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Enhanced Gertsenshtein effect in type-II superconductors

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Abstract

Previously published calculations show that gravitational waves propagate inside superconductors with a phase velocity reduction of ~300 times and a wavenumber increase of ~300 times. This result has major significance for the propagation of gravitational waves. It is shown here that one important consequence may be regarded as a considerably enhanced Gertsenshtein effect for very-high-frequency gravitational waves within type-II superconductors. This arises because type-II superconductors do not always completely expel large magnetic fields; above their lower critical field they allow vortices of magnetic flux to channel the magnetic field through the material. Within these vortices, the superconducting order parameter reduces to zero and so the material has properties approaching those of normal material or non-superconductor. Varying the applied magnetic field varies the proportion of material that is normal, which consequently affects the propagation speed of very-high-frequency gravitational waves through a type-II superconductor.

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1. Introduction

Gravitational waves are now widely recognized as a highly important emerging technology for future communications and propulsion. The initial theoretical work of Einstein [1], Forward and Miller [2], and Dehnen and Romero [3] laid the foundations supporting subsequent experimentation. Confirmed observations of very-low-frequency gravitational waves of astronomical origin eliminated any remaining skepticism concerning their existence and nature [4]. Subsequent development has included the large-scale LISA, LIGO, Virgo, and other prototype projects for low-frequency gravitational wave detection. More recently, the properties of high-frequency gravitational waves (HFGWs, i.e., those containing components at frequencies between 100 kHz and 100 MHz [5]) and very-high-frequency gravitational waves (VHFGWs, i.e., those containing components at frequencies between 100 MHz and 100 GHz [5]) have been investigated theoretically, together with suggested methods of generating and detecting them in a laboratory [6] and of manipulating them for technological purposes [7].

In a classic 1962 paper, Gertsenshtein [8] described how the non-linearity of Einstein's field equations meant that electromagnetic and gravitational waves were coupled in such a way that propagation of one type of wave would simultaneously generate a wave of the other type. The amplitude of the generated wave is, however, so small that practical exploitation of this effect is fraught with apparently insuperable difficulties. Effective manipulation of gravitational waves has previously been thought possible only by events of astronomical scale. The present paper shows that high-frequency gravitational waves will have much stronger interactions with magnetic fields, and in fact strong enough to be readily exploitable, if the interaction takes place inside a type-II superconductor. This lays the foundations for controlling and manipulating

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high-frequency gravitational waves in future gravitational wave systems.

This theoretical breakthrough is important because it means that for the first time there is a clear scientific basis for the effective manipulation of high-frequency gravitational waves. Now there is a simple laboratory-scale recipe available that points the way to focusing, collecting, and distributing gravitational waves in ways not considered before because they were too inefficient to be practical.

2. Li and Torr result

Li and Torr [9,10] published calculations of the propagation behavior of gravitational waves inside a superconductor (SC). They claimed that the phase velocity of gravitational waves in any SC material would be $v_{\rm ph} \approx 10^6$ m s^{-1}, i.e., $\sim$300 times less than the phase velocity of gravitational waves in other materials, generally assumed equal to the speed of light in a vacuum (3×10^8 m s^{-1}). This corresponds to a wavenumber increase of $\sim$300 times in SC materials, and to a *gravitational refractive index*, $n_{\rm g}$, for the bulk material, of $\sim$300.

This result has major consequences for the design of future instruments to generate and detect gravitational waves, which may need to be able to focus, refract, reflect, or manipulate gravitational waves for efficient coupling to detectors, transmitters, generators, resonant chambers, and other transducers. In particular, VHFGWs having frequencies around 3 GHz (typical of VHFGWs predicted to be generated using currently proposed arrangements [6,11]) can have wavelengths on the same order as the dimensions of typical SC components envisaged for VHFGW use. It follows that any suggestion of phase velocity change will affect the design of components typically to be used in a future communications link based upon this technology.

