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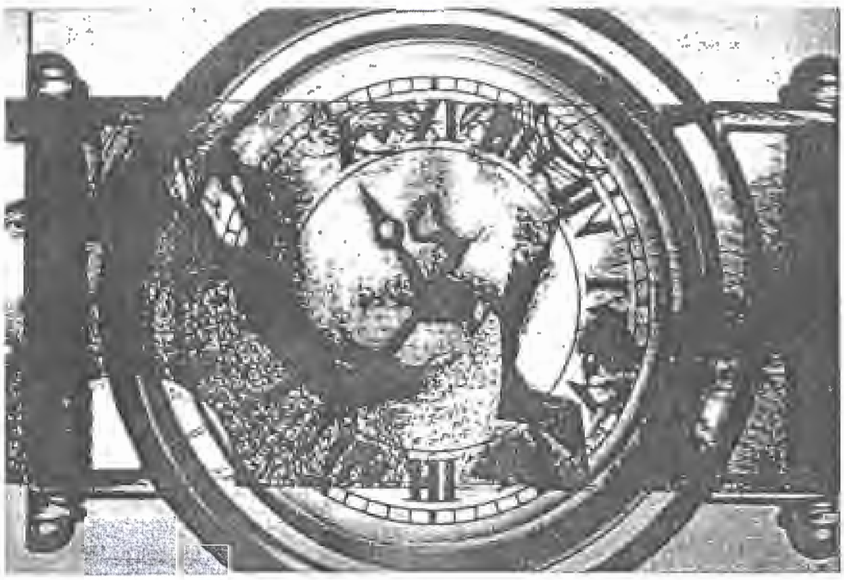

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Clocks and General Relativity

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Abstract

The basic role of the hypothesis of locality in the theory of relativity is discussed. A consequence of this assumption is the accelerated clock hypothesis (ACH). The limitations of ACH are investigated and compared with experimental data. The possibility of using highly accurate clocks to test various aspects of general relativity is emphasized.

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1 Introduction

General relativity (GR) is the most successful relativistic theory of the gravitational field inasmuch as it is in agreement with all available observational data. The basic elements of this theory may be summarized as follows.

Lorentz Invariance connects the measurements of ideal inertial observers in Minkowski spacetime. All actual observers are accelerated. It is therefore necessary to extend physical laws to accelerated observers; this is done in relativity theory via the hypothesis of locality.

The Hypothesis of Locality states that an accelerated observer in Minkowski spacetime is at each instant equivalent to a momentarily comoving inertial observer. *Standard* measuring devices are defined to conform with this assumption; thus a standard clock measures proper time. The laws of physics can be pointwise extended to all observers in Minkowski spacetime via the hypothesis of locality. To extend these laws further to observers in gravitational fields, Einstein's principle of equivalence is indispensable.

Einstein's Principle of Equivalence postulates a pointwise equivalence between an accelerated observer in Minkowski spacetime and an observer in a gravitational field. This cornerstone of GR is based on the equivalence of inertial and gravitational masses. Einstein's principle of equivalence *together with the hypothesis of locality* implies that an observer in a gravitational field is pointwise inertial. The simplest way to connect such local inertial frames is to assume a curved spacetime manifold whose Riemannian curvature is the gravitational field.

The Gravitational Field Equations must connect the spacetime curvature with the energy-momentum tensor of matter and fields, thereby generating a natural generalization of Newtonian gravitation. The simplest possibility is provided by the Einstein field equations of GR [1].

The purpose of this brief description of the foundations of GR has been to place the hypothesis of locality in its proper context in the hierarchy of notions that underlie GR.

2 Accelerated Clock Hypothesis

The hypothesis of locality has its origin in Newtonian mechanics: the two observers in this hypothesis have the same *state* (i.e., position and velocity). The ACH thus refers to this local immateriality of acceleration for standard clocks. However, a realistic accelerated measuring device ("clock") is affected by inertial effects that can be neglected if they do not integrate to any measurable influence over the length and time scales characteristic of the measurement.

It is permissible to replace the curved worldline of an accelerated observer at each instant by its tangent if the intrinsic length scale of the phenomenon under observation (λ) is negligible compared to the acceleration length (L).

Here λ could be the wavelength of electromagnetic radiation or the Compton wavelength of a particle, and L is the natural length that can be formed from the acceleration and the speed of light in vacuum c; thus, $L = c^2/g$ for translational acceleration g, while $L = c/\Omega$ for rotation of frequency Ω. The ACH is exactly valid if $\lambda/L = 0$, i.e. either $\lambda = 0$, so that the phenomena could be expressed in terms of pointlike *coincidences*, or $L = \infty$, so that $g = 0$ and $\Omega = 0$ and hence the observer is inertial. The deviation from the ACH is thus expected to be proportional to λ/L. In practice, such effects are very small; for the Earth, $c^2/g \approx 1$ lyr and $c/\Omega \approx 28$ AU. A detailed analysis reveals that the deviations under consideration here have been negligibly small in all experiments performed thus far that have searched for a direct dependence of clock rate upon acceleration [2].

