Time-Like Extra Dimension and Cosmological Constant in Brane Models

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Abstract

We discuss the general models with one time-like extra dimension and parallel 3-branes on the space-time $M^4 \times M^1$. We also construct the general brane models or networks with n space-like and m time-like extra dimensions and with constant bulk cosmological constant on the space-time $M^4 \times (M^1)^{n+m}$, and point out that there exist two kinds of models with zero bulk cosmological constant: for static solutions, we have to introduce time-like and space-like extra dimensions, and for non-static solutions, we can obtain the models with only space-like extra dimension(s). In addition, we give two simplest models explicitly, and comment on the 4-dimensional effective cosmological constant.

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1   Introduction

Although the Standard Model is very succesful from experiments at LEP and Tevatron, it has some unattractive features which may imply the new physics. One of these problems is that the gauge forces and the gravitational force are not unified. Another is the gauge hierarchy problem. Previously, two solutions to the gauge hierarchy problem have been proposed: one is the idea of the technicolor and compositeness which lacks calculability, and the other is the idea of supersymmetry.

About two years ago, it was suggested that the large compactified extra dimensions may also be the solution to the gauge hierarchy problem [1], because a low $(4+n)$-dimensional Planck scale (M_X) may result in the large 4-dimensional Planck scale (M_{pl}) due to the large physical volume (V_p^n) of extra dimensions: $M_{pl}^2 = M_X^{2+n} V_p^n$. In addition, about one years ago, Randall and Sundrum [2] proposed another scenario that the extra dimension is an orbifold, and the size of extra dimension is not large but the 4-dimensional mass scale in the Standard Model is suppressed by an exponential factor from 5-dimensional mass scale because of the exponential warp factor. Furthermore, they suggested that the fifth dimension might be non-compact [3], and there may exist only one brane with positive tension at origin, but, there exists the gauge hierarchy problem. The remarkable aspect of the second scenario is that it gives rise to a localized graviton field. Recently, a lot of 5-dimensional models with 3-branes were built [4-5]. We constructed the general models with parallel 3-branes on the five-dimensional space-time, and obtained that the 5-dimensional GUT scale on each brane can be identified as the 5-dimensional Planck scale, but the 4-dimensional Planck scale is generated from the low 4-dimensional GUT scale exponentially in our world. Furthermore, the models with codimension-1 brane(s) were constructed on the six-dimensional and higher dimensional space-time [6-8].

In above model buildings, all the models with warp factor in the metric have negative bulk cosmological constant. However, in string theory, it is natural to take the bulk cosmological constant to be zero since the tree-level vacuum energy in the generic critical closed string compactifications (supersymmetric or not) vanishes. And the zero bulk cosmological constant is natural in the scenario in which the bulk is supersymmetric (though the brane need not be), or the quantum corrections to the bulk are small enough to be neglect in a controlled expansion. Therefore, how to construct the models with zero bulk cosmological constant is an important question in the model buildings. One solution was proposed where a scalar ϕ, whose bulk potential is vanished, is introduced [9]. In this scenario, ϕ becomes singular at a finite distance along the extra dimension and the warp factor in the metric vanishes at singularity. The good aspect of this approach is that, the brane tension can be set arbitrary. However, the Z_2 symmetric and 4-dimensional Poincare invariant solution is unstable under the bulk quantum corrections, and any procedure which regularizes the singurality will introduce the fine-tuning which self-tuning is supposed to avoid [10]. Moreover, the time dependent solution might be a saddle point which is unstable to the expansion or contraction of brane world, and might not conserve the energy on the brane [11]. By the way, the quantum solutions of brane worlds in the WKB

approximation were also discussed in [12].

We would like to explore how to construct the brane models or networks with zero bulk cosmological constant and without the bulk scalar ϕ. After we construct the general models or networks with n space-like and m time-like extra dimensions and with constant $(4+n+m)$-dimensional cosmological constant on the space-time $M^4 \times (M^1)^{n+m}$, where M^4 is the four-dimensional Minkowski space-time and M^1 is one-dimensional manifold with or without boundary, we find out that if we introduce the time-like extra dimension(s) or time-dependent solutions, we can obtain the brane models or networks with vanishing $(4+n+m)$-dimensional cosmological constant.

Time-like extra dimension is not a new subject [13]. Kaluza-Klein's six dimensional model with two times and compact extra dimensions was also investigated before [14]. And the experimental lower bounds on the posssible violation of unitarity put a limit on the maximum radius of the internal time-like directions [15]. In addition, F-theory has one time-like extra dimension [16]. Recently, the time-like extra dimension is considered [17] in the brane world scenarios, too. By the way, the two-time physics, suggested by I. Bars et al, has a new sympletic gauge symmetry which indeed removes all the ghosts, establishes the unitarity and causality, and play a role analogous to duality [18].

Here, we do not want to explore the solution to the problems arising from the time-like extra dimension(s): unitarity and causality. We would like to open our mind in the brane model buildings, and hopefully, those problems might be solved in future study.

