TOWARD THE EMERGENCE OF TIME IN QUANTUM GRAVITY

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I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

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Abstract

Space and time are fundamental to our description and understanding of physical laws. But are space and time themselves only approximations to an even more fundamental description of reality? Recent developments in quantum gravity have shown how space can be emergent from more fundamental principles. This dissertation provides a review of tools and results developed over the last five years which shed light on a more fundamental description of time. We first discuss a way of understanding time using quantum information theoretic techniques. These techniques are put to use to study how causality and the arrow of time can emerge from the pattern of entanglement in tensor networks. Next we discuss concrete models of emergent time in quantum gravity, culminating in a non-perturbative model in which both space and time are emergent from random matrices. This model describes low-dimensional de Sitter quantum gravity, which has a positive cosmological constant.

Acknowledgments

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I have also been fortunate to have many other outstanding mentors in physics, including Kristan Jensen, Xiao-Liang Qi, and Frank Wilczek. Kristan has been my kind and patient guide to quantum gravity, emphasizing the importance of precision calculations and diving headfirst into subtle details. Kristan's vast knowledge of the tools and logic of quantum gravity are matched in scope only by his generosity. I am grateful for our friendship, and our collaborations which have opened up new worlds to me (some imagined, some in de Sitter).

Xiao-Liang has been my frequent collaborator and mentor on a variety of subjects. I am always in awe of his lightening-fast physics reflexes and powerful visual thinking. His style of thinking and out-of-the-box approach to interdisciplinary physics projects has helped shape my research more broadly.

Frank has been a guiding influence in my physics career since I was an undergraduate, and his mentorship, often through our frequent Skype discussions, has had a profound impact on me as a physicist and person. Frank's creativity and curiosity are inspiring, and his inclination to work on theoretical problems with a firm base in reality has shaped the physics problems I choose to work on. Perhaps one of the most important lessons I have learned from Frank by his example is that one can view all of physics as a connected whole, and with hard work navigate and solve problems coming from conventionally different subfields.

I would also like to thank all of my collaborators over the last five years – they have each taught me so much, and made the journey of discovery so rewarding. My friends at Stanford and around the Bay Area have made living in California particularly enjoyable. A special thanks to Felipe Hernandez for all our long jogs, long dinners, and endless conversations.

Most importantly, I would like to thank my parents for always supporting me, and always believing in me throughout every step of life. They are not just the kind of parents that wanted me to follow my dreams – they did everything so that I could. My brother Ari has also given me his love, encouragement, and patience every step of the way. My grandparents have been a constant source of love and support. And good practice explaining physics to laymen.

Finally, I would like to thank the Fannie and John Hertz Foundation, as well as the Stanford Graduate Fellowships for their financial support. The Hertz Foundation has also played an important role in my social life, providing get-togethers with other Hertz fellows where I have met some of my closest friends.

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Introduction

Our most incisive analysis of non-perturbative quantum gravity comes from various examples of the AdS/CFT correspondence, originating in Maldacena's seminal paper [1]. The correspondence says that certain kinds of d-dimensional CFTs are non-perturbatively equivalent to string theory in Anti-de Sitter space in $(d+1)$-dimensions.[1] In particular, one of the spatial dimensions of the AdS theory "emerges" from the CFT. Over the last decade, there has been increasing emphasis on the perspective that in AdS/CFT, a quantum gravity theory is encoded in a *non*-gravitational theory, i.e. the CFT. Said simply, within quantum mechanics lies the equations of gravity. Rather than attempting to combine quantum mechanics and gravity into a unified theory, we should instead attempt to unearth quantum gravity hidden in certain quantum mechanical systems.

This perspective has led to intense study of how the quantum information theoretic properties of CFTs and related quantum many-body systems encode the data of hidden quantum gravity theories in one higher spatial dimension. Along the way, previously unknown structures of quantum field theories were elucidated by quantum information-theoretic analyses. Remarkable connections between entanglement and emergent space were proposed and studied theoretically, and many insights into the quantum nature of black holes were realized.

Despite these advances, we still have a very limited understanding of non-perturbative quantum gravity with a *positive* cosmological constant, which would be required to understand our universe. The fact that AdS has a negative cosmological constant is deeply ingrained in the paradigm of AdS/CFT, and is not merely a minor technical point that can be readily modified with a minus sign. In fact, it is believed that with a putative dS/CFT correspondence, or more generally a framework for de Sitter holography, time should be emergent. This is difficult to fathom – if time is emergent, then so too must be quantum mechanics. At stake are the basic structures defining quantum mechanics, namely unitary evolution preserving a positive-definite inner product on a Hilbert space. If de Sitter holography is emergent from a non-quantum mechanical theory, then the paradigm previously mentioned of "gravity hidden in quantum mechanics" may have to be seriously revised to understand our universe. As such, it is not clear which lessons of AdS/CFT will carry

[1] There are also additional compact dimensions that are the size of the AdS scale, but we will not discuss them here.

over to the setting of a positive cosmological constant.

