

Demonstration of universal time-reversal for qubit processes

P. SCHIANSKY,^{1,3,†}  T. STRÖMBERG,^{1,†} D. TRILLO,² V. SAGGIO,¹ B. DIVE,² M. NAVASCUÉS,² AND P. WALTHER^{1,4} 

¹University of Vienna, Faculty of Physics & Research Network Quantum & Aspects of Space Time (TURIS), Boltzmannngasse 5, 1090 Vienna, Austria

²Institute for Quantum Optics and Quantum Information, Boltzmannngasse 3, 1090 Vienna, Austria

³e-mail: peter.schiansky@univie.ac.at

⁴e-mail: philip.walther@univie.ac.at

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In quantum mechanics, the unitary nature of time evolution makes it intrinsically reversible, given control over the system in question. Remarkably, there have been several recent demonstrations of protocols for reverting unknown unitaries in scenarios where even the interactions with the target system are unknown. These protocols are limited by their probabilistic nature, raising the fundamental question of whether time-reversal could be performed deterministically. Here we show that quantum physics indeed allows for this by exploiting the non-commuting nature of quantum operators, and demonstrate a recursive protocol for two-level quantum systems with an arbitrarily high probability of success. Using a photonic platform, we achieve an average rewinding fidelity of over 95%. Our protocol, requiring no knowledge of the quantum process to be rewound, is optimal in its running time, and brings quantum rewinding into a regime of practical relevance.

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1. INTRODUCTION

In the macroscopic world, there is an apparent unidirectionality of processes in time, which stands in contrast to the time-reversal symmetric nature of the underlying laws of physics. This tension was first pointed out by Eddington, who coined the term “arrow of time” to describe the asymmetry [1]. In classical physics, an arrow of time emerges through the second law of thermodynamics, giving rise to processes that cannot be reversed [2]. Due to the statistical nature of the law, and the determinism of classical physics, the irreversibility is not fundamental. Indeed, for classical wave mechanics, it is well known that the time evolution of a system can be reversed without any knowledge of the dynamics through a technique called phase conjugation [3,4]. In the microscopic quantum realm, however, the ability to perform phase conjugation becomes limited by fundamental quantum noise [5], due to the non-unitary nature of the process. It has therefore remained an open question whether or not the dynamics of quantum systems can be reversed in a universal manner.

Recently, there have been several works addressing this question, in which probabilistic protocols for “rewinding” quantum systems were presented [6,7] and demonstrated in a laboratory setting [8,9]. These protocols work independently of both the free Hamiltonian guiding the time evolution of the system in question, and the system's interaction with the experimental apparatus. A major drawback of the protocols in [7] is that they suffer from low success probabilities, typically of the order of 10^{-3} . The scheme

in [6], on the other hand, allows for a form of error correction, whereby the protocol can be repeated when it fails. However, it is not known whether these feed-forward corrections can boost the success probability arbitrarily close to one. Moreover, the protocol cannot rewind a target system in “real time,” instead taking three units of time for every one rewind.

The teleportation based protocol in [10,11], as well as more traditional methods to rewind a quantum system with an unknown free Hamiltonian, such as the refocusing techniques used in nuclear magnetic resonance [12], require the ability to implement controlled operations that are specifically tailored to the target quantum system, and are therefore not universal. The work of [13] combines both quantum theory and general relativity to devise a “time translator,” capable of rewinding or fast-forwarding quantum systems. While this method can time-translate any quantum system, it has two drawbacks: (1) it works only approximately, and under a restriction on the free Hamiltonian of the target; (2) if we demand reasonable precision, the probability of success of the process becomes astronomically small.

In this paper, we demonstrate a novel universal time-reversal protocol (Fig. 1) for which the success probability can be made arbitrarily high, making it, in effect, deterministic. At its heart, the protocol is based on the non-commutativity of quantum operators, a core concept in quantum mechanics. This conceptual simplicity, which translates directly into a straight-forward implementation in the laboratory based on the recently developed