In this paper, first, we discuss the general models with parallel 3-branes and one time-like extra dimension on the space-time $M^4 \times R^1$ in detail, and similarly, one can construct the general models with one time-like extra dimension on the space-time $M^4 \times R^1/Z_2$, $M^4 \times S^1$ and $M^4 \times S^1/Z_2$. In fact, we can obtain the general 3-brane models with one time-like extra dimension from the previous 3-brane models with one space-like extra dimension in [5] by making the following transformaitons for the metric g_{55}, the sectional bulk cosmological constant Λ_i and the brane tension V_i

$$g_{55} \longrightarrow -g_{55} \quad , \tag{1}$$

$$\Lambda_i \longrightarrow -\Lambda_i \ , \ V_i \longrightarrow -V_i \ . \tag{2}$$

Moreover, we construct the general brane models or networks with n space-like and m time-like extra dimensions, and with constant $(4+n+m)$-dimensional cosmological constant on the space-time $M^4 \times (M^1)^{n+m}$. We also include the time (t') term in the conformal metric. Furthermore, we point out that there exist two kinds of the models or networks with warp factor in the metric and zero bulk cosmological constant: one is static, the other is non-static. For static models, in order to obtain zero bulk cosmological constant, we have to introduce space-like and time-like extra dimensions. And if we required that the sum of brane tensions be zero, i. e., the 4-dimensional effective cosmological constant is zero, in order to solve the gauge hierarchy problem, we have to introduce at least one brane with negative tension, which can not be located at fixed point. For non-static models, we can introduce only

space-like extra dimension(s). However, if we required that the brane which includes our world have positive tension, in order to solve the gauge hierarchy problem, we have to introduce at least one brane with negative tension, which can not be located at fixed point. Moreover, in order to have vanishing effective 4-dimensional cosmological constant, we have to fine-tune the parameters and adjust the set up of the branes in the non-static models. In addition, the non-static solutions might not be stable. We also give two simplest models explicitly.

2 General 3-Brane Models with One Time-like Extra Dimension

In this section, we would like to discuss the general models with one time-like extra dimension and parallel 3-branes on the space-time $M^4 \times M^1$. Because those models are similar to the general models with one space-like extra dimension and parallel 3-branes in [5], we only discuss the models whose fifth dimension is R^1 in detail, and then point out the tiny difference between the models with one time-like extra dimension and the models with one space-like extra dimension.

Assuming we have $l + m + 1$ parallel 3-branes, and their fifth coordinates are: $-\infty < \tau_{-l} < \tau_{-l+1} < ... < \tau_{-1} < \tau_0 < \tau_1 < ... < \tau_{m-1} < \tau_m < +\infty$, we obtain the metric in each brane from the five-dimensional metric g_{AB} where $A, B = \mu, \tau$ by restriction

$$g_{\mu\nu}^{(i)}(x^\mu) \equiv g_{\mu\nu}(x^\mu, \tau = \tau_i) . \tag{3}$$

In this paper, we assume that $g_{\mu 5} = 0$ here.

The classical action is given by

$$S = S_{gravity} + S_B , \tag{4}$$

$$S_{gravity} = \int d^4x \ d\tau \ \sqrt{g}\{-\Lambda(\tau) + \frac{1}{2}M_X^3 R\} , \tag{5}$$

$$S_B = \sum_{i=-l}^{m} \int d^4x\sqrt{-g^{(i)}}\{\mathcal{L}_i - V_i\} , \tag{6}$$

where M_X is the 5-dimensional Planck scale, $\Lambda(\tau)$ is the 5-dimensional cosmological constant, and V_i where $i = -l, ..., m$ is the brane tension. The $\Lambda(\tau)$ is defined as

$$\Lambda(\tau) = \sum_{i=1}^{m} \Lambda_i \left(\theta(\tau - \tau_{i-1}) - \theta(\tau - \tau_i)\right) + \Lambda_{+\infty}\theta(\tau - \tau_m)$$

$$+ \sum_{i=-l+1}^{0} \Lambda_i \left(\theta(-\tau + \tau_i) - \theta(-\tau + \tau_{i-1})\right) + \Lambda_{-\infty}\theta(-\tau + \tau_{-l}) , \tag{7}$$

where $\theta(x) = 1$ for $x \geq 0$ and $\theta(x) = 0$ for $x < 0$. So, $\Lambda(\tau)$ is sectional constant.

The 5-dimensional Einstein equations for above action are

$$
\sqrt{g}\left(R_{AB} - \frac{1}{2}g_{AB}R\right) = -\frac{1}{M_X^3}[\Lambda(\tau)\sqrt{g}\ g_{AB} +
$$
$$
\sum_{i=-l}^{m} V_i\sqrt{-g^{(i)}}\ g_{\mu\nu}^{(i)}\ \delta_A^\mu\delta_B^\nu\ \delta(\tau - \tau_i)] . \tag{8}
$$

Assuming that there exists a solution that respects 4-dimensional Poincare invariance in the x^μ-directions, one obtains the 5-dimensional metric

$$
ds^2 = e^{-2\sigma(\tau)}\eta_{\mu\nu}dx^\mu dx^\nu - d\tau^2 . \tag{9}
$$

With this metric, the Einstein equations reduce to

$$
\sigma'^2 = \frac{\Lambda(\tau)}{6M_X^3} , \quad \sigma'' = -\sum_{i=-l}^{m} \frac{V_i}{3M_X^3}\delta(\tau - \tau_i) . \tag{10}
$$

