics.

The spacetime metric is presumably the classical limit of an inherently quantum object and so can be defined only up to some uncertainty. Thus the only properties of the horizon that are physically significant are those that are maintained under small variations of the metric. In Sec. V it will be shown that in general the closed null geodesics in the horizon have this property. That is, if g is a metric such that the Cauchy horizon contains closed null geodesics, then there is a neighborhood U of g such that every metric g' in U has closed null geodesics in its Cauchy horizon. I shall therefore assume that in general E consists of one or more disjoint closed null geodesics. The example given above of the metric with closed timelike curves shows that the Cauchy horizon need not contain more than one.

I shall now concentrate attention on a closed null geodesic γ in the Cauchy horizon. Pick a point p on γ and parallel propagate a frame around γ and back to p. The result will be a Lorentz transformation Λ of the original frame. This Lorentz transformation will lie in the four-parameter subgroup that leaves unchanged the null direction tangent to the generator. It will be generated by an antisymmetric tensor

$$\Lambda = e^{\omega} .$$

The null vector l^a tangent to the null geodesic will be an eigenvector of ω because its direction is left unchanged by Λ:

$$l^a = h\omega^a{}_b l^b .$$

The eigenvalue h determines the change of scale, e^h, of the tangent vector after it has been parallel propagated around the closed null geodesic in the future direction. In Sec. V it is shown that if h were negative, one could move each point of γ to the past to obtain a closed timelike curve. But this curve would be in the Cauchy development of S, which is impossible, because the Cauchy development does not contain any closed timelike curves. This shows that h must be positive or zero. Clearly, $h=0$ is a limiting case. In practice, one would expect h to be positive. This will mean that each time one goes round the closed null geodesic, the parallel-propagated tangent vector will increase in size by a factor e^h. The affine-parameter distance around will decrease by a factor e^{-h}. Thus the closed null geodesic γ will be incomplete in the future direction, although it will remain in the compact set C and so it will not end on any curvature singularity. Because $h \geq 0$, γ will be complete in the past direction.

If $h \neq 0$, there will be another null vector n^a, which is an eigenvector of $\omega^a{}_b$ with eigenvalue $-h$. The Lorentz transformation Λ will consist of a boost e^h in the timelike plane spanned by l^a and n^a and a rotation through an angle θ in the orthogonal spacelike plane.

The quantity h is rather like the surface gravity of a black hole. It measures the rate at which the null cones tip over near γ. As in the black-hole case, it gives rise to quantum effects. However, in this case, they will have imaginary temperature, corresponding to periodicity in real time, rather than in imaginary time, as in the black-hole case.

Another important geometrical quantity associated with the closed null geodesic γ in the Cauchy horizon is the change of cross-sectional area of a pencil of generators of the horizon as one goes round the closed null geodesic. Let

$$f = \ln \left[\frac{A_{n+1}}{A_n} \right] ,$$

where A_n and A_{n+1} are the areas of the pencil on successive passes of the point p in the future direction. The quantity f measures the amount the generators are diverging in the future direction. Because neighboring generators tend toward the closed null geodesic γ in the past direction, f will be greater than or equal to zero. Again, $f=0$ is a limiting case. In general, f will be greater than zero.

The quantity f determines the classical stability of the Cauchy horizon. A small, high-frequency wave packet going round the horizon in the neighborhood of γ will have its energy blueshifted by a factor e^h each time it comes round. This increased energy will be spread across a cross section transverse to γ. On each circuit of γ, the two-dimensional area of the cross section will increase by a factor e^f. The time duration of the cross section will be reduced by a factor e^{-h}. So the local energy density will remain bounded and the Cauchy horizon will be classically stable if

$$f > 2h .$$

This is true of the wormholes that Kim and Thorne consider, provided they are moving slowly. But it seems they will still be unstable quantum mechanically.