Kowitt [12] and Harris [13] independently argued that the Li and Torr results [9,10] are not credible. Kowitt's objection [12] was that Li and Torr [9] assumed a value of magnetic permeability of zero (or, equivalently, a magnetic susceptibility of –1) inside a SC. This is an experimental result, widely accepted, for macroscopic samples but the question appears to be that of whether the macroscopic experimental result is true microscopically. It is perfectly true that a value of magnetic permeability different from that of free space may be obtained macroscopically in magnetically active samples in which individual magnetic moments of atoms align with the applied field. Since this enhanced permeability arises from the aggregate effect of localized magnetic moments, it follows that a renormalized permeability is not definable microscopically. However, zero permeability in a SC results from cancellation of the magnetic field **H** within the SC by surface current loops giving a distributed magnetization **M**. In a typical type-II SC, some circulating supercurrents producing the magnetic moment are also associated with the magnetic vortices, roughly on the scale of one superconducting coherence length, ξ, from the vortex axis. The circulating supercurrents at the material surface clearly have macroscopic included area, and the superconducting coherence length is larger than a typical inter-atomic distance, whereas in a conventional magnetic material the notional currents producing the magnetic moments are those caused by orbiting electrons bound closely to atoms. While it is the case that in a conventional magnetic material it is not possible to define an effective permeability that has local microscopic significance, this argument does not apply to the supercurrents found in SC materials. For these reasons, Kowitt's objection [12] appears not to be entirely tenable. Harris's objection [13] must be taken more seriously; he claims that Li and Torr assume an arbitrary value for the observer's distance of 10^{-6} cm in one place, and 10^{-13} cm at another point in their calculation, and that both values are unreasonable. This objection is harder to refute but applies only to Torr and Li [10]. Li and Torr [9] predict a changed GW phase velocity and establishing its precise value may require an experimental test. However, Fontana (as cited by Baker et al. [11]) has confirmed that the phase velocity of GWs is slowed in a SC, and Modanese and Fontana [14] give details of their own calculation making this point.

The present paper discusses some direct technical consequences of the Li and Torr result, as those consequences suggest further possible tests of this prediction. In particular, such a large mismatch in GW phase velocity results in a huge mismatch in GW propagation wave impedance at SC–air interfaces. This in turn inevitably causes large Fresnel reflections from those interfaces. It is postulated that the presence of magnetic vortices in type-II superconductors will introduce a strong interaction between GW and an applied magnetic field. The classical Gertsenshtein effect [8] in free space is a prediction that any electromagnetic wave must always be accompanied by a corresponding gravitational wave. This presumed interaction between electromagnetic field quantities and gravitational field quantities arises from the non-linear terms in the Einstein equations, and since in practice these non-linear terms are small it follows that the accompanying gravitational wave has a small amplitude for all reasonable amplitudes of electromagnetic waves. However, the arrangement described in the present paper is found (within the postulates of the model used) to affect existing gravitational waves to a considerable extent under the influence of quite modest magnetic fields (such as can easily be obtained in a laboratory), showing that the interaction between the magnetic field quantities and gravitational waves need not always be vanishingly small. This amounts roughly to an enhancement of the Gertsenshtein effect.

3. VHFGW propagation in type-II superconductor

In any superconductor, the superconductivity may be eliminated temporarily by applying a sufficiently large magnetic field. In a "type-I" superconductor, there is a single critical applied magnetic field value which, if exceeded, prevents superconductivity as surely as raising the temper-