The general solution to above differential equations is

$$
\sigma(\tau) = \sum_{i=-l}^{m} k_i|\tau - \tau_i| + k_c\tau + c , \tag{11}
$$

where k_c and c are constants, and $k_i \neq 0$ for $i = -l, ..., m$. The relations between the k_i and V_i, and the relations between the k_i and Λ_i are

$$
V_i = -6k_iM_X^3 , \quad \Lambda_i = 6M_X^3(\sum_{j=i}^{m} k_j - \sum_{j=-l}^{i-1} k_j - k_c)^2 , \tag{12}
$$

$$
\Lambda_{-\infty} = 6M_X^3(\sum_{j=-l}^{m} k_j - k_c)^2 , \quad \Lambda_{+\infty} = 6M_X^3(\sum_{j=-l}^{m} k_j + k_c)^2 . \tag{13}
$$

Therefore, the five-dimensional cosmological constant is positive except the section(s) with zero bulk cosmological constant, then, for any point in $M^4 \times R^1$, which is not belong to any brane and the section(s) with zero bulk cosmological constant, there is a neighborhood which is diffeomorphic to (or a slice of) dS_5 space. Moreover, the five-dimensional cosmological constant and brane tensions should satisfy above equations. In order to obtain the finite 4-dimensional Planck scale, we obtain the constraints: $\sum_{j=-l}^{m} k_j > |k_c|$. So, the sum of brane tensions is negative.

The general bulk metric is

$$
ds^2 = e^{-2\sum_{i=-l}^{m} k_i|\tau-\tau_i|-2k_c\tau-2c}\eta_{\mu\nu}dx^\mu dx^\nu - d\tau^2 . \tag{14}
$$

And the corresponding 4-dimensional Planck scale is

$$
M_{pl}^2 = M_X^3\left(T_{-\infty,-l} + T_{m,+\infty} + \sum_{i=-l}^{m-1} T_{i,i+1}\right) , \tag{15}
$$

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where

$$T_{-\infty,-l} = \frac{1}{2\chi_{-\infty}} e^{-2\sigma(\tau_{-l})} \ , \ \ T_{m,+\infty} = \frac{1}{2\chi_{+\infty}} e^{-2\sigma(\tau_m)} \ , \tag{16}$$

if $\chi_{i,i+1} \neq 0$, then

$$T_{i,i+1} = \frac{1}{2\chi_{i,i+1}} \left(e^{-2\sigma(\tau_{i+1})} - e^{-2\sigma(\tau_i)} \right) \ , \tag{17}$$

and if $\chi_{i,i+1} = 0$, then

$$T_{i,i+1} = (\tau_{i+1} - \tau_i) e^{-2\sigma(\tau_i)} \ , \tag{18}$$

where

$$\chi_{\pm\infty} = \sum_{j=-l}^{m} k_j \pm k_c \ , \ \ \chi_{i,i+1} = \sum_{j=i+1}^{m} k_j - \sum_{j=-l}^{i} k_j - k_c \ . \tag{19}$$

In addition, the four-dimensional GUT scale on the $i-th$ brane $M_{GUT}^{(i)}$ is related to the five-dimensional GUT scale on the $i-th$ brane $M5_{GUT}^{(i)}$ by

$$M_{GUT}^{(i)} = M5_{GUT}^{(i)} e^{-\sigma(\tau_i)} \ . \tag{20}$$

In this paper, we assume that $M5_{GUT}^{(i)} \equiv M_X$, for $i = -l, ..., m$.

These models can be generalized to the models with Z_2 symmetry. Because of Z_2 symmetry, $k_c = 0$. There are two kinds of such models, one has odd number of the branes, the other has even number of the branes. For the first one, we just require that $k_{-i} = k_i$, $\tau_{-i} = -\tau_i$, and $m = l$. For the second one, we just require that $k_{-i} = k_i$, $\tau_{-i} = -\tau_i$, $m = l$, and $k_0 = 0$ (no number 0 brane). Furthermore, these models can also be generalized to the general models whose fifth dimension is R^1/Z_2, one just requires that $k_{-i} = k_i$, $\tau_{-i} = -\tau_i$, $m = l$, then, introduces the equivalence classes: $\tau \sim -\tau$ and $i-th\ brane \sim (-i)-th\ brane$. The only trick point in this case is that the brane tension V_0 is half of the original value, i. e., $V_0 = 3k_0 M_X^3$. And one may notice that, the sum of brane tensions is negative and the sectional bulk cosmological constant is non-negative.

Similarly, we can construct the general models with one time-like extra dimension on the space-time $M^4 \times S^1$ and $M^4 \times S^1/Z_2$, as we have done in [5]. In fact, we can obtain the 3-brane models with one time-like extra dimension from previous 3-brane models with one space-like extra dimension in [5] by making the following transformaitons for the metric g_{55}, sectional bulk cosmological constant Λ_i and brane tension V_i

$$g_{55} \longrightarrow -g_{55} \ , \tag{21}$$

$$\Lambda_i \longrightarrow -\Lambda_i \ , \ V_i \longrightarrow -V_i \ . \tag{22}$$

By the way, if the fifth dimension is compact, the sum of brane tensions is zero. And the gauge hierarchy problem can be solved in all above models, as we have discussed in [5].