One can relate the result of going round γ to integrals of the Newman-Penrose quantities defined in the last section:

$$\oint \rho \, dt = -\tfrac{1}{2} f ,$$

$$\oint \epsilon \, dt = -\tfrac{1}{2}(h + i\theta) ,$$

where e^h is the boost in the l^a-n^a plane and $e^{i\theta}$ is the spatial rotation in the m^a-$\overline{m}^a$ plane of a tetrad that is parallel propagated after one circuit of γ. One can also define the distortion q of an initially circular pencil of generators by

$$\oint \sigma \, dt = -\tfrac{1}{2} q .$$

One can choose the parameter t on γ so that $\epsilon + \overline{\epsilon}$ is constant and so that the parameter distance of one circuit of γ is 1. Then

$$\epsilon + \overline{\epsilon} = -h .$$

One can now integrate the Newman-Penrose equation for ρ around a circuit of γ and use the Schwarz inequality to show

$$\oint R_{ab} l^a l^b dt \leq -[hf + \tfrac{1}{2}(f^2 + q\overline{q})] \leq 0 .$$

This gives a measure of how much the weak energy condition has to be violated on γ. In particular, it cannot be satisfied unless $f = q = 0$.

IV. QUANTUM FIELDS ON THE BACKGROUND

Quantum effects in the spacetime will be determined by the propagator or two-point function

$$\langle T\phi(x)\phi(y)\rangle \ .$$

This will be singular when the two points x and y can be joined by a null geodesic. Thus quantum effects near γ will be dominated by closed or almost-closed null geodesics.

One can construct a simple spacetime that reproduces the Lorentz transformation Λ on going around γ, but not the area increase e^f, in the following way. One starts with Minkowski space and identifies points that are taken into each other by the Lorentz transformation Λ. For simplicity, I will just describe the case where Λ is a pure boost in the n^a-l^a plane. Consider the past light cone of the origin in two-dimensional Minkowski space. The orbits of the boost Killing vector will be spacelike. Identify a point p with its image under the boost Λ. This gives what is called Misner space [12,7] with the metric

$$ds^2=-dt^2+t^2dx^2 \ ,$$

on a half-cylinder defined by $t<0$ with the x coordinate identified with period h. This metric has an apparent singularity at $t=0$. However, one can extend it by introducing new coordinates

$$\tau=t^2, \quad v=\ln t+x \ .$$

The metric then takes the form

$$ds^2=-dv\,d\tau+\tau\,dv^2 \ .$$

This can then be extended through $\tau=0$. This corresponds to extending from the bottom quadrant into the left-hand quadrant. One then gets a metric on a cylinder. This develops from a spacelike surface S. However, at $\tau=0$, the light cones tip over and a closed null geodesic appears. For negative τ, closed timelike curves appear. The full four-dimensional space is the product of this two-dimensional Misner space with two extra flat dimensions. One can identify these other dimensions periodically if one wants to have a spacetime in which the initial surface S and the Cauchy horizon $D^+(S)$ are compact. However, such a compactification will not change the nature of the behavior of the energy-momentum tensor on the horizon.

Misner space has a four-parameter group of isometries and is also invariant under an overall dilation. It is therefore natural to expect the quantum state of a conformally invariant field also to have these symmetries. By the conservation equations and the trace-anomaly equation, the expectation value of the energy-momentum tensor for a conformally invariant field must then have the form

$$\langle T_{ab}\rangle_0=\mathrm{diag}(K,3K,-K,-K), \quad K=\frac{B}{t^4} \ ,$$

in an orthonormal basis along the (t,x,y,z) axes. The coefficient B will depend on the quantum state and spin of the field.

Because the space is flat, it is easy to calculate a propagator $\langle T\phi(x)\phi(y)\rangle_0$ for a particular quantum state of any free field with these symmetries. One just takes the usual Minkowski propagator and puts in image charges under Λ. One can then calculate the expectation value of the energy-momentum tensor by taking the limit of this propagator minus the usual Minkowski propagator. This has been done by Hiscock and Konkowski [13] for the case of a conformally invariant scalar field. They found that B is negative, implying that the expectation value of the energy density is negative and diverges on the Cauchy horizon.