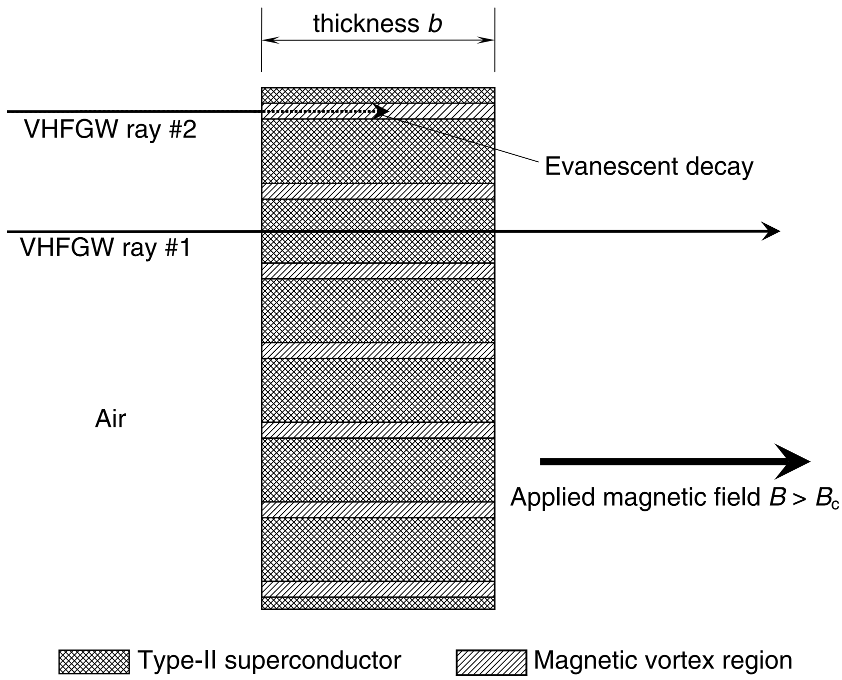

Fig. 1. Interaction of VHFGW with type-II superconductor.

ature higher than the critical temperature T_c. In a "type-II" superconductor (generally, most of the technologically important superconducting alloys and the "high-temperature" superconductors such as $YBa_2Cu_3O_{7-\delta}$ or YBCO), there are two critical field values. In applied magnetic fields greater than the lower critical field, B_c, but lower than the upper critical field, it is more energetically favorable for some parts to quench (become non-superconducting) and allow the field through. These quenched regions or "magnetic vortices" channel the magnetic field through the supercooled material. In detail, what happens is that the superconducting order parameter, Δ, is identically zero on the vortex axis at its center, and it grows to the bulk value smoothly at approximately ξ from the vortex axis [15]. The field B is expelled from material well away from the vortices but the areal density of magnetic vortices depends upon the applied external magnetic field B, since each vortex encloses an amount of flux equal to the lower critical field B_c multiplied by the effective perpendicular area occupied by the vortex.

Now consider the basic geometry shown in Fig. 1, where the applied field B is everywhere greater than B_c. Following the Li and Torr [9,10] predictions confirmed at least partially by Fontana [11] and by Modanese and Fontana [14], the VHFGW phase velocity will be reduced only at a distance greater than ξ from the vortices. In a typical type-II superconductor, the superconducting coherence length ξ (roughly speaking, the internal diameter of a vortex) is on the order of 1 μm or so. The transition between bulk superconducting material and normal material (with order parameter $\Delta = 0$) takes place roughly over a distance comparable to ξ [15]. If this transition were abrupt, there would be extremely large Fresnel reflection [8,11] for any

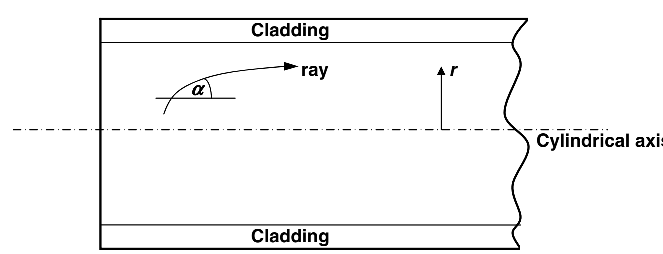

Fig. 2. Path of a meridional ray in a graded-index optical fiber.

VHFGW incident upon the outside (cylindrical curved) surface of a vortex. The ray picture of wave propagation may be used to find the effect, on the VHFGW, of a more gradual transition: the result will be analogous to light rays propagating in a graded-index optical fiber [16]. In the graded-index fiber depicted in Fig. 2, the optical refractive index is $n(r)$, dependent only on radius r from the fiber cylindrical axis. Using geometric ray optics (i.e., assuming that light travels in rays of zero width measured perpendicular to the velocity) and by considering adjacent layers of material having infinitesimal thickness it is straightforward to show that at every point on a meridional ray (i.e., a ray crossing the fiber cylindrical axis), the angle α between the ray path and the cylindrical axis always satisfies the equation

$$n(r)\cos\alpha = n_0, \tag{1}$$

where n_0 is the optical refractive index at the furthest excursion of that particular ray from the cylindrical axis. So, at a given value of refractive index n at specified radius r, the two possible solutions for the ray angle are

$$\alpha = \pm \arccos(n_0/n). \tag{2}$$

Therefore, because of the symmetry about the optical axis normal to the surfaces of constant refractive index, this light ray follows a curved path whose ends coincide with a specular reflection when observed at a distance far from the region of index grading. Similarly, a VHFGW will be effectively subject to specular reflection since the vortex itself is a "fast" structure compared to the bulk material that is a "slowing" structure, and there is therefore no possibility of a transmitted ray component even though the transition between the two regions may not be abrupt.