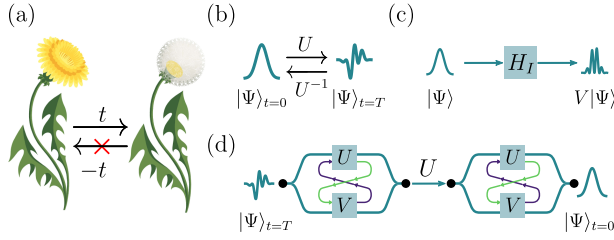


Fig. 1. (a) In the classical world, there is an unmistakable directionality to time, illustrated here through the process of aging, a process that cannot be reversed in practice despite its deterministic nature. In this paper, we show that these same limitations do not apply in the quantum realm. (b) The unitarity of quantum mechanics guarantees that an inverse of a given time evolution U always exists, even though it may be unknown. (c) By letting a target quantum system pass through an interaction region, a perturbed time evolution V can be realized. (d) A quantum switch makes the target system evolve in a superposition of its free evolution U and perturbed evolution V . This superposition of time evolutions can be used to “rewind” the system backwards in time, without requiring any knowledge about either U , V or the state $|\Psi\rangle$.

quantum switch [14,15], allows us to overcome the limitations of previous proposals. The quantum switch is a process that, in its simplest form, transforms two unitaries into a controlled superposition of the two gates being applied in different orders: $(U, V) \rightarrow UV \otimes |0\rangle\langle 0| + VU \otimes |1\rangle\langle 1|_C$.

In this work, the utilization of quantum switches allows us to time-translate the unknown internal degree of freedom of a target system by setting it on a superposition of different trajectories. For some of these trajectories, the free evolution U of the target’s internal degree of freedom is perturbed by an unknown but repeatable interaction, which induces an evolution V on the target. This perturbation can be achieved by any physical interaction and thus can be applied to every possible quantum system. We make these trajectories sequentially interfere in such a way that the final state of the target’s internal degree of freedom is propagated by U^{-n} , for some positive n , independently of the operators U , V . Each quantum switch requires a projection of the target system’s path degree of freedom to induce the desired superposition of time evolutions. An advantage of our scheme is that even in the event that the projection fails, a simple and repeatable error-correction procedure can be applied, yielding an arbitrarily high success rate, as long as $[U, V] \neq 0$. It is also worth emphasizing that the protocol runs in real time, meaning that the time it takes to rewind the system is equal to the amount of time to be rewound, aside from a bounded overhead.

We demonstrate the universality of our protocol by running it on a large set of different time evolutions. Our demonstration utilizes a quantum photonics platform with control of path and polarization degrees of freedom of single photons. We generate a discrete time evolution of a single photon by implementing a “polarization Hamiltonian” using a combination of half- and quarter-wave plates. A superposition of time evolutions is achieved via an interferometric quantum switch in which the propagation direction defines the order of the evolutions U and V . Our setup uses two fast optical switches that allow the quantum switch to be accessed several times.

2. METHODS

A. Protocol

In this section, we will give a description of how the rewinding protocol works in a photonic setting, the basic steps of which are illustrated in Fig. 2. An alternative formulation using a scattering scenario is given in Supplement 1. A full description, as well as the accompanying proofs, can be found in [16]. Given an unknown target system $|\Psi\rangle$, whose time evolution is described by $U = e^{-i\Delta T H_0}$, where H_0 is an unknown Hamiltonian, our goal will be to rewind the system: $|\Psi(t = n\Delta T)\rangle \rightarrow |\Psi(t = 0)\rangle$, where n is the number of discrete time steps to be rewound. The basis of our protocol is the following identity [7]:

$$[U, V]U^n[U, V] \propto U^{-n}. \quad (1)$$

Here U , V are any 2×2 matrices, with U being invertible. When the matrix U describes the time evolution of a system, we see that an experimenter able to implement a commutator can reverse the time evolution, even if U is unknown. The basic protocol is thus as follows: apply the commutator between the time evolution operator U and any other 2×2 matrix V , let the system evolve freely for the amount of time to be rewound, then apply the commutator again. The matrix V represents, in the general setting, a time evolution that is perturbed by any repeatable means, for example, by bringing the target on a trajectory that leads it through some interaction region. This perturbed evolution can also remain unknown; however, the magnitude of the commutator $[U, V]$ affects the success probability of a single-step attempt to rewind the system.

