

## How General Relativity Contains the Standard Model

Peter Gillan

gillan@alumni.fdu.edu

c/o Peter Schaeffer, T-BE2-03

Fairleigh Dickinson University, 1000 River Road, Teaneck, New Jersey 07666, USA

### Abstract

The Standard Model plus gravitation is derived from general relativity with three dimensions of time. When general relativity is calculated with three dimensions of time, a complete unified field theory results. The theory is called 3DT, which stands for three dimensions of time. 3DT is different than other higher-dimensional theories because it allows fields to depend upon the higher-dimensional coordinates. It shows how predictions at the Planck mass can be tested at low energies. Quark confinement and asymptotic freedom are produced. The theory of 3DT provides an explanation for the mass of the electron, the value  $1/137$  of the fine structure constant, the masses of the muon and tau, the masses of the electron's, muon's and tau's neutrinos, the masses of the W, Z and the photon. The calculation of these parameters is made possible by applying the well-known quantum mass correction equation found in QED to 3DT's electroweak sector. 3DT is anomaly-free. The relationship between quantum mechanics and general relativity is demonstrated. 3DT solves all these problems and many more without introducing any new problems or asking any unanswered questions. The known elementary particles are solutions to the field equations generated by the theory's Lagrangian. All fields come naturally from the same place. 3DT predicts that there is no Higgs boson and no sparticles. Instead, it predicts that there are seven new intermediate vector bosons with the masses 4.56 TeV, 7.32 TeV, 27.36 TeV, 29.43 TeV, 31.22 TeV, 33.04 TeV and 38.79 TeV.

## Part I. The Classical Theory

### 1. Introduction

Einstein added one dimension of time to the three dimensions of space. It is only natural to add two more dimensions of time, bringing the number up to three. There are three dimensions of time because there are three dimensions of space. That there are three dimensions of time is the first postulate upon which this work is based. This will finish the job Einstein started of adding dimensions of time and making time symmetrical to space.

To specify a theory of three dimensions of time (3DT), one need look no further than the general theory of relativity. The Lagrangian for any theory contains all of that theory's physical information. The Lagrangian for general relativity is the curvature scalar. The Lagrangian for 3DT is also the curvature scalar. This is the second and last postulate upon which this work is based.

The theory of 3DT is just like relativity because it adds dimensions of time and has the curvature scalar as its Lagrangian. But it is like relativity in a third way. It is a theory of pure gravitation because its Lagrangian is the curvature scalar alone. The theory of 3DT continues the over-300-year-old tradition started by Newton and continued by Einstein of placing gravitation in the position of greatest importance.

The postulate of three dimensions of time implies a six-dimensional spacetime. The six-dimensional curvature scalar  $\hat{R}$  will give a complete description of the elementary particles. We will start off calculating the Christoffel symbols  $\hat{\Gamma}_{\beta\gamma}^{\alpha}$  and wind up describing the elementary particles. This document is at the same time both general relativity and high energy physics, a necessary condition for a unified field theory.

With widespread dissatisfaction with the current state of theoretical physics, one may wonder whether strings and supersymmetry are correct. Indeed, everything that follows from a faulty initial assumption may very well be wrong. One may wonder if there are better starting points. You will never find better starting points than gravitation and three dimensions of time. With the genius of Albert Einstein and Isaac Newton backing them up, they are prime candidates for a unified field theory.

The original Kaluza-Klein (KK) [1,2] theory is not realistic. For example, Kaluza's original theory produced just two terms in its Lagrangian — those for the gravitational and electromagnetic fields. All other fields and terms that we now know exist do not appear in the Lagrangian. Kaluza assumed fields were independent of the fifth coordinate. When this restriction is lifted, one can produce the balance of the Standard Model (SM) [3-6].

Klein postulated that the fifth dimension was microscopic and closed, resulting in an infinite tower of harmonics. This tower constituted a field's higher-dimensional (HD) coordinate dependence. The fundamental was massless. The higher order particles, which represented the dependence on the fifth coordinate, had Planck masses. In 3DT, this dependence will produce the fields and terms that we now know exist. This will be the realization of Einstein's dream [7-9] of accounting for all physical phenomena from the "pure marble" of geometry, without the "base wood" of additional matter fields. Many terms in the Lagrangian for the SM represent separate assumptions. These terms will be produced by just one assumption, the curvature scalar  $\hat{R}$  of 3DT. The SM is a Lagrangian that can be derived from the curvature scalar.

Previously, it was thought that fields could not depend on the HD coordinates because they would be too massive to detect. However, as part of 3DT's complete explanation of masses, I show that quantum field theory reduces the Planck masses of these massive harmonics to ordinary elementary particle masses, producing the usual elementary particles. See Table 1.

$l$	$/m$	-3	$-\frac{5}{2}$	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
3		$A_\mu^{3,-3}$		$A_\mu^{3,-2}$		$A_\mu^{3,-1}$		$A_\mu^{30}$		$A_\mu^{31}$		$A_\mu^{32}$		$A_\mu^{33}$
$\frac{5}{2}$			$\nu_{3\tau}$		$\nu_{2\tau}$		$\tau^-$		$\nu_\tau$		$\tau_2^+$		$\nu_{4\tau}$	
2				$A_\mu^{2,-2}$		$A_\mu^{2,-1}$		$A_\mu^{20}$		$A_\mu^{21}$		$A_\mu^{22}$		
$\frac{3}{2}$					$\nu_{2\mu}$		$\mu^-$		$\nu_\mu$		$\mu_2^+$			
1						$W^-$		$Z^0$		$W^+$				
$\frac{1}{2}$							$e^-$		$\nu_e$					
0								$\gamma$						

Table 1. Tower of harmonics in 3DT. The vectors and leptons have quantum numbers  $|lm\rangle$ .

This conversion of Planck masses to elementary particle masses allows predictions at the Planck mass to be tested at low energies. The conversion is allowed by quantum field theory that is modified by particles that are not points, but are line segments of charge. The line segments spin, producing a three-dimensional distribution of charge. Presumably, these particles avoid the violation of causality or special relativity usually associated with extended particles just like the particles of string theory, but this is conjecture.

Ten masses and coupling constants are derived in Part II. Neither the symmetry group  $SU(3) \times SU(2) \times U(1)$  nor the Higgs mechanism will be derived. These are shown to be superfluous in Part II. They are not needed because particles in 3DT are not points. Point particles require the condition of renormalizability, which is preserved by gauge symmetry and the Higgs mechanism. If particles are extended, as claimed by string theory and 3DT, renormalizability and the gauge symmetry that preserve it become unnecessary. Likewise, the Higgs mechanism is not needed. This is unlike today's Kaluza-Klein theories, where the observed elementary particles lie in the zero-mass sector of a Kaluza-Klein theory and receive a mass from the Higgs mechanism. In this case a determination of the masses of the elementary particles is out of the question. They are completely unexplained. In 3DT, however, complete explanations for masses are given.

The purpose of this document is to establish enough evidence for 3DT to warrant further investigation. This is accomplished by the demonstration that 3DT produces the observable features of the SM. The Standard Model is quite an achievement, but 3DT is far superior. One must allow for the possibility that such a simple and effective theory exists.

The organization of Part I is as follows: Section 2 presents a comparison of 3DT with other Kaluza-Klein theories. Section 3 breaks down the six-dimensional (6D) metric tensor into four-dimensional and HD quantities. Section 4 specifies the Lagrangian. Four-dimensional gravitation is derived in Sec. 5. A cosmological constant is eliminated in this section. In Sec. 6, the term for Maxwell's equations and mass terms for the photon, W and Z are obtained. Methods for dealing with the HD coordinates in each term in the Lagrangian are given in Sec. 7. Section 8 introduces fermions and derives the Dirac equation from the 6D curvature scalar. Terms for leptons interacting with the W, Z and photon are deduced in Sec. 9. Section 10 derives the strong interactions. Quarks, confinement, asymptotic freedom and chiral symmetry breaking are produced in Sec. 11.

## 2. Kaluza-Klein theories

I will now present a comparison of 3DT with other Kaluza-Klein (KK) theories. This will follow closely the review by Overduin and Wesson [10]. The single biggest difference between 3DT and other KK theories is that fields in 3DT depend on the higher-dimensional coordinates  $\theta$  and  $\phi$ , the usual spherical coordinates.

DeWitt [11] was the first to suggest incorporating the non-Abelian symmetry  $SU(2)$  of the SM into a KK theory. Others [12–14] took up the challenge, ending with Cho and Freund [15,16]. The main difference between these KK theories and 3DT is the origin of vectors in the theories. In the former, each dimension produces one vector in the usual way. In 3DT, one dimension produces all vectors needed for the SM. This is made possible by the expansion of the lone vector in terms of spherical harmonics. This represents the vector's dependence upon the HD coordinates  $\theta$  and  $\phi$ . This procedure is superior because it accounts for the  $SU(2) \times U(1)$  structure of electroweak symmetry. Here the photon is associated with the lowest order  $l = m = 0$  spherical harmonic and the  $W^\pm$  and  $Z^0$  are associated with the next higher order  $l = 1, m = -1, 0, 1$  harmonics. See Table 1.

The disadvantage of previous non-Abelian KK theories is that they naively require one dimension for each vector produced. Thus, many higher dimensions are required to produce the group  $SU(3) \times SU(2) \times U(1)$  of the SM. In addition, there is no reason why some of the higher dimensions should be different than others. This would be required to explain the group  $SU(3) \times SU(2) \times U(1)$ . In particular, the question of why the photon is different than the W and Z is left unanswered. Likewise for the difference between the strong and electroweak interactions.

The theory of 3DT and these non-Abelian KK theories, however, are similar in the following respect: All matter fields are contained within the 6D curvature scalar. There are no external, additional matter fields. This embodies Einstein's vision of nature being the result of pure geometry. Likewise, each of these KK theories is pure gravitation with indices allowed to run to values greater than four.

References [17–24] give the physical implications of HD coordinate dependence for general relativity in the macroscopic realm — such as its effect on the advance of the perihelion of Mercury. The theory of 3DT gives its implications for microscopic physics — elementary particles.

Compactification in non-Abelian KK theories can be a problem. However, with only two higher dimensions that form an ordinary 2-sphere, compactification in 3DT is trivial, just

as it is for the original KK theory with one higher dimension. We do not have to worry about compactification as in, for example, the theories of Cho and Freund [25–29], in which the higher dimensions have general curvature. In this case, one runs into difficulties with the consistency of the field equations when the dependence upon the HD coordinates is eliminated (Duff [30–33]). The theory of 3DT does not have this problem. Even if it did we would solve it by allowing fields to depend on the HD coordinates and agree with Cho’s call [34–37] for abandonment of the “zero modes approximation.”

In 3DT, one does not need any of the various compactification mechanisms such as altering Einstein’s equations by incorporating torsion [38–41], adding higher-derivative terms such as  $R^2$  to the Lagrangian [42], or by adding matter fields. This last method, with the right matter terms, is known as spontaneous compactification. The first example of this is that of Cremmer and Scherk [43,44].

All previous unified field theories that include gravity (theories of everything) suffer from the same problem — the inability to make sufficient contact with low-energy phenomenology. In other words, the theories are not realistic. These include the original KK theory in five dimensions, the extensions of this theory with more than one higher dimension (intended to account for non-Abelian symmetry), Supergravity, string theory, and the present-day theory of everything, M-theory.

For example, none of these theories produces the right particle spectrum. The KK theories without supersymmetry have no fermions. The incorporation of fermions in such theories is apparently without explanation. Supersymmetry meliorates the situation. However in this case, the most abundant type of particles is scalars. The theory of d=11, N=8 Supergravity has 70 of them. This is in contradiction to observation, which indicates that there are no scalars. The presently-accepted way out is to assume that the scalars are too massive to be detected. This means 70 additional assumptions. This is unsightly, to say the least.

The solution to this problem is to incorporate the branch of mathematics known as spinor theory [45] into KK theory. Spinor theory dictates that a null vector (the photon) may be equated to a pair of spinors (fermions). When this is combined with 6D KK theory with the proper HD coordinate dependence, all of the observed fermions are produced. And because there is no relation between fermions and scalars as with supersymmetry, this produces no scalars. Agreement with observation is at hand.

The current method of choice for the incorporation of fermions into KK theory is supersymmetry. This is more natural than their incorporation by hand, but there are severe drawbacks. For example, the fermionic superpartners of KK’s bosons have not been detected. Similarly, the bosonic superpartners of the observed fermions are not observed. Thus there is little evidence from experiment for supersymmetry. By comparison, the evidence for 3DT is the fermionic physics of the SM — it will reproduce the observable features of the SM. Spinor theory dictates that the photon may be equated to many pairs of fermions in the Lagrangian. These fermions are the observed ones. Spinor theory in 3DT is proven every day in particle accelerators. It will account for pair production by producing spinor-vector interaction terms in the Lagrangian (the assumption of minimal coupling in the SM). With two spinors and one vector as factors, this term represents a Feynman diagram with two external fermionic lines and one external bosonic line.

In addition, one of the new terms to appear as a result of HD coordinate dependence is the term for the Dirac equation. This equation and all it implies must be assumed in other

theories such as  $d = 11$  Supergravity, for example.

One advantage of Supergravity is that it provides a number for the dimensionality of spacetime. Nahm [46] showed 11 is the maximum number of dimensions of spacetime for supersymmetric gravity. In 11 dimensions, nature is maximally supersymmetric. Witten [47] proved 11 is the minimum number of dimensions required for the incorporation of the group  $SU(3) \times SU(2) \times U(1)$  of the SM. However, this assumes the one-vector-for-one-dimension aspect, along with its disadvantages, of the non-Abelian theories mentioned above. Cremmer, Julia and Scherk [48] showed that in 11 dimensions there is only one choice for additional matter fields. Freund and Rubin [49] showed that  $d=11$  Supergravity naturally compactifies to four macroscopic and seven microscopic dimensions.

3DT also provides a number for the dimensionality of spacetime. Instead of a symmetry between bosons and fermions, it postulates a symmetry between space and time. This implies there are three dimensions of time. There are three dimensions of time because there are three dimensions of space. Spacetime has six dimensions. In six dimensions all the observable features of the Standard Model, in all their complexity and with all their idiosyncracies, will be produced. Thus, we choose six dimensions because it will work.

The theory of  $d = 11$  Supergravity lost its status as the theory of everything in the mid 1980's. There were several reasons for this. First, the extra spacetime did not contain quarks or leptons, nor the gauge group for the SM. Second, one cannot build chirality, necessary for an accurate description of fermions in the Standard Model, into a theory with an odd number of dimensions. Third, it had a large cosmological constant, contradicting observation. Fourth, it had anomalies. 3DT naturally accounts for what appears to be the gauge group for the SM with its expansions in terms of spherical harmonics for electroweak theory and the HD part of the 6D rotation group for the strong interactions. In 3DT, these expansions, together with spinor theory, do indeed produce quarks and leptons. In six dimensions, chirality is possible. The large cosmological constant appearing in the 4D version of Kaluza-Klein theory is naturally eliminated in 3DT. Finally, it will be easy to show 3DT is anomaly-free.

String theory [50] is supposed to be able to predict the parameters of the Standard Model. It does not do this. I will show 3DT does this. There are about 20 unexplained parameters in the SM. 3DT explains ten of these at the present time. String theory explains none.

String theory has three separate uniqueness problems. First, the number of supersymmetric generators can have any value from  $N=1$  to  $N=8$ . Second, there are five different classes of string theory. Finally, within each class there are thousands of different string theories, each corresponding to a different way one can compactify seven higher dimensions. In contrast, there is only one theory of 3DT. 3DT does not have supersymmetry, with its multiple values for  $N$ . Particles in 3DT are not strings, which would be classified into five types. Finally, there is only one way to compactify the two higher dimensions. It is a trivial compactification to the 2-sphere, similar to the compactification of the one HD of original Kaluza-Klein to the 1-sphere or circle. Therefore, 3DT does not suffer from uniqueness problems.

String theory has matter terms known as Chapline-Manton terms [51] that must be added to the Lagrangian. In six dimensions, 3DT has no matter terms.

Green and Schwarz [52] and Gross et al. [53] showed all anomalies vanish for  $SO(32)$  and  $E_8 \times E_8$  string theory, respectively. String theory may provide an anomaly-free path to

quantum gravity [54]. However, I claim quantum gravity is not the unification of gravity and quantum theory. I can produce a unification of the Standard Model and gravitation without it. How can a theory whose effects are entirely negligible be important? The relationship between gravity and quantum theory is given below. But if you must have a quantum theory of gravity, my new way of doing quantum field theory will help.

This being said, I will now show why 3DT is anomaly-free. First, as will be shown below, the 6D metric tensor in this theory contains all types of 4D matter fields. In addition, the 6D curvature scalar contains all types of 4D matter terms. These are accomplished by expanding the vector  $\hat{g}_{5\mu}$  in terms of spherical harmonics, spinors and 6D rotation group generators. Now, the Lagrangian for 3DT is the 6D curvature scalar. There are no other terms, which could be called 6D matter terms. In other words, in six dimensions, spacetime is empty. There are no currents. Thus, there are no conservation laws. Now, anomalies are quantum mechanical violations of conservation laws. Without conservation laws there are no anomalies. Therefore, 3DT is anomaly-free.

Quantum mechanics is not philosophical in origin. It results in part from the properties of waves. For example, it is the fact that one cannot simultaneously determine the position and momentum of a wave that leads to the Uncertainty Principle. This applies to macroscopic waves as well as microscopic ones. The waves follow from wave equations, which are field equations. The field equations originate from the Lagrangian. The Lagrangian is the 6D curvature scalar of 3DT. The curvature scalar is the Lagrangian for general relativity. Therefore, the Uncertainty Principle is derived from general relativity. Proceeding in this manner, it seems to me likely that one can derive all aspects of quantum mechanics from general relativity.

The worst disadvantage of string theory is that it does not make predictions. They reside at the Planck mass. M-theory is worse than string theory because it is nonperturbative as well. The electroweak sector, which contains the testable predictions of 3DT, is perturbative. In addition, 3DT has a mechanism for converting Planck masses to elementary particle masses. Thus, it makes clear-cut physical predictions. 3DT predicts the observable features of the Standard Model as will be shown. In addition, it predicts new intermediate vector bosons with precisely determined masses.

### 3. The 6D metric tensor

In this section, we break down the 6D metric tensor into 4D and HD quantities in preparation for its substitution into the 6D curvature scalar. The postulate of three dimensions of time implies a 6D spacetime. Included in this postulate of six dimensions is the size, shape, spacelike or timelike nature and connection to 4D spacetime of the higher dimensions. Therefore, we will further postulate that the two HD's form the surface of a sphere with radius  $T$ . This radius is the Planck time  $T$ . We have

$$T = 5.39 \times 10^{-44} \text{ sec}, \tag{1}$$

$$cT = L = 1.62 \times 10^{-33} \text{ cm}, \tag{2}$$

where  $L$  is the Planck length and  $c$  is the speed of light. I claim the Planck length is important because the radius of the higher-dimensional sphere is the Planck time. The reason the two

higher dimensions are so small is cosmological in origin. We will not consider cosmology further in this document.

Hawking [55] proposed the use of imaginary time. That is, one uses the coordinate  $it$  instead of  $t$ , where  $i = \sqrt{-1}$ . This is only natural, because one can not see time, unlike space, so it exists only in one's imagination. Therefore, it is imaginary. One is allowed to multiply the coordinate  $t$  by  $i$  because multiplying a coordinate by a constant amounts to choosing one's coordinate system, which one is free to do. The following example illustrates all of this.

Consider a two-dimensional spacetime with coordinates  $x$  and  $t$ . We have the line element

$$ds^2 = g_{ij}dx^i dx^j = dx dx - c^2 dt dt.$$

If instead one uses the coordinates  $x^1 = x$  and  $x^2 = ict$  for the same spacetime, one arrives at the line element

$$ds^2 = g_{ij}dx^i dx^j = dx^1 dx^1 + dx^2 dx^2.$$

As 't Hooft [56] noted, the coordinate  $t$  is timelike because  $g_{22}$  for this coordinate is negative, while the coordinate  $x^2 = ict$  is spacelike because  $g_{22}$  for this coordinate is positive. Thus, we have incorporated the minus sign usually associated with timelike diagonal components of the metric tensor into the coordinate of time, resulting in a positive sign for these components.

It is known that timelike higher dimensions lead to a negative-energy electromagnetic field. In addition, 't Hooft [56] related that timelike higher dimensions lead to unbounded and tachyonic masses for the elementary particles. Clearly, one must have spacelike higher dimensions to avoid these difficulties. So we multiply the HD coordinates of time by  $i$ , rendering the higher dimensions spacelike. We multiply all three coordinates of time in 3DT by  $i = \sqrt{-1}$ . Therefore, like in the above example, the signature of the metric tensor is  $\text{diag.}(+++++)$ . This is another step in making time symmetrical to space. Hawking's idea of imaginary time may solve other important problems. It may eliminate the singularity at the center of a black hole and at the beginning of the universe.

