Gyromagnetic Effect in a Superconductor

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The gyromagnetic ratio of a superconducting tin sphere has been measured by the Einstein-DeHaas method. The result is approximately that to be expected on the picture of perfectly free superconducting electrons and is in agreement with the work of Kikoin and Gubar.

I. INTRODUCTION

FOR pure solid superconducting materials, Meissner and Ochsenfeld[1] have shown that the magnetic induction B inside the metal is zero. This implies either perfect diamagnetism or large surface currents in the presence of a magnetic field. In either case, the gyromagnetic effect for a superconductor should be observable.

The gyromagnetic effect has been most satisfactorily observed by the Einstein-DeHaas method. This method uses a torsion pendulum made of the magnetic material and such that a change of magnetization causes a torque and a change in angular momentum.

Kikoin and Gubar[2] carried out just such an experiment on a small superconducting lead sphere. They drove their torsion pendulum with the impulses produced by reversing a vertical magnetic field at the resonant frequency. They calculated the magnitude of these angular impulses from the resulting steady-state amplitude of the system; and in turn they determined that the ratio of the magnetic moment to the mechanical moment of the superconductor was the same as it would be in ordinary diamagnetic materials with a magnetization arising from orbital electron motions alone.

As has been shown by Meissner,[3] the model of perfect diamagnetism used by Kikoin and Gubar gives the same gyromagnetic effect as the picture of surface currents given by the London theory. According to the London picture,[4] the changing magnetic field acts on the positive charge which remains when the superconducting electrons are disregarded. The experiment then serves to measure the product of this positive charge density and the square of the penetration depth.

The report of Kikoin and Gubar did not make completely clear the direction of the effect. Since the electrons go in one direction, and the material sphere, which is observed, goes in the opposite direction rather than being dragged along by the electrons, it is important that this direction be unambiguous. We have therefore repeated their gyromagnetic experiment with a superconducting sphere to demonstrate again both the magnitude and the direction of the effect.

II. EXPERIMENTAL APPARATUS AND PROCEDURE

Figure 1, shows the details of the experimental equipment. The torsion fiber is a tungsten wire 75 cm long and 0.003 in. in diameter, and is kept at room temperature. To the bottom of the fiber is attached an octagonal Lucite cylinder. On each of the eight faces is a front surface mirror about 1 cm^2 in area. The connection between the Lucite cylinder and the tin sphere is made with a glass tube. The tin sphere is one inch in diameter with variations of less than 0.0004 in., and is made from spectroscopically pure tin supplied by Johnson-Mathey Company, London. The sphere is in contact with the helium vapor, but not in contact with the liquid helium. The Helmholtz coils shown are used to neutralize the horizontal component of the earth's field.

The switching of the solenoid current was controlled by a phototube and two-stage, directly coupled dc amplifier which actuated a mechanical relay. A separate contact on the relay was used to shunt out, on alternate half-cycles, a part of the plate resistance of the phototube, in order to compensate for the difference between the pull current and release current of the relay. This arrangement minimized the errors in switching time which otherwise would have occurred. The time for complete field reversal was less than 3×10^{-3} sec corresponding to less than one six-thousandth of the period of the pendulum.

The earth's field was neutralized and the solenoid field made vertical by adjustments similar to those outlined by Kikoin and Gubar. Precision was obtained by using a torsion pendulum with a one-inch magnetized iron sphere of known magnetic moment.

The amplitude and period of the oscillations were measured by observing a hairline focused on a scale 12.5 meters from the torsion pendulum.

When cooling the tin sphere through the transition temperature the earth's field was neutralized and the sphere was set into oscillation with a large amplitude. This oscillation tended to minimize further any possible frozen in moment.[5]

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III. EXPERIMENTAL RESULTS

The results of the experiment confirm those of Kikoin and Gubar. The directions of the angular impulses are the same as one would observe if the effect were that of Faraday induction operating on a sphere composed of positive ions. Referring to Fig. 1, when the sphere is rotating in the clockwise direction as viewed from above, the field shifts from the down direction to the up direction as the oscillator passes through center. Under these switching conditions, Fig. 2 shows the approach to the steady-state condition with driving fields of 100 gauss or 10^{-2} weber/m^2 (curve A) and 51 gauss or 0.51×10^{-2} weber/m^2 (curve B). Just below these curves the zero field case (curve C) is plotted. Curve D shows the effect of reversing the cycling of the switching in the case of the larger field. Also, for the larger field, the approach to the asymptote was made from the low amplitude side, but these points are not included in the figure.

