

The Meissner effect and massive particles as witnesses of macroscopic entanglement

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(February 1, 2008)

We show how the spatial macroscopic entanglement equivalent to the off diagonal long range order (ODLRO) implies the Meissner effect and flux quantisation for a superconductor. It is argued by analogy with superconductors that the Higgs field must also be entangled in the same way. Internal (spin) entanglement is shown to be irrelevant within this context, although it can of course also be computed.

Introduction. We have recently argued that high temperature macroscopic entanglement is possible and linked it to high T_c superconductivity [1]. We have also discussed the relationship between the notion of long range off diagonal order (ODLRO) [2] in a state and the existence of bipartite and multipartite entanglement in the same state. Now we intend to extend this line of thought and show that the said multipartite entanglement implies two typical superconducting effects: the exclusion of the magnetic field from a simply connected superconductor (the Meissner effect) and the quantization of flux in multiply connected regions. Although we will use the Hubbard related models to aid the discussion in this paper, our results hold for any model which displays ODLRO, i.e. normal superconductors and superfluids. The paper will be finished with a speculation, maintaining that if Higgs bosons are found to exist, and if they are responsible for mass generation through the symmetry breaking mechanism, then they must also be entangled. The reason is that the condensation of Higgs bosons is understood to be the most likely mechanism for mass generation in local gauge field theories (i.e. the Standard Model).

Setting the scene. The model we analyse consists of a number of lattice sites, each of which can be occupied by fermions of spin up or spin down. Since fermions obey the Pauli exclusion principle, we can have at most two fermions attached to one and the same site. Let us introduce fermion creation and annihilation operators, $c_{i,s}^\dagger$ and $c_{i,s}$ respectively, where the subscript i refers to the i -th lattice site and s refers for the value of the spin, \uparrow or \downarrow . The c operators satisfy the anticommutation relations: $\{c_{i,s}, c_{j,t}^\dagger\} = \delta_{ij}\delta_{s,t}$, and c 's and c^\dagger 's anticommute as usual. (Some general features of fermionic entanglement were analysed in [3–5]).

We only need assume that our model has the interaction which favours formation of Cooper pairs of fermions

of opposite spin at each site – these states are known as η states [6] and will be discussed below. The actual Hamiltonian is not relevant for our present purposes. Suffice it to say that the η states are eigenstates of the Hubbard and realted models relevant for superconductivity [6,7].

Introducing η states. Suppose that there are n sites and suppose, further, that we introduce an operator

$$\eta^\dagger = \sum_{i=1}^n c_{i,\uparrow}^\dagger c_{i,\downarrow}^\dagger \quad (1)$$

that creates a coherent superposition of a Cooper pair in each of the lattice sites. This η^\dagger operator can be applied to the vacuum a number of times, each time creating a new coherent superposition. However, the number of applications, k , cannot exceed the number of sites, n , since we cannot have more than one pair per site due to the exclusion principle. We now introduce the following basis

$$|k, n-k\rangle := \binom{n}{k}^{-1/2} (\eta^\dagger)^k |0\rangle, \quad (2)$$

where the factor in front is just the necessary normalisation. Here, the vacuum state $|0\rangle$ is annihilated by all c operators, $c_{i,s}|0\rangle = 0$. We note that the originally defined η operators can also have phase factors dependent on the location of the site on the lattice, like so $\eta_k = \sum_n e^{ikn} c_{n,\uparrow}^\dagger c_{n,\downarrow}^\dagger$. All the states generated with any η_k from the vacuum have the same amount of entanglement so that the extra phases will be ignored in the rest of the paper. However, the phase must be chosen, for otherwise, if we average over all possible phases, the resulting state is no longer entangled. Choosing a phase amounts to “symmetry breaking” and we will have more to say about it below.

We can think of the η states in the following way [1]. Suppose that $k = 2$. Then this means that we will be creating two η -pairs in total, but they cannot be created in the same lattice site. The state $|2, n-2\rangle$ is therefore a symmetric superposition of all combinations of creating two pairs at two different sites. Let us, for the moment, use the label 0 when the site is unoccupied and 1 when it is occupied. Then $|2, n-2\rangle = (|00\dots11\rangle + \dots |11\dots000\rangle) / \sqrt{\binom{n}{2}}$, i.e. the state is an equal superposition of states containing 2 states $|1\rangle$ and $n-2$ states $|0\rangle$. These states, due to their high degree of

symmetry, are much easier to handle than general arbitrary superpositions and we can compute entanglement for them between any number of sites [8]. In this description each site effectively holds one quantum bit, whose 0 signifies that the site is empty and 1 signifies that the site is full.