I claim all coordinates must be spacelike. Of course, the first three coordinates of our 6D spacetime are coordinates of space and are therefore spacelike. It is known that the HD coordinates must be spacelike. This leaves the usual dimension of time. We have shown how even this coordinate can be spacelike. Indeed, it would be well worth it, if this designation were to remove the singularity at the center of a black hole, which has been called by Wheeler to be the greatest problem facing theoretical physics.

The differential line element for the two coordinates of time  $\theta$  and  $\phi$  that form a sphere of radius  $T$  is

$$ds^2 = \hat{g}_{ij}d\hat{x}^i d\hat{x}^j = -c^2 T^2 d\theta^2 - c^2 T^2 \sin^2\theta d\phi^2, \quad (3)$$

where  $i, j$  take the value five or six and  $\theta$  and  $\phi$  are the usual spherical coordinates. A hat over a symbol denotes a 6D quantity. However, instead of choosing  $\hat{x}^5 = \theta$  and  $\hat{x}^6 = \phi$  with  $\hat{g}_{55} = -c^2 T^2$  and  $\hat{g}_{66} = -c^2 T^2 \sin^2\theta$ , we set

$$\hat{x}^5 = icT \theta, \quad (4)$$

$$\hat{x}^6 = icT(\sin\theta) \phi, \quad (5)$$

$$\hat{g}_{55} = \hat{g}_{66} = 1, \quad (6)$$

$$\hat{g}_{56} = \hat{g}_{65} = 0. \quad (7)$$



These assignments for the HD coordinates lead to the same line element Eq. (3) and follow the strict definition of the metric tensor as the dot product of unit vectors.

The incorporation of the factors of  $cT$  into  $\hat{x}^5$  and  $\hat{x}^6$  makes the HD coordinates more like the 4D ones, which have the units of distance. This suggests we incorporate the factor of  $\sin\theta$  into  $\hat{x}^6$  instead of  $\hat{g}_{66}$ . This eliminates a cosmological constant while keeping the higher dimensions small. This is described in Sec. 4. The value 1 for  $\hat{g}_{66}$  gives the appearance of what is called Ricci-flatness of the HD's. This means [31] the cosmological constant is zero and, with the elimination of the scalar field  $\hat{g}_{66}$ , there is no Higgs mechanism. Both these features are not typical of today's KK theories.

For use in calculating the Lagrangian in Sec. 4, We tabulate here the differentials and derivatives of the two HD coordinates  $\hat{x}^5$  and  $\hat{x}^6$  according to Eqs. (3)–(5)

$$d\hat{x}^5 = icT d\theta, \quad (8)$$

$$\partial_5 = \frac{\partial}{\partial\hat{x}^5} = \frac{1}{icT} \frac{\partial}{\partial\theta}, \quad (9)$$

$$d\hat{x}^6 = icT \sin\theta d\phi, \quad (10)$$

$$\partial_6 = \frac{\partial}{\partial\hat{x}^6} = \frac{1}{icT \sin\theta} \frac{\partial}{\partial\phi}. \quad (11)$$

The 6D metric tensor is denoted by  $\hat{g}_{\alpha\beta}$ , where indices in the beginning of the Greek alphabet such as  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  run from one to six. The indices  $\mu$ ,  $\nu$ ,  $\rho$ , and  $\sigma$  in the middle of the Greek alphabet range from one to four. The first four coordinates of the 6D spacetime are those of the ordinary 4D spacetime of experience.

We postulate the connection between the HD sphere and 4D spacetime as follows: The 4D worldline of the particle under consideration is the  $z$ -axis of the 3D embedding space of the HD sphere. This means the  $z$ -axis of the embedding space of the HD sphere is a line of 4D spacetime. This overlap of the 4D and HD spaces is necessary because if one assumes an infinitesimal surface area perpendicular to a line in 4D spacetime is part of the HD sphere, one is inescapably lead to the conclusion that the surface must curve around the line. The line and the sphere then exist in the same 3D space.

The circular sixth dimension, parameterized by the coordinate  $\phi$ , is perpendicular to the  $z$ -axis. Taking this axis to be  $x^1, x^2, x^3$ , or  $x^4$ , we have  $\hat{g}_{6\mu} = 0$ . The coordinate  $\phi$ , along with a line of 4D spacetime, forms a cylinder, and the metric tensor for these perpendicular coordinates is zero. It is not realized that this most common type of HD geometry causes the vector  $\hat{g}_{5\mu}$  in the original KK theory to be zero. The fifth dimension in 3DT is parameterized by the coordinate  $\theta$ . It is measured from the positive  $z$ -axis. It is not, in general, perpendicular to the  $z$ -direction consisting of 4D spacetime. Therefore, the vector  $\hat{g}_{5\mu}$  is not zero. This is responsible for the existence of all elementary particles, which will be contained in the vector  $A_\mu = \hat{g}_{5\mu}$ . It is only because of this peculiar overlap of the 4D and HD spaces that  $A_\mu$  and all matter is not zero.

Allowing fields to depend on the HD coordinates will produce the fields and terms necessary for the Standard Model. At the same time, this makes 3DT more self-consistent and correct because an arbitrary field should depend on all six coordinates, not just the first four. The 2 HD coordinates should be treated like the first 4 coordinates, upon which fields depend. This removes the ad-hoc assumption in Kaluza-Klein theory that fields do not depend on the HD coordinates. This dependence upon the HD coordinates is represented by

a Fourier expansion in terms of spherical harmonics. We have

$$g_{\mu\nu}(\hat{x}^\alpha) = \sum_{l=0}^{\infty} \sum_{m=-l}^l g_{\mu\nu}^{lm}(x^\rho) Y_{lm}(\theta, \phi), \quad (12)$$

$$A_\mu(\hat{x}^\alpha) = \sum_{l=0}^{\infty} \sum_{m=-l}^l A_\mu^{lm}(x^\nu) Y_{lm}(\theta, \phi), \quad (13)$$

where the  $Y_{lm}(\theta, \phi)$  are the spherical harmonics.

The 6D metric tensor is defined in terms of 4D quantities as follows

$$\begin{pmatrix} \hat{g}_{\mu\nu} & \hat{g}_{\mu 5} & \hat{g}_{\mu 6} \\ \hat{g}_{5\nu} & \hat{g}_{55} & \hat{g}_{56} \\ \hat{g}_{6\nu} & \hat{g}_{65} & \hat{g}_{66} \end{pmatrix} = \begin{pmatrix} g_{\mu\nu} + A_\mu A_\nu & A_\mu & 0 \\ A_\nu & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (14)$$

For an example of how to obtain an equation like (14), see Ref. [57].

The contravariant metric tensor, introduced via the relations  $\hat{g}^{\alpha\beta} \hat{g}_{\beta\gamma} = \hat{\delta}^\alpha_\gamma$ , is

$$\begin{pmatrix} \hat{g}^{\mu\nu} & \hat{g}^{\mu 5} & \hat{g}^{\mu 6} \\ \hat{g}^{5\nu} & \hat{g}^{55} & \hat{g}^{56} \\ \hat{g}^{6\nu} & \hat{g}^{65} & \hat{g}^{66} \end{pmatrix} = \begin{pmatrix} g^{\mu\nu} & -A^\mu & 0 \\ -A^\nu & 1 + A^\mu A_\mu & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (15)$$

#### 4. The Lagrangian

The Lagrangian density for 3DT is

$$\mathcal{L} = k \hat{R}, \quad (16)$$

where  $\hat{R}$  is the 6D curvature scalar and  $k = c^3/16\pi G$ , where  $c$  is the speed of light and  $G$  is the constant of gravitation. We have

$$\hat{R} = \hat{g}^{\alpha\beta} \hat{R}_{\alpha\beta}, \quad (17)$$

where  $\hat{R}_{\alpha\beta}$  is defined in the usual way

$$\hat{R}_{\alpha\beta} = \partial_\gamma \hat{\Gamma}^\gamma_{\alpha\beta} - \partial_\beta \hat{\Gamma}^\gamma_{\alpha\gamma} - \hat{\Gamma}^\delta_{\beta\gamma} \hat{\Gamma}^\gamma_{\alpha\delta} + \hat{\Gamma}^\delta_{\alpha\beta} \hat{\Gamma}^\gamma_{\delta\gamma}. \quad (18)$$

Here

$$\hat{\Gamma}^\gamma_{\alpha\beta} = \hat{g}^{\gamma\delta} \hat{\Gamma}_{\delta\alpha\beta}, \quad (19)$$

where

$$\hat{\Gamma}_{\delta\alpha\beta} = \frac{1}{2} (-\partial_\delta \hat{g}_{\alpha\beta} + \partial_\alpha \hat{g}_{\beta\delta} + \partial_\beta \hat{g}_{\delta\alpha}). \quad (20)$$

The action integral for 3DT is

$$I = k \int \hat{R} \sqrt{\hat{g}} d^6 \hat{x}, \quad (21)$$

where  $\hat{g}$  is the determinant of the 6D metric tensor. This determinant may be expressed in terms of the 4D metric tensor  $g_{\mu\nu}$ . If we multiply the middle column of Eq. (14) by  $A_\nu$  and subtract the resulting column from the first, we obtain

$$\sqrt{\hat{g}} = \begin{vmatrix} g_{\mu\nu} & A_\mu & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}^{1/2} = \sqrt{g}. \quad (22)$$

Using Eqs. (22), (8) and (10), the 6D volume element becomes  $-\sqrt{g} c^2 T^2 \sin\theta d\theta d\phi d^4x$ . The constant  $-c^2 T^2$  may be ignored as the entire Lagrangian is multiplied by it and the field equations remain the same after its removal.

The dependence upon the two HD coordinates has a profound effect on 6D Kaluza-Klein theory. Instead of obtaining only the terms for 4D gravitation and electromagnetism (the Kaluza-Klein miracle), one now obtains literally hundreds of new, odd-looking terms. This is because terms with  $\partial_5$  or  $\partial_6$  are not zero now. This embarrassment of riches may be why Kaluza did not allow dependence upon the extra coordinate.

The shortcut to understanding what these terms mean is to consider those with as few factors as possible. The proper procedure starts by determining the free-field terms for each field of interest. These are obtained by setting all other fields to zero in each term in the Lagrangian. Thus, the free-field terms have in them only the type of field under consideration. Interaction terms have more than one type of field in them.

There are three classes of terms in the Lagrangian. First are the terms for the free- $g_{\mu\nu}$  field, which will contain the graviton field. These contain only  $g_{\mu\nu}$  or  $g^{\mu\nu}$ ; they are obtained by setting  $A_\mu = 0$ . These terms are important macroscopically, where the sources of  $g_{\mu\nu}$ , e.g., masses, add and those for the  $A_\mu$  field cancel. Second are the terms for the free- $A_\mu$  field, which will contain the photon field. These are obtained by setting  $g_{\mu\nu} = \delta_{\mu\nu}$ , which is the Kronecker delta. Note one cannot set  $g_{\mu\nu} = 0$ ; however it can be frozen out by setting it equal to a constant. This causes all derivatives of  $g_{\mu\nu}$  and  $g^{\mu\nu}$  to vanish. These terms are important microscopically, where the sources of the  $A_\mu$  field, e.g., charges, do not cancel and those for the  $g_{\mu\nu}$  field are very small. Third are the terms for the interaction of the  $g_{\mu\nu}$  and  $A_\mu$  fields. These contain at least one factor of  $A$  and at least one derivative of  $g_{\mu\nu}$  or  $g^{\mu\nu}$ . These terms vanish both macroscopically and microscopically, where  $A_\mu$  vanishes and  $g_{\mu\nu}$  is constant, respectively. We will not consider this class of terms further.

## 5. Four-dimensional gravitation

There are three types of free-field terms in the Lagrangian for  $g_{\mu\nu}$ : the 4D curvature scalar  $R$  and terms with HD derivatives with two factors of  $g_{\mu\nu}$  and four factors of  $g_{\mu\nu}$ . Neglecting the terms with HD derivatives and four factors of  $g_{\mu\nu}$ , we have

$$k R \quad (23)$$

$$- k g^{\mu\nu} \partial_5^2 g_{\mu\nu} \quad (24)$$

$$- k g^{\mu\nu} \partial_6^2 g_{\mu\nu} \quad (25)$$

$$-\frac{1}{2} k \partial_5 g^{\mu\nu} \partial_5 g_{\mu\nu} \quad (26)$$

$$-\frac{1}{2} k \partial_6 g^{\mu\nu} \partial_6 g_{\mu\nu} \quad (27)$$

Because the size of the HD sphere is of the order of the Planck length, one cannot observe the HD coordinate dependence of  $g_{\mu\nu}$ . To eliminate it, one must integrate over the HD coordinates, thereby averaging over them and obtaining a 4D description of the 6D curvature scalar.

Term (23) contains the set of terms for 4D gravitation. These are obtained by setting  $g_{\mu\nu} = g_{\mu\nu}^{00}|00\rangle$  and  $g^{\mu\nu} = \bar{g}_{00}^{\mu\nu}\langle 00|$ . One may group all the kets together and all the bras together to yield a single bra  $\langle 00|$  and ket  $|00\rangle$ , which when multiplied together yield one. The result is the curvature scalar  $R(g_{\mu\nu}^{00})$ , which contains  $g_{\mu\nu}^{00}$  and  $\bar{g}_{00}^{\mu\nu}$  only. We interpret  $g_{\mu\nu}^{00}$  as the graviton field. Therefore, 3DT contains 4D gravitation.

The procedures for taking the derivatives with respect to the HD coordinates and integrating over  $\theta$  and  $\phi$  for Terms (24)–(27) are the same as those for Terms (33)–(36) described in Sec. 7. Considering only the  $g_{\mu\nu}^{lm}$  with  $l = 1$  or  $0$ , we arrive at the following result for these terms

$$+ \frac{9 - 3\sqrt{3}}{12c^2T^2} k \bar{g}_{00}^{\mu\rho} \bar{g}_{00}^{\nu\sigma} g_{\rho\sigma}^{1,-1} g_{\mu\nu}^{11} \quad (28)$$

$$+ \frac{9 - 3\sqrt{3}}{12c^2T^2} k \bar{g}_{00}^{\mu\rho} \bar{g}_{00}^{\nu\sigma} g_{\rho\sigma}^{11} g_{\mu\nu}^{1,-1} \quad (29)$$

$$+ \frac{2\sqrt{3} + \sqrt{30}}{12c^2T^2} k \bar{g}_{00}^{\mu\rho} \bar{g}_{00}^{\nu\sigma} g_{\rho\sigma}^{10} g_{\mu\nu}^{10}, \quad (30)$$

where we have considered only the  $g_{00}^{\mu\nu}$  term in  $g^{\mu\nu}$  and only the  $g_{\mu\nu}^{00}$  field in the factor of  $\sqrt{g}$  in the differential volume element.

As  $g_{00}^{\mu\nu}$  is the graviton field, Terms (28)–(30) are mass terms for the  $g_{\mu\nu}^{1m}$  fields. The field  $g_{\mu\nu}^{00}$  does not have a mass term because its associated spherical harmonic  $Y_{00} = 1/\sqrt{4\pi}$  does not depend on the HD coordinates. Therefore,  $g_{\mu\nu}^{00}$  is massless, which is consistent with our interpretation of it as the graviton. The factor of  $c^2T^2$  in the denominator of the coefficients of these terms implies that the  $g_{\mu\nu}^{1m}$  have Planck masses. Possibly, quantum corrections of the type in Part II reduce these masses to the usual elementary particle mass scale. In any case, the  $l > 0$  fields are weaker than the massless graviton field in small-scale spacetime because they have masses. This is for the same reason the weak interaction is weaker than the electromagnetic. Thus, they, like the graviton, may be neglected in this arena. In large-scale spacetime, they play no role because their masses limit the range of their interaction. This is where they differ from the graviton and are eliminated from physical importance.

Therefore, one may approximate  $g_{\mu\nu}$  with  $g_{\mu\nu}^{00}$ , which becomes the 4D metric tensor. Thus,  $g_{00}^{\mu\nu}$  and  $g_{\mu\nu}^{00}$  raise and lower 4D indices, respectively. In addition, only  $g_{\mu\nu}^{00}$  appears in the factor of  $\sqrt{g}$  in the volume element. The tensor  $g_{\mu\nu}$  contains only one known elementary particle, the graviton. As will be shown later, all other elementary particles, including the W and Z, leptons, quarks and gluons are contained within  $A_\mu$ . This is accomplished by expanding  $A_\mu$  in terms of spherical harmonics, spinors and/or the strong-interaction-group generators. The various components of  $A_\mu$  thus produced will be identified with the elementary particles.

There arise in the Lagrangian  $k\hat{R}$  the terms

$$-k\hat{g}^{55}\hat{g}^{66}\partial_5^2\hat{g}_{66} - \frac{1}{2}k\hat{g}^{55}\partial_5\hat{g}^{66}\partial_5\hat{g}_{66}. \quad (31)$$

With our choice of  $\hat{x}^5$  and  $\hat{x}^6$ , the field  $\hat{g}_{66} = \hat{g}^{66} = 1$ . Therefore, these terms are zero. However, had we made the usual choice for the HD coordinates  $\hat{x}^5 = \theta$  and  $\hat{x}^6 = \phi$ , then  $\hat{g}_{66} = -c^2 T^2 \sin^2 \theta$ ,  $\hat{g}^{66} = -1/(c^2 T^2 \sin^2 \theta)$  and  $\hat{g}^{55} = -1/(c^2 T^2) + A^\mu A_\mu$ . Terms (31) would then contain the term  $2k/(c^2 T^2)$ , which is a cosmological constant roughly the size of the Planck mass. Therefore, incorporating  $\sin \theta$  into the coordinate  $\hat{x}^6$  instead of the scalar  $\hat{g}_{66}$  eliminates this large cosmological constant.

## 6. The electroweak vectors

In this section we derive Maxwell and mass terms for the photon, W and Z. The free- $A_\mu$ -field terms have anywhere from one to six factors of  $A$ . Of these, we consider here only terms with one or two of these factors. These terms are more important and are easier to obtain. They are

$$+\frac{1}{2}k\delta^{\rho\sigma}\delta^{\mu\nu}(\partial_\rho A_\mu\partial_\nu A_\sigma - \partial_\rho A_\mu\partial_\sigma A_\nu) \quad (32)$$

$$+2k\delta^{\mu\nu}\partial_\mu\partial_5 A_\nu \quad (33)$$

$$-2k\delta^{\mu\nu}A_\mu\partial_5^2 A_\nu \quad (34)$$

$$-2k\delta^{\mu\nu}\partial_5 A_\mu\partial_5 A_\nu \quad (35)$$

$$-\frac{1}{2}k\delta^{\mu\nu}\partial_6 A_\mu\partial_6 A_\nu. \quad (36)$$

A word of warning about this calculation. Like many calculations in general relativity, arriving at these terms is a straightforward, but lengthy calculation. One must devote many hours to it if one is to arrive at the correct result. One should limit oneself to terms with two or less factors of  $A$  as soon as possible in the calculation. Terms (32) are the Kaluza-Klein miracle — they lead to Maxwell's equations for the photon when the HD coordinate dependence of  $A_\mu$  is neglected and  $\partial_5 = \partial_6 = 0$ . In this case the calculation is much simpler and takes less than an hour. Terms (34)–(36) are similar to Terms (24)–(27), which are much easier to obtain. Terms like (33), with only one factor of  $A$ , appear only twice in the Lagrangian before they are added together. Because these terms have one second derivative, one may neglect the terms within the products  $\Gamma\Gamma$ , which have two first derivatives, in the curvature scalar when deriving them.

Terms (32) will lead to Maxwell's equations for the photon, W and Z and predicted intermediate vector bosons. Term (33) will lead to the Dirac equation for fermions. Terms (34)–(36) will lead to mass terms for vectors and interaction terms for spinors with vectors. Terms with three to six factors of  $A$  are self-interaction terms for vectors, similar to those found in the Standard Model.

The procedures for evaluating these terms by differentiating and integrating over the spherical harmonics in them are described in the next section. Because  $g^{\mu\nu} = \delta^{\mu\nu}$ , then  $\delta^{\mu\nu}$  acts as the contravariant 4D metric tensor and is associated with the bra  $\langle 00|$  in these terms. Because the vector  $A^\mu$  is really two separate fields,  $\delta^{\mu\nu}$  and  $A_\nu$ , it should be written as  $\delta^{\mu\nu} A_\nu$  when integrating over  $\theta$  and  $\phi$ ; otherwise the result of integration will be incorrect. This is explained in the next section. The expansion for  $A_\mu$  is carried to  $l = 1$  for now.

