## On the Localized Superluminal Solutions to the Maxwell equations <sup>(†)</sup>

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Abstract - In the first part of this article (after a sketchy theoretical introduction) the various experimental sectors of physics in which Superluminal motions seem to appear are briefly mentioned. In particular, a bird's-eye view is presented of the experiments with evanescent waves (and/or tunneling photons), and with the "localized Superluminal solutions" (SLS) to the wave equation, like the so-called X-shaped beams. In the second part of this paper we present a series of new SLSs to the Maxwell equations, suitable for arbitrary frequencies and arbitrary bandwidths: some of them being endowed with finite total energy. Among the others, we set forth an infinite family of generalizations of the classic X-shaped wave; and show how to deal with the case of a *dispersive* medium. Results of this kind may find application in other fields in which an essential role is played by a wave-equation (like acoustics, seismology, geophysics, gravitation, elementary particle physics, etc.). This e-print, in large part a review, was prepared for the special issue on "Nontraditional Forms of Light" of the IEEE JSTQE (2003); and a preliminary version of it appeared as Report NSF-ITP-02-93 (ITP, UCSB; 2002). Further material can be found in the recent e-prints arVive:0708.1655v2 [physics.gen-ph] and arVive:0708.1209v1][physics.gen-ph]. The case of the very interesting (and more orthodox, in a sense) subluminal Localized Waves, solutions to the wave equations, will be dealt with in a coming paper. [Keywords: Wave equation; Wave propagation; Localized

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### 1. - Introduction.

The question of Superluminal  $(V^2 > c^2)$  objects or waves<sup>\*</sup>, has a long story, starting perhaps in 50 B.C. with Lucretius' *De Rerum Natura* (cf., e.g., book 4, line 201: [<<Quone vides *citius* debere et longius ire/ Multiplexque loci spatium transcurrere eodem/ Tempore *quo Solis* pervolgant *lumina* coelum?>>]). Still in pre-relativistic times, one meets various related works, from those by J.J.Thomson to the papers by A.Sommerfeld. With Special Relativity, however, since 1905 the conviction spread over that the speed *c* of light in vacuum was the upper limit of any possible speed. For instance, R.C.Tolman in 1917 believed to have shown by his "paradox" that the existence of particles endowed with speeds larger than *c* would have allowed sending information into the past. Such a conviction blocked any research about Superluminal speeds. Our problem started to be tackled again essentially in the fifties and sixties, in particular after the papers[1] by E.C.George Sudarshan et al., and, later on,[2] by E.Recami, R.Mignani et al., as well as by H.C.Corben and others (to confine ourselves to the *theoretical* researches). The first experiments looking for faster-than-light objects were performed by T.Alväger et al.[1].

Superluminal objects (particles or wavepackets) were called tachyons, T, by G.Feinberg, from the Greek word  $\tau \alpha \chi \dot{\upsilon} \varsigma$ , quick [and this induced us in 1970 to coin the term bradyon, B, for ordinary subluminal ( $v^2 < c^2$ ) objects, from the Greek word  $\beta \rho \alpha \delta \dot{\upsilon} \varsigma$ , slow]. Finally, objects travelling exactly at the speed of light are called "luxons".

In recent years, terms as "tachyon" and "superluminal" fell unhappily into the (cunning, rather than crazy) hands of pranotherapists and mere cheats, who started squeezing money out of simple-minded people; for instance by selling plasters (!) that should cure various illnesses by "emitting tachyons"... Here we are dealing, however, with Superluminal waves or objects since at least four different experimental sectors of physics seem to indicate the actual existence, in a sense, of Superluminal motions [thus confirming some long-standing theoretical predictions[3]].

In the first part of this article (after a sketchy, non-technical theoretical introduction, which might be useful since it enlightens an original, scarcely known approach<sup>†</sup>) we briefly mention the various experimental sectors of physics in which Superluminal motions seem to appear. In particular, a bird's-eye view is presented of the experiments with evanescent waves (and/or tunneling photons), and with the "localized Superluminal solutions" (SLS)

<sup>\*</sup>It is an old use of ours to write Superluminal with a capital S.

<sup>&</sup>lt;sup>†</sup>For a more detailed reviews, see P.W.Milonni, in ref.[22] below, and the recent e-prints arVive:0708.1655v2][physics.gen-ph] and arVive:0708.1209v1][physics.gen-ph].

to the wave equation, like the so-called X-shaped beams; the shortness of this review is compensated by a number of references, sufficient in some cases to provide the interested readers with reasonable bibliographical information. In the second part of this paper —after having constructed an infinite family of generalizations of the classic X-shaped wave, and having briefly discussed the behavior of some *finite* total-energy SLSs— we propose a series of new SLSs to the Maxwell equations, suitable for arbitrary frequencies and bandwidths, and show moreover how to deal with the case of a *dispersive* medium.

Let us state first that special relativity (SR), abundantly verified by experience, can be built on two simple, natural Postulates: 1) that the laws (of electromagnetism and mechanics) be valid not only for a particular observer, but for the whole class of the "inertial" observers: 2) that space and time be homogeneous and space be moreover isotropic. From these Postulates one can theoretically *infer* that one, and only one, *invariant* speed exists: and experience tells us such a speed to be the one, c, of light in vacuum; indeed, ordinary light possesses the peculiar feature of presenting always the same speed in vacuum, even when we run towards or away from it. It is just that feature, of being invariant, that makes quite exceptional the speed c: no slower, nor faster speed can enjoy the same property.