To illustrate these ideas, let us imagine an observer rotating uniformly with frequency Ω about an axis and a plane monochromatic electromagnetic wave of frequency ω and definite helicity propagating along the axis of rotation. If the observer is assumed to be instantaneously inertial according to the hypothesis of locality, then $\omega' = \gamma\omega$ by the transverse Doppler effect. However, the electromagnetic field appears to rotate with frequency $\omega - \Omega$ or $\omega + \Omega$ about the direction of propagation depending on whether the wave has positive or negative helicity, respectively. A detailed treatment reveals that $\omega' = \gamma(\omega \mp \Omega) = \gamma\omega(1 \mp \Omega/\omega)$, where $\Omega/\omega = \lambda/(2\pi L)$ with $L = c/\Omega$. A peculiar aspect of this phenomenon is that the wave can stand completely still with respect to the rotating observer (i.e., $\omega' = 0$ for positive helicity waves with $\omega = \Omega$). In terms of energy $E' = \gamma(E \mp \hbar\Omega)$, where the "interaction" term is due to the coupling of helicity with rotation. This electromagnetic effect has yet to be observed; however, a similar spin-rotation coupling for spin $\frac{1}{2}$ particles with $H = -\sigma \cdot \Omega$ has been verified experimentally [3]. Such spin-dependent interactions were investigated by Wineland and Ramsey in 1972 [4]. The effect due to Earth's rotation is very small, $\hbar\Omega \sim 10^{-19}$ eV; nevertheless, the coupling of intrinsic spin to the rotation of the Earth has been detected recently via the experiments of Wineland *et al.* [5] and Venema *et al.* [6].

The natural way to think about such effects is to assume that matter waves propagate with respect to an underlying Minkowski spacetime and hence spin keeps its aspect with respect to the inertial frame. From the viewpoint of the rotating observer, the spin would then be precessing in the opposite sense and this apparent motion is expressed in quantum mechanics by the spin-rotation Hamiltonian. In a similar way, one expects that intrinsic spin should precess in the gravitomagnetic field of the Earth just like a GP-B gyroscope, and this spin-rotation-gravity coupling is expressed by $H = -\sigma \cdot \Omega + \sigma \cdot \Omega_{\rm D}$, where $\Omega_{\rm D}$ is the dragging frequency of the local inertial frames and $\hbar\Omega_D \sim 10^{-29}$ eV for the Earth. The dragging frequency is position-dependent; therefore, the spinning particle is subject to a force. This gravitomagnetic Stern-Gerlach force violates the principle of equivalence, since the weight of a particle with spin up would in general be different from its weight with spin down [3]. The violation is proportional to the ratio of the Compton wavelength of the particle and $L = c/\Omega$; for a laboratory test with neutrons or protons, this ratio is extremely small ($\sim 10^{-28}$). One can only hope that such a basic relativistic quantum gravity effect may become measurable in future via improvements in spin-rotation experiments [5, 6] or atom interferometry [7].

3 Standard Clocks and Gravitomagnetism

The spacetime interval contains many aspects of the gravitational field that could be studied using standard clocks; in this connection, a gravitomagnetic effect [8] that is briefly described below is of particular interest.

Imagine a clock in a circular equatorial orbit about a rotating mass. Let τ_+ (τ_-) be the period of this geodesic motion in the same (opposite) sense as the rotation of the mass. If the orbital radius r is much larger than the gravitational radius of the body, $r \gg 2GM/c^2$, then it can be shown that

$$\tau_+ - \tau_- \approx 4\pi \frac{J}{Mc^2} \quad , \tag{1}$$

where the quadrupole and higher moments of the central body (of mass M and angular momentum J) have been neglected. The gravitational effect under consideration becomes independent of the coupling constant G in this approximation; this situation can come about as a result of integrating a small quantity over a long interval. Absence of G in equation(1) indicates that the effect might be "large". Moreover, the effect is independent of orbital radius r in this approximation; in fact, this gravitomagnetic effect is reminiscent of the topological Aharonov-Bohm effect. For Earth orbits, $\tau_+ - \tau_- \approx 2 \times 10^{-7}$ sec. The effect comes about as a result of a coupling of the *azimuthal* orbital motion with the rotation of the central body; thus, the effect vanishes for a polar orbit.

The influence of the gravitomagnetic potential on clock synchronization via light signals has been the subject of investigations by Cohen, Rosenblum, and coworkers [9]. They considered the "synchronization gap", which turns out to be essentially equivalent to the difference in the time that it would take for the rays of light to traverse a path all around a rotating mass in opposite directions [8]. Such experiments in the solar system have been discussed by Davies and Lass [10]. The effect is smaller than in equation (1) by a factor that is proportional to the gravitoelectric potential $\Phi = GM/(c^2 r)$, which is $< 10^{-9}$ for the Earth.

References

[1] A. Einstein, *The Meaning of Relativity* (Princeton University Press, 1955).

[2] D. Newman *et al.*, Phys. Rev. Lett. **40**, 1355 (1978).

[3] B. Mashhoon, Phys. Lett. A **198**, 9 (1995); Phys. Rev. A **47**, 4498 (1993); Phys. Rev. Lett. **61**, 2639 (1988).

[4] D. J. Wineland and N. F. Ramsey, Phys. Rev. A **5**, 821 (1972).

[5] D. J. Wineland *et al.*, Phys. Rev. Lett. **67**, 1735 (1991).

[6] B. J. Venema *et al.*, Phys. Rev. Lett. **68**, 135 (1992).

[7] M. Kasevich and S. Chu, Appl. Phys. B **54**, 321 (1992).

[8] J. M. Cohen and B. Mashhoon, Phys. Lett. A **181**, 353 (1993).

[9] J. M. Cohen, A. Rosenblum and Y. Clifton, Phys. Lett. A **131**, 163 (1988); J. M. Cohen, H. E. Moses and A. Rosenblum, Class. Quantum Grav. **1**, L57 (1984).

[10] R. W. Davies, in: *Experimental Gravitation*, ed. B. Bertotti (Academic Press, 1974) p. 405; R. W. Davies and H. Lass, JPL Technical Memorandum no. 58 (1970).