Terms (32) are the only terms with two factors of  $A$  and two 4D derivatives. The result for these terms is

$$-\frac{k}{4}F_{00}^{\mu\nu}F_{\mu\nu}^{00} - \frac{k}{4\sqrt{3}}F_{1,-1}^{\mu\nu}F_{\mu\nu}^{11} + \frac{k}{4\sqrt{3}}F_{10}^{\mu\nu}F_{\mu\nu}^{10} - \frac{k}{4\sqrt{3}}F_{11}^{\mu\nu}F_{\mu\nu}^{1,-1}, \quad (37)$$

where  $F_{\mu\nu}^{lm} = \partial_\mu A_\nu^{lm} - \partial_\nu A_\mu^{lm}$ . We will identify  $A_\mu^{00}$  with the photon field shortly.

We introduce the reality condition

$$\sum A_{lm}^\mu Y_{lm} = (\sum A_{lm}^\mu Y_{lm})^* \quad (38)$$

and equate coefficients of like  $Y_{lm}$  on both sides of this equation. This produces the relation

$$\bar{A}_{lm}^\mu = (-1)^m A_{l,-m}^\mu, \quad (39)$$

where no sum over  $m$  is implied. Further, we introduce the convention of representing the contravariant fields  $A_{lm}^\mu$  and  $F_{lm}^{\mu\nu}$  as the complex conjugates  $\bar{A}_{lm}^\mu$  and  $\bar{F}_{lm}^{\mu\nu}$ . Terms (37) now become

$$-\frac{k}{4} \bar{F}_{00}^{\mu\nu} F_{\mu\nu}^{00} + \frac{k}{4\sqrt{3}} \bar{F}_{11}^{\mu\nu} F_{\mu\nu}^{11} + \frac{k}{4\sqrt{3}} \bar{F}_{10}^{\mu\nu} F_{\mu\nu}^{10} + \frac{k}{4\sqrt{3}} \bar{F}_{1,-1}^{\mu\nu} F_{\mu\nu}^{1,-1}, \quad (40)$$

where  $\bar{F}_{lm}^{\mu\nu} = \partial^\mu \bar{A}_{lm}^\nu - \partial^\nu \bar{A}_{lm}^\mu$ .

The signs of the Maxwell terms for  $A_\mu^{11}$ ,  $A_\mu^{1,-1}$ , and  $A_\mu^{10}$  are opposite what they should be. This will also be the case for their mass terms. This means these fields have negative energy. However, this is no more of a problem than the existence of the negative energy positron, which follows from the Dirac equation. Both problems are dealt with as follows. Negative energy states were thought to be unstable because there would be no ‘‘barrier’’ preventing them from decaying into negative energy states with increasingly lower energy. However, if the ground state is defined to be the state closest to zero energy instead of lowest energy, then this cannot happen. Particles with negative energy tend to gain energy, not lose it. We adopt this convention so that there is a symmetry around zero energy. Note that the term for the photon has positive energy.

If this interpretation of negative energy is correct, then one could still have an electron decay into a positron, which is no further than an electron from zero energy. The decay of an electron into a positron is prohibited by the apparent impossibility of an elementary particle with charge 2, necessary for the transition. The reason why charge 2 elementary particles cannot exist is described in Part II. The decay could take place with more than one charge 1 particle but terms describing this have more vectors than the minimal coupling term described in Sec. 9 and are therefore much smaller than this term. This is because each vector in the term must be redefined to have the right units, producing factors of the inverse of their normalization factor described immediately below in Eq. (41). One could have terms with more spinors, but these are smaller by a factor greater than  $cT$  for each spinor pair due to their normalization factor in Eq. (72). Therefore, these terms involving more than one intermediary are negligible.

The fields are now redefined to yield the conventional coefficient of the Maxwell term for the photon

$$A_\mu^{lm} = \left( \frac{c^4}{16\pi G} \right)^{1/2} A_\mu^{lm}, \quad (41)$$

where we have included the factor of  $c$  from  $x^4 = ict$  in the differential volume element in the redefinition. Terms (40) now become

$$-\frac{1}{4} \bar{F}'_{00}{}^{\mu\nu} F'_{\mu\nu}{}^{00} + \frac{1}{4\sqrt{3}} \bar{F}'_{11}{}^{\mu\nu} F'_{\mu\nu}{}^{11} + \frac{1}{4\sqrt{3}} \bar{F}'_{10}{}^{\mu\nu} F'_{\mu\nu}{}^{10} + \frac{1}{4\sqrt{3}} \bar{F}'_{1,-1}{}^{\mu\nu} F'_{\mu\nu}{}^{1,-1}, \quad (42)$$

where  $F_{\mu\nu}^{lm} = \partial_\mu A_\nu^{lm} - \partial_\nu A_\mu^{lm}$ . Henceforth, the redefinition of  $A_\mu^{lm}$  will be understood and we will drop their prime. These terms appear in the Lagrangian for the SM. They are the free-field (Maxwell) terms for the photon, W and Z. The photon is identified with  $A_\mu^{00}$ ; the  $W_\mu^-$  is  $A_\mu^{1,-1}$ , the  $Z_\mu^0$  is  $A_\mu^{10}$ , and the  $W_\mu^+$  is  $A_\mu^{11}$ .

We have

$$\hat{g}_{\mu 5} = A_\mu \quad (43)$$

$$= \sum_{l=0}^{\infty} \sum_{m=-l}^l A_\mu^{lm} Y_{lm}(\theta, \phi) \quad (44)$$

$$= A_\mu^{00} Y_{00} + A_\mu^{1,-1} Y_{1,-1} + A_\mu^{10} Y_{10} + A_\mu^{11} Y_{11} + \dots \quad (45)$$

$$= A_\mu^{00} Y_{00} + W_\mu^- Y_{1,-1} + Z_\mu^0 Y_{10} + W_\mu^+ Y_{11} + \dots \quad (46)$$

The photon is denoted by  $A_\mu^{00}$ , so as not to be confused with  $A_\mu$ , which is given in Eq. (43). We use only the physical fields; there is no auxiliary vector  $\mathbf{B}_\mu$  as found in the SM, nor subsequent mixing. Note that we have given the W and Z the same normalization as the photon. This is the simplest thing to do and will be absolutely necessary in order to obtain the masses of the W and Z from the coefficients of their mass terms. This normalization for the W and Z leaves a factor of  $1/\sqrt{3}$  in their Maxwell terms. This does not contradict observation as it would for the photon because one does not detect the W and Z directly; one can only observe their decay products.

The result for Term (33) is  $\frac{\sqrt{3}}{2} \pi (cT)^{-1} k \delta^{\mu\nu} \partial_\mu A_\nu^{10}$ . This term is a divergence, which can be transformed into a surface integral at infinity, where the fields vanish. Therefore, Term (33) is zero for the  $A_\mu^{lm}$ .

Terms (34)–(36) are the only ones with two factors of A and two HD derivatives. The result for these terms is

$$-\frac{9 + 4\sqrt{3}}{12c^2T^2} \delta^{\mu\nu} A_\mu^{1,-1} A_\nu^{11} \quad (47)$$

$$-\frac{9 + 4\sqrt{3}}{12c^2T^2} \delta^{\mu\nu} A_\mu^{11} A_\nu^{1,-1} \quad (48)$$

$$+\frac{\sqrt{30}}{3c^2T^2} \delta^{\mu\nu} A_\mu^{10} A_\nu^{10}. \quad (49)$$

We employ the above-mentioned convention of representing the first factor of  $A$  in each term as a contravariant vector whose complex conjugate is taken. The result for these terms is now

$$+\frac{9 + 4\sqrt{3}}{12c^2T^2} \bar{A}_{11}^\mu A_\mu^{11} \quad (50)$$

$$+\frac{9 + 4\sqrt{3}}{12c^2T^2} \bar{A}_{1,-1}^\mu A_\mu^{1,-1} \quad (51)$$

$$+\frac{\sqrt{30}}{3c^2T^2} \bar{A}_{10}^\mu A_\mu^{10}. \quad (52)$$

These are mass terms for the W and Z. Like  $g_{\mu\nu}^{00}$ , the field  $A_\mu^{00}$  does not have a mass term because its associated spherical harmonic  $Y_{00} = 1/\sqrt{4\pi}$  does not depend on the HD coor-

dinates. Therefore  $A_\mu^{00}$  is massless, which is consistent with our identification of it as the photon.

It is a miracle that the 6D curvature scalar  $\widehat{R}$  leads to the 4D curvature scalar  $R$  plus the Maxwell term  $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$  when fields do not depend on the HD coordinates  $\widehat{x}^5$  and  $\widehat{x}^6$ . The calculation is simple and seems to unify gravitation with electromagnetism. It is miraculous that so little effort leads to such a good result. However, there is a fly in the ointment. There is no reason why fields should not depend on the HD coordinates. If general relativity is truly six-dimensional, fields should depend on all six coordinates, not just the first four. After all, this is how four-dimensional general relativity works. The fields depend on all four coordinates in this case. Therefore, we will make our theory more correct by allowing fields to depend on all six coordinates. Then the miracle continues.

Flowing directly from the HD coordinate dependence of  $A_\mu$  is the existence of the W and Z. And following from terms with two 4D and two HD derivatives are the terms that describe them. Orthogonality of the spherical harmonics produces one Maxwell and one mass term for each of the W's and Z when the Lagrangian density is integrated over  $\theta$  and  $\phi$ . Terms with three to six factors of  $A_\mu$  lead to self-interaction terms for the W and Z similar to the Yang-Mills self-interaction terms found in the SM. This is an example of the details following from properly fixed postulates.

The theory of 3DT explains why there are three weak vectors and one electromagnetic vector. It explains why the weak vectors have charges  $-1, 0, +1$ . It explains why the photon has no charge. (Charge is the momentum canonically conjugate to the coordinate dependence  $e^{i\phi}$ .) It explains why the W and Z have mass while the photon does not. This derivation of the existence of the photon, W and Z, and their properties explains the origin of what appears to be the local symmetry group  $SU(2) \times U(1)$ , which must be assumed in the Standard Model. We have produced all necessary terms for these fields using spherical harmonics and the curvature scalar as Lagrangian density — both natural parts of general relativity with three dimensions of time.

## 7. How to differentiate and integrate over the $Y_{lm}(\theta, \phi)$

This section may be omitted in a first reading. It describes how to go from ‘raw’ terms in the Lagrangian such as Terms (32)–(36) to their more final forms such as Terms (47)–(49). Without an organized procedure for this, one soon runs into expressions that cannot be evaluated because of infinities. In addition, there will be more than one way to do certain calculations, each with a different result. The right way must be specified.

If one attempts to multiply three or more expansions in terms of spherical harmonics found in terms with three or more factors of  $A$  in the Lagrangian, one soon runs into a mess consisting of too many factors of  $\sqrt{\pi}$  as well as a plethora of other square roots. These do not all disappear after the term is integrated over  $\theta$  and  $\phi$  as they do in terms with only two expansions. There must be a better way. Indeed there is: One must convert the spherical harmonics to kets. Then products of two kets may be written as single kets using Clebsch-Gordan coefficients. This process is continued until there is just one ket for the contravariant group of factors and one ket for the covariant group. Then the ket for the contravariant group is written as a bra and the final bra and ket are multiplied together representing the integral over  $\theta$  and  $\phi$ . Or, if one chooses, the final bra and ket may be reconverted to spherical



harmonics. Because we are integrating here over just two spherical harmonics, the excess factors of  $\sqrt{\pi}$  and other square roots appearing in each spherical harmonic will not appear in the answer. Thus, we will have converted integrals over three or more spherical harmonics into those over just two.

Before the procedures for taking the higher-dimensional derivatives and integrating over the spherical harmonics are discussed, I will show how to deal with the bras and kets in each term. As described in Sec. 4,  $g_{00}^{\mu\nu}$  is the 4D contravariant metric tensor and raises 4D indices. Therefore, we have equations like

$$A_{lm}^\mu = g_{00}^{\mu\nu} A_\nu^{lm}. \quad (53)$$

This shows that  $g_{00}^{\mu\nu}$  is the contravariant part of  $A_{lm}^\mu$ . Since every ‘contravariant’ tensor may be written in this fashion, then  $g_{00}^{\mu\nu}$  is the only true contravariant tensor. It matters whether  $A_{lm}^\mu$  or  $g_{00}^{\mu\nu}$  is the contravariant tensor because the results of integrating over  $\theta$  and  $\phi$  depend upon which is chosen. For example, if  $A_{lm}^\mu$  is the contravariant tensor, we would have

$$\overline{A}_{11}^\mu \langle 11 | A_\mu^{11} | 11 \rangle = \overline{A}_{11}^\mu A_\mu^{11}, \quad (54)$$

while if  $g_{00}^{\mu\nu}$  is the contravariant tensor, then the same expression could be written as

$$-g_{00}^{\mu\nu} \langle 00 | A_\mu^{1,-1} | 1, -1 \rangle A_\nu^{11} | 11 \rangle = \frac{1}{\sqrt{3}} \overline{A}_{11}^\mu A_\mu^{11}, \quad (55)$$

where the kets  $|1, -1\rangle$  and  $|11\rangle$  are combined first. There is a factor of  $\sqrt{3}$  difference between these expressions. Equation (55) is correct because  $g_{00}^{\mu\nu}$  is the only true contravariant tensor. This applies, however, only until the integration over  $\theta$  and  $\phi$  is taken in this section.

Because factors are expanded in terms of spherical harmonics, which are then converted into kets, which do not commute, factors in a term must be ordered properly. This order is determined by the above definition of the curvature scalar in terms of the metric tensor. We will be dealing with terms with no more than two nontrivial covariant factors. Therefore the order of the two covariant factors does not matter because integration with  $\langle 00|$ , which is the only bra we will be using, and two kets acts like a dot product between the two kets.

As the kets are nonassociative, the order of the multiplication of factors in a term must be specified. This order is largely implied by the structure of the equations that define the curvature scalar. First, the products of the factors of  $A$  in each of Eqs. (14) and (15) are taken. Then the factors within  $\widehat{\Gamma}_{\alpha\beta}^\delta$ ,  $\widehat{R}_{\alpha\beta}$  and  $\widehat{R}$  are combined in that order. In practice, these rules are not necessary because we will be dealing here with terms with no more than two nontrivial covariant factors. This means there is only one nontrivial multiplication, making the order of multiplications irrelevant.

Note that the order of factors and the order of multiplications in a term pertain only to the covariant factor [for example,  $A_\nu^{lm}$  in Eq. (53)] within the seemingly contravariant factor [ $A_{lm}^\mu$  in Eq. (53)]. The contravariant factors  $g_{00}^{\mu\nu}$  are not tied down to a particular position in the term nor in the order of multiplications like the covariant factors, because they are associated with the ket  $|00\rangle$  only. Therefore, we may group all of the 4D contravariant metric tensors in a term together and place them to the left of the covariant group of factors. The adjoint is then taken of the group of  $g_{00}^{\mu\nu}$ . The reason the  $g_{00}^{\mu\nu}$  are grouped together is that we will want to combine the kets of the contravariant factors and, separately, the kets of the covariant factors. We then write the adjoint of the resultant ket for the contravariant factors

as a bra. This is so that there will be one bra and one ket, representing the contravariant and covariant factors, respectively. The final bra and ket are then combined, representing one integral over  $\theta$  and  $\phi$ . If the contravariant factors were not grouped together, we would have more than one integral over  $\theta$  and  $\phi$  per term. The contravariant group of factors is placed to the left of the covariant group because bras must be placed to the left of kets if their products are to represent integrals. Note that the only bra we will ever have to consider for the contravariant factors is  $\langle 00|$ .

Term (34) has the factor  $\partial_5^2$ , which equals  $-1/(c^2T^2)\partial_\theta^2$ , according to Sec.3. Here  $\partial_\theta = \partial/\partial\theta$ . To see how this term is integrated over  $\theta$  and  $\phi$ , we examine the case where  $l = 0$  or 1 in the expansion for  $A_\mu$ . The derivatives with respect to  $\theta$  are taken:  $\partial_\theta^2 Y_{00} = 0$  and  $\partial_\theta^2 Y_{1m} = -Y_{1m}$ . If  $l > 1$ , this simplification for  $\partial_\theta^2 Y_{1m}$  is not possible and the expression for  $\partial_\theta$  described in the procedure for Term (35) must be used.

The spherical harmonics of the contravariant and covariant factors are now written as kets  $Y_{lm} \rightarrow |lm\rangle$  in preparation for combining them. Next, the two covariant expansions are multiplied and the products of kets are written in terms of single kets using Clebsch-Gordan coefficients. For example,

$$|11\rangle|1, -1\rangle = \frac{1}{\sqrt{6}}|20\rangle + \frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{3}}|00\rangle. \quad (56)$$

Because the metric tensor must be constant for the free- $A_\mu$  fields, the special relativity metric applies to the free- $A_\mu$ -field terms, including Term (34). Therefore, we set  $g_{00}^{\mu\nu} = \delta^{\mu\nu}$ , where  $\delta^{\mu\nu}$  now has the ket  $|00\rangle$ . We are using the Kronecker delta instead of the Minkowski metric because we have incorporated the minus sign of  $\eta_{00} = -1$  into the coordinate  $\hat{x}^4 = ict$ , where  $i = \sqrt{-1}$ . The adjoint of the contravariant factor is taken and its ket is written as a bra. The contravariant and covariant expressions are now multiplied together. The integrals over  $\theta$  and  $\phi$  may be expressed in terms of the orthogonality of the kets:  $\langle 00|lm\rangle = \delta_{0l}\delta_{0m}$ .

If one is doing this calculation for  $A_\mu^{lm}$  with  $l > 1$ , factors of  $e^{\pm 2i\phi}$  must be combined with the kets of the differentiated factor before the kets of the two covariant factors are combined. For example,  $e^{2i\phi}|2, -1\rangle = -|21\rangle$ . Otherwise, a different result is obtained. This procedure must be correct because it leads to the equation  $\partial_\theta^2 Y_{1m} = -Y_{1m}$  for the case  $l = 1$ . (Note, however, that  $e^{2i\phi}|2, -2\rangle$  cannot be combined preliminarily because it does not equal another ket.)

Term (35) contains two factors of  $\partial_5$ . It is dealt with in the same way as Term (34), except that  $\partial_\theta$  is rewritten

$$\partial_\theta = \frac{1}{2\hbar} (e^{-i\phi}L_+ - e^{i\phi}L_-), \quad (57)$$

where  $L_+$  and  $L_-$  are the raising and lowering operators, respectively, for the spherical harmonics. We have

$$L_\pm Y_{lm} = [l(l+1) - m(m \pm 1)]^{1/2} \hbar Y_{l, m \pm 1}. \quad (58)$$

This definition for  $\partial_\theta$  follows from combining  $L_+ = L_x + iL_y$  and  $L_- = L_x - iL_y$ , where

$$L_x = i\hbar(\sin\phi\partial_\theta + \cot\theta\cos\phi\partial_\phi), \quad (59)$$

$$L_y = i\hbar(-\cos\phi\partial_\theta + \cot\theta\sin\phi\partial_\phi), \quad (60)$$

which are the  $x$  and  $y$  components of the angular momentum vector  $\mathbf{L}$  written in spherical coordinates. Rewriting the operator  $\partial_\theta$  is necessary to write  $\partial_\theta Y_{lm}$  properly in terms of spherical harmonics, which can then be written as kets.

When one uses this expression for  $\partial_\theta$ , one winds up with factors of  $e^{i\phi}$  in the term. In order to evaluate these expressions, the terms have their final bra and ket rewritten as spherical harmonics according to the transformations  $\langle 00| \rightarrow Y_{00}$  and  $|lm\rangle \rightarrow Y_{lm}$  and the terms are explicitly integrated over  $\theta$  and  $\phi$  in the usual way. Note that the factors of  $e^{i\phi}$  must cancel if the integral over  $\phi$  and the term is to be nonzero. For terms without functions of  $\phi$ , use may be made of the orthonormality of the kets.

Term (36) contains two factors of  $\partial_6$ , where  $\partial_6 = \partial/(icT \sin\theta\partial\phi)$  from Sec. 3. This term is dealt with in the same way as Term (34). First, the derivatives with respect to  $\phi$  of the  $Y_{lm}$  are taken  $\partial_\phi Y_{lm} = imY_{lm}$ . In order to evaluate these terms with two factors of  $1/\sin\theta$  from the two factors of  $\partial_6$ , they have the spherical harmonics of both covariant factors written in terms of spherical harmonics with their  $l$  and  $|m|$  values decreased by one. (The quantum number  $m$  is increased by one if it is negative.) Some examples of this are

$$Y_{1,\pm 1} = \mp \sqrt{\frac{3}{2}} Y_{00} \sin\theta e^{\pm i\phi}, \quad (61)$$

$$Y_{2,\pm 2} = \mp \sqrt{\frac{5}{4}} Y_{1,\pm 1} \sin\theta e^{\pm i\phi}, \quad (62)$$

$$Y_{3,\pm 1} = \mp \sqrt{\frac{7}{12}} (\sqrt{5} Y_{20} + Y_{00}) \sin\theta e^{\pm i\phi}. \quad (63)$$

The factors of  $\sin\theta$  in the  $\partial_6$ 's cancel with those extracted from the covariant factors in Eqs. (61)–(63). This eliminates the factor of  $1/\sin^2\theta$ , which would otherwise result in infinity when integrating over  $\theta$ . The spherical harmonics are now written as kets and combined.