In a photonic setting, a commutator can be realized using a quantum switch acting on two degrees of freedom of a single photon. The control qubit, defining the order of gate operations, is encoded in the photon’s path, while the target qubit is encoded in the polarization. The two possible gate orders, UV and VU , are superposed by initializing the control qubit in the superposition state $(|0\rangle_C + |1\rangle_C)/\sqrt{2}$ and then applying a controlled operation between the control and target systems [15]:

$$|0\rangle_C \otimes |\Psi\rangle_T \rightarrow \frac{|0\rangle_C + |1\rangle_C}{\sqrt{2}} \otimes |\Psi\rangle_T \rightarrow \frac{1}{\sqrt{2}} \left[|0\rangle_C \otimes UV|\Psi\rangle_T + |1\rangle_C \otimes VU|\Psi\rangle_T \right]. \quad (2)$$

By applying a Hadamard gate to the control qubit, one obtains the following state:

$$|0\rangle_C \otimes \frac{1}{2} \{U, V\}|\Psi\rangle_T + |1\rangle_C \otimes \frac{1}{2} [U, V]|\Psi\rangle_T. \quad (3)$$

A measurement of the control qubit now projects the target state onto either the commutator or the anticommutator, where the latter is denoted by $\{\cdot\}$. If the measurement outcome of the control qubit is $|0\rangle$, the anticommutator is applied by the quantum switch, but the protocol does not necessarily fail. Instead, the following matrix identities can be used to correct the error:

$$\{U, V\}^m [U, V] \{U, V\}^m \propto [U, V], \quad (4)$$

$$[U, V]^2 \propto \mathbb{1}. \quad (5)$$

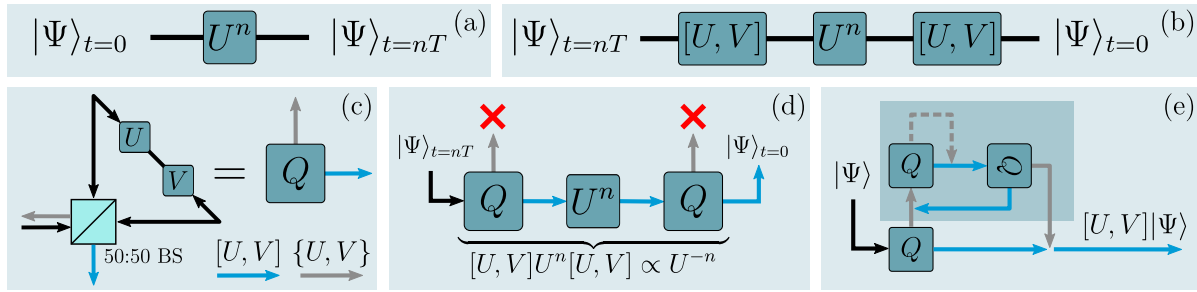


Fig. 2. Interferometric protocol. (a) Circuit diagram for the free evolution U^n of a target quantum from time $t = 0$ to $t = nT$. (b) Symbolic circuit diagram for the rewinding protocol. The target quantum system is made to propagate backwards in time by way of the identity: $U^{-1} \propto [U, V]U[U, V]$, where V is a perturbed evolution. (c) Q is a quantum switch, pictured here as a Sagnac interferometer, and acts as the basic building block of the interferometric scheme, probabilistically applying the commutator $[U, V]$. (d) A full interferometric implementation of the single-step protocol, which succeeds whenever the photon, in which the target state $|\Psi\rangle$ is encoded, exits both quantum switches in the commutator port (blue arrow). Detecting a photon in the anticommutator port (gray arrow) heralds a failure of the quantum switch. (e) Adaptive error correction for achieving an arbitrarily high probability of success. This entire diagram replaces a single quantum switch in (d). Instead of detecting the failure mode of the quantum switch, the photon is made to re-interfere with itself. Whenever it exits in the bottom right, the commutator $[U, V]$ will have been applied (see [16]) The dashed path represents recursive applications of the diagram, through which the success probability can be made arbitrarily high, while the darker shaded area indicates the additional quantum switches needed.

