

Superconductors as quantum transducers and antennas for gravitational and electromagnetic radiation

Raymond Y. Chiao

Department of Physics

University of California

Berkeley, CA 94720-7300

(E-mail: chiao@physics.berkeley.edu)

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Abstract

Superconductors will be considered as macroscopic quantum gravitational antennas and transducers, which can directly convert upon reflection a beam of quadrupolar electromagnetic radiation into gravitational radiation, and vice versa, and thus serve as practical laboratory sources and receivers of microwave and other radio-frequency gravitational waves. An estimate of the transducer conversion efficiency on the order of unity comes out of the Ginzburg-Landau theory for an extreme type II, dissipationless superconductor with minimal coupling to weak gravitational and electromagnetic radiation fields, whose frequency is smaller than the BCS gap frequency, thus satisfying the quantum adiabatic theorem. The concept of “the impedance of free space for gravitational plane waves” is introduced, and leads to a natural impedance-matching process, in which the two kinds of radiation fields are impedance-matched to each other around a hundred coherence lengths beneath the surface of the superconductor. A simple, Hertz-like experiment has been performed to test these ideas, and preliminary results will be reported.

I. INTRODUCTION

In 1966, DeWitt [1] considered the interaction of a superconductor with gravitational fields, in particular with the Lense-Thirring field. Starting from the general relativistic Lagrangian for a single electron with a charge e and a mass m , he derived in the limit of weak gravity and slow particles a nonrelativistic Hamiltonian for a single electron in the superconductor, which satisfied the minimal-coupling rule

$$\mathbf{p} \rightarrow \mathbf{p} - e\mathbf{A} - m\mathbf{h}, \quad (1)$$

where \mathbf{p} is the canonical momentum, \mathbf{A} is the usual vector potential, and \mathbf{h} is a gauge-like vector potential formed from the three space-time components g_{i0} of the metric tensor viewed as an ordinary three-vector. Papini [2] in 1967 considered the possibility of the detection the quantum phase shift induced by \mathbf{h} arising from the Lense-Thirring field generated by a nearby rotating massive body, by means of a superconducting interference device (or SQUID) using Josephson junctions. In 1983, Ross [3] derived the modified London equations for a superconductor in a gravitational field, and showed that these equations are consistent with the modified fluxoid quantization condition in a gravitational field found earlier by DeWitt in 1966.

In a series of papers in the early 1980s, Anandan and I considered the possibility of constructing antennas for time-varying Lense-Thirring fields, and thus for gravitational radiation, using Josephson junctions as transducers, in *neutral* superfluid helium analogs of the SQUID using an antenna geometry in the form of a figure 8 superfluid loop, and also an antenna bent into a the form of a baseball seam [4]. In 1985, Anandan [5] considered the possibility of using superconducting circuits as detectors for astrophysical sources of gravitational radiation, but did not mention the possibility of superconductors being efficient emitters, and thus practical laboratory sources of gravity waves, as is being considered here. In 1990, Peng and Torr used the generalized London equations to treat the interaction of a bulk superconductor with gravitational radiation, and concluded that such a superconducting antenna would be many orders of magnitude more sensitive than a Weber bar [6]. There have also been earlier predictions of a modified Meissner effect in the response of superconductors to time-varying Lense-Thirring fields, and hence to gravitational radiation [7][8]. For recent work along these lines, see [9]. These papers, however, also did not consider the possibility of a transducer action between EM and GR radiation mediated by

the superconductor, as is being considered here. Also, the theoretical approach taken here is quite different, as our approach will be based on the Ginzburg-Landau theory of superconductivity, and the resulting constitutive relation for the gravitomagnetic field, rather than on the modified London equations.

Here, I shall show that Josephson junctions, which are difficult to implement experimentally, are unnecessary, and that a superconductor can by itself be a *direct* transducer from electromagnetic to gravitational radiation upon reflection of the wave from a vacuum-superconductor interface, with a surprisingly good conversion efficiency. By reciprocity, this conversion process can be reversed, so that gravitational radiation can also be converted upon reflection into electromagnetic radiation from the same interface, with equal efficiency. The geometry of a superconducting slab-shaped antenna proposed here is much simpler than some of the earlier proposed antenna geometries. These developments suggest the possibility of a simple, Hertz-like experiment, in which the emission and the reception of gravitational radiation at microwave frequencies can be implemented by means of a pair of superconductors used as transducers. Preliminary results of a first experiment will be reported here.

II. SUPERCONDUCTORS AS ANTENNAS FOR GRAVITATIONAL RADIATION

A strong motivation for performing the quantum calculation to be given below, is that it predicts a large, counterintuitive *quantum rigidity* of a macroscopic wavefunction, such as that in a big piece of superconductor, when it interacts with externally applied gravitational radiation fields. Mathematically, this quantum rigidity corresponds to the statement that the macroscopic wavefunction of the superconductor must remain *single-valued* at all times during the changes arising from adiabatic perturbations due to radiation fields. This implies that objects such as superconductors should be much better gravitational-wave antennas than Weber bars [4][5][6].

The rigidity of the macroscopic wavefunction of the superconductor originates from the instantaneous Einstein-Podolsky-Rosen (EPR) correlations-at-a-distance in the behavior of a Cooper pair of electrons in the Bardeen-Cooper-Schrieffer (BCS) ground state in distant parts of the superconductor viewed as a single quantum system, when there exists some

kind of gap, such as the BCS gap, which keeps the entire system adiabatically in its ground state during perturbations due to radiation. Two electrons which are members of a single Cooper pair are in a Bohm singlet state, and hence are quantum-mechanically entangled with each other, in the sense that they are in a superposition state of opposite spins *and* opposite momenta. This quantum entanglement gives rise to EPR correlations at long distance scales within the superconductor. The electrons in a superconductor in its ground BCS state are not only *macroscopically* entangled, but due to the existence of the BCS gap which separates the BCS ground state energetically from all excited states, they are also *protectively* entangled, in the sense that this entangled state is protected by the presence of the BCS gap from decoherence arising from the thermal environment, provided that the system temperature is kept well below the BCS transition temperature.

The resulting large quantum rigidity is in contrast to the tiny rigidity of classical matter, such as that of the normal metals used in Weber bars, in their response to gravitational radiation. The essential difference between quantum and classical matter is that there can exist macroscopic quantum interference, and hence macroscopic quantum coherence, throughout the entire quantum system, which is absent in a classical system. One manifestation of the tiny rigidity of classical matter is the fact that the speed of sound in a Weber bar is typically five orders of magnitude less than the speed of light. In order to transfer energy coherently from a gravitational wave by classical means, for example, by acoustical modes inside the bar to some local detector, e.g., a piezoelectric crystal glued to the middle of the bar, the length scale of the Weber bar L is limited to a distance scale on the order of the speed of sound times the period of the gravitational wave, i.e., an acoustical wavelength λ_{sound} , which is typically five orders of magnitude smaller than the gravitational radiation wavelength λ to be detected. This makes the Weber bar, which is thereby limited in its length to $L \simeq \lambda_{sound}$, much too short an antenna to couple efficiently to free space.

However, macroscopic quantum objects such as superconductors used as antennas are not limited by these classical considerations, but can have a length scale L on the same order as (or even much greater than) the gravitational radiation wavelength λ . Since the radiation efficiency of a quadrupole antenna scales as the length of the antenna L to the fourth power when $L \ll \lambda$, quantum antennas should be much more efficient in coupling to free space than classical ones like the Weber bar by at least a factor of $(\lambda/\lambda_{sound})^4$. Also, we shall see below that a certain type of superconductor may be a *transducer* for directly converting

gravitational waves into electromagnetic waves, and vice versa; this then dispenses altogether with the necessity of the use of piezoelectric crystals as transducers.

Weinberg [10] gives a measure of the efficiency of coupling of a Weber bar antenna of mass M , length L , and velocity of sound v_{sound} , in terms of a branching ratio for the emission of gravitational radiation by the Weber bar, relative to the emission of heat, i.e., the ratio of the *rate* of emission of gravitational radiation Γ_{grav} relative to the *rate* of the decay of the acoustical oscillations into heat Γ_{heat} , which is given by

$$\eta \equiv \frac{\Gamma_{grav}}{\Gamma_{heat}} = \frac{64GMv_{sound}^4}{15L^2c^5\Gamma_{heat}} \simeq 3 \times 10^{-34}. \quad (2)$$

The quartic power dependence of the efficiency η on the velocity of sound v_{sound} arises from the quartic dependence of the coupling efficiency to free space of a quadrupole antenna upon its length L , when $L \ll \lambda$.

Assuming for the moment that the quantum rigidity of a superconductor allows us to replace the velocity of sound v_{sound} by the speed of light c (i.e., that the typical size L of a quantum antenna bar can become as large as the wavelength λ), we see that superconductors can be more efficient than Weber bars, based on the v_{sound}^4 factor alone, by twenty orders of magnitude, i.e.,

$$\left(\frac{c}{v_{sound}}\right)^4 \simeq 10^{20}. \quad (3)$$

Thus, even if it should turn out that superconducting antennas in the final analysis are still not very efficient *generators* of gravitational radiation, they should be much more efficient *receivers* of this radiation than Weber bars for detecting astrophysical sources of gravitational radiation [5][6]. However, I shall give arguments below as to why under certain circumstances involving “natural impedance matching” between quadrupolar EM and GR plane waves upon a mirror-like reflection at the planar surface of extreme type II, dissipationless superconductors, the efficiency of such superconductors used as simultaneous transducers and antennas for gravitational radiation, might in fact become of the order of unity, so that a gravitational analog of Hertz’s experiment might then become possible.

But why should the speed of sound in superconductors, which is not much different from that in normal metals, not also characterize the rigidity of a superconducting metal when it is in a superconducting state? What is it about a superconductor in its superconducting state that makes it so radically different from the same metal when it is in a normal state? The answer lies in the important distinction between *longitudinal* and *transverse* mechan-

ical excitations of the superconductor. Whereas longitudinal, compressional sound waves propagate in superconductors at normal sound speeds, transverse excitations, such as those induced by a gravitational plane wave incident normally on a slab of superconductor in its superconducting state, cannot so propagate.