## 8. Leptons

Now that we have the four electroweak vectors, we also have fermions. This follows directly from Chapter 41 of Misner, Thorne and Wheeler [45], the classic textbook. This book details standard spinor theory, which states that any vector is equivalent to a second rank spinor. One does not have to invent a fantastic theory like supersymmetry to obtain fermions. They could come from a textbook. For any vector  $W_\mu$ , we have

$$W_\mu = -\frac{1}{2} \sigma_\mu^{B\dot{V}} w_{B\dot{V}}, \quad (64)$$

where  $w_{B\dot{V}}$  is a second rank spinor. A sum is implied over the indices  $B$  and  $\dot{V}$ . These indices take the values 1, 2 and  $\dot{1}$ ,  $\dot{2}$ , respectively. The  $\vec{\sigma}^{B\dot{V}}$  are the Pauli spin matrices and  $\sigma_0^{B\dot{V}}$  is the unit matrix. These matrices are the basis vectors for an expansion of a vector. The spinor field components are its coefficients.

When the vector is null, like for example, the photon, the second rank spinor is equated to the product of two 2-component spinors. This produces the product form  $\bar{\psi}\psi$  of spinors necessary to reproduce the Lagrangian for the Dirac equation. For example, for the photon we have

$$A_\mu^{00}|00\rangle = -\frac{1}{2} \sigma_\mu^{B\dot{V}} \xi_B \eta_{\dot{V}} \frac{1}{\sqrt{2}} \left| \frac{1}{2} \frac{1}{2} \right\rangle \left| \frac{1}{2}, -\frac{1}{2} \right\rangle + \frac{1}{2} \sigma_\mu^{B\dot{V}} \tau_B \omega_{\dot{V}} \frac{1}{\sqrt{2}} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle, \quad (65)$$

where

$$|00\rangle = \frac{1}{\sqrt{2}} \left| \frac{1}{2} \frac{1}{2} \right\rangle \left| \frac{1}{2}, -\frac{1}{2} \right\rangle - \frac{1}{\sqrt{2}} \left| \frac{1}{2}, -\frac{1}{2} \right\rangle \left| \frac{1}{2} \frac{1}{2} \right\rangle. \quad (66)$$

The spinor  $\xi_B$  or  $\omega_{\dot{V}}$  has the ket  $\left|\frac{1}{2}, \frac{1}{2}\right\rangle$ , while  $\eta_{\dot{V}}$  or  $\tau_B$  is in the state  $\left|\frac{1}{2}, -\frac{1}{2}\right\rangle$ . By convention, the first ket in each pair in Eq. (65) is identified with the spinor with an undotted index, while the second is matched with the dotted-index spinor. We have

$$A_{\mu}^{00} = -\frac{1}{2}\sigma_{\mu}^{B\dot{V}}\xi_B\eta_{\dot{V}} \quad (67)$$

for the first pair of kets and

$$A_{\mu}^{00} = -\frac{1}{2}\sigma_{\mu}^{B\dot{V}}\tau_B\omega_{\dot{V}} \quad (68)$$

for the second.

The spinors in this expansion may recombine to form  $A_{\mu}^{10}$  (the  $Z^0$ ). We have

$$A_{\mu}^{10}|10\rangle = -\frac{1}{2}\sigma_{\mu}^{B\dot{V}}\xi_B\eta_{\dot{V}}\frac{1}{\sqrt{2}}\left|\frac{1}{2}, \frac{1}{2}\right\rangle\left|\frac{1}{2}, -\frac{1}{2}\right\rangle - \frac{1}{2}\sigma_{\mu}^{B\dot{V}}\tau_B\omega_{\dot{V}}\frac{1}{\sqrt{2}}\left|\frac{1}{2}, -\frac{1}{2}\right\rangle\left|\frac{1}{2}, \frac{1}{2}\right\rangle, \quad (69)$$

where

$$|10\rangle = \frac{1}{\sqrt{2}}\left|\frac{1}{2}, \frac{1}{2}\right\rangle\left|\frac{1}{2}, -\frac{1}{2}\right\rangle + \frac{1}{\sqrt{2}}\left|\frac{1}{2}, -\frac{1}{2}\right\rangle\left|\frac{1}{2}, \frac{1}{2}\right\rangle. \quad (70)$$

The derivative  $\partial_5$  in Term (33), which will lead to the Dirac equation, eliminates  $A_{\mu}^{00}|00\rangle$  from the expansion of  $A_{\mu}$ . The factors of  $e^{i\phi}$  in the spherical harmonics for  $A_{\mu}^{1,\pm 1}$  eliminate them upon integration over  $\theta$  and  $\phi$ . This leaves  $A_{\mu}^{10}|10\rangle$ . After substituting this only nonzero term into Term (33), the spinor kets are converted to  $|10\rangle$  and  $|00\rangle$  (for example,  $\left|\frac{1}{2}, -\frac{1}{2}\right\rangle\left|\frac{1}{2}, \frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}|10\rangle - \frac{1}{\sqrt{2}}|00\rangle$ ). This results in

$$\frac{\sqrt{3}\pi ik}{8cT}\delta^{\mu\nu}\sigma_{\mu}^{B\dot{V}}(\xi_B\partial_{\nu}\eta_{\dot{V}} + \partial_{\nu}\xi_B\eta_{\dot{V}}), \quad (71)$$

plus a similar set of terms involving the spinors  $\tau_B$  and  $\omega_{\dot{V}}$ . The spinors  $\eta_{\dot{V}}$  and  $\xi_B$  will be identified with the electron field and its complex conjugate, respectively, while  $\omega_{\dot{V}}$  and  $\tau_B$  will be the neutrino and its conjugate, respectively. These identifications are suggested by the associations these fields have with their kets above, according to their quantum numbers  $l$  and  $m$  given in Table 1.

We will ignore the neutrino terms for now and concentrate on deriving the Dirac equation for the electron. The fields are redefined to give them the right units

$$\xi'_B = \left(\frac{\sqrt{3}c^3}{128\hbar GcT}\right)^{1/2}\xi_B, \quad (72)$$

with an identical redefinition for  $\eta_{\dot{V}}$ . The result for Term (33) is now

$$+i\hbar c\delta^{\mu\nu}\sigma_{\mu}^{B\dot{V}}\xi'_B\partial_{\nu}\eta'_{\dot{V}} \quad (73)$$

$$+i\hbar c\delta^{\mu\nu}\sigma_{\mu}^{B\dot{V}}\partial_{\nu}\xi'_B\eta'_{\dot{V}}. \quad (74)$$

Spinor mass terms are not contained directly within the 6D curvature scalar. One must introduce spinors into the Lagrangian with the above-mentioned substitution for vectors, which does not allow terms of the form  $-m\xi'_B\xi'^B$ . However, new terms may result from a particle's motion on the HD sphere. This motion produces energy and must be represented in the Lagrangian. Motion on the HD sphere is isospin. Electric charge will be related to the

$z$ -component of isospin in Part II. Further, spinors acquire mass from their charge according to the classical radius equation considered in Part II. Therefore, we introduce the mass terms

$$-m\xi'_B\xi'^B - m\eta'^{\dot{V}}\eta'_{\dot{V}}. \quad (75)$$

Although these mass terms are not contained directly within  $\widehat{R}$ , they do not produce anomalies because they are derived from  $\widehat{R}$ . It is  $\widehat{R}$  that puts a moving electron on the HD sphere. This is indicated by the electron's HD angular momentum, which is given by the operators  $L_z$  and  $L^2$  operating on its ket.

Each of Terms (73) or (74) leads to the Dirac equation. We will consider only Term (73) in what follows. Lagrange's equations for  $\xi'_B$  and  $\eta'_{\dot{V}}$  are

$$i\hbar c \delta^{\mu\nu} \sigma_{\mu}^{B\dot{V}} \partial_{\nu} \xi'_B = m\eta'^{\dot{V}}, \quad (76)$$

$$i\hbar c \delta^{\mu\nu} \sigma_{\mu}^{B\dot{V}} \partial_{\nu} \eta'_{\dot{V}} = m\xi'^B. \quad (77)$$

Using the equations [45]  $\zeta^B \xi'_B = -\zeta_B \xi'^B$  and  $\eta'^{\dot{V}} = \varepsilon^{\dot{V}\dot{W}} \eta'_{\dot{W}}$ , where the only nonzero components of  $\varepsilon^{\dot{V}\dot{W}}$  are  $\varepsilon^{i\dot{2}} = -\varepsilon^{\dot{2}i} = 1$ , Eq. (76) becomes

$$-i\hbar c \sigma_{B\dot{V}}^{\mu} \partial_{\mu} \xi'^B = m\eta'_{\dot{V}}. \quad (78)$$

Here the  $\sigma_{B\dot{V}}^{\mu}$  are the associated basic spin matrices. They are computed from the  $\sigma_{\mu}^{B\dot{V}}$  via the relations

$$\sigma_{B\dot{V}}^{\mu} \sigma_{\nu}^{B\dot{V}} = -2\delta^{\mu}_{\nu}. \quad (79)$$

The  $\sigma_{B\dot{V}}^{\mu}$  are equal to [45] minus the  $\sigma_{\mu}^{B\dot{V}}$ , except for  $\sigma_{B\dot{V}}^2$ , which equals  $\sigma_2^{B\dot{V}}$ .

Equations (78) and (77) may be put in matrix form

$$(i\partial_0 + i\vec{\sigma} \cdot \vec{\nabla}) \xi = m\eta, \quad (80)$$

$$(i\partial_0 - i\vec{\sigma} \cdot \vec{\nabla}) \eta = m\xi, \quad (81)$$

where the  $\vec{\sigma}$  are the Pauli spin matrices and

$$\xi = \begin{pmatrix} \xi'^1 \\ \xi'^2 \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta'_1 \\ \eta'_2 \end{pmatrix}. \quad (82)$$

We have set  $\hbar = c = 1$ , which applies for the remainder of this section. Adding and subtracting Eqs. (80) and (81), we obtain [58]

$$i\partial_0 \varphi + i\vec{\sigma} \cdot \vec{\nabla} \chi = m\varphi, \quad (83)$$

$$-i\partial_0 \chi - i\vec{\sigma} \cdot \vec{\nabla} \varphi = m\chi, \quad (84)$$

where  $\varphi = \frac{1}{\sqrt{2}}(\xi + \eta)$  and  $\chi = \frac{1}{\sqrt{2}}(\xi - \eta)$ .

Equations (83) and (84) may be recast as

$$i\gamma^{\mu} \partial_{\mu} \psi = m\psi, \quad (85)$$

where

$$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}, \quad (86)$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \quad (87)$$

where each element of the matrices in Eqs. (87) is itself a  $2 \times 2$  matrix. Eq. (85) with (86) and (87) constitute the Dirac equation for the electron in the Dirac representation for  $\psi$ .

A similar derivation of the Dirac equation for the neutrino involving the spinors  $\tau_B$  and  $\omega_V$  may be made. In addition, the photon may be expanded in terms of spinors with  $l > \frac{1}{2}$ . The muon and its neutrino are associated with the kets  $|\frac{3}{2}, -\frac{1}{2}\rangle$  and  $|\frac{3}{2}, \frac{1}{2}\rangle$ , respectively. The tau and its neutrino are in the respective states  $|\frac{5}{2}, -\frac{1}{2}\rangle$  and  $|\frac{5}{2}, \frac{1}{2}\rangle$ . Continuing in this manner, 3DT predicts an infinite number of fermions, each corresponding to a state  $|lm\rangle$ , where  $l$  and  $m$  are half-odd integral. The reason why the number of generations of leptons appears to be limited to three is explained in Part II. Thus the expansion of  $\hat{g}_{5\mu} = A_\mu$  in terms of spherical harmonics produces multiple generations of leptons.

I claim that the fact that the positron appears to have negative energy is not merely appearance but is deep-rooted and cannot be transformed away. The negative energy of the positron is dealt with in the same way as that for the  $Z^0$ : The ground state is defined to be that closest to zero energy instead of lowest energy.

In order to reproduce the SM fully, one must produce the chirality of the weak interactions. Chirality is allowed in 3DT because it has an even number of dimensions. A massless particle must have its right-handed field equal to zero. This is because one cannot slow down a left-handed particle and reverse its direction if it travels at the speed of light. The nearly zero mass of the neutrino is derived in Part II. Therefore,  $\nu_R = 0$  and  $\nu = \nu_L$  because  $\psi = \psi_L + \psi_R$ . This prevents an interaction term involving right-handed neutrinos, the W and electrons, producing the required chirality.

The Dirac equation originates from the 6D curvature scalar of 3DT. This occurs because the divergence  $\partial_\mu A_{10}^\mu$  splits into terms of the form of the Dirac term  $\bar{\psi} \gamma^\mu \partial_\mu \psi$  when the spinor form  $\bar{\psi} \gamma^\mu \psi$  of the vector  $A_{10}^\mu$  is used.

One may wonder why the Maxwell term does not apply to spinors. If a vector is equal to its spinor equivalent, one could substitute its spinor form into the Maxwell Terms (32). However, examination of Terms (32) and (33) reveals that the Dirac term, which contains  $\partial_5$ , is larger by a factor  $1/(cT)$ . Therefore, the spinor form of the Maxwell term is negligible compared to the Dirac term.

We have expanded each  $A_\mu^{lm}$  in terms of the Pauli matrices just like we expanded  $A_\mu$  in terms of spherical harmonics. This was made possible by the existence of the vectors  $A_\mu^{lm}$ , each of which may be equated to the product of two 2-component spinors. This is another example of the details following from properly fixed postulates.

Our method of introducing fermions into a purely bosonic Kaluza-Klein theory has several advantages over supersymmetry. Substituting two spinors for each vector in the theory relates the known spinors to the known vectors. This is highly economical and efficient. It does not introduce entire classes of supersymmetric particles for which there is little evidence. And it is consistent with our program of keeping the curvature scalar as Lagrangian and

therefore pure gravitation as the theory. Finally, one cannot derive the Dirac equation from supersymmetry. It must be assumed, like supersymmetry itself.

## 9. Interaction terms

Interaction terms for intermediate vector bosons and leptons may be derived from the vectors' mass Terms (47)–(49). This is done by expanding one of the vectors in these terms in terms of spinors. For example, Term (49) starts off as

$$+ \frac{\sqrt{30}}{3c^2T^2} k\delta^{\mu\nu} A_\mu^{10} A_\nu^{10}. \quad (88)$$

This term ends up as

$$-\frac{8}{3} \left(\frac{10}{\pi}\right)^{1/2} \sqrt{\hbar c} Z_\mu^0 (\bar{e} \gamma^\mu e + \bar{\nu}_e \gamma^\mu \nu_e), \quad (89)$$

Thus, the bare weak charge of the electron or neutrino interacting with the  $Z^0$  from the 6D curvature scalar is

$$g_{\widehat{R}} = -\frac{8}{3} \left(\frac{10}{\pi}\right)^{1/2} \sqrt{\hbar c} = -4.75766\sqrt{\hbar c}. \quad (90)$$

To see where this coefficient comes from, we write each of the factors that go into its formation. The coefficient is equal to

$$\left(\frac{\sqrt{30}}{3c^2T^2}\right)_1 \left(\frac{c^3}{16\pi G}\right)_2 (c)_3 \left(\frac{16\pi G}{c^4}\right)_4^{1/2} \left(\frac{128\hbar GcT}{\sqrt{3}c^3}\right)_5 \left(-\frac{1}{2}\right)_6 \left(\frac{1}{2}\right)_7, \quad (91)$$

where the factors have been numbered for future reference.

The 1st factor is the coefficient of the Z mass term from Sec. 6. We note that  $cT = L = (\hbar G/c^3)^{1/2}$  is the Planck length. Factor 2 is the constant  $k$  from Sec. 4, by which the entire Lagrangian is multiplied. This gives the Lagrangian the units of energy. The 3rd factor comes from the relativistic time coordinate  $dx^4 = icdt$  in the differential volume element of the action integral for the term. Factor 4 results from the redefinition of one vector field in the term described in Sec. 6. Factor 5 is from the redefinition of two spinors in the Dirac term in Sec. 8. Factor 6 is from Eq. (64), which equates vectors to spinors. The 7th factor arises when  $|10\rangle$  is written in terms of  $l = 1/2$  kets, each pair of which are multiplied by the Clebsch-Gordan coefficient  $1/\sqrt{2}$ . This Clebsch-Gordan coefficient is squared when the spinors'  $l = 1/2$  kets are reconverted to the kets  $|10\rangle$  and  $|00\rangle$ , which is done to combine them with the bra  $\langle 00|$ . Another way of looking at Factor 7 is that the  $Z^0$  is composed of one-half an electron-positron pair and one-half a neutrino-antineutrino pair.

We now consider interaction terms involving the muon  $\mu^-$ . The Clebsch-Gordan coefficient that relates  $|10\rangle$  to the muon-antimuon's  $l = 3/2$  pair of spinor kets is  $1/\sqrt{20}$ . This means  $A_\mu^{10}$  (the Z) may also be considered to be 1/20th a muon-antimuon pair. This produces a factor of 1/20 in Factor 7 in Eq. (91) instead of the factor of 1/2 for the electron. Thus, the coefficient of the weak interaction term from  $\widehat{R}$  for the muon would appear to be 1/10th that of the electron. Because the above-mentioned Clebsch-Gordan coefficient for the muon is 1/10th that of the electron in the Dirac term as well, the resulting Dirac equation for the muon would have a factor of 1/10 as coefficient for the Dirac term as well

as the weak-interaction term. Since, at this point, these are the only two terms in the Dirac equation for the muon, one may multiply the entire equation by 10 and recover the Dirac term with coefficient one as well as a weak charge that is the same as that for the electron. The weak charge of the tau from the 6D curvature scalar remains the same as that for the electron by the same reasoning used for the muon.

Spinors may combine to form the  $W^+$  and  $W^-$

$$A_\mu^{11}|11\rangle = -\frac{1}{2}\sigma_\mu^{B\dot{V}}\xi_B\omega_{\dot{V}}\left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}\frac{1}{2}\right\rangle, \quad (92)$$

$$A_\mu^{1,-1}|1,-1\rangle = -\frac{1}{2}\sigma_\mu^{B\dot{V}}\tau_B\eta_{\dot{V}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle, \quad (93)$$

which, when substituted into Terms (47) and (48), yield

$$+ \frac{16 + 12\sqrt{3}}{3\sqrt{\pi}}\sqrt{\hbar c} W_\mu^+ \bar{\nu}_e\gamma^\mu e \quad (94)$$

$$+ \frac{16 + 12\sqrt{3}}{3\sqrt{\pi}}\sqrt{\hbar c} W_\mu^- \bar{e}\gamma^\mu \nu_e. \quad (95)$$

There is no photon-electron interaction term in  $\widehat{R}$ . This term results from motion on the HD sphere because it depends on the charge of the electron. As noted above, additional terms in the Lagrangian may result from this motion. Therefore, we introduce the interaction term

$$qA_\mu^{00}\bar{e}\gamma^\mu e, \quad (96)$$

where  $q$  is the charge on an electron. This is similar to the introduction of the mass terms for spinors in Sec. 8.

Interaction terms are an important part of the SM, which produces them through a covariant derivative used in the Dirac term. This is done to maintain the symmetry  $SU(2) \times U(1)$ . In 3DT, however, we have a different but equally effective method for generating interaction terms. Here the spinor form  $\bar{\psi}\gamma^\mu\psi$  of the vector  $A^\mu$  is substituted into a mass term of the form  $A^\mu A_\mu$ , creating a term of the form  $\bar{\psi}\gamma^\mu\psi A_\mu$ . One may substitute two spinors for both vectors in terms like (88). Four-spinor interaction terms result. These are negligible compared to vector-spinor interaction terms because they contain an extra factor of  $cT$  from the additional spinor redefinition. [See Factor 5 of Eq. (91).] This explains why there are no four-point Fermi interactions and why the weak interactions are carried by intermediate vector bosons.

## 10. The strong interactions

General relativity can be considered to be the result of local Lorentz transformations in four dimensions. But what about Lorentz transformations in six dimensions? When one makes the transition from four to six dimensions, the Lorentz group requires nine additional generators to describe a Lorentz transformation. Each of these additional generators corresponds to a rotation or boost in a coordinate surface with at least one coordinate  $\hat{x}^5$  or  $\hat{x}^6$ . What force is produced by these nine additional types of Lorentz transformation? It is the strong interaction.



The Lorentz group becomes the rotation group for our coordinates because coordinates of time multiplied by  $i$  are spacelike. This is reflected in the signature diag. ( + + + + + ) of the metric tensor.

