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Rotating cylinders and the possibility of global causality violation *

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In 1936 van Stockum solved the Einstein equations $G_{\mu\nu} = -8\pi T_{\mu\nu}$ for the gravitational field of a rapidly rotating infinite cylinder. It is shown that such a field violates causality, in the sense that it allows a closed timelike line to connect any two events in spacetime. This suggests that a finite rotating cylinder would also act as a time machine.

Since the work of Hawking and Penrose,[1] it has become accepted that classical general relativity predicts some sort of pathological behavior. However, the exact nature of the pathology is under intense debate at present, primarily because solutions to the field equations can be found which exhibit virtually any type of bizarre behavior.[2,3] It is thus of utmost importance to know what types of pathologies might be expected to occur in actual physical situations. One of these pathologies is causality violation, and in this paper I shall argue that if we make the assumptions concerning the behavior of matter and manifold usual in general relativity, then it should be possible in principle to set up an experiment in which this particular pathology could be observed.

Because general relativity is a local theory with no *a priori* restrictions on the global topology, causality violation can be introduced into solutions quite easily by injudicious choices of topology; for example, we could assume that the timelike coordinate in the Minkowski metric is periodic, or we could make wormhole identifications in Reissner-Nordström space.[4] In both of these cases the causality violation takes the form of closed timelike lines (CTL) which are not homotopic to zero, and these need cause no worries since they can be removed by reinterpreting the metric in a covering space (following Carter,[5] CTL removable by such means will be called trivial—others will be called nontrivial).

In 1949, however, Gödel[6] discovered a solution to the field equations with nonzero cosmological constant that contained nontrivial CTL. Still, it could be argued that the Gödel solution is without physical significance, since it corresponds to a rotating, stationary cosmology, whereas the actual universe is expanding and apparently nonrotating.

The low-angular-momentum Kerr field, on the other hand, cannot be claimed to be without physical relevance: It appears to be the unique final state of gravitational collapse,[7] and so Kerr black holes probably exist somewhere, possibly in the center of our galaxy.[8] This field also contains nontrivial CTL, though the region of causality violation is confined within an event horizon; causality violation from this source could never be observed by terrestrial physicists.[9] In addition, since the CTL must thread their way through a region near the singularity, it is quite possible that matter of a collapsing star will replace this region, as matter replaces the past horizon in the case of spherical collapse.[10] The final Kerr field with collapsed star could be causally well behaved, so the CTL pathology might still be eliminated from general relativity's physical solutions.

I doubt this, because nontrivial causality violation also occurs in the field generated by a rapidly rotating infinite cylinder.

The field of such a cylinder in which the centrifugal forces are balanced by gravitational attraction was discovered by van Stockum in 1936.[11] The metric is expressed in Weyl-Papapetrou form:

$$ds^2 = H(dr^2 + dz^2) + Ld\varphi^2 + 2M\,d\varphi\,dt - F dt^2 , \quad (1)$$

where z measures distance along the cylinder axis, r is the radial distance from the axis, φ is the angle coordinate, and t is required to be timelike at $r=0$. ($-\infty < z < \infty$, $0 < r < \infty$, $0 \leq \varphi \leq 2\pi$, $-\infty < t < \infty$.) The metric tensor is a function of r alone, and the coordinate condition $FL + M^2 = r^2$ has been imposed (units $G = c = 1$).

It is clear that since $g = \det g_{\mu\nu} = -r^2H^2$ is negative, the metric signature is $(+++-)$ for all $r > 0$, provided $H \neq 0$. van Stockum assumes the Einstein equations

$$G^{\mu}{}_{\nu} = -8\pi T^{\mu}{}_{\nu}$$
$$= -8\pi\rho \frac{dx^{\mu}}{ds} \frac{dx_{\nu}}{ds} ,$$

where ρ is the particle mass density. Also

$$\frac{dr}{ds} = \frac{dz}{ds} = 0 ,$$

$$\frac{d\varphi}{ds} \Big/ \frac{dt}{ds} = \text{constant} ,$$

$$T = T^{\mu}{}_{\mu} = -\rho$$