3   Brane Models or Networks with Time-like and Space-Like Extra Dimensions

In this section, we will construct the brane models or networks with n space-like and m time-like extra dimensions, and with constant $(4+n+m)$-dimensional cosmological constant on the space-time $M^4 \times (M^1)^{n+m}$. We also include the linear time (t') term in the conformal metric. The brane models or networks with zero bulk cosmological constant are special cases of the general brane models or networks.

Assume we have n space-like extra dimensions, and m time-like extra dimensions, the ordered coordinates for the whole space-time are: t', x^1, x^2, x^3,y^1,y^2, ..., y^n, τ^1, τ^2, ..., τ^m (Note that $(t', x^1, x^2, x^3,y^1,y^2, ..., y^n, \tau^1, \tau^2, ..., \tau^m) \equiv (0,1,2,...,n+m+3)$). Along each extra dimension, we have papallel $(2+n+m)$-branes, so, each brane is the hypersuface which is determined by the algebraic equation $y^i = y^i_j$ or $\tau^i = \tau^i_j$. And we assume that if $j < k$, then, $y^i_j < y^i_k$ or $\tau^i_j < \tau^i_k$. Because we require that the $(4+n+m)$-dimensional cosmological constant is a constant on the whole space-time, along each extra dimension, the brane tensions of parallel branes will have the same magnitudes except the brane tension of the brane at boundary, which is half of that magnitude.

In our notation, the $(4+n+m)$-dimensional metric is g_{AB}, and the metric on each brane is obtained by restriction, for example, the metric of the brane at $y^i = y^i_j$ is

$$(g^{y^i}_j)_{\hat{A}\hat{B}} \equiv g_{\hat{A}\hat{B}}(y^i = y^i_j) \ , \tag{23}$$

where $\hat{A}, \hat{B} \neq 3+i$. And the metric of the brane at $\tau^i = \tau^i_j$ is

$$(g^{\tau^i}_j)_{\hat{A}\hat{B}} \equiv g_{\hat{A}\hat{B}}(\tau^i = \tau^i_j) \ , \tag{24}$$

where $\hat{A}, \hat{B} \neq 3+n+i$.

The classical action is

$$S \ = \ S_{gravity} + S_{BS} + S_{BT} \ , \tag{25}$$

where

$$S_{gravity} \ = \ \int dt' d^3x \ d^n y \ d^m \tau \ \sqrt{(-1)^{1+m}g} \ (\frac{1}{2}M_X^{2+n+m}R - \Lambda) \ , \tag{26}$$

$$S_{BS} \ = \ -\int dt' d^3x \ d^n y \ d^m \tau \ (\sum_{i=1}^{n}\sum_{j_i} \sqrt{(-1)^{1+m}g^{y^i}_{j_i}} \ V^{y^i}_{j_i} \ \delta(y^i - y^i_{j_i})) \ , \tag{27}$$

$$S_{BT} \ = \ -\int dt' d^3x \ d^n y \ d^m \tau \ (\sum_{i=1}^{m}\sum_{j_i} \sqrt{(-1)^{m}g^{\tau^i}_{j_i}} \ V^{\tau^i}_{j_i} \ \delta(\tau^i - \tau^i_{j_i})) \ , \tag{28}$$

where M_X is the $(4+n+m)$-dimensional Planck scale, Λ is the $(4+n+m)$-dimensional cosmological constant, and $V_{j_i}^{y^i}$ and $V_{j_i}^{\tau^i}$ are the brane tensions.

The Einstein equations arising from above action are

$$G_{AB} \equiv R_{AB} - \frac{1}{2}g_{AB}R = \frac{1}{M_X^{2+n+m}}\,T_{AB}\,, \tag{29}$$

where

$$T_{AB} = T_{AB}^{gravity} + T_{AB}^{BS} + T_{AB}^{BT}\,, \tag{30}$$

where

$$T_{AB}^{gravity} = -g_{AB}\Lambda\,, \tag{31}$$

$$T_{AB}^{BS} = -\sum_{i=1}^{n}\sum_{j_i}\sqrt{\frac{g_{j_i}^{y^i}}{g}}\;V_{j_i}^{y^i}\;(g_{j_i}^{y^i})_{\hat{A}\hat{B}}\;\delta_A^{\hat{A}}\;\delta_B^{\hat{B}}\;\delta(y^i - y_{j_i}^i)\,, \tag{32}$$

$$T_{AB}^{BT} = -\sum_{i=1}^{m}\sum_{j_i}\sqrt{-\frac{g_{j_i}^{\tau^i}}{g}}\;V_{j_i}^{\tau^i}\;(g_{j_i}^{\tau^i})_{\hat{A}\hat{B}}\;\delta_A^{\hat{A}}\;\delta_B^{\hat{B}}\;\delta(\tau^i - \tau_{j_i}^i)\,. \tag{33}$$

