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Constraints on Extra Time Dimensions

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Abstract

We discuss phenomenology of extra time dimensions in a scenario where the standard model particles are localized in "our" time, whereas gravity can propagate in all time dimensions. For an odd number of extra times, at small distances, the (real part of the) Newtonian potential is "screened" by tachyonic Kaluza-Klein gravitons. In general, the gravitational self-energy of objects acquires an imaginary part. This complexity may either be interpreted as an amplitude for the disappearance into "nothing", leading to the causality and probability violation in low-energy processes, or as an artifact of the fictitious decay into the unphysical negative energy tachyons. The former case would put the most severe phenomenological restrictions on such theories. In the latter case the size of extra times may be within the reach of the proposed gravitational experiments. In such a case these experiments should observe that the strength of the Newtonian gravity diminishes at short distances.

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Extra space dimensions probed by gravitational forces can be as large as a millimeter without conflicting to any known laboratory or astrophysical bounds [1]. The extra dimensions which are usually considered are space-like. However, there is no *a priory* reason why extra times cannot exist. In this note we will try to study some phenomenological constraints on such hypothetical dimensions. The main problem with time-like compactified dimensions from the point of view of a low-energy field theory is an existence of tachyonic modes.[1] It is not clear whether an adequate quantum field-theoretical description of such states can be formulated [2]. This paper has no goal whatsoever to deal with these problems. We will rather try to find obvious phenomenological constraints contradicting the existence of extra time-like dimensions. We also suggest certain ideas which may weaken these constraints.

Consider for simplicity a five dimensional space-time with a signature $(1, 1, -1, -1, -1)$, with τ being an extra time coordinate. In analogy with extra space coordinates one may attempt to hide τ by compactifying it on a circle of radius L. However, in contrast to the case of extra space-like dimensions this does not do the job efficiently. The standard Kaluza-Klein excitations are now tachyonic states with imaginary masses quantized in units of i/L. In the space-like case, dimensions probed by standard model particles would be practically unobservable for $L >$few TeV or so [3]. However, the time-like case is very different. In fact, the exchange of KK states may lead to causality and probability violations. In [4] it was argued that the probability violation essentially rules out time-like dimension larger than the inverse Planck mass M_P^{-1}. The tree-level potential between two test charges mediated by the exchange of a photon propagating in one extra time has the form

$$V(r) \propto \frac{e^2}{r} \sum_n \exp\left(-in\frac{r}{L}\right) \ . \tag{1}$$

This potential is complex. The complexity of the amplitude may be interpreted in the way that probability is not conserved in the interactions of two charged particles and they can escape into "nothingness". The consideration of the baryon stability in the nuclei was argued to set the bound $L \sim M_P^{-1}$ [4]. Such a straightforward interpretation, however, raises many questions. For instance, disappearance of the particles would signal the charge and energy non-conservation, although the starting theory is gauge and time-translationaly invariant. Another important point is that the extra times may not be experienced by the standard model interactions.

In the present paper we want to study constraints on extra time dimensions, in the light of the possibility that the standard model particles can be localized in the extra time so that the corresponding KK modes are absent. We will also argue that complexity in the potential may have a different interpretation, in which case the bounds would not be as severe. We will be assuming that all the standard model particles are localized at a particular time moment $\tau = 0$ and move freely only in the remaining four coordinates $x_\mu,\ \mu = 0, 1, 2, 3$. They have localized wave-functions $\psi(x_\mu)\delta(\tau)$ in extra times. These particles can be viewed

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[1]The negative norm states can also appear. These will not be discussed here. In our case these modes have no direct couplings to matter. In some cases they can be "projected out" if the translational invariance in the extra time is broken. We thank Massimo Porrati for the discussions on related issues.

as confined to a "time-brane", a 4-dimensional hyper-surface embedded in space-time with q extra time dimensions of size L. The following toy example suggests a possible line of thought how such a localization may come about. Consider a scalar field ϕ in space-time with one extra time-like dimension τ. Take the action

$$S_{2+3} = \int dx^{2+3} \quad \left[(\partial_A \phi)^2 - V(\phi) \right] \ . \tag{2}$$