The quantum state that the propagator $\langle T\phi(x)\phi(y)\rangle_0$ corresponds to is a particularly natural one, but is certainly not the only quantum state of the spacetime. The propagator in any other state will obey the same wave equation. Thus it can be written

$$\langle T\phi(x)\phi(y)\rangle=\langle T\phi(x)\phi(y)\rangle_0 + \sum \tfrac{1}{2}[\psi_n(x)\bar{\psi}_n(y)+\text{c.c.}] \ ,$$

where ψ_n are solutions of the homogeneous wave equation that are nonsingular on the initial surface S. The expectation value of the energy-momentum tensor in this state will be

$$\langle T_{ab}\rangle=\langle T_{ab}\rangle_0+\sum T^{\mathrm{cl}}_{ab}[\psi_n] \ ,$$

where $T^{\mathrm{cl}}_{ab}[\psi_n]$ is the classical energy-momentum tensor of the field ψ_n. One can think of the last term as the energy momentum of particles in modes corresponding to the solutions ψ_n.

One could ask if there was a propagator that gave an energy-momentum tensor that did not diverge on the Cauchy horizon. I have found propagators that give the expectation value of the energy momentum to be zero everywhere, but they do not satisfy the positivity conditions that are required for them to be the time-ordered expectation values of the field operators in a well-defined quantum state. I am grateful to Bernard Kay for pointing this out. One way of getting a propagator that was guaranteed to satisfy the positivity conditions would be to add particle excitations to the $\langle\ \rangle_0$ state. However, no distribution of particles would have a stress in the x direction that is 3 times the energy density. Unless the energy-momentum tensor of the particles had the same form as that of $\langle T_{ab}\rangle_0$, it would not diverge with the same power of distance away from the horizon and so could not cancel the divergence. Thus I am almost sure there is no quantum state on Misner space for which $\langle T_{ab}\rangle$ is finite on the horizon, but I do not have a rigorous proof.

In the general case in which there is a negative Ricci tensor and $f>0$, it is difficult to calculate the expectation value of the energy-momentum tensor exactly because one does not have a closed form for the propagator. However, near the Cauchy horizon the metric and quantum state will asymptotically have the same symmetries and scale invariance as in Misner space. Thus one would still expect the same Bt^{-4} behavior, where the value of t at a point is now defined to be the least upper bound of the lengths of timelike curves from the point to the closed null geodesic γ. If $h>0$, t will be finite on $D^+(S)$.

Again, the coefficient B will depend on the quantum state. Approximate WKB calculations by Kim and Thorne [5] for a wormhole spacetime indicate that there is a quantum state for this spacetime for which B is negative. Because the classical stability condition $f>2h$ is satisfied, it does not seem possible to cancel the negative-energy divergence with positive-energy quanta. Thus it seems that the expectation value of the energy-momentum tensor will always diverge on the Cauchy horizon for any quantum state.

V. GLOBAL RESULTS

If there is a timelike tube T connecting surfaces S and S' of different topology, then the region M_T contains closed timelike curves.

This is a modification of a theorem of Geroch [14]. I shall describe it here because it involves constructions that will be useful later. One first puts a positive-definite metric $\tilde{g}_{ab}$ on the spacetime manifold M. (This can always be done.) Then one can define a timelike vector field V^a as an eigenvector with negative eigenvalue of the physical metric g_{ab} with respect to $\tilde{g}_{ab}$:

$$g_{ab}V^a=-\lambda\tilde{g}_{ab}V^a \ .$$

One can normalize V^a to have unit magnitude in the spacetime metric g_{ab}. With a bit more care, one can choose the vector field V^a so that it is tangent to the timelike tube T. One can define a mapping

$$\mu{:}S_T\rightarrow S'_T \ ,$$

by moving points along the integral curves of V^a. If each integral curve that cuts S_T were also to cut S'_T, μ would be one-to-one and onto. But this would imply that S_T and S'_T have the same topology, which they do not. Therefore there must be some integral curve γ which cuts S_T but which winds round and round inside the compact set M_T and does not intersect S'_T. This implies there will be points $p\in M_T$ that are limit points of γ. Through p there will be an integral curve $\bar{\gamma}$, each point of which is a limit point of γ. But because $\bar{\gamma}$ is timelike, it would be possible to deform segments of γ to form closed timelike curves.