ODLRO. The main characteristic of η states is the existence of ODLRO, which implies its various superconducting features, such as the Meissner effect and the flux quantisation [9]. The ODLRO is defined by the off diagonal matrix elements of the two-site reduced density matrix being finite in the limit when the distance between the sites diverges. Namely,

$$\lim_{|i-j| \rightarrow \infty} \langle c_{j,\uparrow}^\dagger c_{j,\downarrow}^\dagger c_{i,\downarrow} c_{i,\uparrow} \rangle \rightarrow \alpha \quad (3)$$

where α is a constant (independent of n) [2]. We will show that although the existence of off diagonal matrix elements does not guarantee the existence of entanglement between the two sites, it does guarantee the existence of multi-site entanglement between all the sites. Note that here, by ‘‘correlations’’ we mean correlations between the number of electrons positioned at different sites i and j . This is different from spin-spin correlations, which would look at the occurrences of both electron spins being up or down, or one being up and the other being down [4].

The η states are always of the form $|k, n-k\rangle := (\hat{S}|000\dots11\rangle) / \sqrt{\binom{n}{k}}$, where \hat{S} is the total symmetrisation operator. The reason why η states are important for superconductivity is that this phenomenon can be understood to arise through a condensation of Cooper pairs. Condensation means that the temperature is so low that all particles are spread across the whole system – i.e. their wavelengths are as large as the system – so that all their wave functions overlap to a high degree (using the language of the first quantisation). This is why the η states are a good description as they represent equal superpositions of Cooper pairs across all the sites.

We would now like to start to compute the entanglement between every two sites in the state $|k, n-k\rangle$. A simpler and more insightful task would be first to tell if and when every two qubits in a totally symmetric states are entangled. For this, we need only compute the reduced two-qubit density matrix which can be written as:

$$\rho_{12}(k) = a|00\rangle\langle 00| + b|11\rangle\langle 11| + c|\psi^+\rangle\langle \psi^+| \quad (4)$$

where $|\psi^+\rangle = (|00\rangle + |11\rangle) / \sqrt{2}$ and $a = \frac{k(k-1)}{n(n-1)}$, $b = \frac{(n-k)(n-k-1)}{n(n-1)}$ and $c = \frac{2k(n-k)}{n(n-1)}$. We can easily check that $a + b + c = 1$ and so the state is normalised. This density matrix is the same no matter how far the two sites are from each other, since the state is symmetric, and must therefore be identical for all qubits. We can easily test the Peres-Horodecki (partial transposition) condition for

separability of this state [1]. This leads to states $\rho_{12}(k)$ being entangled if and only if $a + b - \sqrt{(a-b)^2 + 4c^2} < 0$, which leads to $(k-1)(n-k-1) < k(n-k)$. This is, of course, satisfied for all $n \geq 2$ and $1 \leq k \leq n-1$. So, apart from the case when the total state is of the form $|000\dots0\rangle$ or $|111\dots1\rangle$, there is always two-qubit entanglement present in symmetric states. Note, however, that in the limit of n and k becoming large – no matter what their ratio may be – the value of the left hand side approaches the value of the right hand side and entanglement thus disappears in the thermodynamical limit [1].

Macroscopic (spatial) entanglement. The two point correlation function used in the calculation of the ODLRO in eq. (3) is, in fact, just one of the 15 numbers we need for the full two-site density matrix. In our simplified case of symmetric states in the η -pairing model, this off diagonal element is equal to c . However, for the density matrix we still need to know a and b , and these numbers clearly affect the amount of entanglement. So, the first lesson is that two-site entanglement is not the same as the existence of ODLRO, and therefore two-site entanglement is not relevant for superconductivity. This does not mean, of course, that there is no entanglement in the whole of the lattice. In fact, the ODLRO implies that the two site density matrix contains classical correlations. This, together with the fact that the overall state of all electrons is pure, means that there is always bipartite entanglement present. For example, entanglement exists in the thermodynamical limit between two bunches of sites containing k and $n-k$ sites respectively, and each bunch containing a site from our two site density matrix [1].