The nine generators corresponding to an infinitesimal 6D rotation involving  $\hat{x}^5$  or  $\hat{x}^6$  are denoted by  $\lambda_a$ , where  $a = 0, \dots, 8$ . Eight of the nine HD generators  $\lambda_1, \dots, \lambda_8$  are colored and produce the strong interaction. These generators correspond to rotations in planes with one HD coordinate ( $\theta$  or  $\phi$ ) and one 4D coordinate. The ninth HD generator  $\lambda_0$  is white and corresponds to the known white elementary particles such as the electron and photon. This white generator corresponds to a rotation in the HD sphere of both HD coordinates  $\theta$  and  $\phi$ , without a 4D coordinate. Fields associated with this generator are analogous to  $\hat{g}_{55} = 1$ , which has two indices equal to five and has no HD angular momentum.

We will construct our theory of the strong interactions based on our electroweak theory. The generators of the HD part of the 6D rotation group are analogous to the spherical harmonics. We will expand fields in terms of these nine generators, just like we expanded the vector  $A_\mu$  in terms of spherical harmonics. The reason for this is the same: it is because the HD's are so small.

We expand  $g_{\mu\nu}^{lm}$  and  $A_\mu^{lm}$  in terms of the nine generators  $\lambda_a$  of the HD part of the 6D rotation group

$$g_{\mu\nu}^{lm} = \sum_{a=0}^8 g_{\mu\nu}^{lma} \lambda_a, \quad (97)$$

$$A_\mu^{lm} = \sum_{a=0}^8 A_\mu^{lma} \lambda_a. \quad (98)$$

The Latin indices  $a, b, c$  range from zero to eight. Henceforth we will observe the convention of summing over these indices when they appear twice in a term. Up until now, we have neglected the strong interactions. With these expansions, this is no longer possible. Previously defined fields like the photon and graviton are now redefined—the photon and graviton are no longer  $A_\mu^{00}$  and  $g_{\mu\nu}^{00}$ . They are denoted now by  $A_\mu^{000}$  and  $g_{\mu\nu}^{000}$ ; the  $W$  and  $Z$  are now the  $A_\mu^{1m0}$ .

In the 4D differential volume element, the  $\sqrt{g}$  refers to  $g_{\mu\nu}^{000}$ , which equals  $\delta_{\mu\nu}$  in microscopic spacetime. The  $\sqrt{g}$  is not expanded in terms of HD rotation group generators because  $\delta_{\mu\nu}$  is constant and after an HD rotation, the field's values are oriented the same way in spacetime. Therefore, it has no strong charge with respect to an HD rotation. Similarly, in microscopic spacetime, the  $A_{lma}^\mu$  are written in terms of the  $A_\mu^{lma}$  using the 4D contravariant metric tensor  $\delta^{\mu\nu}$  according to the formula

$$A_{lma}^\mu = \delta^{\mu\nu} A_\nu^{lma}. \quad (99)$$

Thus, the only contravariant fields we will ever have to consider are the  $\delta^{\mu\nu}$  and the only generator we will ever have to consider for the contravariant fields is  $\lambda_0$ .

Before we proceed to consider terms with factors expanded in terms of HD rotation generators, we describe how to deal with the generators in each term. To eliminate the HD rotation generators from the terms in the Lagrangian and make the terms scalars, we determine one generator for each of the covariant and contravariant groups of factors in a

term and multiply the two generators together in a scalar or dot product. This product is defined such that

$$\lambda_a \cdot \lambda_b = \delta_{ab}. \quad (100)$$

This is similar to integrating over the orthogonal kets representing the covariant and contravariant groups of factors.

Each factor in a term is expanded in terms of the nine generators  $\lambda_a$ , eight of which are colored and one is white. It remains to determine which type of product exists between factors with these generators within each of the covariant and contravariant groups.

There are three possible products for the  $\lambda_a$  vectors of the covariant factors: scalar (dot product), vector (cross product), or a tensor product. In order to produce one  $\lambda_a$  for the covariant factors and yet involve all of them, a cross product seems most appropriate

$$\lambda_a \times \lambda_b = [\lambda_a, \lambda_b] = \sum_c c_{abc} \lambda_c, \quad (101)$$

where the  $c_{abc}$  are the structure constants of the HD part of the 6D rotation group. This is similar to the expression of products of kets in terms of single kets using Clebsch-Gordan coefficients.

Because the generators are not associative under cross product multiplication, parentheses must be used to specify the order of multiplications. This would be the same as that given in Sec. 7.

According to the vector identity  $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$ , the dot product may be interchanged with the last cross product in the term without affecting the value of the term. This would be how to obtain the familiar form of mass and Maxwell terms with two strongly interacting fields  $A_\mu$  — with the dot product between these factors as opposed to between the covariant factors of  $A_\mu$  and contravariant  $\delta^{\mu\nu}$ . The cross product that now exists between the covariant and contravariant groups of factors drops out because the contravariant group of factors has only the generator  $\lambda_0$ . This leaves only a trivial ordinary multiplication between the covariant and contravariant groups.

Expanding Terms (32) first in terms of spherical harmonics and then in terms of HD rotation generators, we arrive at the terms

$$-\frac{1}{4} \overline{F}^{\mu\nu} F_{\mu\nu}^{00a} + \frac{1}{4\sqrt{3}} \overline{F}^{\mu\nu} F_{\mu\nu}^{11a} + \frac{1}{4\sqrt{3}} \overline{F}^{\mu\nu} F_{\mu\nu}^{10a} + \frac{1}{4\sqrt{3}} \overline{F}^{\mu\nu} F_{\mu\nu}^{1,-1a}, \quad (102)$$

where  $F_{\mu\nu}^{lma} = \partial_\mu A_\nu^{lma} - \partial_\nu A_\mu^{lma}$ .

The term with one factor of  $A$ , with only this factor expanded in terms of colored rotation generators, is zero due to the orthogonality of the generators. (With only one factor of colored  $\lambda_a$ , there is no way to make the term a scalar.)

Doubly expanding the factors of  $A$  in Terms (34)–(36), we obtain

$$+ \frac{9 + 4\sqrt{3}}{12c^2 T^2} A_{11a}^\mu A_\nu^{11a} \quad (103)$$

$$+ \frac{9 + 4\sqrt{3}}{12c^2 T^2} A_\mu^{1,-1a} A_\nu^{1,-1a} \quad (104)$$

$$+ \frac{\sqrt{30}}{3c^2 T^2} A_\mu^{10a} A_\nu^{10a}, \quad (105)$$

which are mass terms for the  $A_\mu^{1ma}$ .

The Lagrangian for 3DT should contain terms with anywhere from two to six strongly interacting vectors or gluons. However, strong interactions with diagrams with five or six external gluon lines, corresponding to terms with five or six gluons, are not observed. 3DT provides the explanation for this. If one examines the definition of the curvature scalar described in Sec. 4, one finds that the only way to obtain terms with five or six factors of  $A_\mu$  is through the presence of  $\hat{g}^{55} = 1 + g^{\mu\nu} A_\mu A_\nu$ . Now since the strong interaction cross product between the two factors of  $A$  here is purely antisymmetric, while the multiplication of these two factors by  $g^{\mu\nu}$  symmetrizes them, the resulting terms with five or six gluons are zero.

## 11. Quarks, confinement, asymptotic freedom and chiral symmetry breaking

The main aspects of strong interaction phenomenology are quarks, quark confinement, asymptotic freedom and chiral symmetry breaking. These are not hard to achieve in 3DT. The existence of quarks can be explained by expanding the photon in terms of HD rotation generators and spinors. In final form, for example

$$A_\mu^{00}|00\rangle = -\frac{1}{2}\sigma_\mu^{B\dot{V}}\chi_B^p\varpi_{\dot{V}}^q\lambda_{pq}\frac{1}{\sqrt{2}}\left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}, -\frac{1}{2}\right\rangle + \frac{1}{2}\sigma_\mu^{B\dot{V}}\zeta_B^p\iota_{\dot{V}}^q\lambda_{pq}\frac{1}{\sqrt{2}}\left|\frac{1}{2}, -\frac{1}{2}\right\rangle\left|\frac{1}{2}\frac{1}{2}\right\rangle. \quad (106)$$

We have neglected to consider the terms corresponding to the one white generator in the expansion for  $A_\mu^{00}$ . This is the photon. The index for the eight colored generators of the HD rotation group has been rewritten as  $pq$  where  $p$  and  $q$  take the values r,g,b, which stand for red, green and blue. A sum over  $p$  and  $q$  is implied. The spinors  $\chi_B^q$  and  $\iota_{\dot{V}}^p$  are associated with the ket  $\left|\frac{1}{2}\frac{1}{2}\right\rangle$ , while  $\varpi_{\dot{V}}^p$  and  $\zeta_B^q$  have  $\left|\frac{1}{2}, -\frac{1}{2}\right\rangle$  as their HD angular momentum representation. Transforming the spinors according to Sec. 8, we find the up quark  $u^p$  corresponds to  $\iota_{\dot{V}}^p$  while the down quark  $d^p$  corresponds to  $\varpi_{\dot{V}}^p$ .

By expanding  $A_\mu^{00}$  in terms of rotation generators and spinors with  $l > \frac{1}{2}$ , quarks in other families may be produced. The strange and charmed quarks are associated with the states  $\left|\frac{3}{2}, -\frac{1}{2}\right\rangle$  and  $\left|\frac{3}{2}\frac{1}{2}\right\rangle$ , respectively. The top (bottom) quark is in the state  $\left|\frac{5}{2}\frac{1}{2}\right\rangle$  ( $\left|\frac{5}{2}, -\frac{1}{2}\right\rangle$ ). According to 3DT, there are an infinite number of quarks, each corresponding to a state  $|lm\rangle$ , where  $l$  and  $m$  are half-odd integral. Thus, expanding  $A_\mu$  in terms of spherical harmonics, rotation generators and spinors produces multiple generations of quarks. It is not unlikely that the number of generations appears to be limited by the same reason given in Part II for leptons.

Terms for the interactions of quarks with gluons may be derived from the gluons' mass terms. This occurs by substituting a quark-antiquark pair for one gluon in the gluon mass terms. It is highly unlikely that quarks have charges exactly one-third that of the electron because the value of a particle's charge will be shown in Part II to result from quantum corrections and these are notoriously uneven in value. There does not appear to be any way of achieving one-third integral charges from the methods used in Part II. Thus, we will subscribe to the original theory of the strong interactions, the Han-Nambu model [59,60], in which quarks have integral charges. This is possible because our theory does not have an exact strong interaction symmetry of the Lagrangian. Quarks of different colors have

different charges. The symmetry is broken. Instead of quarks having color, they are non-singlet  $SU(3)'$  states in the Han-Nambu model. Thus, when I use the word ‘color,’ I am referring to this property and not color as it is usually used.

Terms describing free quarks (the Dirac term), quark masses and the electromagnetic interactions of single quarks are zero. This is because the colored generators of the quarks are orthogonal to the white generator of  $\delta^{\mu\nu}$  or  $A_\mu^{000}$  (the only other fields in the terms) and their dot or scalar product is zero. With only one set of colored generators there is no way to make the term a scalar.

The SM assumes the Dirac or free-quark term in its Lagrangian, thereby postulating free quarks. It then goes to great lengths to eliminate the same free quarks. Why not just leave out the free-quark term? In 3DT, the free-quark term is zero. This means there are no free quarks. How can one have free quarks without a free quark term in the Lagrangian to describe them? The only way a quark can exist is if it is interacting with a gluon. Certainly this method of quark confinement is much simpler than the charge-antiscreening-due-to-vacuum-polarization method, which requires the use of supercomputers for its calculations and still does not work. My method requires virtually no calculations, is far simpler and is more effective. And it is just another example of determining the correct Lagrangian for each field of interest. Sometimes it is not what you put in a Lagrangian, but what you leave out.

Although there are no mass terms for quarks in  $\widehat{R}$ , masses for quarks can be generated by their color, in much the same way that the classical radius equation can be used to generate a mass for the electron from its charge. Thus, we would have only strong interaction and mass terms for quarks. New terms for quarks, however, can be generated by expanding the white vectors  $A_\mu^{lm0}|lm\rangle$  in terms of spinors that are white combinations of quarks. In this case, the terms that applied to previously considered spinors now apply to white combinations of quarks. In this way, we arrive at free-field and photon interaction terms for white quark combinations known as hadrons. Because the free-quark terms vanish but the free-hadron terms do not, quarks can only be found in white combinations—hadrons.

The theory of 3DT’s strong interaction sector may be tested by detecting free gluons. These are allowed in 3DT because the free-field (Maxwell) terms for gluons do not vanish. Free gluons are forbidden by confinement in the SM.

The chiral symmetry breaking of the strong interactions is caused by a nonzero pion mass. According to the classical radius equation presented in Part II, a particle that has charge must have mass. The pion has a mass because the quarks and gluons of which it is composed have masses due to their strong interaction charges or color. Therefore, 3DT explains chiral symmetry breaking.

We now describe an effective potential for confinement in 3DT. Although this potential does not really exist, it provides a convenient framework for the discussion of confinement. Each of the gluons with which the quarks interact must have a mass because it has color, which would, like electric charge, yield a mass by the classical radius equation in Part II. Because free quarks are forbidden by the vanishing of the free-quark term they must always be interacting. Now, in order for two quarks to remain interacting, they must be within range of the strong interaction. Therefore, they cannot be separated by a distance greater than the Compton wavelength  $\lambda = h/mc$  of a gluon. The mathematical expression for this is an infinite square-well potential with radius equal to the Compton wavelength of a gluon

surrounding each quark. Inside this infinite square well, the quarks are free to move about because the gluons that influence the quarks have masses due to their color, rendering the interaction weak. Thus, aside from the infinite square-well confining potential, the strong interaction is really weak. This finishes the explanation of quark confinement and asymptotic freedom.

## Part II. The Quantum Theory

### 12. Quantum field theory in 3DT

This part derives the values of ten previously unexplained parameters in the Standard Model (SM) from the theory of three dimensions of time (3DT). Additionally, masses of 3DT's predicted particles will be determined.

Particles in 3DT are extended with sizes of the order of the Planck length. Particles were thought to have to be pointlike in order to avoid violations of either special relativity or causality, but string theory has shown that extended particles can, in fact, be viable. With the properties of elementary particles in 3DT, such as a particle's nonzero size and a tiny mass for the photon, one can replace QED by a simplified theory that does not have regularization, renormalization, and gauge invariance. I will begin to do this here by using one-loop quantum correction equations from present-day QED.

Renormalization is the method for handling infinities when particles are pointlike but if the particles are extended, the infinities never develop. Instead, there are cutoffs. The cutoffs cause quantum corrections to be finite, which allows the calculation of actual quantities from finite bare quantities. Therefore, extended particles eliminate the need for renormalization and allow the calculation of previously unexplained parameters. For example, consider the equation [61]

$$m = m_0 + \delta m \tag{107}$$

from QED. Here  $m$  is the mass of the electron,  $m_0$  its bare mass and  $\delta m$  the quantum mass correction. In a renormalizable model, both  $m_0$  and  $\delta m$  are infinite and cancel and  $m$  must be inserted by hand. Thus,  $m$  here is completely unexplained. However in 3DT, both  $m_0$  and  $\delta m$  are finite and precisely determined, allowing a determination of  $m$  to be made. In this way, one can calculate masses. Similarly, the equation in QED for the quantum charge correction allows one to calculate coupling constants.

We will now show how to determine  $m$ . The equation for the one-loop electron quantum mass correction [61] for electromagnetism is

$$m = m_0 + \delta m, \tag{108}$$

where

$$\delta m = \frac{3\alpha}{4\pi} m \left( \ln \frac{\Lambda^2}{m^2} + \frac{1}{2} \right). \tag{109}$$

Here  $\alpha$  is the fine structure constant. The mass  $m_0$  for the electron is apparently its classical free mass. This is the mass when the charge of the electron is zero and without a quantum mass correction. The mass  $\Lambda$  is the classical interacting mass for the electron. This is the mass of the electron when it has charge, but before the quantum mass correction is made. The mass  $m$  is the quantum-corrected interacting mass. This is the mass of the electron with its charge and after the quantum mass correction has been taken. It is the actual mass of the electron.

In 3DT, the mass  $m_0$  is zero [from Part I]. This is because, unlike vectors and tensors, there are no mass terms in the 6D curvature scalar  $\hat{R}$  for spinors. In addition, because  $m_0$

is calculated with its charge equal to zero, the electron does not acquire a mass from its charge. Producing mass from charge is described in the next section. We have

$$m_0 = 0. \tag{110}$$

One now combines Eqs.(108), (109) and (110) and solves for  $m$ . The result is the mass formula

$$m = \exp\left(\frac{1}{4} - \frac{2\pi}{3\alpha}\right) \Lambda. \tag{111}$$

This is the most important equation in the quantum part of this document. It converts Planck masses  $\Lambda$  to elementary particle masses  $m$ , without fine tuning. This allows predictions at the Planck energy to be tested at low energy. It renders observable the HD coordinate dependence in a Kaluza-Klein theory by converting the masses of the higher order harmonics from Planck masses to elementary particle masses. The massive harmonics become the elementary particles. This equation follows directly from Itzykson and Zuber [61], the classic quantum field theory textbook.

The problem of deriving the mass of the electron on purely theoretical grounds now translates into finding the proper values for  $\Lambda$  and  $\alpha$ . The search for the values of these parameters for each elementary particle occupies the bulk of the remainder of this document. If one substitutes the Planck mass for  $\Lambda$  and sets  $\alpha$  to  $1/137$ , the mass  $m$  obtained is much smaller than the electron mass. This problem is opposite the usual one, where the mass of the elementary particle is too large. The question arises, “What will  $\alpha$  have to be to convert a Planck mass to an ordinary mass?” If Eq.(111) is solved for  $\alpha$ , with  $\Lambda$  the Planck mass and  $m$  the mass of the electron, one finds  $\alpha$  must be larger than  $1/137$ . Perhaps  $\alpha$  in Eq.(111) is the bare fine structure constant. The bare fine structure constant should be larger than  $1/137$ .

As will be shown, the fine structure constant  $\alpha$  starts out at 1, is then reduced to  $1/19$  by charge screening due to vacuum polarization and finally reduced to  $1/137$  by the vertex correction. Because charge screening due to vacuum polarization is allowed by the Feynman diagram for the mass of the electron, while the vertex correction, which has an external photon line, is not, the value for  $\alpha$  in Eq.(111) is  $1/19$ .

Thus, the ordinary mass scale is actually the quantum-corrected Planck mass. Equation (111) solves the mystery of why there are these two widely separated levels of mass in physics. This has been referred to as the hierarchy problem.

Section 13 derives an expression for  $\Lambda$ . This expression contains the photon’s radius, which is calculated in Sec.14. Section 15 determines the value  $1/19$  for  $\alpha$  in the quantum mass correction equation. Section 16 derives the mass of the electron and the value  $1/137$  for the fine structure constant. Masses for the muon, tau and the neutrinos are deduced in Sec.17. New fermions are described in Sec.18. The reason why only the observed elementary particle interactions take place is explained in Sec.19. Section 20 explains the masses of the  $W^\pm$  and  $Z^0$  from scratch (instead of from the weak interaction coupling constant  $G_W$  as is presently done in the Standard Model). The main prediction of the theory — new intermediate vector bosons with precisely determined masses — is given in Sec.21.

### 13. The classical photon mass $\Lambda$

The electron does not have a mass term in the 6D curvature scalar  $\widehat{R}$ . However, the

electron acquires a mass through its electric field. An electric field contains energy and since the electron always carries this field with it, this energy may be considered to be a rest mass. Equivalently, the source of the electron's mass may be ascribed to its charge. It requires energy to keep together an assemblage of electric charges of the same sign, which tend to repel each other. This is represented by the equation for the electron's classical radius

$$mc^2 = \frac{e^2}{r_c}, \quad (112)$$

where  $e$  is the electron's charge and  $r_c$  its classical radius, which is of the order of the size of its charge distribution. Equation (112) shows why a particle that has charge must have mass. This origin of mass is one of the two sources of mass in 3DT. The other is explicit mass terms in the 6D curvature scalar. One reason the classical radius origin of mass is not more accepted is that the classical radius equation concept as it stands is flawed; the situation is fixed in Sec. 14.

The remainder of this section is essentially a derivation of the classical radius equation for the case of the photon. We choose the photon because  $\Lambda$  in QED is defined [61] to be the large mass of a fictitious massive photon. I contend, however, that  $\Lambda$  is the Planck mass of the real photon, that is, its mass before the quantum mass correction. We will have to calculate  $\Lambda$  from the photon's charge because there is no mass term for the photon in  $\hat{R}$ . This section also demonstrates how to construct an elementary particle by determining its distribution of charge.