Suppose that the opposite were true, i.e., that there were *no* substantial difference between the response of a superconductor and a normal metal to gravitational radiation. (Let us assume for the moment the complete absence of any electromagnetic radiation.) Then the interaction of the superconductor with gravitational radiation, either in its normal or in its superconducting state, will be completely negligible, as is indicated by Eq. (2). We would then expect the gravitational wave to penetrate deeply into the interior of the superconductor (see Figure 1).

The motion of a Cooper pair deep inside the superconductor (i.e., deep on the scale of the London penetration depth λ_L given by Eq. (62)), would then be characterized by a velocity $\mathbf{v}_{pair}(t)$ which would not be appreciably different from the velocity of a normal electron or of a nearby lattice ion (we shall neglect the velocity of sound in this argument, since $v_{sound} \ll c$). Hence locally, by the weak equivalence principle, Cooper pairs, normal electrons, and lattice ions (i.e., independent of the charges and masses of these particles) would all undergo free fall together, so that

$$\mathbf{v}_{pair}(t) = -\mathbf{h}(t), \quad (4)$$

where $\mathbf{v}_{pair}(t)$ is the local velocity of a Cooper pair, and where $-\mathbf{h}(t)$ is the local velocity of a classical test particle, whose motion is induced by the presence of the gravitational wave, as seen by an observer sitting in an inertial frame located at the center of mass of the superconductor. Then the *curl* of the velocity field $\mathbf{v}_{pair}(t)$ deep inside the superconductor, as seen by this observer, would be nonvanishing

$$\nabla \times \mathbf{v}_{pair}(t) = -\nabla \times \mathbf{h}(t) = -\mathbf{B}_G(t) \neq 0, \quad (5)$$

since the Lense-Thirring or gravitomagnetic field $\mathbf{B}_G(t)$ of gravitational radiation does not vanish deep in the interior of the superconductor when gravitational radiation is present (we shall see presently that the $\mathbf{h}(t)$ field plays the role of a gravitomagnetic vector potential, just like the vector potential $\mathbf{A}(t)$ in the electromagnetic case).

However, this leads to a contradiction. It is well known that for adiabatic perturbations (e.g., for gravity waves whose frequencies are sufficiently far below the BCS gap frequency,

so that the entire quantum system remains *adiabatically* in its ground state), the superfluid velocity field $\mathbf{v}_{pair}(t)$ deep in the interior of the superconductor (i.e., at a depth much greater than the London penetration depth) must remain *irrotational* at all times [11], i.e.,

$$\nabla \times \mathbf{v}_{pair}(t) = 0. \quad (6)$$

Otherwise, if this irrotational condition were not satisfied in the presence of gravitational radiation, the wavefunction would not remain single-valued. Deep inside the superconductor, $\mathbf{v}_{pair}(t) = \frac{\hbar}{m_2} \nabla \phi(t)$, where m_2 is the mass of the Cooper pair. Thus the superfluid or Cooper pair velocity is directly proportional to the spatial gradient of the phase $\phi(t)$ of the Cooper-pair condensate wavefunction. It would then follow from such a supposed violation of the irrotational condition that

$$\begin{aligned} \Delta\phi(t) &= \oint_C \nabla\phi(t) \cdot d\mathbf{l} = \frac{m_2}{\hbar} \oint_C \mathbf{v}_{pair}(t) \cdot d\mathbf{l} \\ &= \frac{m_2}{\hbar} \int \int_{S(C)} \nabla \times \mathbf{v}_{pair}(t) \cdot d\mathbf{S} = -\frac{m_2}{\hbar} \int \int_{S(C)} \mathbf{B}_G(t) \cdot d\mathbf{S} \neq 0, \end{aligned} \quad (7)$$

where $\Delta\phi(t)$ is the phase difference after one round-trip back to the same point around an arbitrary closed curve C deep inside the superconductor in its ground state, and $S(C)$ is a surface bounded by C (see Figure 1). We shall see that the phase shift $\Delta\phi(t)$ is a gauge-invariant quantity (see Eq.(13)), and that, for small circuits C , it is directly proportional to a nonvanishing component of the Riemann curvature tensor. However, that the round-trip phase shift $\Delta\phi(t)$ is nonvanishing in the ground state wavefunction of any quantum system, is impossible in QM due to the *single-valuedness* of the wavefunction in the local inertial frame of the center-of-mass of the system.

There is extensive experimental evidence that the single-valuedness of the wavefunction in QM is not violated. For example, the observed quantization of orbital angular momentum in atoms and molecules constitutes such evidence on microscopic length scales. Also, the observations of quantization of the circulation of vortices in both superfluids helium of isotope 3 and helium of isotope 4, and of the quantization of flux in superconductors, constitute such evidence on macroscopic length scales. As a special case of the latter when the topological winding number is zero, the Meissner effect is itself evidence for the validity of the principle of the single-valuedness of the macroscopic wavefunction.

Therefore, in the presence of a gravity wave, Cooper pairs *cannot* undergo free fall, and the *transverse* excitations of the Cooper pair condensate must remain rigidly irrotational at all

times in the adiabatic limit. This leads to a Meissner-like effect in which the Lense-Thirring field $\mathbf{B}_G(t)$ is expelled, and would seemingly lead to infinite velocities for the transverse excitations inside the superconductor. (It should be noted here that superluminal, i.e., faster-than- c , infinite, and negative, group velocities for wave packet excitations in a wide variety of classical and quantum settings have been predicted and observed [12]. An analytic function, e.g., a Gaussian wave packet, contains sufficient information in its early tail such that a causal medium can, during its propagation, reconstruct the entire wave packet with a superluminal pulse advancement, and with little distortion. Relativistic causality only forbids the *front* velocity, which connects causes to their effects, from exceeding the speed of light c , but does not forbid a wave packet's *group* velocity from being superluminal.) However, such transverse excitations should be coupled to perturbations of the metric of spacetime through the Maxwell-like equations for the time-varying gravitational fields to be discussed below, and then the speed of such excitations may turn out to be governed by the vacuum speed of light c .

III. MEISSNER-LIKE EFFECT IN THE RESPONSE OF A SUPERCONDUCTOR TO GRAVITATIONAL RADIATION

The calculation of the *quantum* response of large objects, for example, a big piece of superconductor, to weak gravitational radiation, is based on the concept of *wavefunction*, or *quantum state*, for example, the BCS state, and proceeds along completely different lines from the calculation for the *classical* response of a Weber bar to this radiation, which is based on the concept of *geodesic*, or *classical trajectory* [13]. When the frequency of the gravitational radiation is much less than the BCS gap, the entire superconductor should evolve in time in accordance with the quantum adiabatic theorem, and should therefore stay rigidly, i.e., adiabatically, in its ground state. There results a large, diamagnetic-like *linear response* of the entire superconductor to externally applied, time-varying gravitational fields. This Meissner-like effect does not alter the geodesic center-of-mass motion of the superconductor, but radically alters the internal behavior of its electrons, which are all radically delocalized due instantaneous EPR-correlations within the superconductor, even at large distances.

A. Calculation of diamagnetic-like coupling energies: The interaction Hamiltonian

Consider a gravitational plane wave propagating along the z axis, which impinges at normal incidence on a piece of superconductor in the form of a large circular slab of radius r_0 and of thickness d . Let the radius r_0 be much larger than the wavelength λ of the plane wave, so that one can neglect diffraction effects. Similarly, let d be much thicker than λ . For simplicity, let the superconductor be at a temperature of absolute zero, so that only quantum effects need to be considered. The calculation of the coupling energy of the superconductor in the simultaneous presence of both electromagnetic and gravitational fields starts from the Lagrangian for a single particle of rest mass m and charge e (i.e., an electron, but neglecting its spin)

$$L = -m(-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu)^{1/2} + eA_\mu\dot{x}^\mu, \quad (8)$$

from which a minimal-coupling form of the Hamiltonian for an electron in a superconductor, in the limit of *weak* gravitational fields and *low* velocities, has been derived by DeWitt [1]. Here we apply this minimal-coupling Hamiltonian to *pairs* of electrons (i.e, Cooper pairs in spin-zero singlet states),

$$H = \frac{1}{2m_{2eff}} (\mathbf{p} - e_2\mathbf{A} - m_2\mathbf{h})^2 \text{ in SI units,} \quad (9)$$

where $m_2 = 2m_e$ is the rest mass of a Cooper pair, m_{2eff} is its effective mass, $e_2 = 2e$ is its charge, \mathbf{p} is its canonical momentum, \mathbf{A} is the electromagnetic vector potential, and \mathbf{h} is the gravitomagnetic vector potential, which is the gravitational analog of \mathbf{A} in the case of weak gravity. The gravitomagnetic vector potential \mathbf{h} is the three-velocity formed from the space-time components h_{i0} of the small deviations of the metric tensor $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ from flat spacetime (the metric tensor being given by $g_{\mu\nu}$, and the Minkowski tensor for flat spacetime being given by $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$). Thus we shall define

$$\mathbf{h}|_i \equiv h_{i0}c. \quad (10)$$

It is convenient for performing this calculation to choose the radiation gauge for both \mathbf{A} and \mathbf{h} , so that

$$\nabla \cdot \mathbf{A} = \nabla \cdot \mathbf{h} = 0, \quad (11)$$

where the chosen coordinate system is that of an inertial frame which coincides with the freely-falling center of mass of the superconductor at the origin (this is *not* the transverse-

traceless gauge choice). The physical meaning of \mathbf{h} is that, apart from a sign, it is the three-velocity of a local, freely-falling test particle as seen by an observer in an inertial frame located at the center of mass of the superconductor. In Eq. (9), we have neglected for the moment the interactions of the Cooper pairs with each other.