Perhaps these deep questions have seen less attention because it is not at all clear how to proceed. Unlike AdS/CFT, we lack clear and tractable examples of de Sitter holography with which to ground our understanding. Although we know well the rules of AdS/CFT, the rules of de Sitter holography have remained elusive. So progress on de Sitter holography over the last 20 years has been mostly intermittent.

This dissertation summarizes my and my collaborators' attempt over the last five years to grapple with de Sitter holography and more broadly the emergence of time in quantum gravity. This research program is presently ongoing, but we have to date made serious progress on multiple fronts. **Part I** presents a framework for studying quantum information in spacetime. While quantum states encode correlations at a fixed time, we introduce and study *superdensity operators* which encode space*time* correlations in their pattern of entanglement. This is covered in **Chapter 1**. Next, in **Chapter 2**, we put superdensity operators to work to study how tensor networks can encode causal structure and the arrow of time. Here we start shedding away the usual structures of quantum mechanics: we see how space and time can be encoded in the entanglement of a tensor network itself, without any explicit time evolution (i.e., a Hamiltonian, quantum channels, etc.). While the framework and results of **Chapter 1** and **Chapter 2** provide interesting insights for conventional quantum many-body systems and even AdS/CFT, they do not provide us with a model of de Sitter holography. However, the tools are useful for studying models of de Sitter physics.

In **Part II**, we formulate a model of de Sitter holography, and develop techniques to study it. We find that the de Sitter quantum gravity theory is equivalent to a non-quantum theory with neither space nor time. Thus, space, time, and quantum mechanics are emergent in the quantum gravitational description. The way in which this manifests is rather remarkable, and appears to be the first example of its kind. We develop relevant theoretical tools first in the more familiar setting of AdS in **Chapter 3**, followed by dS in **Chapter 4**. The de Sitter holography framework and emergent quantum mechanics, space, and time are analyzed in **Chapter 5**.

Going forward, there is much more work to be done to build on the model of de Sitter holography developed in **Chapter 5**. Our hope is that our findings will shed light on the fundamental nature of time.

Published Contents

The chapters of this thesis are essentially the same as the following published works and preprints:

Chapter 1:

- Cotler, Jordan, Chao-Ming Jian, Xiao-Liang Qi, and Frank Wilczek. "Superdensity Operators for Spacetime Quantum Mechanics." *Journal of High Energy Physics* 2018.9 (2018): 93.

Chapter 2:

- Cotler, Jordan, Xizhi Han, Xiao-Liang Qi, and Zhao Yang. "Quantum Causal Influence." *Journal of High Energy Physics* 2019.7 (2019): 42.

Chapter 3:

- Cotler, Jordan, and Kristan Jensen. "A theory of reparameterizations for AdS_3 gravity." *Journal of High Energy Physics* 2019.2 (2019): 79.

Chapter 4:

- Cotler, Jordan, Kristan Jensen, and Alexander Maloney. "Low-dimensional de Sitter quantum gravity." *arXiv:1905.03780* (2019).

Chapter 5:

- Cotler, Jordan, and Kristan Jensen. "Emergent unitarity in de Sitter from matrix integrals." *arXiv:1911.12358* (2019).

Part I

Spacetime Quantum States

Chapter 1

Superdensity Operators for Spacetime Quantum Mechanics

This chapter is essentially the same as

- Cotler, Jordan, Chao-Ming Jian, Xiao-Liang Qi, and Frank Wilczek. "Superdensity Operators for Spacetime Quantum Mechanics." *Journal of High Energy Physics* 2018.9 (2018): 93.

Abstract

We introduce superdensity operators as a tool for analyzing quantum information in spacetime. Superdensity operators encode spacetime correlation functions in an operator framework, and support a natural generalization of Hilbert space techniques and Dirac's transformation theory as traditionally applied to standard density operators. Superdensity operators can be measured experimentally, but accessing their full content requires novel procedures. We demonstrate these statements on several examples. The superdensity formalism suggests useful definitions of spacetime entropies and spacetime quantum channels. For example, we show that the von Neumann entropy of a superdensity operator is related to a quantum generalization of the Kolmogorov-Sinai entropy, and compute this for a many-body system. We also suggest experimental protocols for measuring spacetime entropies.

1 Introduction

Large parts of the existing formalism of quantum mechanics, and its interpretative apparatus, treat space and time on very different footings. Yet in classical physics it is often advantageous, especially in the analytical theory of dynamical systems, to consider time as an extra dimension on the same footing as spatial dimensions [2]. And of course grossly asymmetric treatment of time and space is, from the point of view of relativity, disturbing and unnatural. In this paper, we propose a formalism for *spacetime* quantum theory which ameliorates the asymmetry. This formalism suggests new ways to analyze the dynamics of quantum information and entanglement in spacetime, and new experiments to elucidate that dynamics.

