General Remarks on the Potential Use of Atomic Clocks in Relativistic Geodesy

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Summary
Current developments of optical atomic clocks and in laser technology make it feasible to geodetically exploit relativistic effects instead of just correcting for it in highly accurate measurements. New observables, like direct observations of potential differences, become available and therefore require the development of novel measurement methods. Geodetic tasks will benefit from a consistent relativistic approach. Modern quantum sensor technology provides the practical means to further develop the emerging field of relativistic geodesy.

Zusammenfassung
Aktuelle Entwicklungen bei optischen Atomuhren und in der Lasertechnologie ermöglichen die geodätische Ausnutzung relativistischer Effekte, anstatt sie nur als notwendige Korrekturen bei hochgenauen Messungen anzubringen. Neue Observable, etwa direkte Potenzialdifferenz-Beobachtungen, werden verfügbar und erfordern deshalb die Entwicklung neuer Messmethoden. Geodätische Problemstellungen werden von einem konsistenten relativistischen Ansatz profitieren. Moderne Technologie zur Quantensensorik stellt die praktischen Mittel bereit, um das aufstrebende Gebiet der relativistischen Geodäsie weiterzuentwickeln.

Keywords: time, atomic clocks, relativistic geodesy, applications

1 Introduction

The theories of relativity and quantum mechanics have an increasing impact on geodesy. Apart from the relativistic modeling of space-geodetic techniques (Müller et al. 2008a), practitioners in the most part still tend to the ideas of Newtonian mechanics and its concepts of space and time. In the framework of physical geodesy it became obvious that relativistic effects no longer should be ignored. In addition to relativity, novel measurement techniques for the determination of time and frequency using atomic clocks are also based on quantum-physical processes. Therefore, geodesy at some point in future will be affected by the on-going search in physics for a unified approach that might bring its both fundamental theories together.

Despite the fact that Newton's classical mechanics was extended by both theories, it still serves as a reasonable approximation in case of moderate relative velocities, short distances, and weak gravitational potentials. But according to Einstein's theory of relativity, concepts like "equal duration", "equal (spatial) distance" or "simultaneity of events at remote places" require a stringent setting of rules to make sense and a corresponding instrumentation to measure it. Terms like "uniform" and "straight", as being used in Newtonian laws of motion, become relative: uniform with respect to which clock, straight with respect to which system of reference? With ongoing progress in technology one can not neglect this kind of fundamental questions.

Regarding the exploitation of relativistic concepts, the Earth system's metric field will become a major subject of investigation in future geodesy. Several quantum sensors are potentially useful for genuine geodetic applications (e.g. Börger 2007, Schilling et al. 2012). Within the domain of physical geodesy, the utilization of highly precise time and frequency instrumentation, e.g. optical atomic clocks and frequency combs, for the direct determination of the gravitational potential in combination with classical gravity field functionals is of special interest. Generally, different types of measurements are available based on the motion of bodies, light propagation, or clocks.

Relativistic geodesy (Müller et al. 2008a) implies the last-mentioned option, e.g. direct potential measurements via atomic clocks. Basic formulae for the determination of heights using atomic clock readings, either by using frequency ratios or clock rate differences, are already given in literature (e.g. Brumberg and Groten 2002). The global unification of national height systems (Colombo 1980, Rummel 2012) is a worthwhile application. In geophysics, for the solution of the inverse problem, i.e., the deduction of sub-surface density variations from ground measurements, the use of atomic clocks may help to reduce the existing degeneracy (Bondarescu et al. 2012).

Exemplarily, the determination equation for the comparison of atomic clock readings (proper time intervals $d\tau_1, d\tau_2$, cf. section 3.1)

$$\frac{d\tau_1}{d\tau_2} = 1 + \frac{V_2 - V_1}{c^2} + \frac{v_2^2 - v_1^2}{2c^2} + O\left(c^{-4}\right) \qquad (1)$$

depends on the relative state of motion (velocities v_1, v_2) of the involved clocks and its respective positions within a (gravitational) potential (V_1, V_2). In return, among others these quantities could be determined by time and frequency comparisons between atomic clocks that are distributed in a (worldwide) network. For instance, geoid height determinations on the cm-level would require atomic clocks with an accuracy of about 10^{-18} (Herrmann and Lämmerzahl 2010, Chou et al. 2010). Specific observation methods and measurement techniques for geodetic application have to be developed.