The Meissner effect. Suppose that we exchange two electron pairs, one at the site 1 and one at the site n . We can imagine doing this adiabatically, although the requirement of adiabaticity is by no means necessary (it is merely convenient, as this evolution then generates no other effect apart from the one we wish to concentrate on). Suppose that whatever the total state is, the reduced density matrix of sites 1 and n has a non-vanishing component of the state

$$|\Psi\rangle = |0_1\rangle|1_n\rangle + |1_1\rangle|0_n\rangle. \quad (5)$$

which, from the above discussion, means that we assume ODLRO. Then, after the swap, this component will look like

$$|\Psi\rangle = |0_1\rangle|1_n\rangle + e^{i\Phi}|1_1\rangle|0_n\rangle, \quad (6)$$

where $\Phi = \int A dl$ is the line integral of the vector potential along the path traversed by the electron pair (with proper units introduced below). The reason why the two states in the superposition acquire different phases is that the electron pairs in two states undergo evolutions in opposite directions of each other – this is, in fact, the well

known Aharonov-Bohm phase [10]. So, if in the first state the pair takes one path (i.e. the electron pair from site n moves to site 1), in the second state it takes the reverse of the same path (i.e. the electron pair from site 1 moves to site n). The off-diagonal element in the $|0\rangle, |1\rangle$ basis of the two site density matrix of sites 1 and n undergoes the following transformation $c \rightarrow e^{i\Phi}c$. However, the overall state must be totally symmetric, and therefore

$$e^{i\Phi} = 1. \quad (7)$$

From this we can conclude that

$$\Phi = \frac{2e}{\hbar c} \int Adl = \frac{2e}{\hbar c} \int \int BdS = 2n\pi. \quad (8)$$

But, in the two (three) dimensional space, the electron pair can take any trajectory, and the only possible choice that satisfies the above is $B = 0$ and $n = 0$. Therefore, in a connected region of a material exhibiting bipartite entanglement there is no magnetic field present. This is the Meissner effect. We note that it is well known that the magnetic field does penetrate the superconductor to a very small degree (falling off exponentially with the distance from the surface), but this cannot be explained with the present simple formalism and we need a more elaborate electrodynamic treatment.

Flux Quantisation. Imagine instead that the region is not simply connected and that there is a hole in the middle pierced through by a magnetic field (there could be more than one hole and the same conclusion will hold for each of them). Then its flux must be quantised. This follows immediately from eq. (8), since now not all paths are allowed. Namely, the field is now confined to the region where electrons cannot go, so that

$$\frac{2e}{\hbar c} \int \int BdS = \frac{2e}{\hbar c} \Phi_c \neq 0. \quad (9)$$

Therefore, we must have that $\frac{2e}{\hbar c} \Phi_c = 2n\pi$, and so the flux is quantised in units of $\hbar c/2e$:

$$\Phi_c = n \frac{\hbar c}{2e}. \quad (10)$$

This is the flux quantisation effect. Note that the denominator contains twice the electron charge and this is a consequence of electrons forming Cooper pairs.

The flux quantisation that we have just derived lies behind the persistent flow of electrical current in a superconductor. The flux is a consequence of the flowing current and any (continuous) dissipation cannot change the current continuously as the flux is discrete. Therefore the current persists indefinitely.

Finally, if there is no ODLRO, meaning that as $n \rightarrow \infty$ we have that $c \rightarrow 0$ in eq. (4), then neither of the above two effects follow. Any phase now gained upon exchange of electrons as described above will not be reflected in

the two site state and therefore we cannot argue that this phase has to have a special value. Therefore, bipartite entanglement is necessary for the Meissner effect and flux quantisation. Note briefly that the converse is not true. Not all entanglement will lead to superconductivity. For example, look at the state of two sites of the form: $|00\rangle + |11\rangle$. When we exchange the pairs we get no extra phase in the second ket (because they have opposite signs and so their product equals identity), so that the state remains the same. Therefore, any magnetic field is allowed to permeate such a state. This is why the ODLRO concerns the coherences between states $|01\rangle$ and $|10\rangle$.

Mass from entanglement between Higgs bosons.

We would now like to talk about the Higgs mechanism as the main explanation for the appearance of mass in local gauge field theories (see Weinberg [11] for a comprehensive introduction). In the modern field theory, gauge invariance of the Hamiltonian (or Lagrangian, which is more typically used) is invoked to explain the appearance of fields and their bosonic mediators. By ‘‘gauge transformation’’ we mean a transformation that acts on the wavefunction (or the field, more precisely) in the following manner $|\Psi(x, t)\rangle \rightarrow e^{\theta(x, t)}|\Psi(x, t)\rangle$, where $\theta(x, t)$ is just a phase that depends both on space and time (i.e. it is local). In order for the Hamiltonian to remain invariant under this local change, we need to introduce an extra (vector) field, whose features exactly cancel out the effects of the local phase change. The necessary field turns out to be the electromagnetic field and its bosons are, of course, photons. The important point is that if we are to derive other forces from local gauge invariance (this requires phases that are non-commuting – i.e. matrices, but the concept is the same), the resulting bosons will always be massless. This result is intuitively clear: the local phase change has to be matched between arbitrary points and times and this therefore requires an infinite range force. The mediators of infinite range forces have to be massless. So, it appears that local gauge invariance cannot explain forces whose mediators are massive.