Why not use the standard transverse-traceless gauge in order to perform these calculations? The answer is given in Figure 1, in which we depict a side view of a snapshot of a gravitational plane wave propagating to the right. The arrows indicate the instantaneous velocity vectors $-\mathbf{h}$ of the test particles induced by the wave, as seen by an inertial observer at the center-of-mass. Note that the gravitational tidal forces reverse in sign after a propagation by half a wavelength to the right along the \mathbf{k} axis. Therefore, by inspection of the diagram, we see that the circulation integrals around circuits C_1 and C_2

$$\oint_{C_1} \mathbf{h} \cdot d\mathbf{l} \neq 0, \text{ and } \oint_{C_2} \mathbf{h} \cdot d\mathbf{l} \neq 0 \quad (12)$$

do not vanish. These circulation integrals are gauge-invariant quantities, since there are related to the gauge-invariant general relativistic time shift Δt (and the corresponding quantum phase shift), where

$$\Delta t = - \oint_C \frac{g_{0i} dx^i}{g_{00}} \approx \frac{1}{c} \oint_C \mathbf{h} \cdot d\mathbf{l} \neq 0. \quad (13)$$

For weak gravity, the time shift Δt is related through Stokes' theorem (see Eq. (7)) to the *flux* of the gravitomagnetic (or Lense-Thirring) field through the circuit C , which, for small circuits C , is directly proportional to the nonvanishing Riemann curvature tensor component R_{0i0j} , where the spatial indices i and j are to be contracted with those corresponding to the small area element enclosed by the circuit C (see Figure 1).

Since in the transverse-traceless gauge, \mathbf{h} is chosen to be identically zero, there would be no way to satisfy Eqs. (12) and (13). In the long-wavelength limit, i.e., in the case where the antenna is much shorter than a wavelength, such as in Weber bars, the transverse-traceless gauge can be a valid and more convenient choice than the radiation gauge being used here. However, we wish to be able to consider the case of superconducting slabs which are thick compared with a wavelength, where the long-wavelength approximation breaks down, and therefore we cannot use the transverse-traceless gauge, but must use the radiation gauge instead.

The electromagnetic vector potential \mathbf{A} in the above minimal-coupling Hamiltonian gives rise to Aharonov-Bohm interference. In like manner, the gravitomagnetic vector potential

\mathbf{h} gives rise to a general relativistic twin paradox for rotating coordinate systems and for Lense-Thirring fields given by Eq. (13). Therefore \mathbf{h} gives rise to Sagnac interference in both light and matter waves. The Sagnac effect has recently been observed in superfluid helium interferometers using Josephson junctions, and has been used to detect the Earth's rotation around its polar axis [14].

From the above Hamiltonian, we see that the minimal coupling rule for Cooper pairs now becomes

$$\mathbf{p} \rightarrow \mathbf{p} - e_2 \mathbf{A} - m_2 \mathbf{h} \quad (14)$$

in the simultaneous presence of electromagnetic (EM) and weak general relativistic (GR) fields. This minimal-coupling rule has been experimentally tested in the case of a uniformly rotating superconducting ring, since it predicts the existence of a London magnetic moment for the rotating superconductor, in which magnetic flux is generated through the center of the ring due to its rotational motion with respect to the local inertial frame. The consequences of the above minimal coupling rule for the slightly different geometry of a uniformly rotating superconducting sphere can be easily worked out as follows: Due to the single-valuedness of the wavefunction, the Aharonov-Bohm and Sagnac phase shifts deep inside the bulk of the spherical superconductor (i.e., in the interior far away from the surface) arising from the \mathbf{A} and the \mathbf{h} terms, must cancel each other exactly. Thus the minimal coupling rule leads to a relationship between the \mathbf{A} and the \mathbf{h} fields inside the bulk given by

$$e_2 \mathbf{A} = -m_2 \mathbf{h} . \quad (15)$$

This relationship in turn implies that a uniform magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$, where $\mathbf{A} = \frac{1}{2} \mathbf{B} \times \mathbf{r}$ in the symmetric gauge, will be generated in the interior of the spherical superconductor due to its uniform rotational motion at an angular velocity $\boldsymbol{\Omega}$ with respect to the local inertial frame, where $\mathbf{h} = \boldsymbol{\Omega} \times \mathbf{r}$ in the rotating frame. Thus the London moment effect will manifest itself here as a uniform magnetic field \mathbf{B} in the interior of the rigidly rotating sphere, which can be calculated by taking the curl of both sides of Eq. (15), and yields

$$\mathbf{B} = -\frac{2m_2}{e_2} \boldsymbol{\Omega} , \quad (16)$$

which is consistent with Larmor's theorem. In general, the proportionality constant of the London moment will be given by the inverse of the charge-to-mass ratio e_2/m_2 , where m_2

has been experimentally determined to be the *vacuum* value of the Cooper pair rest mass, apart from a small discrepancy of the order of ten parts per million, which has not yet been completely understood [15].

However, in the above argument, we have been assuming rigid-body rotation for the entire body of the superconductor, which is obviously not valid for microwave-frequency gravitational radiation fields, since the lattice cannot respond to such high frequencies in such a rigid manner. Hence the above analysis applies only to time-independent (i.e., magnetostatic) and spatially homogeneous (i.e., uniform) magnetic fields and steady rotations, and is not valid for the high-frequency, time-dependent, and spatially inhomogeneous case of the interaction of gravitational and electromagnetic radiation fields near the surface of the superconductor, since the above magnetostatic analysis ignores the boundary-value and impedance-matching problems for radiation fields at the vacuum-superconductor interface, which will be considered below.

One can generalize the above time-independent minimal-coupling Hamiltonian to adiabatic time-varying situations as follows:

$$H = \frac{1}{2m_{2eff}} (\mathbf{p} - e_2\mathbf{A}(t) - m_2\mathbf{h}(t))^2 , \quad (17)$$

where $\mathbf{A}(t)$ and $\mathbf{h}(t)$ are the vector potentials associated with low-frequency electromagnetic and gravitational radiation fields, for example. (This time-dependent Hamiltonian can also of course describe low-frequency time-varying tidal and Lense-Thirring fields, as well as radiation fields, but the adiabatic approximation can still be valid for radiation fields oscillating at high microwave frequencies, since the BCS gap frequency of many superconductors lie in the far-infrared part of the spectrum.) Again, it is natural to choose the radiation gauge for both $\mathbf{A}(t)$ and $\mathbf{h}(t)$ vector potentials, Eq. (11), in the description of these time-varying fields. The physical meaning of $\mathbf{h}(x, y, z, t) \equiv \mathbf{h}(t)$ is that it is the *negative* of the time-varying three-velocity field $\mathbf{v}_{test}(x, y, z, t)$ of a system of noninteracting, locally freely-falling classical test particles as seen by the observer sitting in an inertial frame located at the center of mass of the superconductor. At first, we shall treat both $\mathbf{A}(t)$ and $\mathbf{h}(t)$ as classical fields, but shall treat the matter, i.e., the superconductor, quantum mechanically, in the standard semiclassical approximation.

The time-dependent Hamiltonian given by Eq. (17) is, I stress, only a “guessed” form of the Hamiltonian, whose ultimate justification must be an experimental one. In case of

the time-dependent vector potential $\mathbf{A}(t)$, there have already been many experiments which have justified this “guess,” but there have been no experiments which have tested the new term involving $\mathbf{h}(t)$. However, one justification for this new term is that in the static limit, this “guessed” Hamiltonian goes over naturally to the magnetostatic minimal-coupling form, which, as we have seen above, *has* been tested experimentally.

From Eq. (17), we see that the time-dependent generalization of the minimal-coupling rule for Cooper pairs is

$$\mathbf{p} \rightarrow \mathbf{p} - e_2 \mathbf{A}(t) - m_2 \mathbf{h}(t). \quad (18)$$

It would be hard to believe that one is allowed to generalize \mathbf{A} to $\mathbf{A}(t)$, but that somehow one is *not* allowed to generalize \mathbf{h} to $\mathbf{h}(t)$.

One important consequence that follows immediately from expanding the square in Eq. (17) is that there exists a cross-term [16]

$$H_{int} = \frac{1}{2m_{2eff}} \{2e_2 m_2 \mathbf{A}(t) \cdot \mathbf{h}(t)\} = \left(\frac{m_2}{m_{2eff}} \right) e_2 \mathbf{A}(t) \cdot \mathbf{h}(t). \quad (19)$$

It should be emphasized that Newton’s constant G does not enter here. The physical meaning of this interaction Hamiltonian H_{int} is that there should exist a *direct* coupling between electromagnetic and gravitational radiation mediated by the superconductor that involves the charge e_2 as its coupling constant, in the quantum limit. Thus the strength of this coupling is electromagnetic, and not gravitational, in its character. Furthermore, the $\mathbf{A} \cdot \mathbf{h}$ form of H_{int} implies that there should exist a *linear and reciprocal* coupling between these two radiation fields mediated by the superconductor. This implies that the superconductor should be a *quantum-mechanical transducer* between these two forms of radiation, which can, in principle, convert power from one form of radiation into the other, and vice versa, with equal efficiency.

We can see more clearly the significance of the interaction Hamiltonian H_{int} once we convert it into second quantized form and express it in terms of the creation and annihilation operators for the positive frequency parts of the two radiation fields, as in the theory of quantum optics, so that in the rotating-wave approximation

$$H_{int} \propto a^\dagger b + b^\dagger a \quad (20)$$

where the annihilation operator a and the creation operator a^\dagger of a single classical mode of the electromagnetic radiation field, obey the commutation relation $[a, a^\dagger] = 1$, and where

the annihilation operator b and the creation operator b^\dagger of a matched single classical mode of the gravitational radiation field, obey the commutation relation $[b, b^\dagger] = 1$. (This represents a crude, first attempt at quantizing the gravitational field, which applies only in the case of weak gravity.) The first term $a^\dagger b$ then corresponds to the process in which a graviton is annihilated and a photon is created inside the superconductor, and similarly the second term $b^\dagger a$ corresponds to the reciprocal process, in which a photon is annihilated and a graviton is created inside the superconductor. Energy is conserved by both of these processes. Time-reversal symmetry, and hence reciprocity, is also respected by this interaction Hamiltonian.

B. Calculation of diamagnetic-like coupling energies: The macroscopic wavefunction

At this point, we need to introduce the purely quantum concept of *wavefunction*, in conjunction with the quantum adiabatic theorem. To obtain the response of the superconductor, we must make explicit use of the fact that the ground state wavefunction of the system is unchanged (i.e., “rigid”) during the time variations of both $\mathbf{A}(t)$ and $\mathbf{h}(t)$. The condition for validity of the quantum adiabatic theorem here is that the frequency of the perturbations $\mathbf{A}(t)$ and $\mathbf{h}(t)$ must be low enough compared with the BCS gap frequency of the superconductor, so that no transitions are permitted out of the BCS ground state of the system into any of the excited states of the system. However, “low enough” can, in practice, still mean quite high frequencies, e.g., microwave frequencies in the case of high T_c superconductors, so that it becomes practical for the superconductor to become comparable in size to the microwave wavelength λ .