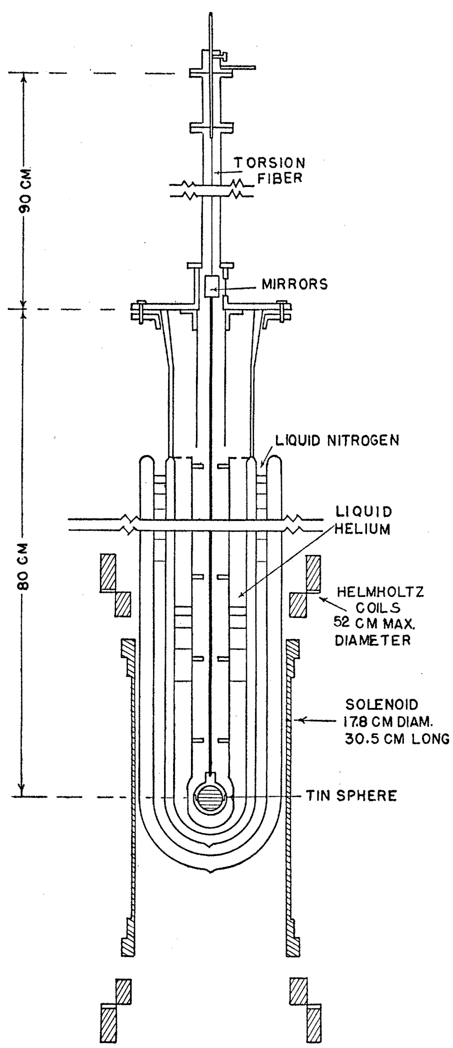

FIG. 1. Schematic outline of the cryostat and torsion pendulum.

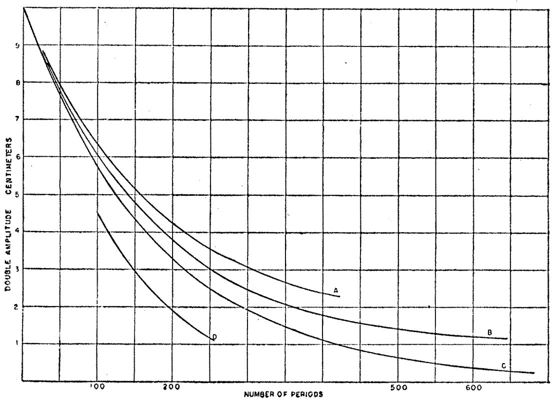

FIG. 2. Decay of the amplitude of vibration toward the limiting values.

The experimental curves show an asymptotic approach to a double amplitude of 0.95 cm at 0.51×10^{-2} weber/m^2 and 1.65 cm with 10^{-2} weber/m^2.

The observed value may be compared with the results of a simple calculation based on consideration of the impulse and momentum change of the damped oscillator.

In the steady state the system behaves like a damped oscillator for $\frac{1}{2}$ cycle. It then receives an angular impulse which just makes up for the frictional loss of the absolute value of the momentum. This frictional loss of momentum is equated to the angular impulse given the oscillator because of the magnetic field change from $+B$ to the value $-B$. The equation of motion for $\frac{1}{2}$ period is

$$\theta=\theta_0 e^{-\delta t}\cos wt,$$

with momentum: $I\dot{\theta}=-I\theta_0(we^{-\delta t}\sin wt+\delta e^{-\delta t}\cos wt)$, where the moment of inertia $I=69.4\times10^{-7}$ kg m^2 and the measured value of $\delta=2.67\times10^{-4}$ sec^{-1}. The loss in momentum in one half-period occurs in the time interval $-\pi/2w$ to $+\pi/2w$, so that the momentum change is

$$\Delta|P|=-I\theta_0\delta\pi+\text{small terms.}$$

Further detailed analysis shows that the steady state is approached exponentially and that the maximum amplitude of the nth oscillation θ_n is given by

$$\theta_n=\theta_0+Ae^{-2\pi n\delta/w}.$$

In the magnetic field B the sphere has a magnetic moment[4]

$$M=2\pi R^3B/\mu_0,$$

so that switching the field from $+B$ to $-B$ causes a change of magnetic moment $4\pi R^3B/\mu_0$. Then the change of angular momentum must be $(2m/e)(4\pi R^3B/\mu_0)$, if $2m/e$ is taken to be the ratio between the angular momentum and the magnetic moment.