This paper provides a short review of selected general aspects of relativistic geodesy.

2 A question of time

Time can be discussed from different perspectives. In physics, we distinguish between parametric (or coordinate) time t (absolute), proper time τ (general relativistic) and a unified concept of time that still needs to be developed. The time specified by the one time coordinate of a fourdimensional space-time coordinate system is referred to as coordinate time t, whereas in these terms proper time τ analogously refers to the spatial arc length in threedimensional space. For the investigation of possible geodetic applications we will focus on τ, which corresponds to local (atomic) clock readings, and its relation to t (Guinot 1986, Arias 2006). The more sensitive our instrumentation gets the more obvious the need of a consistent treatment of space and time concepts will become. For the time being, in view of the reached precision of instrumentation, the well-established concepts of coordinate and proper time are fine.

2.1 Measurement of time

Realization of a clock does not clear up the phenomenon of time, it rather defines a time scale in order to compare the occurrence of events. Time in the sense of a coordinate time could only be measured by an idealized clock. Timing or pulsing means counting of certain recurring phenomena, e.g., phases or periods of an oscillatory motion. Harmonic oscillators are vibratory systems with ideal properties, and a clock simply is a device to create such a periodic process. In practice, we have to draw on real representatives of a clock, i.e., real world motions like pendulums, oscillatory crystals, planetary revolutions, atomic state superpositions and so on. Such devices, i.e., non-idealized clocks, do not provide coordinate time. On the other hand, the only tools needed to experimentally test whether a given clock is a standard clock (providing proper time) or not, are light rays and freely falling particles (Perlick 1987).

On a large astronomical scale, remote celestial objects such as pulsars (Hartnett and Luiten 2011) may also provide timing signals. Using astronomical objects requires a careful transformation between terrestrial and the objects' proper frames taking into account a variety of relativistic effects (Hellings 1986). The treatment of any (astrometric) observation comprises the solution of the equations of motion representing the world lines of the observer (observing subject) and the observed object, and of the equations of light propagation. Additionally, the observation process itself has to be modeled in a consistent relativistic framework.

The aspect of measurability is important because time is not identical to motion itself. Instead, the perception of time relies on the observable order of periodic motion. A good clock selection then requires a well-countable motion which is as regularly as possible. Of course, regularity is just another relative term – we simply compare periodic motions against each other. A comparison of two clocks will not reveal, which one is more accurate. Today it is quite common to reject the idea of being able to measure absolute time and space at all. All we can possibly detect is a change in the relative positioning of material objects.

2.2 Different notions of time

Parametric time t as a paradigm of classical, i. e. Newtonian, mechanics also enters quantum mechanics and even the concept of special relativity, where space-time is still treated as an invariant entity. The introduction of general relativity changes the character of time. Now time dynamically depends on matter and/or energy and will retroact upon it. Time and motion are strongly coupled, and space-time itself becomes an object such that time no longer can be applied externally. Instead, we introduce the concept of proper time τ and therefore time changes from a global to a local entity – any motion bears its own, i. e. proper, time.

Relativistic time scales and their relations to more classical approaches were considered in various IERS conventions (2012). Relativistic geodesy rests on the consistent implementation and distribution of those revised time scales (Seidelmann and Fukushima 1992, McCarthy and Seidelmann 2009).

3 Applying the theory of relativity

The definition of space and time affects three fundamental concepts that are of great relevance to relativistic geodesy, namely the clock hypothesis (clocks measure proper time), the length hypothesis (whether or not to introduce a fundamental length to fix the scale), and the concept of a field quantity (considering different special cases of a field, e. g. stationary versus static, which has some implications on the arrow of time problem). The use of a specific metric, i. e., the formulation of the invariance of space-time intervals, is eventually based upon an empirical basis. These intervals can be described by a combination of spatial and temporal coordinates, which results in an invariant spatiotemporal line element

$$\mathrm{d}s^2 = \mathrm{d}r^2 - c^2\mathrm{d}t^2, \tag{2}$$

where $\mathrm{d}r^2$ is the usual spatial element of arc, e. g., expressed in Cartesian coordinates.