Using the quantum adiabatic theorem, one obtains in first-order perturbation theory the coupling energy $\Delta E_{int}^{(1)}$ of the superconductor in the simultaneous presence of both $\mathbf{A}(t)$ and $\mathbf{h}(t)$ fields, which is given by

$$\Delta E_{int}^{(1)} = \left(\frac{m_2}{m_{2eff}} \right) \langle \psi | e_2 \mathbf{A}(t) \cdot \mathbf{h}(t) | \psi \rangle =$$

$$\left(\frac{m_2}{m_{2eff}} \right) \int_V dx dy dz \psi^*(x, y, z) \mathbf{A}(x, y, z, t) \cdot \mathbf{h}(x, y, z, t) \psi(x, y, z) \quad (21)$$

where

$$\psi(x, y, z) = \left(N / \pi r_0^2 d \right)^{1/2} = \text{Constant} \quad (22)$$

is the Cooper-pair condensate wavefunction (or Ginzburg-Landau order parameter) of a homogeneous superconductor of volume V [17], the normalization condition having been imposed that

$$\int_V dx dy dz \psi^*(x, y, z) \psi(x, y, z) = N, \quad (23)$$

where N is the total number of Cooper pairs in the superconductor. Assuming that both $\mathbf{A}(t)$ and $\mathbf{h}(t)$ have the same (“+”) polarization of quadrupolar radiation, and that both plane waves impinge on the slab of superconductor at normal incidence, then in Cartesian coordinates,

$$\mathbf{A}(t) = (A_1(t), A_2(t), A_3(t)) = \frac{1}{2}(x, -y, 0)A_+ \cos(kz - \omega t) \quad (24)$$

$$\mathbf{h}(t) = (h_1(t), h_2(t), h_3(t)) = \frac{1}{2}(x, -y, 0)h_+ \cos(kz - \omega t). \quad (25)$$

One then finds that the time-averaged interaction or coupling energy in the rotating-wave approximation between the electromagnetic and gravitational radiation fields mediated by the superconductor is

$$\overline{\Delta E_{int}^{(1)}} = \frac{1}{16} \left(\frac{m_2}{m_{2eff}} \right) N e_2 A_+ h_+ r_0^2. \quad (26)$$

Note the presence of the factor N , which can be very large, since it can be on the order of Avogadro’s number N_0 .

The calculation for the above coupling energy $\overline{\Delta E_{int}^{(1)}}$ proceeds along the same lines as that for the Meissner effect of the superconductor, which is based on the diamagnetism term H_{dia} in the expansion of the *same* time-dependent minimal-coupling Hamiltonian, Eq. (17), given by

$$H_{dia} = \frac{1}{2m_{2eff}} \{e_2 \mathbf{A}(t) \cdot e_2 \mathbf{A}(t)\}. \quad (27)$$

This leads to an energy shift of the system, which, in first-order perturbation theory, again in the rotating-wave approximation, is given by

$$\overline{\Delta E_{dia}^{(1)}} = \frac{1}{32m_{2eff}} N e_2^2 A_+^2 r_0^2. \quad (28)$$

Again, note the presence of the factor N , which can be on the order of Avogadro’s number N_0 . From this expression, we can obtain the *diamagnetic susceptibility* of the superconductor. We know from experiment that the size of this energy shift is sufficiently large to cause a complete expulsion of the magnetic field from the interior of the superconductor, i.e., a Meissner effect. Hence there must also be a complete reflection of the electromagnetic wave

from the interior of the superconductor, apart from a thin surface layer of the order of the London penetration depth. All forms of diamagnetism, including the Meissner effect, are purely quantum effects.

Similarly, there is a “gravitodiamagnetic” term H_{Gdia} in the expansion of the *same* minimal-coupling Hamiltonian given by

$$H_{Gdia} = \frac{1}{2m_{2eff}} \{m_2 \mathbf{h}(t) \cdot m_2 \mathbf{h}(t)\}. \quad (29)$$

This leads to a gravitodiamagnetic energy shift of the system given in first-order perturbation theory in the rotating-wave approximation by

$$\overline{\Delta E_{Gdia}^{(1)}} = \frac{1}{32m_{2eff}} N m_2^2 h_+^2 r_0^2. \quad (30)$$

From this expression, we can obtain the *gravitodiamagnetic susceptibility* of the superconductor.

IV. THE IMPEDANCE OF FREE SPACE FOR GRAVITATIONAL PLANE WAVES

It is not enough merely to calculate the coupling energy arising from the interaction Hamiltonian given by Eq. (26). We must also compare how large this coupling energy is with respect to the free-field energies of the uncoupled problem, in particular, that of the gravitational radiation, in order to see how big an effect we expect to see in the gravitational sector. To this end, I shall introduce the concept of *impedance matching*, both between the superconductor and free space in both forms of radiation, and also between the two kinds of waves inside the superconductor viewed as a transducer. The impedance matching problem determines the *efficiency of power transfer* from the antenna to free space, and from one kind of wave to the other. It is therefore useful to introduce the concept of the *impedance of free space* Z_G for a gravitational plane wave, which is analogous to the concept of the impedance of free space Z_0 for an electromagnetic plane wave (here SI units are more convenient to use than Gaussian cgs units)

$$Z_0 = \frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 \text{ ohms}, \quad (31)$$

where μ_0 is the magnetic permeability of free space, and ε_0 is the dielectric permittivity of free space.

The physical meaning of the “impedance of free space” in the electromagnetic case is that when a plane wave impinges on a large, but thin, resistive film at normal incidence, due to this film’s ohmic losses, the wave can be substantially absorbed and converted into heat if the resistance per square element of this film is comparable to 377 ohms. In this case, we say that the electromagnetic plane wave has been approximately “impedance-matched” into the film. If, however, the resistance of the thin film is much lower than 377 ohms per square, as is the case for a superconducting film, then the wave will be reflected by the film. In this case, we say that the wave has been “shorted out” by the superconducting film, and that therefore this film reflects electromagnetic radiation like a mirror. By contrast, if the resistance of a normal metallic film is much larger than 377 ohms per square, then the film is essentially transparent to the wave. As a result, there will be almost perfect transmission.

The boundary value problem for Maxwell’s equations coupled to a thin resistive film with a resistance per square element of $Z_0/2$, yields a unique solution that this is the condition for the *maximum* possible fractional absorption of the wave energy by the film, which is 50%, along with 25% of the wave energy being transmitted, and the remaining 25% being reflected (see Appendix A) [19]. Under such circumstances, we say that the film has been “*optimally* impedance-matched” to the film. This result is valid no matter how thin the “thin” film is.

The gravitomagnetic permeability μ_G of free space is [20][21]

$$\mu_G = \frac{16\pi G}{c^2} = 3.73 \times 10^{-26} \frac{\text{m}}{\text{kg}}, \quad (32)$$

i.e., μ_G is the coupling constant which couples the Lense-Thirring field to sources of mass current density, in the gravitational analog of Ampere’s law for weak gravity. Ciufolini *et al.* have recently measured, to within $\pm 20\%$, a value of μ_G which agrees with Eq. (32), in the first observation of the Earth’s Lense-Thirring field by means of laser-ranging measurements of the orbits of two satellites [22]. From Eq. (32), I find that the impedance of free space is [18]

$$Z_G = \frac{E_G}{H_G} = \sqrt{\frac{\mu_G}{\varepsilon_G}} = \mu_G c = \frac{16\pi G}{c} = 1.12 \times 10^{-17} \frac{\text{m}^2}{\text{s} \cdot \text{kg}}, \quad (33)$$

where the fact has been used that both electromagnetic and gravitational plane waves propagate at the same speed

$$c = \frac{1}{\sqrt{\varepsilon_G \mu_G}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}. \quad (34)$$

Therefore, the gravitoelectric permittivity ε_G of free space is

$$\varepsilon_G = \frac{1}{16\pi G} = 2.98 \times 10^8 \frac{\text{kg}^2}{\text{N} \cdot \text{m}^2}. \quad (35)$$

Newton's constant G now enters explicitly through the expression for the impedance of free space Z_G , into the problem of the interaction of radiation and matter. Note that Z_G is an extremely small quantity. Nevertheless, it is also important to note that it is not strictly zero. Since nondissipative quantum fluids, such as superfluids and superconductors, can in principle have strictly zero losses, they can behave like “short circuits” for gravitational radiation. Thus we expect that quantum fluids, in contrast to classical fluids, can behave like perfect mirrors for gravitational radiation. That Z_G is so small explains why it is so difficult to couple *classical* matter to gravity waves. It is therefore natural to consider using nondissipative quantum matter instead for achieving an efficient coupling.

By analogy with the electromagnetic case, the physical meaning of the “impedance of free space” Z_G is that when a gravitational plane wave impinges on a large, but thin, *viscous* fluid film at normal incidence, due to this film's dissipative losses, the wave can be substantially absorbed and converted into heat, if the dissipation per square element of this film is comparable to Z_G . Again in this case, we say that the gravitational plane wave has been approximately “impedance-matched” into the film. If, however, the dissipation of the thin film is much lower than Z_G , as is the case for nondissipative quantum fluids, then the wave will be reflected by the film. In this case, we say that the wave has been “shorted out” by the superconducting or superfluid film, and that therefore the film should reflect gravitational radiation like a mirror. By contrast, if the dissipation of the film is much larger than Z_G , as is the case for classical matter, then the film is essentially transparent to the wave, and there will be essentially perfect transmission.

The same boundary value problem holds for the Maxwell-like equations coupled to a thin viscous fluid film with a dissipation per square element of $Z_G/2$, and yields the same unique solution that this is the condition for the *maximum* possible fractional absorption (and the consequent conversion into heat) of the wave energy by the film, which is 50%, along with 25% of the wave energy being transmitted, and the remaining 25% being reflected (see Appendix A). Under such circumstances, we again say that the film has been “*optimally* impedance-matched” to the film. Again, this result is valid no matter how thin the “thin” film is.