A compactly generated Cauchy horizon $D^+(S)$ contains a set E of generators which have no past or future end points and which are contained in the compact set C.

Let λ be a generator of the Cauchy horizon. This means that it may have a future end point (where it intersects another generator), but it can have no past point. Instead, because the horizon is compactly generated, in the past direction λ will enter and remain within a compact set C. This means that there will be points q in C which are such that every small neighborhood of q is intersected by λ an infinite numbers of times. Let B be a normal coordinate ball about a limit point q. There will be points p and r on ∂B to the future and past of q which will be limit points of where λ intersects ∂B. It is easy to see that p and r must lie on a null geodesic segment γ through q. By repeating this construction about p and r, one can extend γ to a null geodesic without future or past end points, each point of which is a limit point for λ. Because λ enters and remains within C, γ must remain within C in both past and future directions. the set E consists of all such limit geodesics γ.

If γ is a closed null geodesic with $h<0$, then γ can be deformed to give a closed timelike curve λ to the past of γ.

Let $l^a=dx^a/dt$ be the future-directed vector tangent to γ and let a be defined by

$$l^a{}_{;b}l^b=al^a \ .$$

Then $a=(\epsilon+\bar{\epsilon})$, and so

$$\oint a\,dt=-h \ .$$

Let V^a be a future-directed timelike vector field normalized so that $l^aV^bg_{ab}=-1$. Then one can find a one-parameter family of curves $\gamma(t,u)$ such that

$$\gamma(t,0)=\gamma(t) \ ,$$

$$\frac{\partial x^a}{\partial t}=l^a \ ,$$

$$\frac{\partial x^a}{\partial u}=-xV^a \ ,$$

where x is a given function on γ. Then

$$\begin{aligned}\frac{\partial}{\partial u}(l^al^bg_{ab})&=-2xl^a{}_{;c}V^cl^bg_{ab}\\&=-2(xV^a)_{;c}l^cl^bg_{ab}\\&=-2(xV^al^bg_{ab})_{;c}l^c+2xV^al^b{}_{;c}l^cg_{ab}\\&=2\frac{\partial x}{\partial t}-2ax \ .\end{aligned}$$

Let

$$x=\exp\left[\int_0^t a\,dt+htb^{-1}\right] ,$$

where $b=\oint dt$. Then, for sufficiently small $v>0$, $\gamma(t,v)$ will be a closed timelike curve to the past of γ.

If the metric g is such that the Cauchy horizon $H^+(S)$ contains a closed null geodesic γ with $h>0$ and $f-|q|\neq 0$, then the property of having a closed null geodesic is stable; i.e., g will have a neighborhood U such that for any metric $g'\in U$, there will also be a closed null geodesic in the Cauchy horizon.

Let p be a point on γ. A point q in $I^-(p)$, the chronological past of p, will lie in the Cauchy development $D^+(S)$, and $J^-(p)\cap J^+(S)$, the intersection of the causal past of p with the causal future of S, will be compact. This means that a sufficiently small variation of g will leave q in the Cauchy develpment of S. On the other hand, because $h>0$, the previous result implies there is a closed timelike curve λ through a point r just to the future of p. A sufficiently small variation of the metric will leave λ a closed timelike curve and hence will leave r not in the Cauchy development. Thus the existence of a Cauchy horizon will be a stable property of the metric g. Similarly, the positions, directions, and derivatives of the generators will be continuous functions of the metric g in a neighborhood of γ.

Let W be a time like three-surface through p transverse to the Cauchy horizon. Then the generators of the horizon near γ define a map

$$\nu: W \cap D^+(S) \to W \cap D^+(S) \ ,$$

by mapping where they intersect W to where they intersect it again the next time round. If $f - |q| \neq 0$, the eigenvalues of $d\nu$ will be bounded away from 1. It then follows that the existence of a closed orbit is a stable property.