In order to avoid an imaginary quantity, a purely formal factor i (imaginary unit) is being introduced together with the observers' real-valued proper time τ via

$$\mathrm{d}s =: ic\mathrm{d}\tau, \tag{3}$$

such that $\mathrm{d}s^2 = -c^2\mathrm{d}\tau^2$.

Denoting the spatial velocity of the observer as $v = \mathrm{d}r/\mathrm{d}t$, we find a fundamental relationship between t and τ:

$$-c^2\mathrm{d}\tau^2 = \left(v^2 - c^2\right)\mathrm{d}t^2 \quad \Rightarrow \quad \mathrm{d}\tau = \sqrt{1-\left(\frac{v}{c}\right)^2}\,\mathrm{d}t. \tag{4}$$

This equation indicates that a time dilatation will occur whenever an object, e. g., an atomic clock, is moving with respect to the t-related coordinate system, i. e., the rest frame $(x, y, z; t)$. Because of $v < c$, always $\mathrm{d}\tau < \mathrm{d}t$.

The minus sign in equation (2) epitomizes the different character of space and time – the dimension of time is geometrically mapped onto a spatial axis. In principle, any spatial reference system and time scale is possible, but the minus sign further implies that space-time can not be split up in an entirely arbitrary way. Time is separating and joining events while maintaining a consistent causal ordering. Depending on the actual chosen splitting, space will change in time and vice versa. This kind of arbitrariness, as a key property of the theory of relativity, eliminates the existence of a single time.

Regarding space, one often introduces (quasi-)inertial systems, but non-inertial systems are by no means unphysical or forbidden (Giulini 2004). It is just a question of convenience, because non-inertial motion of a reference system causes some extra terms describing apparent forces within the equations of motion. Special relativity theory distinguishes between real and apparent forces and therefore admits a special role to inertial systems. Going one step further by geometrizing gravity, the theory of general relativity (Schröder 2011) abandons the existence of distinguished reference systems altogether. The fundamental difference between apparent or inertia forces and gravitational forces vanishes, at least locally. In essence, this theory renounces the concept of (a gravitational) force altogether. Instead, any mass causes a negative curvature of the non-Euclidean space-time. The straightest possible world line of an observer, e. g., a clock or just a single photon, equals the path of its actual motion. In space-time it is a geodesic, but (in general) not in three-dimensional space. The concept of gravity as a force is thus replaced physically by the geometric structure of a four-dimensional system of space-time.

According to the relativity principle, physical laws are equally valid in all inertial systems. In the sense of Newton, this principle is restricted to classical mechanics. Einstein extended its scope of application to electrodynamics and therefore to the propagation of electromagnetic waves, e.g. light rays. He combined this general principle with some special properties of light. According to his theory, the speed of light in vacuum c should be of limited constant value, isotropic, and independent of the relative velocity of the light source.

In special relativity inertial systems are still marked reference systems. Changing from one inertial system to another, only the light's frequency (Doppler effect) and propagation direction (aberration effect) may change. A finite c poses a limit for the transmission of information and action/energy. Signals can not be transmitted instantly. Nevertheless, looking at visual phenomena in the propagation of light waves, phase velocities and group velocities may be of larger value than c, well in accordance to special relativity (Giulini 2004). Furthermore, among any set of inertial systems, there exist no prominent ones. These fundamental statements lead to a number of derivable effects, e.g., time dilation, length contraction, relativistic Doppler shift and aberration, that all have to be accounted for in the interpretation of highly precise observations (Günther 2007).

In general, relativity has to be exploited in two alternative ways. Single time measurements make use of clock readings, and clock comparisons additionally make use of electromagnetic signals. Both means are affected by relativistic effects, the causes of which can be described by parameters that are of interest in geodetic applications.