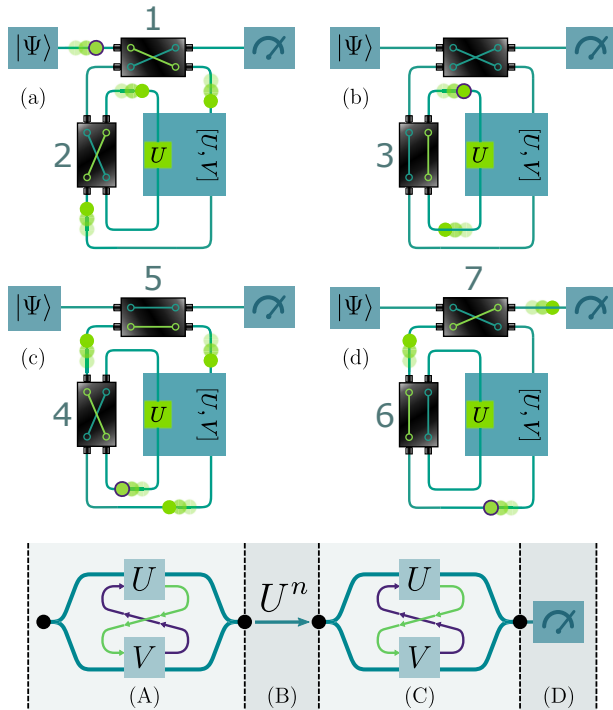


Fig. 3. Active photon routing. The use of electro-optical (EO) switches enables active routing of a single photon (green dot) encoding the target quantum state. The settings of the EO-switches determine whether the photon passes through the quantum switch, evolves freely, or is sent to a detector. Sub-diagrams (a)–(d) indicate the states of EO-switches at different steps of the protocol (A)–(D), illustrated at the bottom of the figure. The numbers index the order in which the EO-switches are traversed. In each sub-diagram, the photon's initial position is indicated by a contour; the subsequent dots represent the photon at a slightly later time, and the green trace shows the photon path through a given switch. (a) The photon passes through the quantum switch for the first time. (b) For $n \geq 2$, the photon is trapped in a loop until the free time-evolution operator U has been applied a total of n times. (c) The photon passes through the quantum switch a second time. (d) The EO-switches direct the photon to a quantum tomography stage.

Through recursive application of these identities, an anticommutator can always be turned into a commutator. This process can be described using a virtual road map, illustrated in Fig. 2(e). In [16], some of us prove that when U, V are unitary and $[U, V] \neq 0$, the protocol always terminates in a finite number of steps. Note that for random U, V , the probability of the commutator vanishing is zero. We also point out that Eqs. (1), (4), and (5) hold even for non-unitary matrices. Remarkably, the protocol can thus be used to rewind, for example, a two-level system undergoing a continuous decay governed by a non-Hermitian Hamiltonian.

From Eq. (1), it can be seen that to rewind a free evolution of time T , our protocol runs for $T + O(1)$ units of time, which is asymptotically optimal [7], and where the $O(1)$ term accounts for the constant overhead introduced by the adaptive error correction. In comparison, the protocol demonstrated in [8,9] takes $3T + O(1)$ units of time for the same task, making our protocol superior not only in terms of success probability, but also in terms of running time, at the cost of requiring coherent control over the time evolution.

The above description of the protocol involves placing the target quantum system in a spatial superposition; however, we note that the alternative, but equivalent, description of the protocol provided in Supplement 1 does not require this.

B. Experiment

The rewinding protocol described in the previous section is applied to a qubit state encoded in the polarization degree of freedom of a single photon, while the path degree of freedom of the same photon is used at two points to encode a second qubit that acts as a control system, thereby enabling the application of a commutator through a controlled unitary inside a quantum switch. The photons are generated using spontaneous parametric downconversion (SPDC). The SPDC process produces pairs of photons denoted signal and idler, the former of which is sent straight to a detector and is used to herald the presence of the idler photon. Upon such a heralding event, a trigger signal is transmitted in optical fiber to a field programmable gate array (FPGA) controlling two active electro-optical (EO) switches to permit the idler photon to pass through parts of the setup multiple times. The active routing of