At this point it is important to note that for typical VHFGW frequencies (3 GHz) expected from laboratory generation, typical VHFGW wavelengths [8,11] in free space will be on the order of ~10 cm and inside a superconductor on the order of ~300 μm, so that treatment of VHFGW using the ray picture is justified approximately as it is for long-wavelength optical radiation. (Note that although the wavelength is reduced inside a superconductor, the frequency is unchanged; the wavelength reduction arises entirely from a reduction in GW phase velocity rather than from a frequency increase. The GW amplitude will also be changed inside a superconductor, but allowing for energy loss at the interfaces the energy density is unchanged, because the amplitude change results from the changed wave impedance related to the changed phase velocity.) The picture described here is certainly very different from that typically used for describing low-frequency gravitational waves, where wavelengths of many kilometers and detectors similarly sized are required. This difference is due entirely to the vast difference in frequency between low- and high-frequencies; similar differences in electromagnetic waves dictate corresponding technological approaches to handling audio and microwave frequencies.

This means that whichever picture is preferred, the vortex produces an effective reflection coefficient of unity for rays incident from outside. It follows that the vortex may be regarded as an "anti-waveguide". Moreover, its small effective diameter means that it will be well-beyond waveguide cutoff for VHFGWs having wavelengths significantly larger than 1 μm or so. So, there will be essentially no VHFGW propagation along the vortices (ignoring evanescent excitations which will decay after a short distance).

Hence, the incident VHFGWs can logically only propagate through the remaining superconducting matrix. (The vortices will extend through the superconductor material in a regular lattice except where they are pinned by a dislocation or grain boundary, so that their properties may be analyzed by examining a perfect vortex lattice.) To find the effective phase velocity of VHFGWs through the superconductor we must therefore find the VHFGW phase velocity through a material containing impenetrable obstructions parallel to the propagation direction. This is a difficult problem to solve accurately and here the approximation is made that this material will act as a waveguide having an effective diameter equivalent to the mean distance between adjacent vortices. Of course, the waveguide properties, including the guide propagation constant, depend upon the lateral dimensions of the guide. The mean vortex separation depends upon the applied magnetic field. Therefore, it is possible to adjust the effective phase velocity through the superconductor by adjusting the applied magnetic field. Increasing the applied field increases the area density of vortices; decreasing the applied field reduces the area density of vortices. In other words, increasing the field reduces the effective guiding diameter and increases the VHFGW phase velocity; decreasing the field increases the effective guiding diameter and reduces the VHFGW phase velocity. It follows that the effective VHFGW phase velocity through the material is closer to c/n_g at lower applied fields, provided the VHFGW wavelength is sufficiently large that the granularity of the vortex structure is not a serious issue.

For VHFGW wavelengths significantly smaller than the typical vortex diameter, the behavior is best described differently. There are two types of paths through the material, i.e. through a vortex or not through a vortex. The paths through a vortex will either have velocity c or will still be beyond waveguide cutoff, so that an overall continuously variable phase velocity will not occur. Instead, the resultant GW will, on passing through the material, and in the near-field, produce a silhouette (or phase silhouette) of the vortex pattern. In other words, the GW merely images the vortex pattern. This situation is not considered further here, since this seems to be of little technological interest, though it may afford a novel technique for mapping the vortex positions in a superconductor.

4. VHFGW propagation in type-II superconductor with applied magnetic field

In the basic structure (Fig. 1) considered here, VHFGW ray #1 travels through type-II superconducting material, while VHFGW ray #2 is incident upon a vortex acting as a GW waveguide beyond cutoff, and becomes evanescent. Therefore, the intensity of ray #2 decays exponentially and will be assumed to play no further part in the device. Ray #1 has phase velocity dependent upon the disposition of vortices in the material. The thickness of superconductor, b, is fixed and is assumed to be significantly greater than the evanescent decay length of VHFGWs within each vortex (this requirement is easy to satisfy since the typical vortex diameter is so small).