3.1 Atomic clocks as relativistic sensors

The availability of precise atomic clocks make the theory of relativity a practical tool. Special relativity states that a clock's ticking rate in comparison to another ones' depends on their relative states of motion. Likewise, following general relativity, clock readings are affected by the clocks' position within a gravitational potential, such that equation (4) has to be supplemented by the influence of the potential $V_k = V(\mathbf{r}_k)$. For different clocks ($k = 1, 2, \ldots$) at positions $\mathbf{r}_k$ we have (Mai 2013a,b)

$$\frac{d\tau_k}{dt} = \sqrt{1 - \frac{2V_k}{c^2} - \frac{v_k^2}{c^2}} = 1 - \frac{V_k}{c^2} - \frac{v_k^2}{2c^2} + O\left(c^{-4}\right). \tag{5}$$

Taking the ratio $(d\tau_1/dt)/(d\tau_2/dt)$ of two clock readings yields

$$\frac{d\tau_1}{d\tau_2} \approx \left(1 - \frac{V_1}{c^2} - \frac{v_1^2}{2c^2}\right)\left(1 + \frac{V_2}{c^2} + \frac{v_2^2}{2c^2}\right), \tag{6}$$

which leads to equation (1). In case of frequency standards at rest ($v_k = 0$), or provided that their velocities were determined precisely enough (e.g. via GNSS methods) and corrected for, we get the simplified expression

$$\frac{d\tau_1}{d\tau_2} - 1 = \frac{f_2 - f_1}{f_1} = \frac{V_2 - V_1}{c^2}, \tag{7}$$

where f_k denote the respective proper frequencies. Remark: any gravitational potential values V_k are assumed to be positive here (geodetic sign convention).

Representing the inverse problem, velocities and potential values can be determined by clock comparisons. Starting from these primary quantities, secondary quantities may be derived, e.g., potential differences or heights. All these quantities are relative ones. Therefore, as in classical geodesy, the question of reference frames (Soffel et al. 1988a) is essential for relativistic geodesy.

3.2 Increasing sensitivity

Electromagnetic signals traveling through space will be affected in multiple ways (frequency shift, time delay, path deflection) due to several properties of disturbing massive bodies (mass, spin, irregular shape). Reversely, highly precise measurements of these effects on a signal, in principle, allow for the determination of the massive objects' properties (Ciufolini and Ricci 2002).

In the early days of relativistic testing, experimental settings relied on comparatively high relative velocities and big changes in elevation, i.e., changes of the gravity potential. Thus the theory of relativity was first respected on astronomical scales. With the growing precision and stability of frequency standards it became apparent that the annual motion of the Earth about the Sun imposes significant variations in the readings of clocks that are attached to Earth's surface (Clemence and Szebehely 1967). Later, observational equations accounted for many different physical causes of these variations.

Today, with the advent of 10^{-17} to 10^{-18} (frequency inaccuracy level, $\delta f/f$) frequency standards, relativistic effects can even be applied in experiments that make use of only moderate to small earthbound velocities (Chou et al. 2010) and changes in position (Pavlis and Weiss 2003). Nonetheless, relativistic geodesy nowadays still owns the status of a somewhat experimental science. Proving its applicability to contemporary open questions (e.g., a unified world geodetic height system), the beneficial use of relativistic geodetic techniques should become evident.

4 Transition from classical to relativistic geodesy

For the ease of practical application within geodesy, the task of taking relativistic measurements should not be superimposed with the task of performing relativistic tests. In fact, several measurement campaigns and satellite missions were especially dedicated to the determination of refined confidence intervals for various parameters of candidate theories of gravitation. Instead, depending on the attainment of a chosen adequate precision level of our instrumentation, for the time being, we take for granted the validity of the theory of relativity. It simply should be regarded as a new tool (Will 1989). Different post-Newtonian approximations were examined but Einstein's theory of gravity remains the simplest alternative, which so far still passed all tests (e.g. Müller et al. 2008b, 2012). Applying his general theory of relativity, we assume that the metric tensor is the only gravitational field variable. Nonetheless, Einstein's theory has its own limitations, which acts as a motivation to develop more elaborate theories of gravity in fundamental physics.

4.1 Technological issues

The relativistic framework of actual instrumentation is presented in mathematical detail in Bordé (2002). Atomic clocks are based upon certain atomic state transitions. Gill (2005) describes several suitable transitions for optical clocks.

Frequency standards periodically generate pulses at a rate as regular as possible. Getting a clock requires the additional implementation of a counting unit, which continually records the number of pulses. It should be possible to turn on and off a clock easily. Another desirable property would be the mobility of the whole device.