We assume the metric to be conformally flat and write it as

$$ds_{4+n+m}^2 = \Omega^2(-dt'^2 + \sum_{i=1}^{3}dx^{i2} + \sum_{i=1}^{n}dy^{i2} - \sum_{i=1}^{m}d\tau^{i2})\,, \tag{34}$$

where $\Omega \equiv \Omega(t',y,\tau)$. The simplest way to proceed is to transform the metric to a conformally related metric, *i.e.*

$$g_{AB} = \Omega^2\,\tilde{g}_{AB}\,. \tag{35}$$

The Einstein tensors in the two metrics are related by

$$\begin{aligned}G_{AB} =\ &\tilde{G}_{AB} + (2+n+m)\Big[\tilde{\nabla}_A\ln\Omega\;\tilde{\nabla}_B\ln\Omega - \tilde{\nabla}_A\tilde{\nabla}_B\ln\Omega \\ &+\tilde{g}_{AB}\Big(\tilde{\nabla}^2\ln\Omega + \frac{1+n+m}{2}(\tilde{\nabla}\ln\Omega)^2\Big)\Big]\,,\end{aligned} \tag{36}$$

where the covariant derivatives $\tilde{\nabla}$ are evaluated with respect to the metric $\tilde{g}$. Since the metric is conformally flat, the covariant derivatives are identical to the ordinary derivatives and $\tilde{G}_{AB} = 0$. So, the above equation can be recast to

$$G_{AB} = (2+n+m)\Big[\Omega\tilde{\nabla}_A\tilde{\nabla}_B\Omega^{-1} + \tilde{g}_{AB}\Big(-\Omega\tilde{\nabla}^2\Omega^{-1} + \frac{3+n+m}{2}\,\Omega^2(\tilde{\nabla}\Omega^{-1})^2\Big)\Big]\,. \tag{37}$$

Using above form of Einstein tensor, the Einstein equations can be written as

$$\frac{\partial^2}{\partial t'^2}\Omega^{-1} = 0 , \tag{38}$$

$$\frac{\partial^2}{\partial y^{i2}}\Omega^{-1} = \sum_{j_i} \frac{1}{M_X^{2+n+m}(2+n+m)} V_{j_i}^{y^i} \, \delta(y^i - y_{j_i}^i) , \tag{39}$$

$$\frac{\partial^2}{\partial \tau^{i2}}\Omega^{-1} = -\sum_{j_i} \frac{1}{M_X^{2+n+m}(2+n+m)} V_{j_i}^{\tau^i} \, \delta(\tau^i - \tau_{j_i}^i) , \tag{40}$$

$$(\tilde{\nabla}\Omega^{-1})^2 = -\frac{2\Lambda}{M_X^{2+n+m}(2+n+m)(3+n+m)} . \tag{41}$$

We can relate the fundamental scale M_X to the four-dimensional Planck scale M_{Pl} by integrating over the extra dimensions

$$M_{Pl}^2 = M_X^{2+n+m} \int d^n y \, d^m \tau \, \Omega^{2+n+m} . \tag{42}$$

The four-dimensional "Grand Unification Scale" $(M_{GUT}^4)_{(i_1,\ldots,i_n,j_1,\ldots,j_m)}$ at each brane junction is

$$(M_{GUT}^4)_{(i_1,\ldots,i_n,j_1,\ldots,j_m)} = M_X \Omega(y^1 = y_{i_1}^1, \ldots, y^n = y_{i_n}^n, \tau^1 = \tau_{j_1}^1, \ldots, \tau^m = \tau_{j_m}^m) , \tag{43}$$

where we have assumed that the $(4+n+m)$-dimensional Planck scale is equal to the $(4+n+m)$-dimensional GUT scale at each brane junction. Thus, it is possible to solve the gauge hierarchy problem by choosing Ω appropriately.

Noticing that Ω^{-1} is a linear combination of the solutions to Eqs. (38-40), we obtain

$$\Omega^{-1} = h\, t' + \sum_{i=1}^n \sigma^{Si}(y^i) + \sum_{j=1}^m \sigma^{Tj}(\tau^j) + c , \tag{44}$$

where $\sigma^{Si}(y^i)$ and $\sigma^{Tj}(\tau^j)$ satisfy Eqs. (39) and (40), respectively.

Along each extra dimension, the brane tensions of the parallel $(2+n+m)$-branes have the same magnitudes $|V^{y^i}|$ or $|V^{\tau^i}|$ except the brane tension of the brane at boundary, which is half of that value, so, we define

$$k^{y^i} = \frac{1}{2(2+n+m)M_X^{2+n+m}}|V^{y^i}| , \tag{45}$$

and

$$k^{\tau^j} = \frac{1}{2(2+n+m)M_X^{2+n+m}}|V^{\tau^j}| , \tag{46}$$

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where $i = 1, ..., n$ and $j = 1, ..., m$.

Because in our discussion, the manifold of extra dimensions is the product of $n + m$ one-dimensional manifold, each extra dimension can only be R^1, R^1/Z_2, S^1 and S^1/Z_2. The functions of $\sigma^{Si}(y^i)$ and $\sigma^{Tj}(\tau_j)$ are the functions discussed in the second section in [5], so, we just write down the results here.