Suppose that the magnetic flux through each vortex is Φ_0. For the coupled-electron wavefunction to be single-valued and continuous, it is well known that the value of Φ_0 must be quantized in units of the flux quantum, $h/(2e) = 2.08 \times 10^{-15}$ Wb, where h is Planck's constant and e is the fundamental charge. If Φ_0 takes the same value in vortices anywhere within the entire superconductor, then in an applied field B the local areal density of vortices is given by B/Φ_0 and so the average area occupied by a single vortex is Φ_0/B. Since the vortices generally form a regular

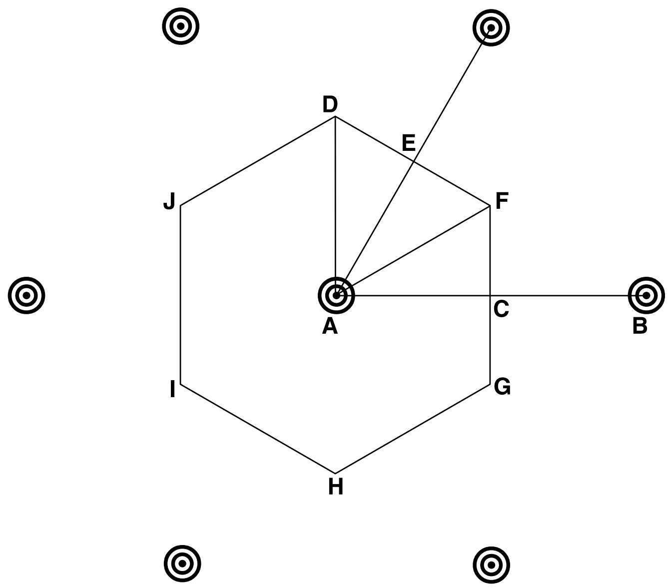

Fig. 3. Idealized magnetic vortex geometry in a typical superconductor. The magnetic field is normal to the page, and vortex positions are indicated by ◎.

triangular lattice (see Fig. 3 [17]), the relation between the vortex separation, d, and the applied magnetic field may be found simply as follows. In Fig. 3, d is the distance AB, and the distances AC and AE are both equal to $d/2$. Since angle EAD is 30°, distance AD is $d/(2\cos 30°) = d/\sqrt{3}$. Now, triangle FAD is equilateral, so distance DF is also $d/\sqrt{3}$, and hence the area of triangle FAD is (AE)(DF)/2 = $d^2/(4\sqrt{3})$. The vortex A is located at the center of the regular hexagon DFGHIJ whose area is six times the area of triangle FAD, or $d^2(\sqrt{3})/2$. This hexagon is the two-dimensional unit cell of the vortex lattice, so the external applied field B must be given by

$$B = 2\Phi_0/(d^2\sqrt{3}). \tag{3}$$

Solving for d gives the distance between vortices,

$$d = \sqrt{(2/\sqrt{3})}\sqrt{(\Phi_0/B)} \approx 1.075\sqrt{(\Phi_0/B)}, \tag{4}$$

which will be much greater than a typical internal vortex diameter.

Accurate calculation of the effective phase velocity of ray #1 appears to be a difficult task since simpler related problems, such as optical propagation through cylindrical optical fibers, have solutions known only approximately or empirically. However, those empirical approximations can be employed in this case.

The V-parameter, or normalized frequency, is used to describe wave propagation through cylindrical fibers [16] and this approach is invoked approximately here for a guiding structure having a diameter approximately given by $d = \sqrt{(2/\sqrt{3})}\sqrt{(\Phi_0/B)}$. The definition of V is

$$V = \pi\frac{d}{\lambda_0}\sqrt{n_g^2 - n_{g2}^2} \approx \frac{\pi d}{\lambda_0}\sqrt{2n_g^2\Delta}, \tag{5}$$

where n_g is the gravitational refractive index of the guiding material, n_{g2} is the effective gravitational refractive index of the material surrounding the guide, λ_0 is the free-space VHFGW wavelength, and Δ is the normalized gravitational refractive index difference defined by

$$\Delta = (n_g - n_{g2})/n_g. \tag{6}$$

Because of the averaged effect of the presence of vortices comprised of magnetically quenched superconductor, n_{g2} is slightly less than n_g and so Δ is small but positive. In general, finding the wave velocity in a waveguide is a difficult problem, but we can use the approximate result for the phase velocity [16]

$$v_{ph} \approx \frac{c/n_g}{1 - \pi^2\Delta/V^2}, \tag{7}$$

so that by using the definition of V, Eq. (5),

$$v_{ph} \approx \frac{c/n_g}{1 - \lambda_0^2/(2n_g^2 d^2)} = \frac{2cn_g}{2n_g^2 - \lambda_0^2/d^2} \tag{8}$$

and substituting the value of the effective guiding diameter, approximately $d = \sqrt{(2/\sqrt{3})}\sqrt{(\Phi_0/B)}$, gives

$$v_{ph} \approx \frac{4cn_g\Phi_0}{4n_g^2\Phi_0 - \sqrt{3}B\lambda_0^2}. \tag{9}$$