Now choose the potential so that ϕ has a soliton-type solution $\phi_{cl}(\tau)$ localized at say $\tau = 0$ and independent of x_μ. This solution should satisfy the boundary conditions $\phi_{cl}(\pm\infty) = \text{const}$. If the approach to the asymptotic values is fast enough there always exists a normalizable zero mode

$$\kappa = \partial_\tau \phi_{cl} \ g(x_\mu) \tag{3}$$

which is localized at $\tau = 0$. From the four-dimensional perspective this may be viewed as a massless particle satisfying the equation of motion

$$\partial^2_{3+1} \ g(x_\mu) = 0 \ . \tag{4}$$

This state propagates with the speed of light in our space-time, although its wave-function in the extra time is peaked at $\tau = 0$ moment and vanishes away from it. This mode is a Goldstone boson of spontaneously broken translation invariance in the τ direction and simply reflects the fact that shifts of the soliton in τ does not cost any energy. Similar considerations can be extended to states with different spins using the line of arguments of Refs. [5]. This will not be attempted here. Unfortunately, this method cannot completely get rid of tachyonic states which can propagate in the extra time and also couple to the ordinary matter. In particular such are KK gravitons (the "time bulk gravitons") which can propagate in all space-time dimensions and are viewed as tachyonic states by us. On the other hand all the standard model particles can be localized in the extra time. Again, the above example should not be considered as a recipe for the realistic model building, instead it just gives a crude idea of the localization in time. Below we will simply assume that the standard model particles are localized without further investigation of the precise mechanism behind it. More details will be given elsewhere. Let us compute the corrections to the gravitational energy due to the existence of extra time dimensions. In the leading order these arise due to the tree-level exchange of tachyonic KK states. Each mode with mass n/L generates a potential (for two test point masses m)

$$V_n(r) = G_N \frac{m^2}{r} \exp\left(-in\frac{r}{L}\right) \ . \tag{5}$$

The contribution to the self-energy per unit mass of a gravitating body with an uniform density $\rho = M/R^3$ and size R can be estimated as

$$\frac{E}{M} = 4\pi G_N \int_0^R dr \ r\rho \sum_{n=0}^{n_{max}} \exp\left(-i\frac{rn}{L}\right) \ , \tag{6}$$

where for q extra times $n = \sqrt{n_1^2 + .. + n_q^2}$ and summation goes over all n's up to a maximal value, which we will take as $n_{max} \sim (M_{Pf}L)^N \sim (M_P/M_{Pf})^2$ (here M_{Pf} is the fundamental

Planck scale [1]). For small L one can expand each integral in a series of Ln/R. In the leading order this gives the following corrections to the real and imaginary parts of the energy

$$\frac{\mathrm{Re}E}{M} \sim 4\pi G_N \frac{ML^2}{R^3} \sum_{n=1}^{n_{max}} \frac{1}{n^2} \ , \tag{7}$$

and

$$\frac{\mathrm{Im}E}{M} \sim 4\pi G_N \frac{ML^3}{R^4} \sum_{n=1}^{n_{max}} \frac{1}{n^3} \ . \tag{8}$$

The expression (7) is a correction to the gravitational self-energy and is negligible for L even as large as 1 millimeter (this is true even for a neutron star [1]).

However, the physical meaning of expression (8) is somewhat less clear. Under the normal circumstances the complexity of the self-energy would signal a non-zero decay amplitude. In the present context, however, there is no obvious candidate to decay into (note that the tachyons with negative or imaginary energy should be regarded as unphysical, see below). Thus, one option is that (8) signals that probability is not conserved and can be interpreted as a decay width into "nothing". An analogous estimate for the neutron inside a nucleus gives the following life-time for $q < 3$ (for $q = 3$ and higher, the sum is divergent and the contribution is enhanced by a coefficient $\sim n_{max}^{q-3}$)

$$\tau_n \simeq 10^{38} \ (L\mathrm{GeV})^{-3} \ \mathrm{GeV}^{-1} \ . \tag{9}$$

One can set the bound on L from the experiments that look for the invisible decays of nucleons. The most sensitive are the searches for $n \to \nu\nu\bar{\nu}$ channel [6]. This experiments look for the γ-rays emitted in the deexcitation of the nucleon hole produced by nucleon decay (or disappearance). The corresponding lower limit on a partial lifetime is $\sim 5 \times 10^{26}$ years. This implies that the period of an extra time must not exceed $L \sim 10^{12} \ M_{\mathrm{P}}^{-1}$.