The photon must have some type of charge because anything that interacts with the photon can be said to have charge and one photon interacts with another during photon-photon scattering [61]. The Feynman diagram for photon-photon scattering has four external photon lines, four internal electron lines, four vertices and one loop. In this interaction, photons do not interact with each other directly, but indirectly through electrons. Thus the actual charge of each photon is zero. Nevertheless, it is instructive to assign to each of the two photons involved an effective charge as if the two photons had interacted directly. The four vertices in the diagram for this interaction imply that the coupling constant is proportional to  $e^4$ , where  $e$  is the charge on an electron. I claim the effective charge  $q_\gamma$  for each of the two photons in this interaction is

$$q_\gamma = \frac{e^2}{4\pi\sqrt{\hbar c}}. \quad (113)$$

In analogy with the electron, the classical interacting mass for the photon will be equal to its electrostatic energy  $H$ . Equation (112) for the classical radius of the electron is derived from classical electrodynamics but is valid for quantum electrodynamics as well. Therefore, we will derive the classical interacting mass of the photon from classical electrodynamics [62]. We have

$$\Lambda c^2 = H, \quad (114)$$

where the electrostatic energy is derived from the electric field  $\mathbf{E}$

$$H = \frac{1}{8\pi} \int |\mathbf{E}|^2 d^3x. \quad (115)$$



The electric field is obtained from the potential  $\Phi(\mathbf{x})$

$$\mathbf{E} = -\vec{\nabla}\Phi(\mathbf{x}). \quad (116)$$

Finally, the potential is derived from the charge density  $\rho(\mathbf{x})$

$$\Phi(\mathbf{x}) = \int \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x'. \quad (117)$$

A prime on a coordinate means that is the coordinate for a point of the charge density. Because  $\mathbf{x}'$  is the variable of integration and  $\mathbf{x}$  is the observation point, it is useful to separate the variables in  $1/|\mathbf{x} - \mathbf{x}'|$

$$\frac{1}{|\mathbf{x} - \mathbf{x}'|} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^l \frac{1}{(2l+1)} \frac{r'^l}{r^{l+1}} Y_{lm}^*(\theta', \phi') Y_{lm}(\theta, \phi), \quad (118)$$

where  $r = |\mathbf{x}|$  and the  $Y_{lm}(\theta, \phi)$  are the spherical harmonics. The next step is to determine the charge density  $\rho(\mathbf{x}')$ .

The charge is located, of course, where the particle is located. The particle is located in ordinary three-dimensional space (3DS). However, one cannot localize the photon field to a distance smaller than  $2r_\gamma$ , where  $r_\gamma$  is the radius of the photon, along its direction of motion because this would mean that it would be localized on the HD sphere. This localization on the HD sphere is impossible because the photon's higher-dimensional wave function, which is the spherical harmonic  $Y_{00}(\theta, \phi) = 1/\sqrt{4\pi}$ , is by definition spread out uniformly over the HD sphere. The coordinate  $\theta$  of the sphere projects onto the photon's 4D world line, which is the  $z$ -axis of the embedding space of the HD sphere. (The coordinate  $\theta$  is the ordinary spherical coordinate measured from the positive  $z$ -axis or 4D world line.) The photon's quantum mechanical spin causes the length  $2r_\gamma$  to form a sphere of radius  $r_\gamma$  in 3DS. The photon is therefore a line segment of a certain type of charge of length  $2r_\gamma$  spinning about its midpoint.

The charge density of the photon will be proportional to the magnitude squared of its wave function. This is because the charge is located where the particle is located and the probability of finding the photon at a given point is equal to the magnitude squared of its wave function. We have

$$\rho(\mathbf{x}') = q_\gamma |\psi(\mathbf{x}')|^2. \quad (119)$$

The wave function  $\psi(\mathbf{x}')$  of a photon as localized as possible in 3DS is equal to its radial wave function  $R(r')$  times its spin wave function  $S(\theta', \phi')$

$$\psi(\mathbf{x}') = R(r')S(\theta', \phi'). \quad (120)$$

The radial wave function is given by the projection of the HD wave function onto the 4D world line parameterized by the coordinate  $z$ . As with any projection of an object, this projection will be proportional to the size of the original object, in this case the HD wave function. Therefore, the radial wave function  $R(r')$  in 3DS is proportional to the higher-dimensional wave function  $Y_{00}$ , which is constant. This result applies to any  $Y_{lm}$  — the projection of  $Y_{lm}$  onto the  $z$ -axis is proportional to  $Y_{lm}$ .

After normalization, the radial function is given by a new constant  $c$

$$R(r') = c, \quad r_\gamma \geq r' \geq 0 \quad (121)$$

$$R(r') = 0, \quad \infty > r' > r_\gamma. \quad (122)$$

The spin wave function is given by

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \rightarrow \frac{1}{\sqrt{3}} Y_{11}(\theta', \phi') + \frac{1}{\sqrt{3}} Y_{1,-1}(\theta', \phi') = S(\theta', \phi'), \quad (123)$$

where we have omitted the longitudinal polarization state  $Y_{10}(\theta', \phi')$  of the photon. The constant  $c$  is determined by the condition

$$\int \rho(\mathbf{x}') d^3x' = q_\gamma. \quad (124)$$

We find

$$\rho(\mathbf{x}') = \frac{9q_\gamma}{8\pi r_\gamma^3} \sin^2\theta', \quad r_\gamma \geq r' \geq 0 \quad (125)$$

$$\rho(\mathbf{x}') = 0, \quad \infty > r' > r_\gamma. \quad (126)$$

Substituting this into Eq. (117), it follows from Eqs. (115)–(118) that

$$H = \frac{q_\gamma^2}{2r_\gamma}. \quad (127)$$

This is the monopole contribution in Eq. (117). The higher order terms are negligible. Therefore, the dependence of  $H$  upon the angular and radial dependence of  $\rho(\mathbf{x}')$  is negligible. The energy behaves as if the charge were uniformly distributed throughout a sphere of radius  $r_\gamma$ .

Equations (127) and (114) yield for the classical mass of the photon

$$\Lambda = \frac{q_\gamma^2}{2c^2 r_\gamma} = \frac{\beta \hbar}{2c r_\gamma}, \quad (128)$$

where

$$\beta = \frac{q_\gamma^2}{\hbar c} = \frac{e^4}{(4\pi)^2 \hbar^2 c^2} \quad (129)$$

is the fine structure constant for the photon. Equation (128) is the classical radius equation. The distance appearing in the classical radius equation is that between the two most widely separated points of charge in the distribution. For a spherical distribution it is therefore the diameter. This is why a factor of two multiplying the radius appears in our equation. This factor does not appear in the usual definition of the classical radius equation because this is only an order-of-magnitude estimate.

The effective charge on a photon is proportional to the square of the charge on an electron. It turns out that the fine structure constant for the photon  $\beta$  is always equal to the square of

the fine structure constant for the electron. This holds true at every point in the calculation for  $\alpha$ , the fine structure constant for the electron, done later. Using the relation

$$\beta = \alpha^2, \quad (130)$$

we have

$$\Lambda = \frac{\alpha^2 \hbar}{2c r_\gamma}. \quad (131)$$

It remains to determine the radius  $r_\gamma$  of the photon.

#### 14. The radius of the photon

In order to eliminate the infrared divergences in QED, the photon must have a small mass. Therefore, we treat it in what follows as a massive particle, like the electron. Among other things, this means the photon may be brought to rest. If a particle is at rest and located at the origin of ordinary three-dimensional space (3DS), then its 4D world line is the time axis. However, particles are never really at rest because they are spinning. Setting the photon's spin angular momentum equal to  $\Lambda v \langle r \rangle$ , we have

$$[s(s+1)]^{1/2} \hbar = \Lambda v \langle r \rangle. \quad (132)$$

A simple way of determining the radius at which the particle rotates is to place its entire mass at a particular average radius  $\langle r \rangle$ . The first step in determining  $\langle r \rangle$  is to determine  $R(r)$  because

$$\langle r \rangle = \int_0^{r_\gamma} R^*(r) r R(r) dr. \quad (133)$$

The quantity  $R(r)$  will be the projection of the photon's higher-dimensional wave function  $Y_{00}$ , which is constant, onto its 4D world line. Any projection is proportional to the size of the original object so  $R(r)$  must be proportional to a constant. The second requirement on  $R(r)$  is that it be normalized to unity. We find

$$R(r) = \frac{1}{\sqrt{r_\gamma}}. \quad (134)$$

This radial wave function for the photon is constant and satisfies the normalization criterion  $\int_0^{r_\gamma} R(r)^* R(r) dr = 1$ . Substituting Eq. (134) into Eq. (133), we find

$$\langle r \rangle = \frac{1}{2} r_\gamma. \quad (135)$$

Setting  $s = 1$  in Eq. (132), we obtain

$$\Lambda v \frac{1}{2} r_\gamma = \sqrt{2} \hbar. \quad (136)$$

Substituting Eq. (128) into Eq. (136), we have

$$v = \frac{4\sqrt{2}c}{\beta}. \quad (137)$$

Note that if  $\beta = \alpha^2 = (1/137)^2$ , the velocity of spin for the photon is much greater than the speed of light. This is the main objection to the concept of the classical radius equation. There is a way around this problem, however. The trick is to use the bare fine structure constant  $\beta_0 = \alpha_0^2$ , which will later be determined to be unity, in the denominator of Eq. (137) instead of the actual fine structure constant. As we will show, this larger denominator will bring the velocity of spin for the photon to something below the speed of light. The justification for substituting the bare fine structure constant for the actual one is that only the bare photon is spinning. Its surrounding outer shell of virtual particles caused by charge screening and the vertex correction does not spin along with it.

We will define the bare fine structure constant  $\beta_0$  for the photon to be  $\beta_0 = \alpha_0^2$  while the actual fine structure constant  $\beta$  is

$$\beta = 4\pi\alpha_V^2, \quad (138)$$

where  $\alpha_V$  is the fine structure constant for the electron after all quantum corrections have been taken. This equation will be derived in Sec. 16. This boosts the value of the actual fine structure constant by a factor of  $4\pi$  in relation to the bare value and introduces a factor of  $4\pi$  into the denominator of Eq. (137) when the actual fine structure constant is converted to the bare one. Thus

$$v = \frac{\sqrt{2}c}{\pi\alpha_0^2}. \quad (139)$$

We now show why  $\alpha_0 = 1$ . We start with the z-component of angular momentum operator  $L_z$  operating on the neutrino's and electron's wave functions  $\psi_\nu$  and  $\psi_e$

$$L_z \begin{pmatrix} \psi_\nu \\ \psi_e \end{pmatrix} = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \psi_\nu \\ \psi_e \end{pmatrix} \quad (140)$$

This shows the electron has z-component of angular momentum  $-\frac{1}{2}\hbar = m\hbar$  in the three-dimensional embedding space of the HD sphere. This angular momentum is more commonly known as isospin. Here  $m$  does not represent a mass but rather the quantum number for the z-component of isospin. The electron's charge quantum number, which is integral and  $m$  are related by elementary quantum mechanics according to the relation [63]

$$m = c + \text{int.}, \quad (141)$$

where  $c$  is a constant for the multiplet and int. is an integer. According to Ref. [63],  $c = \frac{1}{2}$  for spinors. Equation (141) is the Gell-Mann relation  $Q = I_3 + \frac{1}{2}y$ , with the following identifications: charge  $Q = \text{int.}$ , the z-component of weak isospin  $I_3 = m$  and one-half the weak hypercharge  $\frac{1}{2}y = -c$ . Therefore, the origin of the Gell-Mann relation is explained by 3DT.

If

$$Q = m - 1/2, \quad (142)$$

which is a restatement of the Gell-Mann relation, and  $m = -1/2$  corresponds to an angular momentum of  $m\hbar$ , then  $Q = -1$  must correspond to an angular momentum of  $Q\hbar$ . We have

$$L_Q = Q\hbar, \quad (143)$$

where  $L_Q$  is the angular momentum due to the electron's charge. The equation defining the magnitude of this angular momentum is

$$L_Q = |(\vec{r} \times \vec{p})|_Q = rp_Q \sin \theta_Q, \quad (144)$$

where  $r$  is the radius of motion,  $p_Q$  is the linear momentum and  $\theta_Q$  is the angle between  $\vec{r}$  and  $\vec{p}$ . The subscript  $Q$  means the quantity pertains only to the angular momentum of the electron's charge. Combining Eqs. (143) and (144), we have

$$L_Q = cTmv_Q \sin \theta_Q = Q\hbar, \quad (145)$$

where  $cT$  is the radius of the HD sphere and is the radius of motion and  $mv_Q$  is the magnitude of the linear momentum. The quantity  $cT$  is the Planck length. We have

$$cT = \sqrt{\hbar G/c^3}. \quad (146)$$

Combining Eqs. (146) and (145) we have

$$mv_Q \sin \theta_Q = QMc = p', \quad (147)$$

where

$$M = \sqrt{\hbar c/G} \quad (148)$$

is the Planck mass. The energy for this type of motion is of the form  $p'^2/(2m)$ . In our case it follows from Eq. (147) and is

$$E = \frac{p'^2}{2m} = \frac{Q^2Mc^2}{2}. \quad (149)$$

From the classical radius equation we have

$$E = \frac{e_0^2}{2cT}, \quad (150)$$

where  $e_0$  is the electron's bare charge and  $2cT$  is the diameter of the HD sphere.

The energy of motion of the electron on the HD sphere that causes its charge is equal to its electrostatic self-energy. Therefore, equating the two energies in Eqs. (149) and (150) and using Eqs. (146) and (148), we have

$$e_0^2 = Q^2\hbar c, \quad (151)$$

The definition of the bare fine structure constant  $\alpha_0$  is

$$\alpha_0 = \frac{e_0^2}{\hbar c}. \quad (152)$$

Combining Eqs. (151) and (152) produces the result

$$\alpha_0 = Q^2. \quad (153)$$

The electron has  $Q = -1$ , so for it,  $\alpha_0 = 1$  as was to be shown. A similar derivation may be made involving the total angular momentum operator squared  $L^2$  instead of  $L_Q$ . This leads

to the result  $\alpha_0 = l(l + 1)$ . This is the coupling constant for the weak interactions as they couple to isospin.

Substituting this into Eq. (139), we have

$$v = \frac{\sqrt{2}}{\pi} c. \quad (154)$$

Because  $\sqrt{2}$  is less than  $\pi$ , the velocity of spin is less than the speed of light. This is the velocity of spin of a point midway out from the center of a spinning photon. The velocity of a point on its surface is twice as great but is still less than the speed of light because  $2\sqrt{2}$  is less than  $\pi$ . As will be implied shortly, this velocity of spin applies only to an image of the photon, not the actual photon itself. The velocity of spin of the actual photon is considerably less.

The size of the charge distribution in 3DS is given by its projection from the 4D world line onto a spatial axis in 3DS. Therefore  $r_\gamma = cT \sin \psi$ , where  $\psi$  is the angle between the 4D world line and the ordinary  $t$ -axis. Thus the radius for the photon is less than  $cT$ . However, we now convert the photon's charge density of a solid sphere in 3DS into the surface charge density of a hollow sphere. The two are equivalent from our discussion of the classical radius equation. The energy behaves as if the internal distribution of charge does not matter. Now, the HD sphere forms a boundary in 3DS because the higher dimensions overlap 3DS. A spherical surface charge density inside the HD sphere plus a boundary at the HD sphere is equivalent to a spherical surface charge density outside the HD sphere. This follows from the method of images in classical electrodynamics. Therefore, we have

$$r'_\gamma = \frac{c^2 T^2}{r_\gamma}, \quad (155)$$

where the effective radius of the photon is  $r'_\gamma$ . Substituting  $r_\gamma = cT \sin \psi$  into this equation, we find

$$r'_\gamma = \frac{cT}{\sin \psi}. \quad (156)$$

Henceforth we will drop the prime from  $r'_\gamma$

$$r_\gamma = cT / \sin \psi \quad (157)$$

and when the radius of a particle is referred to, we will mean its effective radius.

From the photon's effective velocity of spin determined above, one may calculate the angle  $\psi$  from the following equation

$$\tan \psi = \frac{\Delta x}{c \Delta t} = \frac{v}{c} = \frac{\sqrt{2}}{\pi}, \quad (158)$$

where  $x$  is the spatial path in which the photon moves as it spins and the time axis is labeled  $ct$ . Therefore,

$$r_\gamma = 2.4361cT. \quad (159)$$

Combining Eqs. (2), (159) and (131) we obtain the value for the classical mass  $\Lambda$  of the photon

$$\Lambda = 2.37 \times 10^{-10} \text{ g}. \quad (160)$$

## 15. The screened fine structure constant

We now consider the effects of charge screening due to vacuum polarization on  $\alpha_0$ . The one-loop expression for the fine structure constant  $\alpha_\gamma$  after charge screening via the photon is [61]

$$\alpha_\gamma = \frac{\alpha_0}{1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda'^2}{m^2}}. \quad (161)$$

The cutoff  $\Lambda'$  is defined according to [61]

$$\sum_{s=1}^S C_s \ln \lambda_s^2 = -\ln \frac{\Lambda'^2}{m^2}, \quad (162)$$

where the  $\lambda_s m$  are the large masses of the  $S$  spinors. The quantum mass correction Eq. (111) suggests that the large mass of each spinor is  $\Lambda$ . Therefore,

$$\lambda_s m = \Lambda. \quad (163)$$

Combining Eqs. (162) and (163), we have

$$\sum_{s=1}^S C_s \ln \frac{\Lambda^2}{m^2} = -\ln \frac{\Lambda'^2}{m^2}. \quad (164)$$

According to Ref. 61,

$$\sum_{s=1}^S C_s = -1. \quad (165)$$

Substituting this into Eq. (164), we find

$$\Lambda' = \Lambda. \quad (166)$$

In addition to photons producing an electron-positron pair, one might have vacuum polarization involving the  $Z^0$ . To calculate the effect of this process on  $\alpha_\gamma$ , we use the equation

$$\alpha_{\gamma Z} = \frac{\alpha_\gamma}{1 + \frac{\alpha_\gamma}{3\pi} \ln \frac{\Lambda_Z^2}{m^2}}, \quad (167)$$

which is similar to Eq. (161). Here  $\alpha_{\gamma Z}$  is the bare fine structure constant  $\alpha_0$  after screening via the photon and  $Z^0$ . The mass  $\Lambda_Z$  is the classical mass of the  $Z^0$  due to its weak charge. The classical mass  $\Lambda_Z$  is determined by the classical radius equation

$$\Lambda_Z = \frac{g^2}{2c^2 r_Z}, \quad (168)$$

where the weak charge  $g$  of the  $Z^0$  is given by the definition of the weak interaction coupling constant  $G_W$

$$g^2 = 2^{5/2} \frac{G_W}{(\hbar c)^3} M_W^2 \hbar c, \quad (169)$$

where  $M_W$  is the mass of the  $W$ .

The procedure for determining the radius  $r_Z$  of the  $Z^0$  is similar to that which determined the radius of the photon. One difference is that the higher-dimensional wave function for the  $Z^0$  is  $Y_{10}$ , which is proportional to  $\cos\theta$ . The projection of this function onto the 4D world line or  $z$ -axis should therefore be proportional to  $\cos\theta$ . This, in turn, is equal to  $z/r_Z$  according to the definition of spherical coordinates. The line segment along the coordinate  $z$  rotates in 3DS to become the radial coordinate  $r$ . Therefore the coordinate  $z$  is replaced by the radial coordinate  $r$ . The radial function for the  $Z^0$  is

$$R(r) = \left(\frac{3}{r_Z}\right)^{1/2} \frac{r}{r_Z}. \quad (170)$$

This satisfies the criteria of being proportional to  $\cos\theta$  and normalized to one. The average value for  $r$  is given by

$$\langle r \rangle = \int_0^{r_Z} R^*(r) r R(r) dr = \frac{3}{4} r_Z. \quad (171)$$

In the velocity calculation,  $\alpha_0$  appears instead of  $\alpha_0^2$  because self interactions with two vertices are allowed for the  $Z^0$  as opposed to the case of the photon, where only photon-photon scattering (a self interaction with four vertices) is permitted. Here the  $Z^0$ 's bare fine structure constant is denoted by  $\alpha_0$ . In addition, the value of the bare fine structure constant  $\alpha_0$  is given by

$$\alpha_0 = l(l+1), \quad (172)$$

where, from its spherical harmonic  $Y_{10}$ ,  $l = 1$  for the  $Z^0$ . The above considerations lead to a radius for the  $Z^0$

$$r_Z = 6.7389cT. \quad (173)$$

Equations (168), (169), (173), and (2) yield for the classical mass of the  $Z^0$  due to its weak charge

$$\Lambda_Z = 6.87 \times 10^{-7} \text{ g}. \quad (174)$$

Substituting the actual mass of the electron for  $m$  in Eqs. (161) and (167) and  $\alpha_0 = 1$  into Eq. (161) and using Eqs. (166), (160), (167) and (174), we obtain a value for the screened fine structure constant

$$\alpha_{\gamma Z} = \frac{1}{19.71}. \quad (175)$$

## 16. The electron

We have yet to consider the effect of the quantum charge vertex correction on  $\alpha_{\gamma Z}$ . However this is unnecessary for quantum mass corrections because it requires one external photon line, which does not appear in the quantum mass correction diagram. Thus, the fine structure constant used in the quantum mass correction formula (111) is  $\alpha_{\gamma Z}$ . To summarize, the equations that determine the electron's mass are

$$\alpha_\gamma = \frac{\alpha_0}{1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{m^2}}, \quad (176)$$



$$\alpha_{\gamma Z} = \frac{\alpha_\gamma}{1 + \frac{\alpha_\gamma}{3\pi} \ln \frac{\Lambda_Z^2}{m^2}}, \quad (177)$$

$$m = \Lambda \exp\left(\frac{1}{4} - \frac{2\pi}{3\alpha_{\gamma Z}}\right), \quad (178)$$

where  $\alpha_0 = 1$  and  $\Lambda$  and  $\Lambda_Z$  are given by Eqs.(160) and (174), respectively. Solving the three Eqs.(176)–(178) for the three unknowns  $\alpha_\gamma$ ,  $\alpha_{\gamma Z}$  and  $m$ , we obtain

$$m = \Lambda^5 \Lambda_Z^{-4} \exp\left(\frac{9}{4} - \frac{6\pi}{\alpha_0}\right). \quad (179)$$

The value for  $m$  obtained from Eq.(179) is  $1.16 \times 10^{-4}$  MeV, which is much less than the actual electron mass of 0.511 MeV. This is because we have calculated the mass of the electron due to its electric charge. To this we must add the contribution due to its weak charges.