When the superconductor is viewed as a transducer, the conversion from electromagnetic to gravitational wave energy, and vice versa, can be viewed as an *effective* dissipation mechanism, where instead of being converted into heat, one form of wave energy is converted into the other form, whenever impedance matching is achieved within a thin layer inside the superconductor. As we shall see, this can occur naturally when the electromagnetic wave impedance is *exponentially* reduced in extreme type II superconductors as the wave penetrates into the superconductor, so that a layer is automatically reached in its interior where the electromagnetic wave impedance is reduced to a level comparable to Z_G . Under such circumstances, we should expect efficient conversion from one form of wave energy to the other.

V. MAXWELL-LIKE EQUATIONS FOR GRAVITY WAVES

For obtaining the impedance of free space Z_0 for electromagnetic plane waves, we recall that one starts from Maxwell's equations

$$\nabla \cdot \mathbf{D} = +\rho_e \quad (36)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (37)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (38)$$

$$\nabla \times \mathbf{H} = +\mathbf{j}_e + \frac{\partial \mathbf{D}}{\partial t}, \quad (39)$$

where ρ_e is the electrical free charge density (here, the charge density of Cooper pairs), and \mathbf{j}_e is the electrical current density (due to Cooper pairs), \mathbf{D} is the displacement field, \mathbf{E} is the electric field, \mathbf{B} is the magnetic induction field, and \mathbf{H} is the magnetic field intensity. The constitutive relations (assuming an isotropic medium) are

$$\mathbf{D} = \kappa_e \varepsilon_0 \mathbf{E} \quad (40)$$

$$\mathbf{B} = \kappa_m \mu_0 \mathbf{H} \quad (41)$$

$$\mathbf{j}_e = \sigma_e \mathbf{E}, \quad (42)$$

where κ_e is the dielectric constant of the medium, κ_m is its relative permeability, and σ_e is its electrical conductivity. We then convert Maxwell's equations into wave equations for free space in the usual way, and conclude that the speed of electromagnetic waves in free

space is $c = (\varepsilon_0\mu_0)^{-1/2}$, and that the impedance of free space is $Z_0 = (\mu_0/\varepsilon_0)^{1/2}$. The impedance-matching problem of a plane wave impinging on a thin, resistive film is solved by using standard boundary conditions in conjunction with the constitutive relation $\mathbf{j}_e = \sigma_e \mathbf{E}$.

Similarly, for *weak* gravity and *slow* matter, Maxwell-like equations have been derived from the linearized form of Einstein's field equations [21][23][24][25]. The gravitoelectric field \mathbf{E}_G , which is identical to the local acceleration due to gravity \mathbf{g} , is analogous to the electric field \mathbf{E} , and the gravitomagnetic field \mathbf{B}_G , which is identical to the Lense-Thirring field, is analogous to the magnetic field \mathbf{B} ; they are related to the vector potential \mathbf{h} in the radiation gauge as follows:

$$\mathbf{g} = -\frac{\partial \mathbf{h}}{\partial t} \text{ and } \mathbf{B}_G = \nabla \times \mathbf{h} , \quad (43)$$

which correspond to the electromagnetic relations in the radiation gauge

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \text{ and } \mathbf{B} = \nabla \times \mathbf{A} . \quad (44)$$

The physical meaning of \mathbf{g} is that it is the three-acceleration of a local, freely-falling test particle induced by the gravitational radiation, as seen by an observer in a local inertial frame located at the center of mass of the superconductor. The local three-acceleration \mathbf{g} is the local time derivative of the local three-velocity $-\mathbf{h}$ of this test particle, which is a member of a system of noninteracting, locally freely-falling, classical test particles (e.g., interstellar dust) with a velocity field $\mathbf{v}_{test}(x, y, z, t) = -\mathbf{h}(x, y, z, t)$ as viewed by an observer in the center-of-mass inertial frame (see Eq. (43a)). Similarly, the physical meaning of the gravitomagnetic field \mathbf{B}_G is that it is the local angular velocity of an inertial frame centered on the same test particle, with respect to the same observer's inertial frame, which is centered on the freely-falling center-of-mass of the superconductor. Thus \mathbf{B}_G is the Lense-Thirring field induced by gravitational radiation.

The Maxwell-like equations for weak gravitational fields (upon setting the PPN ("Parametrized Post-Newton") parameters to be those of general relativity) are [23]

$$\nabla \cdot \mathbf{D}_G = -\rho_G \quad (45)$$

$$\nabla \times \mathbf{g} = -\frac{\partial \mathbf{B}_G}{\partial t} \quad (46)$$

$$\nabla \cdot \mathbf{B}_G = 0 \quad (47)$$

$$\nabla \times \mathbf{H}_G = -\mathbf{j}_G + \frac{\partial \mathbf{D}_G}{\partial t} \quad (48)$$

where ρ_G is the density of local rest mass in the local rest frame of the matter, and \mathbf{j}_G is the local rest-mass current density in this frame (in the case of classical matter, $\mathbf{j}_G = \rho_G \mathbf{v}$, where \mathbf{v} is the coordinate three-velocity of the local rest mass; in the quantum case, see Eq. (64)). Here \mathbf{H}_G is the gravitomagnetic field intensity, and \mathbf{D}_G is the gravitodisplacement field.

Since the forms of these equations are identical to those of Maxwell's equations, the same boundary conditions follow from them, and therefore the same solutions for electromagnetic problems carry over formally to the gravitational ones. These include the solution for the optimal impedance-matching problem for a thin, dissipative film (see Appendix A).

The constitutive relations (assuming an isotropic medium) analogous to those in Maxwell's theory are

$$\mathbf{D}_G = 4\kappa_{GE}\varepsilon_G\mathbf{g} \quad (49)$$

$$\mathbf{B}_G = \kappa_{GM}\mu_G\mathbf{H}_G \quad (50)$$

$$\mathbf{j}_G = -\sigma_G\mathbf{g} \quad (51)$$

where ε_G is the gravitoelectric permittivity of free space given by Eq. (35), μ_G is the gravitomagnetic permeability of free space given by Eq. (32), κ_{GE} is the gravitoelectric dielectric constant of a medium, κ_{GM} is its gravitomagnetic relative permeability, and σ_G is the gravitational analog of the electrical conductivity of the medium, whose magnitude is inversely proportional to its viscosity. It is natural to choose to define the constitutive relation, Eq. (51), with a minus sign, so that for *dissipative* media, σ_G is always a *positive* quantity. The factor of 4 on the right hand side of Eq. (49) implies that Newton's law of universal gravitation emerges from Einstein's theory of GR in the correspondence principle limit.

The phenomenological parameters κ_{GE} , κ_{GM} , and σ_G must be determined by experiment. Since there exist no negative masses which can give rise to a gravitational analog of the polarization of the medium, we expect that at low frequencies, $\kappa_{GE} \rightarrow 1$. However, because of the possibility of large Meissner-like effects such as in superconductors, κ_{GM} need not approach unity at low frequencies, but can approach zero instead. The gravitodiamagnetic susceptibility calculated from Eq. (30) should lead to a value of κ_{GM} close to zero. Also, note that κ_{GM} can be spatially inhomogeneous, such as near the surface of a superconductor.

Again, converting the Maxwell-like equations for weak gravity into a wave equation for

free space in the standard way, we conclude that the speed of GR waves in free space is $c = (\epsilon_G \mu_G)^{-1/2}$, which is identical in GR to the vacuum speed of light, and that the impedance of free space for GR waves is $Z_G = (\mu_G/\epsilon_G)^{1/2}$, whose numerical value is given by Eq. (33).

It should be stressed here that although the above Maxwell-like equations look formally identical to Maxwell's, there is an elementary physical difference between gravity and electricity, which must not be overlooked. In electrostatics, the existence of both signs of charges means that both repulsive and attractive forces are possible, whereas in gravity, only positive signs of masses, and only attractive gravitational forces between masses, are observed. One consequence of this experimental fact is that whereas it is possible to construct Faraday cages that completely screen out electrical forces, and hence electromagnetic radiation fields, it is impossible to construct gravitational analogs of such Faraday cages that screen out gravitoelectric forces, such as Earth's gravity.

However, the gravitomagnetic force can be either repulsive or attractive in sign, unlike the gravitoelectric force. For example, the gravitomagnetic force between two parallel current-carrying pipes changes sign, when the direction of the current flow is reversed in one of the pipes, according to the Ampere-like law Eq. (48). Hence *both* signs of this kind of gravitational force are possible. One consequence of this is that gravitomagnetic forces *can* cancel out, so that, unlike gravitoelectric fields, gravitomagnetic fields *can* in principle be screened out of the interiors of material bodies. A dramatic example of this is the complete screening out of the Lense-Thirring field by superconductors in a Meissner-like effect, i.e., the complete expulsion of the gravitomagnetic field from the interior of these bodies, which is predicted by the Ginzburg-Landau theory given below. Therefore the expulsion of gravitational radiation fields by superconductors can also occur, and thus mirrors for this kind of radiation, although counterintuitive, are not impossible.

VI. POYNTING-LIKE VECTOR AND THE POWER FLOW OF GRAVITATIONAL RADIATION

In analogy with classical electrodynamics, having obtained the impedance of free space Z_G , we are now in a position to calculate the time-averaged power flow in a gravitational plane through a gravitational analog of Poynting's theorem in the weak-gravity limit. The

local time-averaged intensity of a gravitational plane wave is given by the time-averaged Poynting-like vector

$$\overline{\mathbf{S}} = \overline{\mathbf{E}_G \times \mathbf{H}_G} . \quad (52)$$

For a plane wave propagating in the vacuum, the local relationship between the magnitudes of the \mathbf{E}_G and \mathbf{H}_G fields is given by

$$|\mathbf{E}_G| = Z_G |\mathbf{H}_G| . \quad (53)$$

From Eq.(43a), it follows that the local time-averaged intensity, i.e., the power per unit area, of a harmonic plane wave of angular frequency ω is given by

$$|\overline{\mathbf{S}}| = \frac{1}{2Z_G} |\mathbf{E}_G|^2 = \frac{\omega^2}{2Z_G} |\mathbf{h}|^2 = \frac{c^3 \omega^2}{32\pi G} |h_{0i}|^2 . \quad (54)$$

For a Gaussian-Laguerre mode of a quadrupolar gravity-wave beam propagating at 10 GHz with an intensity of a milliwatt per square centimeter, the velocity amplitude $|\mathbf{h}|$ is typically

$$|\mathbf{h}| \simeq 2 \times 10^{-20} \text{ m/s}, \quad (55)$$

or the dimensionless strain parameter $|h_{0i}| = |\mathbf{h}|/c$ is typically

$$|h_{0i}| \simeq 8 \times 10^{-31} , \quad (56)$$

which is around ten orders of magnitude smaller than the typical strain amplitudes observable in the earlier versions of LIGO. At first sight, it would seem extremely difficult to detect such tiny amplitudes. However, if the natural impedance matching process in dissipationless, extreme type II superconductors to be discussed below can be achieved in practice, then both the generation and the detection of such small strain amplitudes should not be impossible.