However, the following discussion suggests that the applicability of such constraints is far less obvious. Let us try to understand better the origin of the complexity. For this it is useful to compute a one-loop correction to the self-energy of an ordinary fermion (e.g. electron) due to KK tachyon exchange. For simplicity, we shell consider a spin-1 tachyon of mass μ. One can do computation for real μ and then perform analytic continuation. Using for instance the Pauli-Villars regularization, one gets a well known result (for zero external momentum)

$$\frac{\Delta m}{m} \sim \int_0^1 dx(2-x)log\left(\frac{x\Lambda^2}{(x-1)^2m^2 + x\mu^2}\right) \ . \tag{10}$$

The logarithm has a branch cut for imaginary μ, which gives complexity. Formally this corresponds to an electron decaying into electron and a tachyon with a *negative* energy, $E_{tachion} = -\sqrt{\overrightarrow{p}^2 - \mu^2}$. Since this decay is in principle allowed by the energy-momentum conservation, it creates a branch cut in (10). Note the difference between the tachyon and a non-tachyonic boson with negative energy $E = -\sqrt{\overrightarrow{p}^2 + \mu^2}$. The energy conservation alone would be enough to prevent a similar decay of the electron into such a boson with negative energy. However, it is not enough to prevent decay into the tachyon. Thus, if the negative energy tachyons were "real", the complexity in the self-energy could have been interpreted as the decay into these states. However, existence of such states would contradict to the energy-positivity condition. Thus, any sensible quantum theory of tachyons is expected in some way to eliminate such states from the physical spectrum. Perhaps most trivially, by imposing the energy positivity condition and reinterpreting the negative energy tachyons as the positive energy anti-tachyons. We do not intend here to suggest any approach towards building such theories. However, it is reasonable to think that whatever mechanism eliminates negative and imaginary ($\overrightarrow{p}^2 < -\mu^2$) energy states from the physical spectrum, must also render (10) unphysical. If this is the case then the above constraints from the nucleon decay should presumably be disregarded.

Note that in our estimates it was crucial that we cut-off the number of possible KK excitations by n_{max}. However, if one sums an infinite tower of KK excitations the situation becomes more delicate. For instance, consider the case of a single extra time. The potential mediated by the infinite sum of KK modes takes the form

$$V(r) \propto -i\cot\left(\frac{r}{2L}\right)\frac{1}{r} = -\frac{i}{r}\left(\frac{2L}{r} + \sum_{k=1}^{\infty}\frac{4rL}{r^2 - 4k^2\pi^2L^2}\right) . \tag{11}$$

Let us compute the gravitational self-energy of a spherical body of the radius R. This energy is proportional to the integral of the potential (11) from zero to R. The answer depends on how many poles of (11) contribute to the integral. In the simples case when all the poles of (11) are outside of the volume of the body, i.e. $R < 2\pi L$, the expression for the energy is complex and does not have a real part. This means that classical gravity is "screened" at very small distances by the corresponding KK states. In the case when $R > 2\pi kL$ (for some finite set of k's) the poles of (11) would produce a nonzero real part in the expression for the self-energy. This can be calculated by doing the integral explicitly and estimating the corresponding finite sum over the poles. The result takes the following form:

$$\frac{\mathrm{Re}E}{M} \sim G_N\rho\left[2\pi RL + R^2\right] . \tag{12}$$

The second term in this expression comes from the zero mode graviton that mediates a normal Newtonian potential. The corrections due to KK modes, however, are only linearly suppressed in L. Let us now turn to the imaginary part of the self-energy. It is defined by the regular part of (11) and can be calculated. Along with the standard terms proportional to either iRL or iR^2 there appear terms proportional to $iRL\log[(R-2\pi kL)/(R+2\pi kL)]$. These are singular as poles of (11) coincide with R, i.e. when $R = 2\pi kL$. This type of singularities could indicate that decoupling in the theory might not be happening, and that a possible right way to deal with this models is to consider them as effective theories with a cutoff. Even if we are away from the singularities mentioned above, the model with an infinite number of KK states would put a much more severe constraint on the size of extra times. The reason for such a different behavior is the same as before. When an infinite number of KK states are taken into account the potential develops an infinite number of poles (11). Some of these contribute to the imaginary part of self-energy, thus making "decay" width of the object much bigger. As a result one needs to impose a stronger bound on the size of extra time dimensions.