The electron has two weak charges: that from weak isospin and one from an explicit weak interaction term in the 6D curvature scalar  $\hat{R}$  of 3DT. The mass of the electron due to its weak charges is obtained from Eq.(179), but with  $\alpha_0$  determined by the electron's weak charge from its weak isospin  $I$  or its weak charge from the 6D curvature scalar  $\hat{R}$ . The electron's weak charge  $g_{\hat{R}}$  from  $\hat{R}$  in Part I is given by Eq.(90) and is  $-4.75766\sqrt{\hbar c}$ . Therefore

$$\alpha_0 = \frac{g_{\hat{R}}^2}{4\pi\hbar c} = 1.8013. \quad (180)$$

Substituting this into Eq.(179), we obtain for the weak mass of the electron 0.510 MeV, which has an error of three tenths of one percent. In general, there should not be a factor of  $4\pi$  in  $\alpha_0$ . This factor should only be retained in Eq.(180) if this  $\alpha_0$  is substituted into Eq.(179), which includes the effects of vacuum polarizations via the photon and the  $Z^0$ . This is because Eqs.(176) and (177) are incorporated into it. As explained below, each of these one-loop quantum corrections introduces a factor of  $\sqrt{4\pi}$  into the denominator of the fine structure constant for the electron.

The electron's remaining weak charge  $g_I$  due to its weak isospin  $I = L = [l(l+1)]^{1/2} \hbar$  is  $[l(l+1)4\pi\hbar c]^{1/2}$ , where from Table 1,  $l = \frac{1}{2}$  for the electron. Exchanging  $g_I$  for  $g_{\hat{R}}$  in Eq.(180) we find the mass of the electron due to its weak isospin is negligible. Thus, the prediction for the electron's mass stands at an accurate 0.510 MeV. We have determined our first previously unexplained parameter — the mass of the electron.

The observed value  $1/137$  of the fine structure constant  $\alpha$  is determined from  $\alpha_{\gamma Z}$  by the final quantum charge correction, known as the vertex correction. The one-loop expression for the fine structure constant  $\alpha_V$  after the vertex quantum correction is [61]

$$\alpha_V = \frac{\alpha_{\gamma Z}}{\left(1 + \frac{\alpha_{\gamma Z}}{3\pi} \ln \frac{m_e}{\mu}\right)^2}, \quad (181)$$

where  $\alpha_{\gamma Z}$  is given by Eq.(175),  $m_e$  is the mass of the electron, and  $\mu$  is the actual mass of the photon.

To calculate  $\mu$ , we adapt the mass formula Eq. (178) for use by the photon. We have

$$\mu = \Lambda \exp\left(\frac{1}{4} - \frac{2\pi}{3\beta_{\gamma Z}}\right), \quad (182)$$

Next, we make use of the fact that the fine structure constant for the photon is the square of that for an electron. We have

$$\beta_{\gamma Z} = \alpha_{\gamma Z}^2 = \left(\frac{1}{19.27}\right)^2. \quad (183)$$

This value for  $\beta_{\gamma Z}$  contains  $1/19.27$  instead of  $1/19.71$  because we are using the electromagnetic  $\alpha_0 = 1$  for the electron in Eq. (176) instead of the weak  $\alpha_0 = 1.8013$ .

Substituting Eq. (183) into Eq. (182), we obtain

$$\mu = 5.31 \times 10^{-348} \text{ g}. \quad (184)$$

This mass for the photon is effectively zero. It is far below the experimental limit of  $4 \times 10^{-48}$  g. The mass of the photon is generally thought to be zero, although no one knows why. There is no currently accepted explanation for this mass. Here we have a complete and consistent explanation for the mass of the photon, right down to the fact that it must be small but nonzero in order to eliminate infrared divergences from QED. This mass is the second previously unexplained parameter of the SM to be explained by 3DT.

Because  $\mu \neq 0$  the quantum vertex correction in Eq. (181) is not infinite. Equation (181) then yields

$$\alpha_V = \frac{1}{486}, \quad (185)$$

which is multiplied by a factor of  $\sqrt{4\pi}$  to obtain  $\alpha$ .

To see where the factor of  $\sqrt{4\pi}$  comes from, we make the following observations. Factors of  $4\pi$  arise from forces that vary as the inverse square of the distance. For example,

$$F = \frac{q^2}{4\pi r^2}. \quad (186)$$

They have to do with the surface area, which is  $4\pi r^2$ , of a sphere. The further away from a point source of charge one is, the less dense are the field lines that intersect the surface of the sphere and the weaker is the force. Each one-loop quantum correction produces one factor of  $\sqrt{4\pi}$  in the denominator of a particular fine structure constant for the electron because each of these Feynman diagrams is surrounded by one sphere. The constant  $\alpha_V$  is the result of three quantum corrections: vacuum polarization via the photon, vacuum polarization via the  $Z^0$ , and the vertex correction. Thus it will have three factors of  $\sqrt{4\pi}$  in its denominator. We have

$$\alpha_V = \frac{e^2}{(4\pi)^{3/2} \hbar c}. \quad (187)$$

Each one-loop quantum correction produces a factor of  $\sqrt{4\pi}$  instead of  $4\pi$  because we are only considering half the diagram for photon-photon scattering. The whole diagram corresponds to  $\beta$ , the fine structure constant for the photon, while half this diagram represents  $\alpha$ , the

fine structure constant for the electron. Half the diagram has two external photon lines, two external electron lines, one internal electron line, two vertices and no loops.

Combining Eqs. (187) and (129), we have

$$\beta = 4\pi\alpha_V^2, \quad (188)$$

or

$$\sqrt{\beta} = \sqrt{4\pi} \alpha_V. \quad (189)$$

Thus we have two choices for  $\alpha$ :  $\alpha_V$  or  $\sqrt{\beta}$ , corresponding to whether the diagram for  $\alpha$  is important alone or should always be considered to represent half the photon-photon scattering diagram. Apparently, the latter is the case because the photon is a more important particle than the electron. It is the fundamental in a series of harmonics. With its spinor expansions, the photon represents all spinors, of which the electron is only one. Finally, all spinors take their Planck masses from the photon's  $\Lambda$ . Therefore, we choose  $\alpha = \sqrt{\beta}$  over  $\alpha = \alpha_V$ . This is consistent with  $\beta = \alpha^2$ .

Combining Eqs. (189) and (185), we have

$$\alpha = \sqrt{\beta} = \frac{\sqrt{4\pi}}{486}. \quad (190)$$

Thus, our derived value for  $\alpha$  is 1/137.1, which differs from the actual value of 1/137.036 by less than one tenth of one percent. This is the third previously unexplained parameter in the SM explained by 3DT.

## 17. The muon, tau and neutrinos

The masses of the muon and tau are calculated in a manner similar to that of the electron. Their electromagnetic masses remain the same because  $\alpha_0$  stays at unity. Their weak masses, however, increase drastically because their weak  $\alpha_0$ 's due to isospin increase from  $l(l+1) = \frac{1}{2}(\frac{1}{2}+1)$  for the electron to  $\frac{3}{2}(\frac{3}{2}+1)$  for the muon and  $\frac{5}{2}(\frac{5}{2}+1)$  for the tau. The mass depends on the weak  $\alpha_0$  exponentially. It is this exponential dependence of the mass upon the square of weak charge that causes the muon to be so much more massive than the electron.

Like the electron, the mass of the muon will be given by the sum of its masses due to its weak charges from isospin and from  $\hat{R}$ . The weak charge from isospin yields  $\alpha_0 = l(l+1)$ , where  $l = 3/2$ . We have  $\alpha_0 = 3.75$ . Equation (179) yields  $m = 117.3$  MeV, which is 11% higher than the actual muon mass of 105.7 MeV. The weak charge for the muon  $g_{\hat{R}}$  from the 6D curvature scalar is the same as that for the electron [Part I]. The mass contribution from this weak charge for the muon is the same as that for the electron (one electron mass) and is therefore negligible. The mass of the muon is the fourth previously unexplained parameter of the SM to be explained by 3DT.

The mass of the muon comes from its motion on the HD sphere. At first glance this appears to be something apart from the curvature scalar  $\hat{R}$ , but it is  $\hat{R}$  that puts a moving muon on the HD sphere. This follows from its HD angular momentum representation  $|\frac{3}{2}, -\frac{1}{2}\rangle$ .

Like the muon, the mass of the tau due to its weak charge from  $\hat{R}$  is one electron mass and is negligible. Therefore, its weak  $\alpha_0$  from isospin is given by  $l(l+1)$ , where  $l = \frac{5}{2}$ . Its

calculated mass is 2072 MeV, which is 16% higher than its actual mass of 1784 MeV. This is the fifth previously unexplained parameter to be explained by 3DT.

Note the error in the calculation of mass increases as one goes from the electron to the muon to the tau. This is as one would expect for a first-order, one-loop approximation to a series in powers of fractions of  $\alpha_0$ . The higher-order terms do not drop off as fast for increasing  $\alpha_0$ , resulting in increased error for the first-order approximation.

We will now derive the masses of the neutrinos. Solving Eq. (141) for the integer and substituting  $c = \frac{1}{2}$  and  $m = \frac{1}{2}$  for the neutrino, we find the charge on the neutrino is zero. This means neutrinos do not couple to the photon. Therefore, their electromagnetic masses are zero. Neutrinos have the same weak charges  $g_I$  and  $g_{\widehat{R}}$  as their charged leptonic counterparts. However, since the photon is absent, their weak  $\alpha_0$  is screened only via the  $Z^0$ . Thus, there will be only one vacuum polarization equation. In addition, the electromagnetic cutoff  $\Lambda$  will be absent from the quantum mass correction equation. It is replaced by a weak cutoff described below.

The equations that determine the masses of neutrinos are

$$\alpha_Z = \frac{\alpha_0}{1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda_Z^2}{m_\nu^2}}, \quad (191)$$

$$1 = \frac{3\alpha_Z}{4\pi} \left( \ln \frac{\Lambda_Z \Lambda'_Z}{m_\nu^2} + \frac{1}{2} \right), \quad (192)$$

where Eq. (192) is Eqs. (108) and (109) after  $m$  cancels when  $m_0 = 0$ . One of the  $\Lambda$ 's in this equation must be  $\Lambda'_Z = \Lambda_Z m_\nu^2 / M_W^2$ . In other words,  $\Lambda'_Z$  is simply  $\Lambda_Z$  with the  $W$  mass squared replaced by the neutrino mass squared. This is necessary in order to allow for the weakness of the weak interaction where the neutrino emits a  $Z^0$  at the first vertex of the quantum mass correction diagram. This process is a weak one due to the large mass of the  $Z^0$ . However, all subsequent processes described by the vacuum polarization and quantum mass correction diagrams should not be ‘tagged’ as weak because the  $Z^0$  has already been produced — the process is not further hindered by the production of a more massive particle. This means that all  $\Lambda$ 's in the vacuum polarization and mass correction equations besides the  $\Lambda'_Z$  mentioned above must be  $\Lambda_Z$ .

In the electron's quantum mass correction equation, the classical mass of the electron was given by the sum of its classical electromagnetic and weak masses, but the classical weak mass  $\Lambda'_Z = \Lambda_Z m_e^2 / M_W^2 = 2.78 \times 10^{-17}$  g was negligible compared to the electromagnetic  $\Lambda = 2.37 \times 10^{-10}$  g. Now that the photon's effects are absent from the quantum mass equation,  $\Lambda$  is replaced by  $\Lambda'_Z$ .

Note that the neutrino mass  $m_\nu$  cancels in Eq. (192). The relative smallness of  $\Lambda'_Z$  is the reason why neutrinos have such small masses. And the only reason we need to consider  $\Lambda'_Z$  is because the neutrino has zero charge. Therefore, neutrinos have such small masses because they have no charge, as one might expect.

The solution of Eqs. (191) and (192) is

$$m_\nu = \Lambda_Z^{-1/8} \left( \frac{\Lambda'_Z}{m_\nu^2} \right)^{-9/8} \exp\left( \frac{3\pi}{2\alpha_0} - \frac{9}{16} \right). \quad (193)$$

Because the fraction involving  $\alpha_0$  in the exponential in Eq. (193) does not have a minus sign in front of it as it does for charged leptons, the masses of neutrinos decrease with increasing  $\alpha_0$ . Thus, the smaller of  $|g_I|$  or  $|g_{\widehat{R}}|$  instead of the larger will yield the masses of the neutrinos. The larger of these two weak charges will yield a mass that is negligible.

Each neutrino will have the same weak charges  $g_I$  and  $g_{\widehat{R}}$  as its charged leptonic counterpart. For the electron neutrino  $g_I = 3.07\sqrt{\hbar c}$  (leading to  $\alpha_0 = 3/4$ ) and  $g_{\widehat{R}} = -4.758\sqrt{\hbar c}$  (leading to  $\alpha_0 = 1.8013$ ). Substituting these separately into Eq. (193), we find that the mass of the electron neutrino is due to  $g_I$  and is  $6.16 \times 10^{-7}$  eV, which is well below the experimental limit of 20 eV. The masses of the muon and tau neutrinos are determined from  $g_{\widehat{R}} = -4.758\sqrt{\hbar c}$ . They are the same and are  $1.57 \times 10^{-8}$  eV, far lower than their experimental limits of 0.2 MeV and 35 MeV, respectively. The masses of the neutrinos are the sixth, seventh and eighth previously unexplained parameters of the SM to be determined by 3DT.

## 18. New fermions

According to Table 1, other leptons exist in the muon's and tau's multiplets besides the known particles. These particles turn out to have properties identical to the known particles but have different quantum numbers. Apparently, we have been detecting them but have been mistaking them for the muon, tau, and their neutrinos. The usual muon is symbolized by  $\mu^-$ . The new charged particle in the muon's multiplet has the symbol  $\mu_2^+$ . Table 1 gives the quantum numbers for the new particle as  $l = \frac{3}{2}$  and  $m = \frac{3}{2}$ . According to the Gell-Mann relation (141), its bare charge quantum number is  $Q = 1$ . The  $\mu_2^+$  has the same electromagnetic and weak bare fine structure constants as the usual muon  $\mu^-$ . Therefore, it has the same mass as the muon. Because its charge is reversed it is similar to the antimuon  $\bar{\mu}^+$ . However, it is not the same as the antimuon because its quantum number  $m = \frac{3}{2}$  is different than the antimuon's  $m = \frac{1}{2}$ . Because this particle has the same mass as the usual muon, there is no reason why it should not have been detected already. I believe we have been detecting it all along but have been mistaking it for the antimuon.

The particle with  $l = \frac{3}{2}$  and  $m = -\frac{3}{2}$  in Table 1 has  $Q = -2$  according to the Gell-Mann Relation (141). Therefore,  $\alpha_0 = Q^2 = 4$ . This would put its electromagnetic mass at about the muon's weak mass because the muon's weak  $\alpha_0 = 3.75$ . Remember that a spinor's mass comes from its charge and that a particle may have more than one type of charge. The mass contributions from the different types of charge add. Because this particle has  $l = \frac{3}{2}$ , which is the same value for  $l$  as the muon, its weak mass would be that of the muon. Its total mass would therefore, be about double that of the muon (266 MeV to be exact). However, a lepton with this mass is not observed. This must be due to the fact that the electromagnetic  $\alpha_0$  is too large for the higher-loop series [61] in terms of  $\alpha_0$  to converge. This electromagnetic series for vacuum polarization becomes infinite, resulting in an infinite denominator for  $\alpha_\gamma$ . The parameter  $\alpha_\gamma$  is therefore zero. This means the charge is completely screened by the photon. In effect, the particle has no charge and does not couple to the photon. Therefore, it is a neutrino. Because it is in the muon's multiplet, it is a muon neutrino. It would have the same mass as, but be different than, the ordinary muon neutrino, which has  $m = \frac{1}{2}$ .

The new lepton in the tau's multiplet have  $l = \frac{5}{2}$  and  $m = \pm\frac{5}{2}, \pm\frac{3}{2}$ , according to Table 1. These  $m$  quantum numbers distinguish the new particles from the tau and its neutrino, which

have  $m = \pm\frac{1}{2}$ . The bare charge quantum numbers of these new particles are  $Q = -3, -2, 1, 2$ . By the same reasoning used above for the muon's multiplet, the particle with  $Q = 1$  has properties similar to the antitau, while the others are new tau neutrinos.

Fermions in the next spinor multiplet beyond the tau's have  $l = \frac{7}{2}$  and  $\frac{7}{2} \geq m \geq -\frac{7}{2}$ . Because none of these particles are observed, the weak  $\alpha_0$  must be too large for the higher-loop series in terms of this  $\alpha_0$  to converge. Because this series in terms of the weak  $\alpha_0$  does not converge, the weak charges of these  $l = \frac{7}{2}$  particles are zero, by the same argument used above for the vanishing of the electric charge. Therefore, they do not couple to the  $Z^0$ . This means they would not affect the decay width, which is the test for their existence, of the  $Z^0$ . This is the reason why there appears to be only three families of leptons.

The particles with  $Q = 0$  or  $|Q| \geq 2$  also have no electric charge and therefore, have no electromagnetic masses because  $\alpha_\gamma = 0$ . These particles would acquire a small mass through their superweak couplings to  $A_\mu^{lm}$ ,  $l \geq 2$ . These superweak masses should be smaller than ordinary neutrino masses, which are small due to the weakness of the weak interaction.

The particle with  $Q = 1$  has electric charge but no weak charge and is screened by the photon only. The equations that determine the mass of this particle are

$$\alpha_\gamma = \frac{\alpha_0}{1 + \frac{\alpha_0}{3\pi} \ln \frac{\Lambda^2}{m^2}}, \quad (194)$$

$$m = \Lambda \exp\left(\frac{1}{4} - \frac{2\pi}{3\alpha_\gamma}\right). \quad (195)$$

The solution of Eqs. (194) and (195) is

$$m = \Lambda \exp\left(\frac{9}{20} - \frac{6\pi}{5\alpha_0}\right). \quad (196)$$

Substituting  $\alpha_0 = 1$  into Eq. (196), we find  $m = 4.81 \times 10^9$  TeV.

Because their electric and weak charges are the same, spinors with  $l > \frac{7}{2}$  will have masses similar to those with  $l = \frac{7}{2}$ . That is, particles with  $Q = 0$  or  $|Q| \geq 2$  will have neutrino-like masses and those with  $|Q| = 1$  will have the mass  $4.81 \times 10^9$  TeV.

## 19. Elementary particle interactions

Just as a given particle decays only into certain decay products, a given ket may only have certain other kets as its factors. In fact, the relationship is one-to-one, with the ket for each particle determining which decays can take place. For example, one may ask why the  $W^+$  decays into the particles it does, instead of the decay products for the  $W^-$  or the  $Z^0$ . To be sure, these decays can be ruled out by the law of conservation of charge, but then one must ask why the law of conservation of charge holds. I claim this law follows from the rules for combining kets given by Clebsch-Gordan coefficients. These rules uphold the law of conservation of angular momentum.

One may know which elementary particle interactions can take place by determining whether or not the relevant Clebsch-Gordan coefficient is zero. An elementary particle interaction can take place if the coefficient is not zero. This is the case if and only if

$m = m_1 + m_2$  and  $|l_1 + l_2| \geq l \geq |l_1 - l_2|$ . I claim the relation  $m = m_1 + m_2$  is the law of conservation of charge. In terms of the bare charge quantum number  $Q$  for vectors, we have  $Q = m$ ; in addition, for spinors  $Q = m - \frac{1}{2}$ , and for antispinors  $Q = m + \frac{1}{2}$ .

A short cut to obtaining the spinor expansion of a vector is to write the vector's ket in terms of half-odd integral  $l$  kets using Clebsch-Gordan coefficients. One then writes the particle symbols of the spinors with the quantum numbers of the spinor kets below the spinor kets. The quantum numbers of spinors are given in Table 1. Remember that the first spinor ket in each pair is associated with an antiparticle by convention.