To give an estimate of the size of the magnetic field amplitudes which correspond to the above gravitational wave amplitudes, one uses energy conservation in a situation in which the powers in the EM and GR waves become comparable to each other in the natural impedance-matching process described below, where, in the special case of perfect power conversion (i.e., perfect impedance matching) from EM to GR radiation,

$$\frac{\omega^2}{2Z_0} |\mathbf{A}|^2 = \frac{\omega^2}{2Z_G} |\mathbf{h}|^2 , \quad (57)$$

from which it follows that

$$\frac{|\mathbf{A}|}{|\mathbf{h}|} = \frac{|\mathbf{B}|}{|\mathbf{B}_G|} = \frac{|\mathbf{B}|}{|\boldsymbol{\Omega}_G|} = \left(\frac{Z_0}{Z_G}\right)^{1/2}, \quad (58)$$

where the ratio is given by the square-root of the impedances of free space $(Z_0/Z_G)^{1/2}$ instead of the mass-to-charge ratio $2m_2/e_2$ ratio implied by Eq. (16). Thus for the above numbers

$$|\mathbf{B}| \simeq 5 \times 10^{-3} \text{ Tesla} . \quad (59)$$

VII. GINZBURG-LANDAU EQUATION COUPLED TO BOTH ELECTROMAGNETIC AND GRAVITATIONAL RADIATION

A superconductor in the presence of the electromagnetic field $\mathbf{A}(t)$ alone is well described by the Ginzburg-Landau (G-L) equation for the complex order parameter ψ , which in the adiabatic limit is given by [26]

$$\frac{1}{2m_{2eff}} \left(\frac{\hbar}{i} \nabla - e_2 \mathbf{A}(t) \right)^2 \psi + \beta |\psi|^2 \psi = -\alpha \psi. \quad (60)$$

When \mathbf{A} is time-independent, this equation has the same form as the time-independent Schrödinger equation for a particle (i.e., a Cooper pair) with mass m_{2eff} and a charge e_2 with an energy eigenvalue $-\alpha$, except that there is an extra nonlinear term whose coefficient is given by the coefficient β , which arises at a microscopic level from the Coulomb interaction between Cooper pairs [26]. The values of these two phenomenological parameters α and β must be determined by experiment. There are two important length scales associated with the two parameters α and β of this equation, which can be obtained by a dimensional analysis of Eq. (60). The first is the *coherence length*

$$\xi = \sqrt{\frac{\hbar^2}{2m_{2eff}|\alpha|}}, \quad (61)$$

which is the length scale on which the condensate charge density $e_2|\psi|^2$ vanishes, as one approaches the surface of the superconductor from its interior, and hence the length scale on which the electric field $\mathbf{E}(t)$ is screened inside the superconductor. The second is the *London penetration depth*

$$\lambda_L = \sqrt{\frac{\hbar^2}{2m_{2eff}\beta|\psi|^2}} \rightarrow \sqrt{\frac{\varepsilon_0 m_{2eff} c^2}{e_2^2 |\psi_0|^2}}, \quad (62)$$

which is the length scale on which an externally applied magnetic field $\mathbf{B}(t) = \nabla \times \mathbf{A}(t)$ vanishes due to the Meissner effect, as one penetrates into the interior of the superconductor away from its surface. Here $|\psi_0|^2$ is the pair condensate density deep inside the superconductor, where it approaches a constant.

The G-L equation represents a mean field theory of the superconductor at the macroscopic level, which can be derived from the underlying microscopic BCS theory [27]. The meaning of the complex order parameter $\psi(x, y, z)$ is that it is the Cooper pair condensate wavefunction. Since $\psi(x, y, z)$ is defined as a complex field defined over *ordinary* (x, y, z) *space*, it is difficult to discern at this level of description the underlying quantum entanglement present in the BCS wavefunction, which is a many-body wavefunction defined over the *configuration space* of the many-electron system. Nevertheless, quantum entanglement, and hence instantaneous EPR correlations-at-a-distance, shows up indirectly through the non-linear term $\beta|\psi|^2\psi$, and is ultimately what is responsible for the Meissner effect. The G-L theory is being used here because it is more convenient than the BCS theory for calculating the response of the superconductor to electromagnetic, and also to gravitational, radiation.

I would like to propose that the Ginzburg-Landau equation should be generalized to include gravitational radiation fields $\mathbf{h}(t)$, whose frequencies lie well below the BCS gap frequency, by using the minimal-coupling rule, Eq. (18), to the following equation:

$$\frac{1}{2m_{2eff}} \left(\frac{\hbar}{i} \nabla - e_2 \mathbf{A}(t) - m_2 \mathbf{h}(t) \right)^2 \psi + \beta |\psi|^2 \psi = -\alpha \psi. \quad (63)$$

Again, the ultimate justification for this equation must come from experiment. With this equation, one can predict what happens at the interface between the vacuum and the superconductor, when both kinds of radiation are impinging on this surface at an arbitrary angle of incidence (see Figure 2). Since there are still only the two parameters α and β in this equation, there will again be only the same two length scales ξ and λ_L that we had *before* adding the gravitational radiation term $\mathbf{h}(t)$. Since there are no other length scales in this problem, one would expect that the gravitational radiation fields should vanish on the same length scales as the electromagnetic radiation fields as one penetrates deeply into the interior of the superconductor. Thus one expects there to exist a Meissner-like expulsion of the gravitational radiation fields from the interior of the superconductor.

Both $\mathbf{B}(t)$ and $\mathbf{B}_G(t)$ fields *must* vanish into the interior of the superconductor, since both $\mathbf{A}(t)$ and $\mathbf{h}(t)$ fields *must* vanish in the interior. Otherwise, the single-valuedness of

ψ would be violated. Suppose that $\mathbf{A}(t)$ did not vanish deep inside the superconducting slab, which is topologically singly connected. Then Yang's nonintegrable phase factor $\exp((ie_2/\hbar) \oint \mathbf{A}(t) \cdot d\mathbf{l})$, which is a gauge-independent quantity, would also not vanish [28], which would lead to a violation of the single-valuedness of ψ . Similarly, suppose that $\mathbf{h}(t)$ did not vanish. Then the nonintegrable phase factor $\exp((im_2/\hbar) \oint \mathbf{h}(t) \cdot d\mathbf{l})$, which is a gauge-independent quantity, would also not vanish, so that again there would be a violation of the single-valuedness of ψ (see Figure 1).

The $\mathbf{A}(t)$ and $\mathbf{h}(t)$ fields are coupled strongly to each other through the $e_2\mathbf{A} \cdot \mathbf{h}$ interaction Hamiltonian. Since the electromagnetic interaction is very much stronger than the gravitational one, the exponential decay of $\mathbf{A}(t)$ on the scale of the London penetration depth λ_L should also govern the exponential decay of the $\mathbf{h}(t)$ field. Thus both $\mathbf{A}(t)$ and $\mathbf{h}(t)$ fields decay exponentially with the *same* length scale λ_L into the interior of the superconductor. This implies that both EM and GR radiation fields will also be expelled from the interior, so that a flat surface of this superconductor should behave like a plane mirror for both EM and GR radiation.

The Cooper pair current density \mathbf{j} , which acts as the source in Ampere's law in both the Maxwell and the Maxwell-like equations, can be obtained in a manner similar to that for the Schrödinger equation

$$\mathbf{j} = \frac{\hbar}{2im_{2eff}} (\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{e_2}{m_{2eff}} |\psi|^2 \mathbf{A} - \frac{m_2}{m_{2eff}} |\psi|^2 \mathbf{h} . \quad (64)$$

Note that \mathbf{j} is nonlinear in ψ , but linear in \mathbf{A} and \mathbf{h} . Near the surface of the superconductor, the gradient terms dominate, but far into the interior, the \mathbf{A} and the \mathbf{h} terms dominate. We shall now use \mathbf{j} for calculating the sources for both Maxwell's equations for the electromagnetic fields, and also for the Maxwell-like equations for the gravitational fields. The electrical current density, the electrical free charge density, the rest-mass current density, and the rest mass density, are, respectively,

$$\mathbf{j}_e = e_2 \mathbf{j} , \rho_e = e_2 |\psi|^2 , \mathbf{j}_G = m_2 \mathbf{j} , \rho_G = m_2 |\psi|^2 . \quad (65)$$

I have not yet solved the generalized Ginzburg-Landau equation, Eq. (63), coupled to both the Maxwell and Maxwell-like equations through these currents and densities. These coupled equations are nonlinear in ψ , but are linear in \mathbf{A} and \mathbf{h} for weak radiation fields. However, from dimensional considerations, I can make the following remarks. The electric

field $\mathbf{E}(t)$ should be screened out exponentially towards the interior of the superconductor on a length scale set by the coherence length ξ , since the charge density $\rho_e = e_2|\psi|^2$ vanishes exponentially on this length scale near the surface of the superconductor. Similarly, the magnetic field $\mathbf{B}(t)$ should vanish exponentially towards the interior of the superconductor, but on a different length scale set by the London penetration depth λ_L . Both fields vanish exponentially, but on different length scales.

At first sight, it would seem that similar considerations would apply to the gravitational fields $\mathbf{g}(t)$ and $\mathbf{B}_G(t)$. However, since there exists only one sign of mass for gravity, the gravitoelectric field $\mathbf{g}(t)$ *cannot* be screened out. Nevertheless, the gravitomagnetic field $\mathbf{B}_G(t)$ *can* be, indeed *must* be, screened by the quantum-mechanical currents \mathbf{j} , in order to preserve the single-valuedness of ψ . The quantum-current source terms responsible for the screening out of the $\mathbf{B}_G(t)$ field in Meissner-like effects are not coupled to spacetime by means of Newton's constant G through the right-hand side of the Ampere-like law, Eq. (48), but are coupled directly without the mediation of G through the gravitomagnetic constitutive relation, Eq. (50), and through the Faraday-like law, Eq. (46), whose right-hand side does not contain G . For the reasons given above, $\mathbf{B}_G(t)$ must decay exponentially on the same scale of length as the gauge field $\mathbf{A}(t)$ and the magnetic field $\mathbf{B}(t)$, namely the London penetration depth λ_L .