The quality of a frequency standard can be characterized by certain quantities: accuracy, precision, and stability. Precise frequency standards are often still based on atomic hydrogen maser oscillators (Vessot 2005) and atomic fountain clocks (Wynands and Weyers 2005). Microwave clocks are progressively being replaced by so-called optical clocks. Their development (Peik and Sterr 2009) required the availability of ultra-stable interrogation lasers (Sterr at al. 2009). The realization of superradiant lasers (Bohnet et al. 2012) may lead to even more stable clocks (Meiser et al. 2009). Optical lattices can be created to trap (statistically) countable sets of individual particles (atoms or ions) (Katori et al. 2011), the interrogation of which is suitable for constructing precise frequency standards. The application of quantum logic in precision spectroscopy indicated the feasibility to construct optical clocks based on single ions (Schmidt et al. 2005).

Ultra-high performance can be achieved by reducing perturbation effects as far as possible. In order to reduce thermal noise, isolated atoms or ions are put nearly at perfect rest by means of laser cooling techniques (e.g. Schmidt 2003). This improves the signal-to-noise ratio and even more so the overall performance. Reaching the cooling limits takes a certain amount of time, where this procedure is part of a comparatively long total preparation process which is necessary for the entire laboratory instrumentation setup. In this respect, ion clocks are a bit easier to handle. Wineland et al. (1987) discuss how to achieve a requested measurement precision within a reasonable time frame. The performance can also be increased by another technological improvement. For instance, a better laser beam guidance (Kleine Büning et al. 2010) enables longer interrogation times and possibly even continuous interferometric measurements.

In quantum optics, the ability to create coherent laser light with high frequency stability is a prerequisite for precise measurements (Lisdat and Tamm 2009). Femtosecond laser frequency combs (Hollberg et al. 2005) show great potential for the development of new instrumentation. An alternative to atomic clocks and gravimeters (using a free falling atomic sample) is given by comprising the levitation of a Bose-Einstein condensate which shows enhanced measurement sensitivity (Impens and Bordé 2009).

Another promising advancement in the transition of atomic clocks towards a practical geodetic instrument comes from successful miniaturization attempts. Frequency reference devices with a total volume of only a few cubic centimeters with reasonable precision and stability seem to be technically feasible (Kitching et al. 2005).

4.2 Clock networks

In view of potential geodetic applications, e.g., intercontinental height transfer by chronometric leveling, we rely on relative measurements of time, instead of "absolute" measurements at a single location. Consequently, we have to compare clock readings in a suitable way. Ashby and Allan (1979) present theoretical and practical aspects on how to set up a clock network. Exploiting its full capability requires the ability to intercompare frequency standards at the same level (Abbas et al. 1997). Synchronization of networks is a major issue, e.g., for combined measurement campaigns within networks of globally distributed observation sites. Without it clocks could not be successfully distributed to globally read time and attain a causally consistent temporal order of remote events. Wong (1997) discusses general network topologies for different synchronization methods, whereas Bregni (2002) provides more technical details.

4.3 Time and frequency transfer

Synchronization (Klioner 1992) or syntonization (Wolf and Petit 1995) of clocks in a relativistic sense can practically be achieved by various means. Clocks are either located nearby at the same site or separated at remote sites. In the first case, clock readings may easily be compared in a direct way. In case of remote clocks, one could bring them together in one location by physical transportation. This requires portable clocks in the sense that they are moveable.

Clocks in motion potentially bear information on its state of motion and the environment along its path of transportation. To gather and exploit this information, we need the clock(s) to be continuously ticking during transportation phases. This requirement is much more challenging than simple movability, where the clockwork will be switched off and on. In case of a running clock during transportation we call it a mobile clock. Clock synchronization by (slow) transport is equivalent to Einstein's rule for synchronization. There already exist several suggestions for specific measurement procedures in case one can use mobile clocks (Bjerhammar 1986). Currently, mobile optical clocks are still under development. Till this day, aside from validating earthbound or airborne experiments (e.g., round-the-world flying clocks aboard a plane) to dedicated space missions concerning certain predictions of the theory of relativity, there is only limited practical experience with mobile frequency standards on a routinely basis, e.g. with a passive hydrogen maser (Feldmann 2011).