VI. CONCLUSIONS

As one approaches a closed null geodesic γ in the Cauchy horizon, the propagator will acquire extra singularities from null geodesics close to γ that almost return to the original point. In the Misner-space example in Sec. IV, these extra contributions came from the image charges under the boost. When one approached the Cauchy horizon, which corresponded to the past light cone of the origin in two-dimensional Minkowski space, these image charges became nearly null separated and their light cones became nearly on top of each other. It was therefore natural to find that the expectation value of the energy-momentum tensor diverged as one approached the Cauchy horizon.

If the boost h on going round γ is zero, the distance t from γ to any point to the past of γ in the Cauchy development will be infinite. This is rather like the fact that there is an infinite spatial distance to the horizon of a black hole with zero surface gravity. If the expectation value were of the form of Bt^{-4} with finite B, it would therefore be zero. Even if the energy-momentum tensor of individual fields did not have this form and still diverged on the Cauchy horizon, one might expect that the total energy-momentum tensor might vanish in a supersymmetric theory, because the contributions of bosonic and fermionic fields might have opposite signs. However, one would not expect such a cancellation unless the spacetime admitted a supersymmetry at least on the horizon. This would require that the tangent vector to the horizon corresponded to a Killing spinor, which would imply

$$\theta = \rho = \sigma = 0 \ ,$$

in addition to

$$h = 0 \ .$$

These conditions will not hold on a general horizon, but it is possible that the back reaction could drive the geometry to satisfy them, as the back reaction of black-hole evaporation can drive the surface gravity to zero in certain circumstances.

If one assumes that the expectation value of the energy-momentum tensor diverges on the horizon, one can ask what effect this would have if one fed it back into the field equations. On dimensional grounds one would expect the eigenvalues of the energy-momentum tensor to diverge as Bt^{-4}, where B is a constant that depends on the quantum state and t is the distance function to the horizon. However, because of boost and other factors, the energy density measured by an observer who crosses the Cauchy horizon on a timelike geodesic will go as $Bd^{-1}s^{-3}$, where s is proper distance along the observer's world line until the horizon and d is some typical length in the problem. In Misner space, d is the length of the spacelike geodesic from the origin orthogonal to the observer's world line.

To get the metric perturbation generated by this energy-momentum tensor, one has to integrate with respect to s twice. Thus the metric perturbation will diverge as $GBd^{-1}s^{-1}$. Kim and Thorne [5] agree that the metric perturbation diverges, but claim that quantum-gravitation effects might cut it off when the observer's proper time before crossing the Cauchy horizon, s, is the Planck time. This would give a metric perturbation of order

$$Bl_p d^{-1} \ .$$

If d were of order 1 m, the metric perturbation would be of order 10^{-35}. This is far less than about 10^{-19}, which is the best that can be detected with the most sensitive modern instruments.

It may be that quantum gravity introduces a cutoff at the Planck length. But one would not expect any cutoff to involve the observer-dependent time s. If there is a cutoff, one would expect it to occur when the invariant distance t from the Cauchy horizon was of order the Planck length. But t^2 is of order ds. So a cutoff in t at the Planck length would give a metric perturbation of order 1. This would completely change the spacetime and probably make it impossible to cross the Cauchy horizon. One would not therefore be able to use the region of closed timelike curves to travel back in time.

If the coefficient B is negative, the energy-momentum tensor will have a repulsive gravitational effect in the equation for the rate of change of the volume. This will tend to prevent the spacetime from developing a Cauchy horizon. The calculations that indicate B is negative therefore suggest that spacetime will resist being warped so that closed timelike curves appear. On the other hand, if B were positive, the graviational effect would be attractive, and the spacetime would develop a singularity, which would prevent one reaching a region of closed timelike curves. Either way, there seem to be theoretical reasons to believe the chronology protection conjecture: *The laws of physics prevent the appearance of closed timelike curves.*

There is also strong experimental evidence in favor of the conjecture from the fact that we have not been invaded by hordes of tourists from the future.

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