All of this is made clear by a simple example: The ket  $|11\rangle$  is equal to the product of kets  $|\frac{1}{2} \frac{1}{2}\rangle |\frac{1}{2} \frac{1}{2}\rangle$  because the relevant Clebsch-Gordan coefficient is not zero. It is one. We have

$$|11\rangle = |\frac{1}{2} \frac{1}{2}\rangle |\frac{1}{2} \frac{1}{2}\rangle. \quad (197)$$

As in Sec. 8, we identify a spinor with a bar over it with the first half-odd integral  $l$  ket. After identifying  $|11\rangle$  above with the  $W^+$ , the first  $|\frac{1}{2} \frac{1}{2}\rangle$  with  $\bar{e}^+$  (a positron) and the second  $|\frac{1}{2} \frac{1}{2}\rangle$  with  $\nu_e^0$  (a neutrino), we have the decay  $W^+ \rightarrow \bar{e}^+ + \nu_e^0$ . Another way of looking at this is that the above interaction is justified by substituting a spinor expansion for one factor of  $W_\mu^+$  in its mass term in the Lagrangian, thereby generating an interaction term as described in Part I.

The  $W^+$  (ket  $|11\rangle$ ) cannot decay into two neutrinos (kets  $|\frac{1}{2}, -\frac{1}{2}\rangle |\frac{1}{2} \frac{1}{2}\rangle$ ) because the Clebsch-Gordan coefficient for this process is zero. Similarly, the  $W^+$  cannot decay into two electrons, two positrons, a positron and an electron, or an electron and a neutrino because the Clebsch-Gordan coefficients for all these processes are zero. These violate  $m = m_1 + m_2$ , the law of conservation of charge.

There is one more rule for obtaining nonzero Clebsch-Gordan coefficients involving leptons. As it stands, there is nothing to prevent a ket  $|lm\rangle$  from being expanded in terms of kets with different values for half-odd integral  $l_1$  and  $l_2$ . For example, if  $l = 1$ ,  $l_1 = \frac{1}{2}$  and  $l_2 = \frac{3}{2}$ , then it would appear one could have  $|11\rangle = \sqrt{\frac{1}{4}} |\frac{1}{2} \frac{1}{2}\rangle |\frac{3}{2} \frac{1}{2}\rangle - \sqrt{\frac{3}{4}} |\frac{1}{2}, -\frac{1}{2}\rangle |\frac{3}{2} \frac{3}{2}\rangle$ , with the corresponding interactions  $W^+ \rightarrow \bar{e}^+ + \nu_\mu^0$  for the first pair of kets and  $W^+ \rightarrow \bar{\nu}_e^0 + \mu_2^+$  for the second. Both of these decays violate electron and muon number conservation. One needs a reason why this cannot occur — it contradicts observation. The reason is as follows. All spinors in 3DT are ultimately derived from the photon because this is the only vector that is null. As explained in Ref. (45), this is a requirement for writing a vector in terms of two spinors. Now one cannot expand the ket for the photon  $|00\rangle$  in terms of kets with different values for  $l_1$  and  $l_2$  because then the minimum value for  $l$  will be greater than zero, while the photon must have  $l$  equal to zero. The minimum value for  $l$  in the above example was  $|\frac{1}{2} - \frac{3}{2}| = 1$ , which is not zero. (Remember the minimum value for  $l$  is given by  $|l_1 - l_2|$  from the relation  $|l_1 + l_2| \geq l \geq |l_1 - l_2|$ ). This means that we must have  $l_1 = l_2$  in order to achieve  $l = 0$  and that  $|00\rangle$  can only be expanded in terms of kets from one family of leptons at a time. This means one could expand  $|00\rangle$  in terms of  $l_1 = l_2 = \frac{1}{2}$  kets or  $l_1 = l_2 = \frac{3}{2}$  kets but not both in the same expansion. In effect, only one family of leptons may exist at one time. This adds the requirement  $l_1 = l_2$  to the above rules for obtaining nonzero Clebsch-Gordan coefficients involving leptons and ensures that electron and muon number is conserved.

The elementary particles are described by the Lagrangian for 3DT. The Lagrangian for 3DT, in turn, is derived from 6D spacetime. It may be that conjugation of charge, parity and time, known as CPT conjugation, is an invariance because reversal of all six dimensions leads to an identical spacetime.

## 20. The W and Z

One source of mass in 3DT is the appearance of explicit mass terms in the 6D curvature scalar  $\hat{R}$ . For spinors, the graviton and the photon this mass is zero, while for the  $Z^0$  and  $W^\pm$  these masses [from Part I] are  $1.9109 \hbar/c^2 T$  and  $1.6293 \hbar/c^2 T$ , respectively. (Note that  $cT$  is the Planck length and  $\hbar/c^2 T$  is the Planck mass.) To these masses we should add the mass due to the particle's charges. One obtains this type of mass by substituting the particle's charge into the classical radius equation. The predicted masses of the  $W$  and  $Z$  without these masses will turn out to be close enough to the actual values to allow us not to bother with this calculation. Therefore, the masses of the  $Z^0$  and  $W^\pm$  are those from  $\hat{R}$  given in Part I.

Now that we have the Planck masses for the  $W^\pm$  and  $Z^0$ , the question arises, "How do we reduce these masses to their observed values?" We noted above that a vector can be considered to be a combination of two spinors. Thus, we can try the quantum mass correction developed for spinors for vectors. Both the wave functions for the  $Z^0$  and the  $W^\pm$  are made up of those for the electron and electron neutrino. From its spinor expansion, one can say that the  $Z^0$  is one half an electron-positron pair and one half a neutrino-antineutrino pair. The factors of one half arise from the Clebsch-Gordan coefficients of  $1/\sqrt{2}$  multiplying each of the two pairs of spinor kets. The mass of the neutrino-antineutrino pair is negligible compared to that of the electron-positron pair. Thus, the  $Z^0$ 's mass contributions are from the electron-positron pair and are one-half of twice the electron's mass or simply one electron mass. The  $W^+$  is a positron-neutrino pair and the  $W^-$  is an electron-antineutrino pair. Because the neutrino masses are negligible, each  $W$  mass is based on one electron mass.

The quantum mass corrections of the  $Z^0$  and  $W^\pm$  will be that of the electron because the masses are based on one electron mass. Therefore,

$$M_W = \exp\left(\frac{1}{4} - \frac{2\pi}{3\alpha_{\gamma Z}}\right) \Lambda(W_\mu^\pm), \quad (198)$$

where we have denoted the above-mentioned Planck mass of the  $W$  by  $\Lambda(W_\mu^\pm)$ . In addition,  $\alpha_{\gamma Z} = 1/19.71$ , which is the value that yields the mass of the electron in Eq. (178). Dividing Eq. (198) by Eq. (178) when  $\alpha_{\gamma Z}$  equals  $1/19.71$ , we obtain

$$\frac{M_W}{m_e} = \frac{\Lambda(W_\mu^\pm)}{\Lambda}. \quad (199)$$

Using Eq. (199) to solve for the mass of the  $W^\pm$ , we find  $M_W = 76.48$  GeV. Similarly,

$$\frac{M_Z}{m_e} = \frac{\Lambda(Z_\mu^0)}{\Lambda}, \quad (200)$$

where  $\Lambda(Z_\mu^0)$  is the classical Planck mass of the  $Z$ . This leads to a  $Z^0$  mass of 89.70 GeV.



These masses are in fairly good agreement with experiment [64], where  $M_W = 80.40 \pm 0.2$  GeV and  $M_Z = 91.1867 \pm 0.0021$  GeV. The errors are presumably due to lack of consideration of the mass contributions from their charges. The SM does not explain the masses of the W and Z. It derives them from the weak interaction coupling constant  $G_W$ , which comes from experiment. 3DT explains the masses of these two particles from scratch, allowing, instead, one to derive  $G_W$  from them. These are parameters nine and ten of the SM. These are the final parameters of the SM to be derived for now.

It should not be surprising that we have arrived at the correct values for the masses and charges of the elementary particles. If your classical theory is correct, then applying quantum mechanical corrections to it should result in the correct values for physical quantities.

## 21. Predicted intermediate vector bosons

The vector  $A_\mu$  in 3DT is expanded in terms of spherical harmonics  $Y_{lm}$

$$A_\mu = \sum_{l=0}^{\infty} \sum_{m=-l}^l A_\mu^{lm} Y_{lm}. \quad (201)$$

We neglect the strong interactions in what follows. The  $l = 0$  coefficient  $A_\mu^{00}$  in this expansion is the photon. The  $l = 1$  coefficients  $A_\mu^{1m}$  are the W and Z. The  $l = 2$  and  $l = 3$  coefficients  $A_\mu^{2m}$  and  $A_\mu^{3m}$  are vectors for two new superweak interactions.

The procedures used to obtain the masses of the  $A_\mu^{2m}$  from the spinors of which they are composed are the same as those described for the  $A_\mu^{1m}$  in the last section. Remember that in order to obtain nonzero Clebsch-Gordan coefficients for leptons,  $l_1 = l_2$ , which means that one cannot expand  $l = 2$  kets in terms of  $l_1 = \frac{3}{2}$  and  $l_2 = \frac{1}{2}$  kets as one might expect, but rather kets with  $l_1 = l_2 = \frac{3}{2}$  only. The wave function of the vector  $A_\mu^{2m}$  is composed of those of the muon and the muon neutrino. The mass contributions of these particles add. The neutrino contribution is negligible. An exact prediction for the mass of  $A_\mu^{2m}$  can be made if we multiply the classical mass ratio  $\Lambda(A_\mu^{2m})/\Lambda$  by the mass of the muon [similar to Eq. (199)].

From  $\Lambda(A_\mu^{20}) = 3.4032 \hbar/c^2 T$  (where  $\hbar/c^2 T$  is the Planck mass), we obtain for  $A_\mu^{20}$  a mass of 33.04 TeV. Similarly,  $\Lambda(A_\mu^{2,\pm 1}) = 3.2159 \hbar/c^2 T$ , which leads to a mass of 31.22 TeV for  $A_\mu^{2,\pm 1}$ . The classical mass  $\Lambda(A_\mu^{2,\pm 2}) = 0.46994 \hbar/c^2 T$  yields the actual mass 4.56 TeV for the  $l = 2, m = \pm 2$  particle.

The vectors  $A_\mu^{3m}$  are composed of a muon and a muon neutrino. Their masses are obtained by multiplying their classical mass ratios by the mass of the muon. We have  $\Lambda(A_\mu^{30}) = 3.9957 \hbar/c^2 T$ ,  $\Lambda(A_\mu^{3,\pm 1}) = 3.0315 \hbar/c^2 T$ ,  $\Lambda(A_\mu^{3,\pm 2}) = 2.8175 \hbar/c^2 T$  and  $\Lambda(A_\mu^{3,\pm 3}) = 0.75385 \hbar/c^2 T$ . The masses of these particles are 38.79 TeV, 29.43 TeV, 27.36 TeV and 7.32 TeV, respectively.

The charges of the  $A_\mu^{2m}$  are not, in general, given by their quantum number  $m$ . This is because  $m$  is the vector's bare charge quantum number, which is not necessarily its actual charge quantum number. The vector's actual charge quantum number is given by the sum of the actual charges of the fermions in its spinor expansion. This is because the vector may be thought to come about from the combination of the spinors from which it is composed. Another way of looking at this is that a vector and its spinor expansion must be mathematically and physically identical. For example, consider the  $Z^0$ . From its spinor expansion,

we find the  $Z^0$  is composed of one-half an electron-positron pair and one-half a neutrino-antineutrino pair. Thus, the charge of the  $Z^0$  is given by  $\frac{1}{2}(-1 + 1) + \frac{1}{2}(0 + 0)$ , which equals zero. Had one of the numbers in one set of the parentheses been different, the  $Z^0$  would have wound up with a different charge — and a fractional one at that. One of the numbers will be different for the case of the spinor with bare charge quantum number  $-2$  and actual charge quantum number  $0$ . This spinor is in the muon's multiplet and is described in Sec. 18 (it has  $Q = -2$ ). Therefore, instead of having a  $-2$  here we will have a  $0$ . This is why the charge of  $A_\mu^{lm}$  is not, in general, given by  $m$  and why it may have a fractional charge.

The charges of the next two generations of intermediate vector bosons — the  $A_\mu^{lm}$  where  $l = 2$  or  $3$  are as follows. The charge of the superweak vector  $A_\mu^{2,\pm 2}$  is  $\pm 1$ . The vector  $A_\mu^{20}$  is neutral, while  $A_\mu^{2,\pm 1}$  has a charge of  $\pm \frac{3}{4}$ . The charge of the superweak vector  $A_\mu^{3,\pm 3}$  is  $\pm 1$ . The vector  $A_\mu^{30}$  is neutral, while  $A_\mu^{3,\pm 1}$  has a charge of  $\pm \frac{3}{5}$ . The charge of  $A_\mu^{3,\pm 2}$  is  $\pm 1$ . Remember, one can only detect a superweak vector's decay products so these fractional charges can not be observed. (Each of its decay products has charge  $-1$ ,  $0$ , or  $+1$ .)

The predictions 3DT makes are clear cut. 3DT predicts no Higgs boson and no sparticles. Instead, it predicts new intermediate vector bosons. The least massive of these is  $A_\mu^{2\pm 2}$  with a mass of  $4.56$  TeV. This is too massive for the Large Hadron Collider or the proposed International Linear Collider to detect. Therefore, these machines will detect no Higgs boson, no sparticles, and no new vectors.

## 22. Conclusion

Quantum field theory in the Standard Model requires gauge symmetry groups, spontaneous symmetry breaking, and renormalization. In 3DT these are not needed and are unnecessary. The theory works much better without them. They are vestiges of an attempt to build a unified field theory from the bottom up. The unification problem is so difficult, however, that one must also proceed from the top down if one is to have any hope of succeeding. This attacks the problem from both ends and recognizes the importance of having the correct initial assumptions. In place of these tenets of quantum field theory, 3DT has non-zero sized particles, which produce cutoffs. These cutoffs eliminate infinities before they appear and allow the calculation of masses and coupling constants.

The Standard Model is just that — a model that attempts to imitate a larger and more comprehensive theory like 3DT. For example, the Standard Model does not explain why there are three massive electroweak vectors while one remains massless (the photon). 3DT naturally produces one massless and three massive vectors because mass terms in the 6D curvature scalar have the form, e.g.  $A^\mu \partial_5^2 A_\mu$ . This is zero for the photon, which is associated with the lowest order  $l = m = 0$  spherical harmonic that does not depend on the HD coordinates  $\hat{x}^5$  or  $\hat{x}^6$ . This singles out the photon for special treatment and rightly so, as it is the fundamental in a series of harmonics. This mass term is not zero for all other spherical harmonics, which do depend on the higher-dimensional coordinates. It produces Planck masses for them. In the past, these Planck masses were the reason why dependence on the higher-dimensional coordinates was forbidden, but in 3DT these Planck masses are converted to ordinary elementary particle masses by the quantum mass correction equation in QED. When used properly, this equation converts the Planck mass scale to the usual elementary particle mass scale.

The reason 3DT can calculate masses is that we no longer have point particles. With extended particles come precisely calculated cutoffs of the order of the Planck mass. The logarithm in the quantum mass correction equation converts these Planck masses to ordinary elementary particle masses. In addition, we now have the complete classical electroweak theory, which is ready to be quantized. This theory says the bare mass  $m_0$  of the electron is zero. This greatly simplifies the quantum mass correction equation and allows this conversion of Planck masses.

The Standard Model must assume a local  $SU(2) \times U(1)$  symmetry. 3DT explains this with the first four terms of an expansion of the vector  $A_\mu$  in terms of spherical harmonics. This takes place inside a Lagrangian density that is integrated over the spherical coordinates, producing the required terms.

3DT is highly organized, self-consistent, systematic, efficient and compact. This is unlike string theory, which seems to never end, and unlike the Standard Model, which must introduce many of the terms in its Lagrangian ad hoc. These include the terms for Maxwell's equations and the Dirac equation. These equations have their roots in experiment. The Standard Model provides no explanation for why they should exist. It only assumes them. In contrast, 3DT naturally produces these terms as part of the 6D curvature scalar.

The Standard Model is based on QED. QED is accepted because it works, explaining properties of the electron such as its gyromagnetic ratio. This despite the appearance of infinities. 3DT must be accepted because it also works, explaining 10 parameters of the Standard Model. It does this without the appearance of infinities. Not only this, but it actually eliminates the infinities in QED!

One criticism of string theory is that it does not make predictions. String theorists claim it does make predictions. But how can we have faith in its predictions if it does not explain or postdict anything? It explains little about the known elementary particles. In contrast, 3DT explains or postdicts a great deal about the elementary particles. It explains what came before the LHC, predicts what the LHC will find (no new particles), and will predict what comes after the LHC (more intermediate vector bosons). One can have faith in its predictions because what it predicts will come out of the LHC is merely a continuation of what came before it. It is not enough that a theory makes predictions. It must make predictions that come true. Otherwise the theory is wrong. String theory does not make predictions one can have faith in because it has not explained anything in the past. In contrast, 3DT has explained the Standard Model. Thus we can have faith in its predictions.

String theory makes the claim that it quantizes gravity. I claim that it does not even quantize electromagnetism. QED, the standard for this, has infinities, which proves something is wrong. By contrast, 3DT properly quantizes electroweak theory. It has no infinities. Everything is finite and calculable the way it should be. As proof that it works it produces the masses and coupling constants of nine elementary particles. This is the example that should be set for quantum gravity.

One does not need 10 or 11 dimensions to produce all the vectors needed for the Standard Model. If just one vector  $\hat{g}_{\mu 5} = A_\mu$  depends on the higher dimensional coordinates, it will contain an infinite number of component vectors  $A_\mu = \sum_{l,m} A_\mu^{lm} Y_{lm}$ , the first few of which,  $A_\mu^{00}$ ,  $A_\mu^{1,-1}$ ,  $A_\mu^{10}$ ,  $A_\mu^{11}$ , can be used to describe the electroweak interaction. The remaining vectors with  $l$  greater than one, are the theory's predictions.

String theory must assume 10 or 11 dimensions, supersymmetry, strings and a large

symmetry group. Of these, 3DT assumes only 2 extra dimensions. That is all. And adding dimensions of time has worked before. There was a time when all the talk was about the fourth dimension. Unlike other unified field theories, there is an enormous historical precedent for adding dimensions of time — the theory of relativity.

The bottom line is 3DT assumes much less than either the Standard Model or string theory and explains far more.

There is already more than enough evidence to suggest 3DT may be correct. As proof that 3DT is correct, I offer the fact that the solutions to its field equations are the known elementary particles. This means the theory explains why we see what we see for a vast body of observations. These include identifying the known elementary particles with specific harmonics with well-defined quantum numbers; the absence of fundamental scalars, none of which has ever been seen; the absence of superpartners, which have never been detected; where the Uncertainty Principle comes from; why there is minimal coupling in the Standard Model; the existence of the Dirac equation; the absence of a cosmological constant; the appearance of the symmetry group  $SU(2) \times U(1)$ ; why the strong interactions are different than the electroweak interactions; why the photon is massless while the  $W$  and  $Z$  have mass; why the observed quarks and leptons exist; where four-dimensional gravitation comes from (the 6D curvature scalar); where Maxwell's equations come from; the existence of the  $W$  and  $Z$  along with the terms that describe them; why there are three weak vectors and one electromagnetic vector; interaction terms for vectors with spinors; why there appears to be three generations of leptons; why there are no four-point Fermi interactions; why there are no Feynman diagrams with five or six external gluon lines; quark confinement; asymptotic freedom; chiral symmetry breaking; how to make observations at the Planck distance; why there are two widely separated levels of mass in physics (the hierarchy problem); the origin of the Gell-Mann equation; how to construct an elementary particle; whether the electron, muon, tau, the neutrinos, photon,  $W$  and  $Z$  have electric charge or not; the mass of the electron; the mass of the photon; the value  $1/137$  for the fine structure constant; the masses of the muon and tau; the masses of the electron's, muon's and tau's neutrinos; why only the observed particle decays take place and not others; the masses of the  $W$  and  $Z$ ; and the masses of predicted particles allowing for a definite test of the theory. This list of 47 phenomena explained by 3DT is a preponderance of the evidence that 3DT is certainly useful and could very well be correct. No other theory explains so much. String theory does not explain any of these 47 important phenomena. Despite a huge effort, String theory still seems to make no contact with reality.

The short-distance behavior of the elementary particles is described by quantum mechanics. Therefore, one could say quantum mechanics is a property of the elementary particles. We have seen how general relativity produces the elementary particles. As a result, it may be that general relativity produces quantum mechanics.

Newton showed the force that pulls an apple from a tree is the same as that which keeps the moon in its orbit. We have now shown that 6D gravitation also drives the elementary particles and their interactions. The inference to be drawn is that general relativity is the source of everything. From quark confinement to gravitation, everything would come from general relativity.

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