In an indirect approach one compares frequency standards via an exchange of signals. Mathematically, relativistic formulae for time and frequency transfer up to order c^{-3} have been derived (e.g., Blanchet et al. 2001). Perlick (2006) discusses geometrical subtleties of clock synchronization using electromagnetic signals. The transfer medium could either be "free air" or "by wire". The former means going through the atmosphere with all its subtle effects on the electromagnetic signal. Environmental influences are much more manageable, if we use solid material connections. Optical fiber networks for time and frequency transfer (Foreman et al. 2007) and dissemination (Amemiya et al. 2006) have already been used to achieve an optical frequency transfer with 10^{-19} relative accuracy over a distance of more than 100 kilometers (Grosche et al. 2009). Other experiments demonstrated the feasibility of signal comparisons using telecommunication fiber links for extended distances up to 920 kilometers (Predehl et al. 2012).

If intercontinental distances have to be bridged then fiber networks are of limited use. Besides technological problems, their cost efficiency probably loses against free air transmission techniques. If there is no direct line of sight between remote clock sites, one may set up a signal chain. Depending on its wavelength, atmospheric layers could be used to bounce off and forward the electromagnetic signal. More accurate transfers require shorter wavelengths and therefore the use of artificial reflectors. Aiming at highest accuracies for very long distances, laser ranging techniques using two-way signals require active or at least passive transponders instead of reflectors (Degnan 2006). Depending on the actual application and its constraints, these reflectors or transponders may have to be mounted on balloons, air vessels, stratosphere planes, satellites or even the Moon and other celestial bodies (e.g., if interplanetary distances have to be bridged).

Depending on the sites' geometry and available instrumentation, several transfer methods have been established. These can be classified using different categories. For instance, there are single view or common view methods (Schmidt and Miranian 1997), and one-way or two-way methods (Petit and Wolf 1994), respectively. Stable clock readings require comparatively long averaging times. This poses some topological constraints on the use of moving reflectors or transponders. In this respect, using a geostationary orbit would be of great benefit.

Despite the fact that two-way methods are of advantage, e.g. for the elimination of unwanted systematic effects, the one-way method is most commonly used. A variety of active artificial satellites carries passive retro-reflectors on-board, mainly for the reason of orbit determination. These reflectors can readily be applied for time transfer and dissemination. As long as the two way satellite time and frequency transfer approach is more expensive, the application of GNSS (Levine 2002) with its established geodetic phase and code measurements is of advantage (Ray and Senior 2003). For example, the GPS time signal is exploited by customized receivers, and one way GPS carrier phase time transfer (Delporte et al. 2008) has been used in the past.

For highly precise signal transfers, connections to the satellites will be established via stable microwave or laser links (Fridelance et al. 1997). Estimating the standard uncertainty in frequency transfer (Douglas and Boulanger 1997) is essential. Kleppner (2006) remarks that, at high levels of precision, uncontrollable fluctuations might act upon atomic clocks such that it will be impossible to select a master clock for keeping true time. Furthermore, for ground-to-space time transfers with picosecond-accuracy via laser link, as with the "Atomic Clock Ensemble in Space" (Heß et al. 2011), the signal detection process has to factor in optical to electrical detection delays within the instrumentation setup (Prochazka et al. 2011). One aims at a few picoseconds time stability for the comparison of distant clocks and one hundred picoseconds time accuracy in the distribution of time scales.

Obviously, to accomplish highly precise time transfers, many effects have to be taken into account. One way of checking the achieved performance is a loop-wise use of multiple transfers, similar to the leveling method in classical geodesy. By doing so, one could study resulting non-zero closures, e.g., for triplets of two-way satellite time and frequency transfers (Bauch et al. 2008).