In the estimates presented above L was assumed to be smaller than the size of the body in question. One might wonder about the cases when $R << L$. To study this issue let us compute the Newtonian potential between two point-like masses m which are localized at $\tau = 0$ at $r << L$ distance apart. In the infinite volume approximation the potential can be defined as follows:

$$V(r)t = \frac{1}{M_{Pf}^{2+q}} \int dx^{4+q} dx'^{4+q} T^{AB}(x)\ G_{AB,CD}(x - x')\ T^{CD}(x')\ , \tag{13}$$

where $G_{AB,CD}$ is the graviton propagator, $T^{AB}(y) = (0, 0, ..0, m, 0, 0, 0)\delta^q(\tau)\delta(\overrightarrow{y} - \overrightarrow{x})$ and t is interaction time. For the static case this gives the following expression:

$$V(r) \sim (i)^q \frac{m^2}{M_{Pf}^{2+q}} \frac{1}{r^{1+q}}\ . \tag{14}$$

The unusual thing about this expression is that it is pure imaginary for odd q, meaning that Newtonian potential is "screened" at $r << L$ distances. For $q = 1$ this can be directly read off (11) by taking the $L \to \infty$ limit.

Above should mean that the Newtonian gravity shuts-off at small distances. The gravitation self-energy of a spherical body of size $R << L$ for finite L can be directly computed from (11), since in this case we do not encounter any poles

$$\frac{E}{M} \sim iG_N \rho LR \left[1 + O(\frac{R}{L})^2\right] \sim iG_{N(5)}\rho R \left[1 + O(\frac{R}{L})^2\right]\ . \tag{15}$$

where $G_{N(5)} \sim M_{Pf}^{-3}$ is a five-dimensional Newtonian constant. Note that an analogous estimate for one extra space dimension would give

$$\frac{E}{M} \sim G_N \rho LR \left[1 + O(\frac{R}{L})^2\right] \sim G_{N(5)}\rho R \left[1 + O(\frac{R}{L})^2\right]\ . \tag{16}$$

since in this case the analog of (11) is

$$V(r) \propto -\coth\left(\frac{r}{2L}\right)\frac{1}{r} = -\frac{1}{r}\left(1 + 2\sum_{n=1}^{\infty} \exp\left(-\frac{rn}{L}\right)\right)\ . \tag{17}$$

If we cut-off the number of KK modes by $n_{max} \sim M_{Pf}L$, then a nonzero real part will appear in eq. (15)

$$\mathrm{Re}\frac{E}{M} \sim G_N \rho \frac{L}{M_{Pf}} \tag{18}$$

This is suppressed by a factor $\sim (RM_{Pf})$ relative to (16) as one naively could have guessed, since for bodies as small as an inverse ultraviolet cut-off M_{Pf}^{-1}, there are no KK modes available for "screening" gravity. How large L can be in such a case? As we explained above, if one disregards complexity in the potential as being unphysical, the probability-violation bounds can be ignored. In such a case the size of extra times can be within the reach of proposed submillimeter measurements [7]. Thus for an odd number of extra times these experiments should see the strength of the Newtonian gravity diminishing at short distances!

However, if complexity may really signal the probability non-conservation, L presumably can not be in submillimeter range, even in the case of even q (in which case the potential is real for infinite L). If complexity appears in the amplitude, on the dimensional grounds, it should be of order one at distances $r \sim L$. Since the four- and high-dimensional Newtonian laws must match at the size of extra dimensions, the potential at $r \sim L$ should have imaginary part comparable to the (four-dimensional) gravitational strength

$$\mathrm{Im}V \sim \frac{m^2}{M_P^2}\,\frac{1}{L}\ . \tag{19}$$

If regarded as probability violation, for two neutrons separated by a distance L this would imply a disappearance rate $\Gamma \sim 10^{-38}/(L\mathrm{GeV})$ GeV, which for $L \sim 1$ mm gives the lifetime $t \sim 10^{51}$ GeV^{-1} clearly contradicting with an above derived bound.

Finally, we would like to quote somewhat less severe bounds which may come from the production of the tachyonic KK gravitons in the stars. A single graviton production rate is $\sim T(T/M_P)^2$. The total rate is enhanced by the multiplicity of final states

$$\sim T(T/M_P)^2(TL)^q\ , \tag{20}$$

where T is a temperature in the star. Expressing this in terms of the fundamental Planck length we get a suppression factor analogous to the one obtained in the case of extra spatial dimensions [1]

$$\sim T(T/M_{Pf})^{2+q}\ . \tag{21}$$

The two-graviton production rate is suppressed by extra powers of M_P

$$\sim T(T/M_P)^4(M_{Pf}L)^q \sim T^5/(M_{Pf}M_P)^2\ , \tag{22}$$

and is sub-dominant.

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