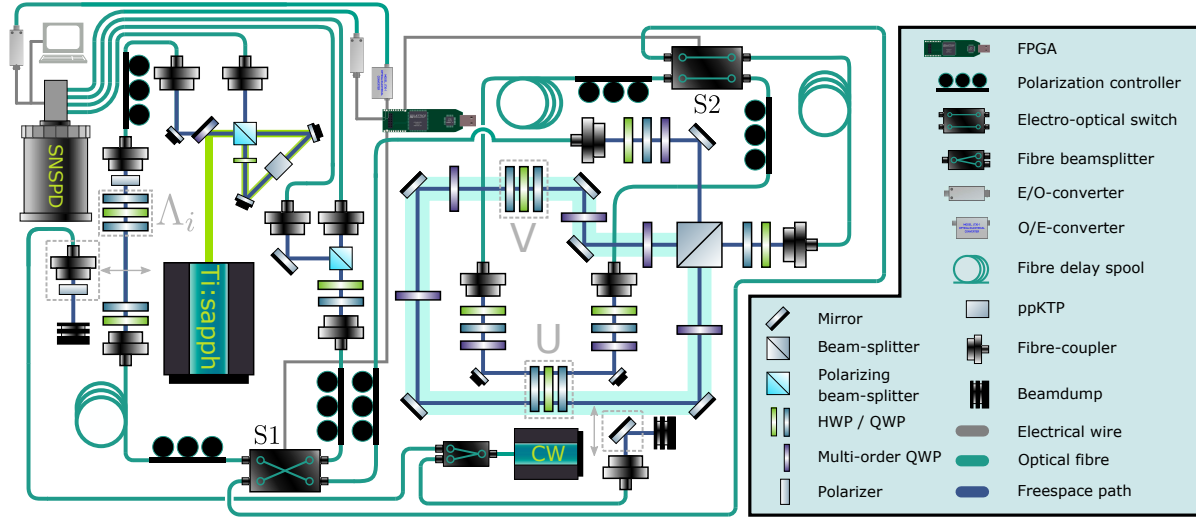


Fig. 4. Experimental setup. A pulsed Ti:sapphire laser pumps a spontaneous parametric downconversion source to generate pairs of single photons in a type-II process using a ppKTP crystal (top left). The signal photon is directed to the high efficiency superconducting nanowire single-photon detectors (SNSPDs), and a successful detection event triggers an FPGA to initiate a pulse sequence for the EO-switches (see Fig. 3). The approximately 400 ns rise time of the EO-switches is compensated for by three fiber spools, each around 100 m long, adding the needed optical delay. The target state, encoded in the idler photon, is initialized using a state-preparation stage after which it is sent to the quantum switch, realized using a free-space Sagnac interferometer (highlighted in blue). The unitaries U and V are implemented using a combination of half- and quarter-wave plates. Additional wave plates are used in conjunction with the fiber polarization controllers to compensate for unwanted polarization rotations induced by the fibers and mirrors. Two additional fiber couplers placed inside the Sagnac allow the photons to propagate through U separately. A tomography stage at the output of EO-switch S1 is used to measure the photons' polarization. The CW laser is used during the pre-measurement polarization compensation procedure.

the photons by the EO-switches is shown in Fig. 3, while a detailed schematic of the setup is displayed in Fig. 4.

The unitary Λ_i initializes the idler photon into the polarization state $|\Psi_i\rangle$ chosen from a tomographically complete set, after which an EO-switch (S1) routes the photon into the quantum switch. The unitaries V and U inside the quantum switch are implemented using two sets of three wave plates [17]. Note that there is only one physical realization of U and V , and they could thus in principle remain unknown without compromising the protocol. Depending on whether the photon exits in the backwards or forwards propagating port of the interferometer, either $[U, V]$ or $\{U, V\}$ is applied. The backpropagating port corresponding to $\{U, V\}$ [Fig. 2(c)] is disregarded in our implementation, and no adaptive error correction is applied, but photons exiting in this port could be used to increase the success probability of the protocol. Any photon leaving the interferometer in the forward propagating direction passes through a second EO-switch (S2), which traps the photon in a loop, allowing it to propagate through U a total of n times. Upon exiting the loop, the photon is directed back to S1, which sends the photon through the quantum switch a second time where $[U, V]$ is probabilistically applied once more. Finally, the photon is routed to a quantum tomography stage by S1, where its polarization is measured, post-selecting on successful application of the commutators. These measurements are then used to reconstruct the density matrix ρ . In a successful run of the experiment, the state $U^{-n}|\Psi_i\rangle$ is recorded.