5 Making the case for the geodetic use of atomic clocks

In traditional physical geodesy, one of the main tasks is to establish and monitor the global geodetic observing system (GGOS, Plag and Pearlman 2009). Such a GGOS comprises three major parts, namely the geometry, rotation, and gravity field or geoid of the Earth. Geodesists contribute to the detection of the actual state of all three parts and its changes. A functional GGOS requires an accuracy level for its individual observation techniques and reference systems of at least 10^{-9}. Regarding Earth dimensions with radius $R_\oplus \approx 6378$ km and a gravity value at the surface of about $g_\oplus \approx 9.81$ m/s^2, an order of 10^{-9} means that we want to determine metric quantities on the mm-level and gravitational quantities on the μgal-level. Present Earth models do not meet these demands. Current gravity field space missions strive for a geoid accuracy at the cm-level with a spatial resolution of about 100 km (Pail et al. 2011). Stations of the international reference system of gravity (epoch year 1971) typically have an accuracy of approximately 100 μgal, whereas modern classical gravimeters can reach the μgal (absolute gravimeter) or even sub-μgal (superconducting gravimeter) level (Torge and Müller 2012).

5.1 Decorrelation of physical effects by means of clock readings

Changes in the system Earth involve various interacting spheres, namely the biosphere, geosphere, atmosphere, cryosphere, and hydrosphere. The gravity field connects different geo-disciplines and thus plays a dominant role in Earth system research. There exist models to all parts of the system, the parameters of which are monitored via corresponding time series. Changes in form of variations, deformations, and rotations are induced by a variety of correlated causes. One can separate these to a certain extent, because they show a broad spectrum in amplitude and frequency. Spatially, we distinguish between local and global effects. All these space-wise and time-wise differences imply corresponding requirements on geodetic instrumentation.

Different time scales are involved, which has consequences on the necessary frequency stability of the clock. Short intervals are associated with signal travel times or epoch distances. Intermediate intervals are needed for comparisons of atomic time scales (related to elementary particles, i.e. micro-scale objects like electrons or photons) with ephemeris or astronomical time scales (related to celestial bodies, i.e. macro-scale objects, like Earth, Moon, Sun). Long intervals are associated with large-scale Earth system processes, e.g. plate tectonics.

In practice, atomic clock readings are affected by several effects (state of motion, gravity field, magnetic field, any possibly imperfect shielding from environment, etc.). Their output can be written as a functional of different parameters describing these effects. Observation equations can be set up to determine those parameters. In order to separate effects, differently or equally constructed clocks at various locations in space-time have to be operated.

5.2 Applications of highly precise atomic clocks

Bjerhammar (1985, 1986) can be viewed as one of the pioneers of relativistic geodesy. Unfortunately, previous occasional work on the same subject (Bjerhammar 1975) has been hardly appreciated by the geodesy community. Allan and Ashby (1986) also discussed a wide range of possible applications. The chronometric leveling idea came up decades ago (Vermeer 1983, Schüler 1991) by reasoning that clock rates are shifted by changing gravitational potentials. Differences in potential values relate with differences in height. As in the classical approach, relativistic height determination is inevitably tied to the definition of reference surfaces. Correspondingly, one can introduce the concept of a relativistic geoid (Kopejkin et al. 2011), which can act as a reference for global clock comparisons. Basic formulae for relativistic gravimetry and gradiometry have been derived already (Soffel et al. 1988b, Kopejkin 1991, Gill et al. 1992, Kusche 1996). Švehla and Rothacher (2005) discussed global gravity field determination by using atomic clocks in space.

Atomic clocks may be used for the comparison and alignment of reference systems, e.g., stemming from different positioning and timing systems based on various satellite constellations. The same holds true for the unification of geometric (GNSS) and gravimetric (optical clocks) positioning. The timing aspect itself is another important issue. Accurate clocks in Earth orbit may provide a lasting and reliable global time scale which could be used for instance to monitor GNSS time signals. GNSS would be designed in a new way, based upon two-way connections. This allows for the elimination of first order Doppler effects and enables real-time frequency dissemination. The estimation of clock parameters and ambiguities would become unnecessary. Precise timing is also essential in Earth measurement techniques like reflectometry, radio occultations, scatterometry, atmospheric, or ocean sounding. Popular research topics on climate change (e.g., separation of mass portions and volume portions in sea level changes), the atmosphere (e.g., atmospheric remote sensing by signal detection using zero differences), or oceanography (e.g., sea level heights, dynamic ocean topography, tsunami early warning systems) offer plenty of potential applications.