For the space-like extra dimension, we use y^i as an example:

(I) The 1-dimensional manifold for y^i is R^1. Along y^i, we will have odd number of parallel $(2 + n + m)$-branes. Assume we have $2L + 1$ branes, with coordinates on y^i: $-\infty < y^i_1 < ... < y^i_{2L} < y^i_{2L+1} < +\infty$, we obtain

$$\sigma^{Si}(y^i) = \sum_{j_i=1}^{2L+1} (-1)^{j_i+1} k^{y^i} |y^i - y^i_{j_i}| \ , \tag{47}$$

and the brane tensions are

$$V^{y^i}_{j_i} = (-1)^{j_i+1} 2(2 + n + m) M_X^{2+n+m} k^{y^i} \ . \tag{48}$$

(II) The 1-dimensional manifold for y^i is R^1/Z_2. Assume along y^i, we have $L + 1$ parallel $(2 + n + m)$-branes, with coordinates on y^i: $y^i_0 = 0 < y^i_1 < ... < y^i_{L-1} < y^i_L < +\infty$, we have

$$\sigma^{Si}(y^i) = \frac{1}{2}(1 + (-1)^L) k^{y^i} |y^i| + \sum_{j_i=1}^{L} (-1)^{j_i+L} k^{y^i} |y^i - y^i_{j_i}| \ . \tag{49}$$

And the brane tensions for $j_i \neq 0$ are

$$V^{y^i}_{j_i} = (-1)^{j_i+L} 2(2 + n + m) M_X^{2+n+m} k^{y^i} \ , \tag{50}$$

and

$$V^{y^i}_0 = (-1)^L (2 + n + m) M_X^{2+n+m} k^{y^i} \ . \tag{51}$$

(III) The 1-dimensional manifold for y^i is S^1. Along y^i, we will have even number of parallel $(2 + n + m)$-branes. Assume we have $2L$ branes, with coordinates on y^i: $0 = y^i_1 < ... < y^i_{2L-1} < y^i_{2L} < 2\pi\rho^i$, where ρ^i is the radius, we obtain

$$\sigma^{Si}(y^i) = \pm \sum_{j_i=2}^{2L} (-1)^{j_i+1} k^{y^i} |y^i - y^i_{j_i}| \ , \tag{52}$$

and the brane tensions are

$$V^{y^i}_{j_i} = \pm(-1)^{j_i+1} 2(2 + n + m) M_X^{2+n+m} k^{y^i} \ . \tag{53}$$

In addition, there is one constraint equation

$$\pm \sum_{j_i=2}^{2L} (-1)^{j_i+1} y^i_{j_i} = -\pi\rho^i \ . \tag{54}$$

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(IV) The 1-dimensional manifold for y^i is S^1/Z_2. Assume along y^i, we have $L+1$ parallel $(2+n+m)$-branes, with coordinates on y^i: $0 = y_0^i < ... < y_{L-1}^i < y_L^i = \pi\rho^i$, where ρ^i is the radius, we have

$$\sigma^{Si}(y^i) = \pm(\sum_{j_i=1}^{L-1}(-1)^{j_i}k^{y^i}|y^i - y_{j_i}^i| + \frac{1}{2}(1 + (-1)^{L+1})k^{y^i}y^i) \; . \tag{55}$$

And the brane tensions for $j_i \neq 0$ and L are

$$V_{j_i}^{y^i} = \pm(-1)^{j_i}2(2+n+m)M_X^{2+n+m}k^{y^i} \; , \tag{56}$$

and

$$V_0^{y^i} = \pm(2+n+m)M_X^{2+n+m}k^{y^i} \; , \tag{57}$$

$$V_L^{y^i} = \pm(-1)^L(2+n+m)M_X^{2+n+m}k^{y^i} \; . \tag{58}$$

Similarly, for the time-like extra dimension, we use τ^i as an example:

(I) The 1-dimensional manifold for τ^i is R^1. Along τ^i, we will have odd number of parallel $(2+n+m)$-branes. Assume we have $2L+1$ branes, with coordinates on τ^i: $-\infty < \tau_1^i < ... < \tau_{2L}^i < \tau_{2L+1}^i < +\infty$, we obtain

$$\sigma^{Ti}(\tau^i) = \sum_{j_i=1}^{2L+1}(-1)^{j_i+1}k^{\tau^i}|\tau^i - \tau_{j_i}^i| \; , \tag{59}$$

and the brane tensions are

$$V_{j_i}^{\tau^i} = (-1)^{j_i}2(2+n+m)M_X^{2+n+m}k^{\tau^i} \; . \tag{60}$$

(II) The 1-dimensional manifold for τ^i is R^1/Z_2. Assume along τ^i, we have $L+1$ parallel $(2+n+m)$-branes, with coordinates on τ^i: $\tau_0^i = 0 < \tau_1^i < ... < \tau_{L-1}^i < \tau_L^i < +\infty$, we have

$$\sigma^{Ti}(\tau^i) = \frac{1}{2}(1 + (-1)^L)k^{\tau^i}|\tau^i| + \sum_{j_i=1}^{L}(-1)^{j_i+L}k^{\tau^i}|\tau^i - \tau_{j_i}^i| \; . \tag{61}$$