The exponential decay into the interior of the superconductor of both EM and GR waves on the scale of λ_L means that a flat superconducting surface should behave like a plane mirror for both electromagnetic and gravitational radiation. However, the behavior of the superconductor as an efficient *mirror* is no guarantee that it should also be an efficient *transducer* from one type of radiation to the other. For efficient power conversion, a good transducer impedance-matching process from one kind of radiation to the other is also required.

VIII. NATURAL IMPEDANCE MATCHING IN EXTREME TYPE II SUPERCONDUCTORS

Impedance matching in a natural transduction process between EM and GR waves could happen near the surface of type II superconductors, where the electric field decays more quickly than the magnetic field into the interior of the superconductor, since $\xi < \lambda_L$. For

extreme type II superconductors, where $\xi \ll \lambda_L$, the electric field is screened out much more quickly on the scale of the coherence length ξ , than the magnetic field, which is screened much more slowly on the scale of the penetration depth λ_L . The EM wave impedance for extreme type II superconductors should therefore decay on the scale of the coherence length ξ much more quickly than the GR wave impedance, which should decay much more slowly on the scale of the penetration depth λ_L . The high-temperature superconductor YBCO is an example of an extreme type II superconductor, for which ξ is less than λ_L by three orders of magnitude [29].

The wave impedance $Z = E/H$ of an EM plane wave depends exponentially as a function of z , the distance from the surface into the interior of the superconductor, as follows:

$$Z(z) = \frac{E(z)}{H(z)} = Z_0 \exp(-z/\xi + z/\lambda_L). \quad (66)$$

The GR wave impedance Z_G , however, behaves very differently, because of the absence of the screening of the gravitoelectric field, so that $E_G(z)$ should be constant independent of z near the surface, and therefore

$$Z_G(z) = \frac{E_G(z)}{H_G(z)} = Z_G \exp(+z/\lambda_L). \quad (67)$$

Thus the z -dependence of the ratio of the two kinds of impedances should obey the exponential-decay law

$$\frac{Z(z)}{Z_G(z)} = \frac{Z_0}{Z_G} \exp(-z/\xi). \quad (68)$$

We must at this point convert the two impedances Z_0 and Z_G to the same units for comparison. To do so, we express Z_0 in the natural units of the quantum of resistance $R_0 = h/e^2$, where e is the electron charge. Likewise, we express Z_G in the corresponding natural units of the quantum of dissipation $R_G = h/m^2$, where m is the electron mass. Thus we get the *dimensionless* ratio

$$\frac{Z(z)/R_0}{Z_G(z)/R_G} = \frac{Z_0/R_0}{Z_G/R_G} \exp(-z/\xi) = \frac{e^2/4\pi\epsilon_0}{4Gm^2} \exp(-z/\xi). \quad (69)$$

Let us define the ‘‘depth of natural impedance-matching’’ z_0 as the depth where this dimensionless ratio is unity, and thus natural impedance matching occurs. Then

$$z_0 = \xi \ln \left(\frac{e^2/4\pi\epsilon_0}{4Gm^2} \right) \approx 97\xi. \quad (70)$$

This result is a robust one, in the sense that the logarithm is very insensitive to changes in numerical factors of the order of unity in its argument. From this, we conclude that it is necessary to penetrate into the superconductor a distance of z_0 , which is around a hundred coherence lengths ξ , for the natural impedance matching process to occur. When this happens, transducer impedance matching occurs automatically, and we expect that the conversion from electromagnetic to gravitational radiation, and vice versa, to be an efficient one. For example, the London penetration depth of around 6000 \AA in the case of YBCO is very large compared with $97\xi \simeq 400 \text{ \AA}$ in this material, so that the electromagnetic field energy has not yet decayed by much at this natural impedance-matching plane $z = z_0$, although it is mainly magnetic in character at this point. Therefore the transducer power-conversion efficiency could be of the order of unity, provided that there is no appreciable parasitic dissipation of the EM radiation fields in the superconductor before this point.

The Fresnel-like boundary value problem for plane waves incident on the surface of the superconductor at arbitrary incidence angles and arbitrary polarizations (see Figure 2) needs to be solved in detail before these conclusions can be confirmed.

IX. RESULTS OF A FIRST EXPERIMENT AND FUTURE PROSPECTS

However, based on the above crude dimensional and physical arguments, the prospects for a simple Hertz-like experiment testing these ideas appeared promising enough that I have performed a first attempt at this experiment with Walt Fitelson at liquid nitrogen temperature. The schematic of this experiment is shown in Figure 3. Details will be presented elsewhere.

Unfortunately, we did not detect any observable signal inside the second Faraday cage, down to a limit of more than 70 dB below the microwave power source of around -10 dBm at 12 GHz. (We used a commercial satellite microwave receiver at 12 GHz with a noise figure of 0.6 dB to make these measurements; the Faraday cages were good enough to block any direct electromagnetic coupling by more than 70 dB). We checked for the presence of the Meissner effect in the samples *in situ* by observing a levitation effect upon a permanent magnet by these samples at liquid nitrogen temperature.

Note, however, that since the transition temperature of YBCO is 90 K, there may have been a substantial ohmic dissipation of the microwaves due to the remaining normal electrons

at our operating temperature of 77 K, so that the EM wave was absorbed before it could reach the impedance-matching depth at z_0 . It may therefore be necessary to cool the superconductor down very low temperatures before the normal electron component freezes out sufficiently to achieve such impedance matching. The exponential decrease of the normal electron population at very low temperatures due to the Boltzmann factor, and thereby an exponential “freezing out” of the ohmic dissipation of the superconductor, may then allow this impedance matching process to take place, if no other parasitic dissipative processes remain at these very low temperatures. Assuming that the impedance-matching argument given in Eq. (69) is correct, and assuming that in the normal state, the surface resistance of YBCO is on the order of $h/e^2 = 26$ kilohms in its normal state, one needs a Boltzmann factor of the order of e^{-100} in order to freeze out the dissipation due to the normal electrons down to an impedance level comparable to Z_G . This would imply that temperatures around a Kelvin should suffice.

However, there exist unexplained residual microwave and far-infrared losses in YBCO and other high T_c superconductors, which are independent of temperature and have a frequency-squared dependence [30]. It may therefore be necessary to cool the superconductor down extremely low temperatures, such as millikelvins, before the normal electron component freezes out sufficiently to achieve such impedance matching, but more research needs first to be done to understand the mechanism for these microwave residual losses before they can be eliminated. An improved Hertz-like experiment using extreme type II superconductors with extremely low losses, perhaps at millikelvin temperatures, is a much more difficult, but worthwhile, experiment to perform.

Such an improved experiment, if successful, would allow us to communicate through the Earth and its oceans, which, like all classical matter, are transparent to GR waves. Furthermore, it would allow us to directly observe for the first time the CMB (Cosmic Microwave Background) in GR radiation, which would tell us much about the very early Universe.

X. ACKNOWLEDGMENTS

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birthday. I dedicate this paper to my teacher, John Archibald Wheeler, whose vision helped inspire this work. I would like to thank S. Braunstein, C. Caves, A. Charman, M. L. Cohen, P. C. W. Davies, J. C. Davis, F. Everitt, W. Fitelson, J. C. Garrison, L. Hall, J. M. Hickmann, J. M. Leinaas, R. Laughlin, C. McCormick, R. Marrus, M. Mitchell, J. Moore, J. Myrheim, R. Packard, R. Ramos, P. L. Richards, R. Simmonds, D. Solli, A. Speliotopoulos, C. H. Townes, W. Unruh, S. Weinreb, E. Wright, W. Wootters, and A. Zetzl for stimulating discussions. I would especially like to thank my father-in-law, the late Yi-Fan Chiao, for his financial and moral support of this work. This work was partly supported also by the ONR.

XI. APPENDIX A: OPTIMAL IMPEDANCE MATCHING OF A GRAVITATIONAL PLANE WAVE INTO A THIN, DISSIPATIVE FILM

Let a gravitational plane wave given by Eq. (25) be normally incident onto a thin, dissipative (i.e., viscous) fluid film. Let the thickness d of this film be arbitrarily thin compared to the gravitational analog of the skin depth $(2/\kappa_{GM}\mu_G\sigma_G\omega)^{1/2}$, and to the wavelength λ . The incident fields calculated using Eqs. (43) (here I shall use the notation \mathbf{E}_G instead of \mathbf{g} for the gravitoelectric field) are

$$\mathbf{E}_G^{(i)} = -\frac{1}{2}(x, -y, 0)\omega h_+ \sin(kz - \omega t) \quad (71)$$

$$\mathbf{H}_G^{(i)} = -\frac{1}{2Z_G}(y, x, 0)\omega h_+ \sin(kz - \omega t). \quad (72)$$

Let ρ be the amplitude reflection coefficient for the gravitoelectric field; the reflected fields from the film are then

$$\mathbf{E}_G^{(r)} = -\rho\frac{1}{2}(x, -y, 0)\omega h_+ \sin(kz - \omega t) \quad (73)$$

$$\mathbf{H}_G^{(r)} = +\rho\frac{1}{2Z_G}(y, x, 0)\omega h_+ \sin(kz - \omega t). \quad (74)$$