Space geodetic techniques like Very Long Baseline Interferometry (VLBI) or Satellite Laser Ranging (SLR) are based on the precise determination of signals' time of arrival, time delay, or travel time. Improved start/stop detection relies on better clocks. Using highly accurate clocks one could switch from relative to absolute epoch allocation and possibly decorrelate geometrically, atmospherically, and instrumentally induced time delays. Present time scale realizations at the ground stations are no better than on the 100 ns-level. So far, time scale synchronization between stations is done via GNSS. In general, a closer tie of temporal and spatial referencing is highly desirable for existing and upcoming space geodetic techniques, e. g., in performing VLBI also with solar system sources instead of only quasars, or space based VLBI in addition to ground based interferometry.

Earth system research in total with all its differential or time delay techniques will benefit from more accurate clock readings. Besides potential/height measurements and positioning, the determination of the rotational behavior of the Earth remains a major task in geodesy. Comparing the possible use of atom interferometers against customary laser technology shows that there is still lot of room for improvements regarding large ring lasers (Stedman et al. 2007). Besides space geodetic techniques ring laser gyroscopes are currently being used to detect variations in Earth's rotation. Appropriate atomic clock networks may provide an alternative device to sense it.

A general advantage of using clocks is the fact, that we do not necessarily rely on available line of sights. This may be of great benefit if we think of underwater or tunnel applications. In the long term the sensitivity of clocks will probably reach a level where very moderate velocities become detectable using the relativistic Doppler effect. In this case a lot more applications come in reach. In addition to the above mentioned examples of Earth measurements, relativistic effects are routinely to be applied in satellite geodesy (Müller et al. 2008a).

6 Outlook

Future geodesy will benefit from a consistent incorporation of relativistic aspects. Technological progress opens the field of relativistic geodesy, where atomic clocks are sensitive enough to exploit relativistic effects even in case of only moderate velocities and potential differences as usually experienced in earthbound applications.

Successful implementation in practice requires a sound theoretical basis. Regarding practical aspects, the optimal configuration of clock networks and clock reading comparisons has to be investigated, depending on the needs of proposed geodetic applications. Highly precise frequency and time transfer methods are already being tested between established sites of remote metrological institutes or laboratories of physics. Experience with advanced optical fiber networks and satellite based connections, either via microwave or laser link, can now be gathered from past experiments and will grow in future. Various experiments on these issues are running, others are at least proposed or in planning phase.

Certain geodetic applications rely on mobile measurement devices. Consequently, the mobility of (optical) clocks is a prerequisite for their widespread use in geodesy. Miniaturization is another important request. Various applications imply different requirements on such parameters as short/long term stability (i. e. necessary averaging time), power consumption, weight, robustness, manageability of the whole instrumentation, costs, and so on. Ultra-stable oscillators readily available for short period applications are much cheaper than long-term stable hydrogen masers. In future, multiple miniature clocks or even chip traps (clocks on a chip) with sufficient stability might become realizable. This would enable quite different measurement setups like clock swarms, etc.

As long as an area-wide use of atomic clocks remains out of reach, its selective or point-wise application stays in focus. In this sense, highly precise frequency standards are predestined for creating references in space-time. Terminologically, we should speak of reference events in space-time instead of reference points (in space). Geodetic quantities like coordinates or heights refer to a certain reference system or surface. Furthermore, in today's classical geodesy, near real-time availability of final results plays an increasingly important role. Relativistic geodesy, based on consistent redefinitions of reference systems and surfaces, e. g. the relativistic geoid, would offer a clear and transparent way of meeting these present requirements.

This new approach even might be more cost effective, especially in comparison to high-budget space missions. On the other hand, the success of relativistic geodesy relies on continuous progress in quantum engineering in order to release the long-desired instrumentarium from the laboratories to the geodetic community and on the widespread use of a consistent mathematical framework among the practitioners. The latter issue will be addressed in a subsequent article (Mai 2013b).

Close cooperation between geodesists and clock operators/developers at the (metrological) laboratories should enable us to eliminate significant sources of error that may possibly hamper the successful application of clocks in relativistic geodesy. Different views of perspective provide a chance to identify various beneficial applications of clocks beyond the quite obvious chronometric leveling idea.

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