3. RESULTS

To demonstrate that the performance of the protocol is independent of the initial state $|\Psi_i\rangle$, the free evolution U , the perturbed evolution V , and the number of time steps n , a large set of combinations of these parameters was realized. More specifically, the unitary

operators U and V were chosen from the set

$$U_p = e^{-i\arcsin(\alpha)\sigma_z}, \quad \alpha = \frac{p}{10}, \quad p = 1, \dots, 10,$$

$$V_q = \cos(\theta)\sigma_z + \sin(\theta)\sigma_y, \quad \theta = \frac{q\pi}{11}, \quad q = 1, \dots, 10.$$

Depending on the choice of p and q , the degree to which the operators U, V commute changes. As a measure of the commutativity, we define

$$N_c = 1 - \left(\frac{\| [U, V] \|_2}{2} \right)^2, \quad (6)$$

which is normalized to be one when the unitaries are fully commuting and is equal to the probability of applying the commutator in a single step. For our implementation, we select 50 pairs of U, V for which $N_c \leq 0.9$. Choosing a finite set of unitaries generated by fixed Hamiltonians, rather than sampling them randomly, allows us to compare our results to those of a classical experimenter given access to the same resources, but who cannot implement quantum superpositions of time evolutions. The optimal strategy for such a classical experimenter is discussed in Supplement 1. While polarization rotations implemented by wave plates alone are in general not invariant under counterpropagation, the specific set above is since it does not contain any σ_x terms. As was recently shown in [18], the restriction on the set of implementable unitaries in a Sagnac quantum switch can be lifted if one includes time-reversal symmetry breaking elements, making the implementation equivalent to previous Mach-Zehnder based quantum switches.

To benchmark the fidelity of the protocol, we ran it on the four input states $\{|H\rangle, |+\rangle, |-\rangle, |R\rangle\}$, corresponding to horizontally, diagonally, anti-diagonally, and right-handed circularly polarized light, respectively. This was independently repeated

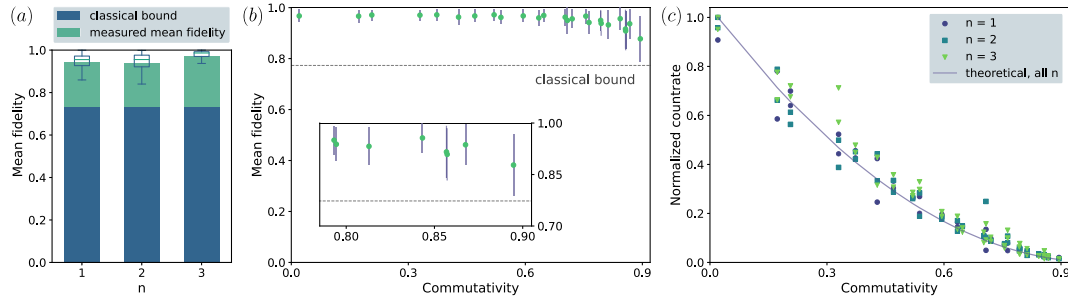


Fig. 5. Experimental results. (a) Experimental state fidelities. Each bar shows the measured state fidelity averaged over all 50 pairs of U , V , the four different input states, and three independent experimental runs, giving a total of 600 different reconstructed density matrices for each value of n . The combined measurement time for all n was approximately three weeks. The exact fidelities are $\mathcal{F}_1 = (0.94234 \pm 0.00023)$, $\mathcal{F}_2 = (0.93803 \pm 0.00041)$, $\mathcal{F}_3 = (0.97336 \pm 0.00043)$, with an average of $\mathcal{F}_m = (0.95129 \pm 0.00021)$. The superimposed box plot indicates the median and spread of the fidelities for each n . The higher fidelity for $n = 3$ can be attributed to higher polarization contrast in the setup (see Supplement 1). The dark blue bars show the highest theoretical fidelity for an experimenter unable to implement superpositions of time evolutions. (b) Fidelities to $U^{-n}|\Psi_i\rangle$ as a function of the commutativity (N_c). For a given N_c , the plotted fidelity is averaged over all runs and input states, with a total of 72 samples per point as several pairs of unitaries commute to the same degree. The error bars show the standard deviations of the fidelities, and not the uncertainty in the estimated mean fidelity, which is too small to be visible. At high commutativity, the experiment becomes more sensitive to several noise sources, such as detector dark counts, background photons, and the leakage of the interferometer due to finite visibility, whereas in the regime of high commutativity, the fidelity is limited by constant effects such as finite polarization contrast through the setup. (c) Commutativity (N_c) versus the normalized total event count rate for all implementations of V and U . Count rates are normalized to the maximal event rate separately for each n (to account for additional losses at higher n). The rates are averaged over all four input states and all three runs. $n = 1$ (circles), $n = 2$ (rectangles), $n = 3$ (triangles). Error bars (Poissonian standard deviation) are too small to be visible. The theoretically ideal behavior, depicted by the solid line, is given by N_c^2 and has a quadratic behavior due to the commutator being applied twice. The biggest deviation from the overall good agreement to the theory appears in the central region of the curve, where the interferometer has a higher sensitivity to noise.