Before attempting to put space and time on similar footing in quantum theory, it is instructive to recall how space is treated in quantum theory – in other words, the relationship between physical space and the Hilbert space of (single-time) states. In simple cases, we can decompose Hilbert state space into tensor factors

$$\mathcal{H} = \bigotimes_{\mathbf{x}} \mathcal{H}_{\mathbf{x}} \tag{1.1}$$

with each factor corresponding to a particular point in space $\mathbf{x}$. In this way, the tensor factor decomposition identifies local degrees of freedom.

Given a quantum state, we can analyze it relative to the tensor factors $\mathcal{H}_{\mathbf{x}}$, using a basis of the Hilbert space which is the tensor product of bases of the individual tensor factors. We learn about spatial correlations by considering entanglement between partial traces of the quantum state (i.e., spatial entanglement) or, in practice, by measuring correlation functions of operators that act on different tensor factors (i.e., local observables). Another, more general way to identify local degrees of freedom is to provide a "net of observables," essentially defining which operators on the Hilbert space are local [3]. Also, notoriously, important subtleties arise in taking infinite products of Hilbert spaces. In this paper we will prioritize simplicity over maximum generality.

To upgrade time from its parametric manifestation, it is natural to consider an expanded Hilbert space, containing tensor products for different times, which we will call the history Hilbert space $\mathcal{H}_{\text{hist.}}$. Supposing a tensor factorization into spacetime points is possible, then the history Hilbert space takes the form

$$\mathcal{H}_{\text{hist.}} = \bigotimes_{\mathbf{x},t} \mathcal{H}_{\mathbf{x},t}\,. \tag{1.2}$$

More compactly, if $\mathcal{H}_t$ is the Hilbert space of states at time t, then $\mathcal{H}_{\text{hist.}} = \bigotimes_t \mathcal{H}_t$. Now we require a spacetime generalization of quantum states. It is tempting to think that the right notion of spacetime states would simply be states in $\mathcal{H}_{\text{hist.}}$ with the standard inner product, but this turns out not to be useful. In fact, finding a suitable notion of spacetime states is rather subtle.

History Hilbert space has appeared before in discussions of quantum theory, perhaps most notably in Griffiths' foundational work on consistent histories [4–6]. In the consistent histories framework and its variations the central objects are projection operators on $\mathcal{H}_{\text{hist.}}$ [7–16]. In the entangled histories formalism developed by two of the authors of the present paper, the consistent histories formalism was generalized to define quantities which are more akin to spacetime states (in particular, allowing superposition) and some of their characteristic phenomenology was explored [17–20]. Other attempts at defining spacetime states include multi-time states [21–23] and pseudo-density matrices [24]. We should also mention that the multi-time correlation operator, which was developed as a tool to study dynamical entropies in quantum systems [25–31], embodies a special case of our superdensity operator. We will return to comparative discussion of some of these approaches later; for now, let us only mention that each of them can be expressed within the superdensity formalism, which appears to us more systematic and comprehensive. We have been inspired by the elegance and power that Dirac's transformation theory achieves for quantum states, and have attempted to achieve something analogous for spacetime analogs of quantum states. There has also been related work on spacetime quantum circuits and quantum measurements [32–41] which can be interfaced with our formalism.

In the approach pursued here, superdensity operators play a central role. Mathematically, superdensity operators are *quadratic forms on the space of operators on the history Hilbert space*. Physically, the superdensity operator of a physical system codifies its response to experimental probes, allowing that those probes may be applied at different times (and places). More abstractly, just as a standard density operator represents a state and encodes the data of all correlation functions at a fixed time, we propose that the superdensity operator represents a spacetime state, and encodes the data of all *spacetime* correlation functions. As the name suggests, superdensity operators share many formal properties with density operators. We will demonstrate, in particular, that the information theoretic properties of superdensity operators – such as their entanglement, entropies, mutual informations, etc. – are meaningful notions with operational physical significance. Thus the superdensity operator provides a compelling definition of the spacetime state of a system, which also appears to be fruitful.

The paper is organized as follows:

- In Section 2 we will further motivate, define, and exemplify superdensity operators, and discuss their formal properties. Within this framework, the concepts of spacetime observables and spacetime entanglement arise as naturally as do the corresponding concepts for (single-time) states.

- In Section 3 we show that the superdensity operator is in principle observable, and discuss the interesting, novel kinds of measurements its full exploration requires.

- In Section 4 we show how the superdensity operator suggests a definition of quantum dynamical