And the brane tensions for $j_i \neq 0$ are

$$V_{j_i}^{\tau^i} = (-1)^{j_i+L+1}2(2+n+m)M_X^{2+n+m}k^{\tau^i} \; , \tag{62}$$

and

$$V_0^{\tau^i} = (-1)^{L+1}(2+n+m)M_X^{2+n+m}k^{\tau^i} \; . \tag{63}$$

(III) The 1-dimensional manifold for τ^i is S^1. Along τ^i, we will have even number of parallel $(2+n+m)$-branes. Assume we have $2L$ branes, with coordinates on τ^i: $0 = \tau_1^i < ... < \tau_{2L-1}^i < \tau_{2L}^i < 2\pi\rho^i$, where ρ^i is the radius, we obtain

$$\sigma^{Ti}(\tau^i) = \pm\sum_{j_i=2}^{2L}(-1)^{j_i+1}k^{\tau^i}|\tau^i - \tau_{j_i}^i| \; , \tag{64}$$

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and the brane tensions are

$$V_{j_i}^{\tau^i} = \pm(-1)^{j_i} 2(2+n+m) M_X^{2+n+m} k^{\tau^i} . \tag{65}$$

Moreover, there is one constraint equation

$$\pm \sum_{j_i=2}^{2L} (-1)^{j_i+1} \tau_{j_i}^i = -\pi \rho^i . \tag{66}$$

(IV) The 1-dimensional manifold for τ^i is S^1/Z_2. Assume along τ^i, we have $L+1$ parallel $(2+n+m)$-branes, with coordinates on τ^i: $0 = \tau_0^i < ... < \tau_{L-1}^i < \tau_L^i = \pi \rho^i$, where ρ^i is the radius, we have

$$\sigma^{Ti}(\tau^i) = \pm\Big(\sum_{j_i=1}^{L-1} (-1)^{j_i} k^{\tau^i} |\tau^i - \tau_{j_i}^i| + \frac{1}{2}(1+(-1)^{L+1}) k^{\tau^i} \tau^i\Big) . \tag{67}$$

And the brane tensions for $j_i \neq 0$ and L are

$$V_{j_i}^{\tau^i} = \pm(-1)^{j_i+1} 2(2+n+m) M_X^{2+n+m} k^{\tau^i} , \tag{68}$$

and

$$V_0^{\tau^i} = \pm(-1)(2+n+m) M_X^{2+n+m} k^{\tau^i} , \tag{69}$$

$$V_L^{\tau^i} = \pm(-1)^{L+1}(2+n+m) M_X^{2+n+m} k^{\tau^i} . \tag{70}$$

Furthermore, we obtain the $(4+n+m)$-dimensional cosmological constant

$$\begin{aligned}
\Lambda &= -\frac{1}{2}(2+n+m)(3+n+m) M_X^{2+n+m} \\
&\quad (-h^2 + \sum_{i=1}^{n}(k^{y^i})^2 - \sum_{j=1}^{m}(k^{\tau^j})^2) .
\end{aligned} \tag{71}$$

Requiring that $-h^2 + \sum_{i=1}^{n}(k^{y^i})^2 - \sum_{j=1}^{m}(k^{\tau^j})^2 = 0$, we obtain the brane models or networks whose $(4+n+m)$-dimensional cosmological constant is zero. Therefore, there exist two kinds of models or networks with warp factor in the metric and vanishing bulk cosmological constant: one is static ($h = 0$), the other is non-static ($h \neq 0$). For static models ($h = 0$), we have to introduce space-like and time-like extra dimensions to obtain zero bulk cosmological constant. And if one required that the sum of brane tensions be zero, i.e., the 4-dimensional effective cosmological constant is zero, in order to solve the gauge hierarchy problem, one has to introduce at least one brane which has negative tension and can not be located at fixed point. For non-static models ($h \neq 0$), we can introduce only space-like extra dimension(s). However, if one required that the observable brane have positive tension, in order to solve the gauge hierarchy problem, we have to introduce at least one brane which has negative tension and can not be located at fixed point. Moreover, in order to have vanishing 4-dimensional effective cosmological constant, we need to fine-tune the parameters

h, k^{y^i}, k^{τ^j} and adjust the set-up of the branes in the non-static models. In addition, the non-static solutions might not be stable.

By the way, using the σ functions in the third section in [5], one can easily construct the general brane models or networks with space-like and time-like extra dimensions whose $(4 + n + m)$-dimensional cosmological constant is not constant on the whole space-time.

4   Two Simplest Models

In this section, we will give two explicit simplest models with warp factor in the metric and zero bulk cosmological constant.