(It is interesting to note that the phase velocity is independent of the precise geometry of the vortices.) Finally, the phase shift after traveling through a superconductor of thickness b is given by (wavenumber) × (distance) which, within the approximations described, is

$$\phi = \frac{\omega b}{v_{ph}} \approx \omega\left(\frac{n_g}{c} - \frac{\sqrt{3}\pi^2 Bc}{n_g\omega^2\Phi_0}\right)b \tag{10}$$

$$= \left(\frac{n_g\omega}{c}\right)b - \left(\frac{\sqrt{3}\pi^2 c}{n_g\omega\Phi_0}\right)Bb. \tag{11}$$

As expected, for $B = 0$ this gives

$$\phi = n_g\omega b/c, \tag{12}$$

and the phase shift ϕ can be varied continuously by varying the magnetic field B. (At this point, note that it is not possible to produce a variable phase-shifter simply by making an array of fine SC filaments immersed in a non-SC matrix, since the areal density of the filaments would then be fixed. The field would be expelled by the superconducting filaments and would pass instead through the inert matrix, accomplishing no more than a different fixed gravitational refractive index [7] though with greater fabrication complexity arising from the use of superconducting filaments instead of particles.) The specific excess phase shift (i.e., caused solely by the applied field B, per unit applied magnetic field and per unit propagation length) is given by $-(\sqrt{3})\pi^2 c/(n_g\omega\Phi_0)$. Assuming one flux quantum per vortex, $\Phi_0 = h/(2e)$, the specific phase shift becomes $-2(\sqrt{3})\pi^2 ce/(n_g\omega h)$. At VHFGW frequency 3 GHz, taking $n_g = 300$ [9], and other fundamental physical constants taking their usual values, this expression gives a specific excess phase shift of –0.4 rad μT^{-1} μm^{-1}. This can be regarded as amounting to an increase over the free-space interaction between the electromagnetic and gravitational field quantities, or an "enhanced Gertsenshtein effect" [8] within a type-II superconductor (see above), since the coupling predicted by Eq. (11) between the GW and the applied magnetic field is considerably greater than occurs in free space due to the non-linearity of Einstein's field equations.

There may be some loss of GW power flow through Fresnel reflection at the air–superconductor interfaces. To minimize this in practice an effective anti-reflection coating [7] would be required over both the front and back faces of the superconductor.

The waveguide analysis itself is an approximation since the exact waveguide result is extremely difficult to calculate; in the purely optical case this introduces small errors. There will be a rather larger error associated with approximating the lattice of vortices as a waveguiding structure. However, by far the dominant error source is likely to be the current lack of precision established in the Li and Torr [9,10] result. A very conservative estimate of the systematic error likely in this result is that the slowing factor may be uncertain by as much as $\pm 50\%$, and this will lead to a corresponding uncertainty in the final result for the field-induced phase shift, Eq. (11).

5. Conclusions

Li and Torr [9,10] predicted a substantial reduction in phase velocity for GWs traveling within a SC material. This result has been disputed but currently has not been conclusively disproved.

Since the SC effect in a type-II superconductor is quenched in a sufficiently large magnetic field, the effective phase velocity of VHFGWs is deduced to depend upon the applied field in a SC. Accurate analysis is difficult and here an approximate treatment based upon the well-known analysis of fiber-optic waveguides has been used. This should be accurate to within the limits of the Li and Torr [9,10] result. Numerical simulation may provide a more sophisticated treatment, though probably such a detailed analysis will not be productive until the magnitude of the Li and Torr effect has been established beyond doubt more precisely than it has been at the moment.

The result shows a significant change in the VHFGW propagation properties within the SC as a result of this interaction. Gertsenshtein originally predicted coupling between electromagnetic waves and GW in free space, and so within a SC the Gertsenshtein effect may be regarded as considerably enhanced. The implications of this study will be important in the design of future communications links based upon VHFGW technology, and other applications of VHFGW based upon the Gertsenshtein effect.

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