A solution to this problem was found by Higgs [12] (and a number of other people, but Higgs was most prominent). The idea is that in addition to specifying the Hamiltonian of the fields, we also need to specify their actual physical state. This state need not possess the same symmetry properties of the Hamiltonian (hence the phrase ‘‘symmetry breaking’’ [11]). Suppose now that our local gauge invariance leads to several interacting massless fields. Suppose also, that one of the fields – known as the Higgs field – has condensed. In a mechanism that is completely analogous to the Meissner effect, the other fields will now be ‘‘expelled’’ from the Higgs field and will therefore become short range. In other words, their mediators will become massive. The condensation of the Higgs field therefore provides a mechanism to maintain local gauge invariance and have massive gauge fields at the same time, thereby circumventing previously men-

tioned limitation (the Higgs boson also acquires a mass in this process). Whether this is the correct way of explaining the origin of mass in the Universe is still unclear as the search for Higgs bosons has so far been fruitless. However, one conclusion we can draw with more confidence (following this paper) is that if the Higgs field exists, then its bosons must be entangled. The reason is that the ODLRO, necessary for condensation, also implies existence of entanglement, and this is also true for Higgs condensation. The (obvious) fact that there are massive objects in the Universe would then be an entanglement witness of the purely quantum correlations in the underlying Higgs field.

Here we have to exercise some caution. The entanglement in the Higgs field is something that we refer to as “continuous variable entanglement”, as opposed to the discrete degrees of freedom of the Hubbard model above, and this quantity can become infinite. However, this is not a serious problem because with enough care this infinity can always be controlled.

We now show in a very simple example the connection between mass and entanglement that is meant to substantiate the above discussion, but is by no means a proof of it. Suppose we have a massive bosonic free field, ϕ , with the usual Lagrangian density $1/2((\partial_\mu\phi)^2 + m^2\phi^2)$ (this is in $1 + 1$ dimensions), where m is the (fixed finite) mass. This is an infinite continuous system and we divide it into two halves (arbitrarily). Tracing one part out and computing the von Neuman entropy of the remaining part results in the entropy of entanglement of $E \approx \ln 1/m^2 a^2$ [13], where a is some cutoff used to avoid the ultraviolet infinity (this divergence may also be avoided by using the relative entropy with respect to some coarse graining [14], much in the same way as Gibbs did in classical statistical mechanics). The amount of entanglement clearly depends on the mass which could therefore be said to witness it.

Spin Entanglement. We would like to finally point out an interesting curiosity that clarifies the notion of entanglement we have analysed in this paper. Namely, as we mentioned before, the relevant entanglement for superconductivity (and Higgs bosons) is the spatial entanglement between numbers of electrons at different space points. What about the entanglement between the spin degrees of freedom? If the electrons occupy the same site, then they have to be anticorrelated (in the singlet state) because of Pauli’s exclusion principle. If the electrons are on different sites, then they are not spin correlated in the η state (in the BCS model they would be, for a sufficiently small distance [15]). This is because if we measure an electron in one site and then in another all four possibilities for their internal states are equally likely and so there can be no spin correlation present. There is a limit, however, in which the spin entanglement becomes relevant. This is when the interaction between sites dominates the hopping amplitude in the Hubbard

model and in the state where we have one electron per site. Here there is no ODLRO and the state is not superconducting. However, the effective interaction between electrons at neighbouring sites is now of the Heisenberg type, as electrons can still exchange their locations and could at the same time have opposite spins. Therefore, at low temperatures there would be some two site spin entanglement present in the model, which is albeit not important for superconductivity.

Conclusions. In one of our previous publications [1] we argued that macroscopic entanglement exists at high temperatures and is related to high temperature superconductivity. In the present work we showed that the consequences of that entanglement are the standard features of superconductors: the Meissner effect and flux quantisation. Therefore any experiments confirming these two effects are also automatically offering evidence for macroscopic entanglement. We have speculated that if the Higgs mechanism for mass generation is proven to be correct, then the resulting Higgs bosons will be found to be entangled. Be that as it may, one question remains open, both for superconductors, or for any other more general field. Can we extract this existing entanglement and use it for information processing? This would be very useful in practice, and it would seem that natural macroscopic entanglement could offer an infinite amount of quantum non-locality for genuine quantum information processing. This is the subject of an ongoing research.

Acknowledgements. We thank the European Commission and Elsas Spa for financial support.

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