(I) We consider the static model with one space-like extra dimension y^1 and one time-like extra dimension τ^1 on the space-time $M^4 \times R^1 \times R^1$. For simplicity, we can write y and τ for the space-like extra dimension and time-like extra dimension coordinates, respectively, i. e., $y \equiv y^1$, $\tau \equiv \tau^1$. Because the solution is static, we have $t' \equiv t \equiv x^0$ and $h = 0$. And we only consider two 4-branes, one brane with tension V^y is the hypersuface determined by the equation $y = 0$, the other brane with tension V^τ is the hypersuface determined by the equation $\tau = 0$. Zero bulk cosmological constant implies that $V^y = -V^\tau$. In short, this is the simplest static model with warp factor in the metric and zero bulk cosmological constant. The conformal metric is

$$ds^2 \;=\; \Omega(\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 - d\tau^2) , \tag{72}$$

where

$$\Omega = \frac{1}{k|y| + k|\tau| + c} , \tag{73}$$

where c is a positive real number. The brane tensions are

$$V^y = 8 \; M_X^4 \; k \; , \; V^\tau = -8 \; M_X^4 \; k \; , \tag{74}$$

so, the 4-dimensional effective cosmological constant is zero. The 4-dimensional Planck scale is

$$M_{pl}^2 = M_X^4 \int \; dy \; d\tau \; \Omega^4 = \frac{2M_X^4}{3k^2 c^2} . \tag{75}$$

Assuming the Standard Model lives at the intersection of two branes, we obtain the 4-dimensional GUT scale M_{GUT} in our world

$$M_{GUT} = \frac{M_X}{c} , \tag{76}$$

therefore,

$$M_{pl} = \frac{\sqrt{2}M_X}{\sqrt{3}k} M_{GUT} . \tag{77}$$

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We can not naturally solve the gauge hierarchy problem in this model. Of course, if we introduce more 4-brane(s), we can solve the gauge hierarchy problem.

(II) The second model we consider is a non-static model with one space-like extra dimension $y \equiv y^1$ on the space-time $M^4 \times R^1$. For simplicity, we just consider one 3-brane with tension V, which is the hypersuface determined by the equation $y = 0$. Of course, this is the simplest non-static model with warp factor in the metric and zero bulk cosmological constant. The conformal metric is

$$ds^2 = \Omega(-dt'^2 + \sum_{i=1}^{3} dx^i dx^i + dy^2) , \tag{78}$$

where

$$\Omega = \frac{1}{ht' + k|y| + c} , \tag{79}$$

where c is a real number. Requiring that the 5-dimensional cosmological constant is zero, we obtain $h = \pm k$. The brane tension is

$$V = 6 \ M_X^3 \ k , \tag{80}$$

so, the 4-dimensional effective cosmological constant is non-zero. The 4-dimensional Planck scale is

$$M_{pl}^2 = M_X^3 \int \ dy \ \Omega^3 = \frac{M_X^3}{k(ht' + c)^2} . \tag{81}$$

Assuming the Standard Model lives at 3-brane, we obtain the 4-dimensional GUT scale M_{GUT} in our world

$$M_{GUT} = \frac{M_X}{ht' + c} , \tag{82}$$

therefore,

$$M_{pl}^2 = \frac{M_X}{k} M_{GUT}^2 . \tag{83}$$

We can not naturally solve the gauge hierarchy problem in this model. Of course, if one introduce more 3-brane(s), one can solve the gauge hierarchy problem.

In addition, this model is unstable, in other words, the universe either begins or ends in a singularity, depending on whether $h > 0$ or $h < 0$. But, if there exist higher order correction ($< t' >$) terms (non-linear t' terms) in the Ω^{-1}, for example $\epsilon \ t'^2$, the solution might be stable.

5   Conclusion and Discussion

We construct the general models with parallel 3-branes and one time-like extra dimension on the space-time $M^4 \times R^1$ in detail, and similarly, one can discuss the general models with one time-like extra dimension on the space-time $M^4 \times R^1/Z_2$,

$M^4 \times S^1$ and $M^4 \times S^1/Z_2$. In addition, we construct the general brane models or networks with n space-like and m time-like extra dimensions, and with constant $(4 + n + m)$-dimensional cosmological constant on the space-time $M^4 \times (M^1)^{n+m}$. Time (t') dependent term is also included in the conformal metric. We point out that there exist two kinds of models or networks with warp factor in the metric and zero $(4 + n + m)$-dimensional cosmological constant: one is static, the other is non-static. For static models, we have to introduce space-like and time-like extra dimensions to obtain vanishing bulk cosmological constant. For non-static models, we can introduce only space-like extra dimension(s). We also give two simplest models explicitly.

Although taking zero bulk cosmological constant ($\Lambda = 0$) is natural in the string theory at tree-level or in the scenario where the bulk is supersymmetric, one might expect the bulk quantum corrections to correct Λ in a power series in the couplings, for example g_s. But, our solutions might still be interesting if the bulk corrections to Λ are very small, which can happen for instance if the supersymmetry breaking is localized in a small neighborhood of the branes, or if the supersymmetry breaking scale in the bulk is small enough. Moreover, if all the gauge fields and matter fields were confined to the branes, the quantum corrections of these fields to the brane tensions might not affect the solutions with $\Lambda = 0$, for example, in the model I in section 4, if the quantum corrections to the two brane tensions are equal, or in the model II in section 4, if the variation of h is equal to the quantum corrections to the brane tension. These results are similar to those in the self-tuning models [9]. Of course, we have fine-tuning in our solutions, but we do not have singularities.

Acknowledgments

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