Similarly the transmitted fields on the far side of the film are

$$\mathbf{E}_G^{(t)} = -\tau\frac{1}{2}(x, -y, 0)\omega h_+ \sin(kz - \omega t) \quad (75)$$

$$\mathbf{H}_G^{(t)} = -\tau\frac{1}{2Z_G}(y, x, 0)\omega h_+ \sin(kz - \omega t), \quad (76)$$

where τ is the amplitude transmission coefficient. The Faraday-like law, Eq. (46), and the Ampere-like law, Eq. (48), when applied to the tangential components of the gravitoelectric and gravitomagnetic fields parallel to two appropriately chosen infinitesimal rectangular loops which straddle the thin film, lead to two boundary conditions

$$\mathbf{E}_G^{(i)} + \mathbf{E}_G^{(r)} = \mathbf{E}_G^{(t)} \text{ and } \mathbf{H}_G^{(i)} + \mathbf{H}_G^{(r)} = \mathbf{H}_G^{(t)}, \quad (77)$$

which yield the following two algebraic relations:

$$1 + \rho - \tau = 0 \quad (78)$$

$$1 - \rho - \tau = (Z_G \sigma_G d) \tau \equiv \zeta \tau \quad (79)$$

where we have used the constitutive relation, Eq. (51),

$$\mathbf{j}_G = -\sigma_G \mathbf{E}_G$$

to determine the current enclosed by the infinitesimal rectangular loop in the case of the Ampere-like law, and where we have defined the positive, dimensionless quantity $\zeta \equiv Z_G \sigma_G d$. The solutions are

$$\tau = \frac{2}{\zeta + 2} \text{ and } \rho = -\frac{\zeta}{\zeta + 2}. \quad (80)$$

Using the conservation of energy, we can calculate the absorptivity A , i.e., the fraction of power absorbed from the incident gravitational wave and converted into heat

$$A = 1 - |\tau|^2 - |\rho|^2 = \frac{4\zeta}{(\zeta + 2)^2}. \quad (81)$$

To find the condition for maximum absorption, we calculate the derivative $dA/d\zeta$ and set it equal to zero. The unique solution for maximum absorptivity occurs at

$$\zeta = 2, \text{ where } A = \frac{1}{2} \text{ and } |\tau|^2 = \frac{1}{4} \text{ and } |\rho|^2 = \frac{1}{4}. \quad (82)$$

Thus the optimal impedance-matching condition into the thin, dissipative film, i.e., when there exists the maximum rate of conversion of gravitational wave energy into heat, occurs when the dissipation in the fluid film is $Z_G/2$ per square element. At this optimum condition, 50% of the gravitational wave energy will be converted into heat, 25% will be transmitted, and 25% will be reflected. This is true independent of the thickness d of the film, when

the film is very thin. This solution is formally identical to that of the optimal impedance-matching problem of an electromagnetic plane wave into a thin ohmic film [19].

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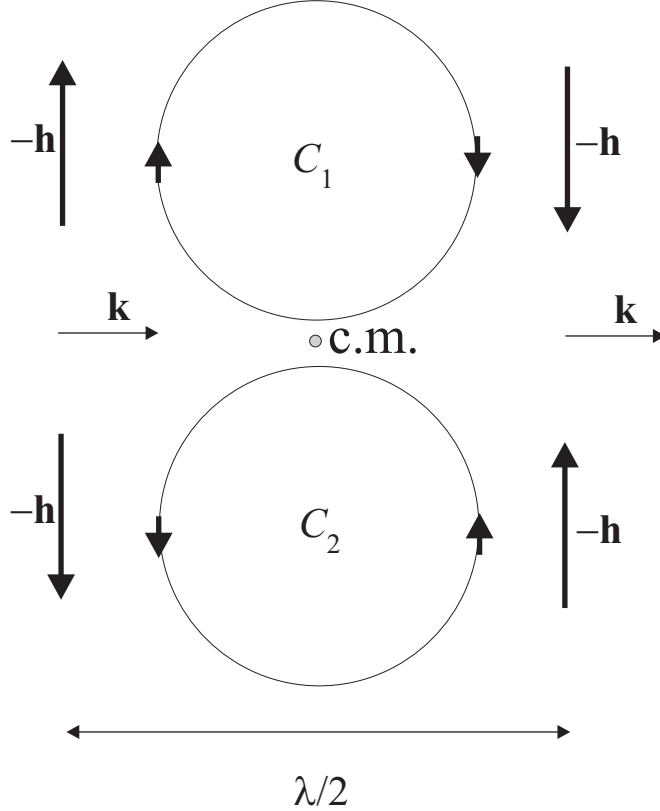
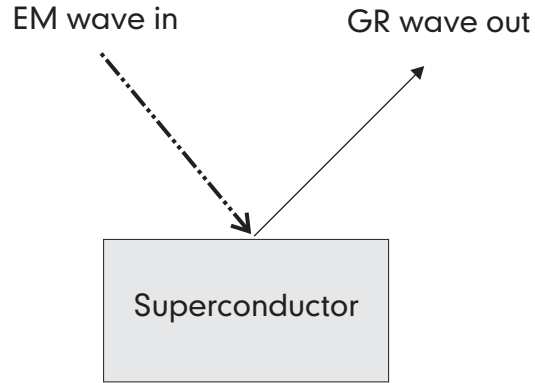
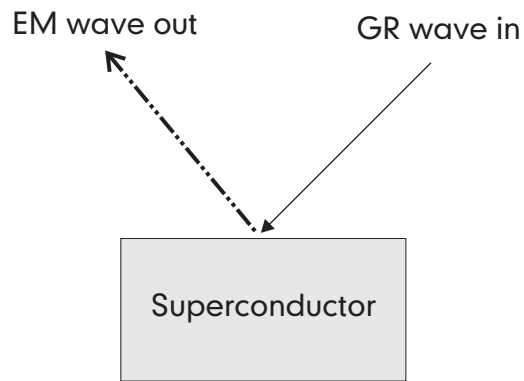


FIG. 1: A side-view snapshot of a monochromatic gravitational plane wave inside a thick superconducting slab, propagating to the right with a wave vector \mathbf{k} along the z axis, which induces tidal motions along the x axis resulting in the velocity field $-\mathbf{h}$ of test particles, as seen by an observer sitting in an inertial frame centered on the center-of-mass (c.m.) of the superconductor. After half a wavelength of propagation, these tidal motions reverse sign. Hence there exists nonvanishing circulations around the circuits C_1 and C_2 , i.e., *fluxes* of the gravitomagnetic (or Lense-Thirring) field inside C_1 and C_2 . These fluxes are directly proportional to the quantities given by Eqs. (7) and (13), and are gauge-invariant. For small circuits, they are also directly proportional to the nonvanishing Riemann curvature tensor component R_{0x0z} . The propagation depicted here of a GR wave penetrating deeply into the interior of a thick slab of superconductor (which is thick compared to the wavelength λ) would violate the single-valuedness of the wavefunction. However, the existence of a Meissner-like effect, in which all the radiation fields of the GR wave, including its Lense-Thirring fields, are totally expelled from the superconductor (apart from a thin surface layer of the order of the London penetration depth), would prevent such a violation.



(a)



(b)

FIG. 2: Superconductor as an impedance-matched transducer between electromagnetic (EM) and gravitational (GR) radiation. (a) An EM plane wave is converted upon reflection into a GR plane wave. (b) The reciprocal (or time-reversed) process in which a GR plane wave is converted upon reflection into an EM plane wave. Both EM and GR waves possess the same *quadrupolar* polarization pattern.

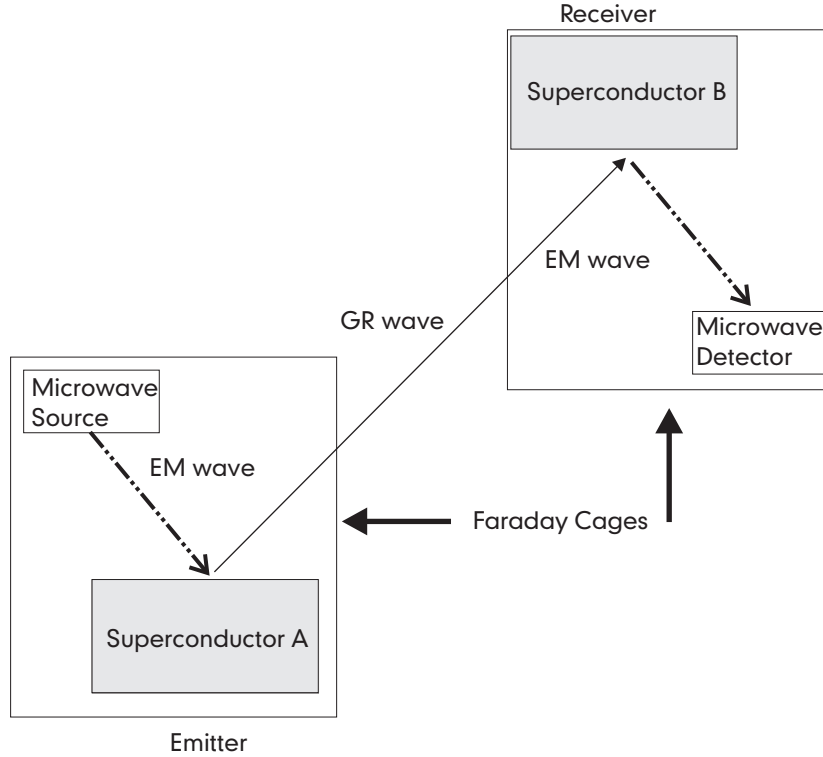


FIG. 3: Schematic of a simple Hertz-like experiment, in which gravitational radiation at 12 GHz could be emitted and received using two superconductors. The “Microwave Source” generated, by means of a T-shaped quadrupole antenna, quadrupolar-polarized electromagnetic radiation at 12 GHz (“EM wave”), which impinged on Superconductor A (a 1 inch diameter, 1/4 inch thick piece of YBCO), which was placed inside a dielectric Dewar (a stack of styrofoam cups containing liquid nitrogen), and would be converted upon reflection into gravitational radiation (“GR wave”). The GR wave, but not the EM wave, could pass through the “Faraday Cages,” i.e., normal metal cans which were lined on the inside with Eccosorb microwave foam absorbers. In the far field of Superconductor A, Superconductor B (also a 1 inch diameter, 1/4 inch thick piece of YBCO in another stack of styrofoam cups containing liquid nitrogen) would reconvert upon reflection the GR wave back into an EM wave at 12 GHz, which could then be detected by the “Microwave Detector,” which was a sensitive receiver used for microwave satellite communications, again coupled by another T-shaped quadrupole antenna to free space. The GR wave, and hence the signal at the microwave detector, should disappear once either superconductor was warmed up above its transition temperature (90 K), i.e., after the liquid nitrogen boiled away.