three times for all 50 choices of time evolutions, and for three different sizes of time steps ($n = 1, 2, 3$), yielding a total of 1800 experimental runs with a combined measurement time of more than 500 h. In each experimental run, full quantum state tomography was performed on the output states ρ , and the fidelity $\langle \Psi_i | U^n \rho U^{-n} | \Psi_i \rangle$ was calculated. The density matrices of the output states were reconstructed using a maximum likelihood fit [19], and a background contribution originating from the detector dark counts was accounted for using a Monte Carlo simulation, which is how the uncertainties in the fidelities were calculated (see Supplement 1). The average fidelities for $n = 1, 2, 3$ were $\mathcal{F}_1 = (0.94234 \pm 0.00023)$, $\mathcal{F}_2 = (0.93803 \pm 0.00041)$, $\mathcal{F}_3 = (0.97336 \pm 0.00043)$. These fidelities, along with the classical bound, are shown in Fig. 5(a); it can be seen that the quantum protocol clearly outperforms the classical strategy, achieving a high fidelity independent of the length of the time evolution.

In our implementation, the fidelity of the final state is not fully independent of the choice of U , V . This is due to the fact that for pairs of unitaries that almost commute, photons are most likely to exit in the anticommutator port of the interferometer, which in turn makes the protocol more sensitive to experimental imperfections such as finite interferometric visibility and detector dark counts. In Fig. 5(b), the relationship between the degree of commutativity N_c and fidelity is illustrated. The mean fidelity stays at high levels over a broad range of N_c ; only when the degree of commutativity approaches 0.9 can a small drop in the fidelity be seen.

Since it is expected that the event rate will drop with increasing values of N_c , we verify that our setup produces the correct scaling by comparing N_c to our normalized detected photon rate, separately for each n . The comparison is visualized in Fig. 5(c) where good agreement between relative rate and degree of commutativity can be seen. We attribute the undesired variance in rate to imperfect polarization compensation inside the Sagnac interferometer, as

well as phase shifts originating from slight interferometer misalignment. The largest variance is seen in the neighbourhood around $N_c = 0.5$, where the sensitivity to phase noise is highest, due to the sinusoidal relationship between phase and output intensity in an interferometer.

4. DISCUSSION

In this work, we have demonstrated a universal time-rewinding protocol for two-level quantum systems. Unlike previously proposed protocols, ours can reach an arbitrarily high probability of success and is asymptotically optimal in the time required to perform the rewinding, answering the question of whether or not such processes are permitted by the laws of quantum mechanics. Remarkably, the experimenter performing the rewinding does not need any knowledge about the target quantum system, its internal dynamics, or even the specifics of the perturbed evolution. The optimality of the protocol is demonstrated in our implementation, where the total elapsed time (equivalent to the number of applications of U) grows linearly with the length of time to be rewound, with an optimal proportionality constant of one. We find that the experimental quantum protocol significantly outperforms the optimal classical strategy in terms of the resulting state fidelity.

We emphasize that our results are in principle not restricted to photonic quantum systems, since the concepts used do not make any assumptions about the physical system the protocol is applied to. We note that, while experiments using cold atom interferometers have demonstrated the necessary building blocks for the protocol [20], implementations utilizing massive particles would still likely prove challenging. In contrast, our photonic implementation offers a particularly simple and robust approach that utilizes a mature technological platform, in particular for implementing the commutator of the time evolutions through a quantum switch. Given the recent progress in integrated quantum photonics [21,22], we envision that fully monolithic architectures

capable of higher fidelity operations will facilitate demonstrations of the active error correction [Fig. 2(e)] in the near future. Additional follow-up investigations could include non-optical implementations of the protocol as well as extensions to higher dimensions, as described in [7].

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Disclosures. The authors declare no conflicts of interest.

Data availability. All data used in the manuscript are made publicly available in Ref. [23].

Supplemental document. See Supplement 1 for supporting content.

*These authors contributed equally to this paper.

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