Equating the frictional loss of momentum to the gain from the field change, we get a steady-state angular amplitude:

$$\theta_0 = (2m/e)(4\pi R^3 B/\mu_0)(1/I\delta\pi) = 3.87\times10^{-2}B \text{ radian},$$

where B is expressed in webers/m^2.

This angular momentum is that of the superconducting electrons, but since the total angular momentum around the vertical axis does not change, the angular momentum of the positive ions and remaining electrons composing the sphere must change in just the opposite sense and by the same amount. It is this latter change that is observed.

Another way of looking at this phenomenon was suggested by Meissner.[3] The magnetic field penetrates only a short distance below the surface of the sphere, but as it changes, an electric field is present, which acts both on the superconducting electrons and on the remaining positive ions. Since the superconducting electrons do not drag the ions with them, the two systems move independently with equal and opposite angular momenta. The motion of the positive ions is the one observed by the experimental arrangement used.

IV. DISCUSSION OF ERRORS

In spite of much care the tin sphere had a slightly ellipsoidal character, the field was not exactly uniform over the volume of the sphere, and the switching of the magnetic field did not always occur precisely at the same point. In zero field a certain rest point or zero point about which the oscillator swung was noted. With a steady upward vertical field of 10^{-2} weber/m^2, the zero point shifted 0.3 mm to the right on the scale corresponding to 1.2×10^{-5} radian of angle. With a steady downward vertical field of the same amount, the zero point shifted 0.9 mm to the right. These errors might be further reduced with greater effort should greater accuracy justify such action, but differences between the experimental and expected values may be attributed to these experimental errors.

V. CONCLUSION

The experiments gave rough agreement with the theory as to the magnitude of the effect. The result was 15 percent low for the 10^{-2} weber/m^2 driving field and 4 percent low for the 0.51×10^{-2} weber/m^2 driving field. The direction of driving torque was such as to produce an angular impulse on the superconducting sphere just as if the Faraday induction were operating on the positive ion lattice.

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Angular Momentum Distributions in the Thomas-Fermi Model

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The Hartree and Thomas-Fermi models of the atom and nucleus are compared in their predictions for the angular momentum distributions. It is found that the most natural quantity to compare is not the "first appearance" of a given orbital angular momentum, but the mean squared angular momentum.

I. INTRODUCTION

RECENTLY the subject of the distribution of angular momentum in nuclei has been given considerable impetus by the discovery of certain "magic numbers"[1] which indicate a shell structure in nuclei. The interpretation of the magic numbers in terms of a "shell model"[2] enables us to make detailed statements about the angular momentum distribution in nuclei. Several authors[3] have attempted to find an interpretation of this shell structure in terms of a Thomas-Fermi model of the nucleus. Now, apart from objections to their detailed handling of the Thomas-Fermi model, it seems to us that these authors have also interpreted the appearance of closed shells in an incorrect way. They have identified the closing of a shell with the first appearance of a particle with a higher orbital angular momentum than was previously present. It is very easy to see that even in the atomic case these two phenomena are not related. In atoms the closed shells (noble gases) occur at $Z=2$, 10, 18, 36, 54, and 86, while the first appearance of electrons with orbital angular momentum 1, 2, 3 occurs at $Z=5$, 21, and 58, respectively. In the nuclear shells, say for neutrons, the shells close at $N=2$, 8, 20, 28, 50, 82, and 126 while the first appearance of $l=1$, 2, 3, 4, and 5 occur at $N=3$, 9, 21, 41, $\sim$64, $\sim$100. The closing of a shell is a phenomenon which is connected with the filling up of a group of energy levels which are relatively isolated

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