Another (known) consequence of our Postulates is that the total energy of an ordinary particle increases when its speed v increases, tending to infinity when v tends to c. Therefore, infinite forces would be needed for a bradyon (= slower than light object) to reach the speed c. However, as speed c photons exist which are born live and die always at the speed of light (without any need of accelerating from rest to the light speed), so objects can exist<sup>[4]</sup> always endowed with speeds V larger than c (see Fig.1). [This circumstance has been pictures quely illustrated by George Sudarshan (1972) with reference to an imaginary demographer studying the population patterns of the Indian subcontinent: << Suppose a demographer calmly asserts that there are no people North of the Himalayas, since none could climb over the mountain ranges! That would be an absurd conclusion. People of central Asia are born there and live there: they did not have to be born in India and cross the mountain range. So with faster-than-light particles>>.] Let us add that, still starting from the above two Postulates (besides a third postulate, even more obvious), the theory of relativity can be generalized [3,4] in such a way to accommodate also Superluminal objects; a large part of such an extension being contained in a series of works which date back to the sixties-seventies. Also within "Extended Relativity" [3] the speed c, besides being invariant, is a limiting velocity: but every limiting value has two sides, and one can a priori approach it both from the left and from the right.

Actually, the ordinary formulation of SR is restricted too much. For instance, even leaving Superluminal speeds aside, it can be easily so widened as to include antimatter[5]. Then, one finds space-time to be a priori populated by normal particles P, and by dual particles Q "which travel backwards in time carrying negative energy": The latter shall appear to us as antiparticles, i.e., as particles —regularly traveling forward in time with



Figure 1: Energy of a free object as a function of its speed. [2-4]

positive energy, but— with all their "additive" charges (e.g., the electric charge) reversed in sign: see Fig.2. and refs.[3-5]. To clarify this point, one could here recall only what follows: We, macroscopic observers, have to move in time along a single, well-defined direction, to such an extent that we cannot even see a motion backwards in time...; and every object like Q, travelling backwards in time (with negative energy), will be *necessarily* reinterpreted by us as an anti-object, with opposite charges but travelling forward in time (with positive energy): cf. again Fig.2 and refs.[3-5].

But let us forget about antimatter and go back to "tachyons" (= faster-than-light waves or objects). A strong objection against their existence is based on the opinion that by tachyons it would be possible to send signals into the past, owing to the fact that a tachyon T which, say, appears to a first observer O as emitted by A and absorbed by B, can appear to a second observer O' as a tachyon T' which travels backwards in time with negative energy. However, by applying the same "reinterpretation rule" or switching procedure seen above (Fig.2), T' will appear to the new observer O' just as an antitachyon  $\overline{T}$  emitted by B and absorbed by A, and therefore traveling forward in time, even if in the contrary *space* direction.<sup>‡</sup> In such a way, every travel towards the past, and every negative energy, disappear: Cf. refs.[3-5].

Starting from this observation, it is possible to solve[5] the so-called causal paradoxes associated with Superluminal motions: paradoxes which result to be the more instructive, the more sophisticated they are; but that cannot be re-examined here (some of them having been proposed by R.C.Tolman, J.Bell, F.A.E.Pirani, J.D.Edmonds and others).[6,3] [Let us only mention here the following. The reinterpretation principle, according to which, in simple words, signals are carried only by objects which appear to be endowed with positive energy does eliminate any information transfer backwards in time; but this has

<sup>&</sup>lt;sup>‡</sup>In simple words, the reinterpretation rule implies that signals are carried only by objects which appear to be endowed with positive energy.

$$(P); -q; E < 0; \bar{T}; p < 0$$

$$(Q); +q; E > 0; \bar{T}; p > 0$$

Figure 2: Depicting the "switching rule" (or reinterpretation principle) by Stueckelberg-Feynman-Sudarshan-Recami[3-5]: Q will appear as the antiparticle of P. See the text.

a price: That of abandoning the ingrained conviction that the judgement about what is cause and what is effect be independent of the observer. In fact, in the case examined above, the first observer O considers the event at A to be the cause of the event at B. By contrast, the second observer O' will consider the event at B as causing the event at A. All the observers will however see the cause to happen *before* its effect.]<sup>§</sup>

Let us observe that last century theoretical physics led us in a natural way to suppose the existence of various types of objects: magnetic monopoles, quarks, strings, tachyons, besides black-holes: and various sectors of physics could not go on without them, even if the existence of none of them is certain (also because attention has not yet been paid to some links existing among them: e.g., a Superluminal electric charge is expected to behave as a magnetic monopole; and a black-hole a priori can be the source of tachyonic matter). According to Democritus of Abdera, everything that was thinkable without meeting contradictions had to exist somewhere in the unlimited universe. This point of view —which was given by M.Gell-Mann the name of "totalitarian principle"— was later on expressed (T.H.White) in the humorous form "Anything not forbidden is compulsory"...

Let us add here the information that the case of the very interesting (and more orthodox, in a sense) *subluminal* Localized Waves, solutions to the wave equations, will be dealt with in a coming paper.

### 2. A glance at the experimental status-of-the-art.

Extended Relativity can allow a better understanding of many aspects also of *ordinary* 

 $<sup>^{\$}</sup>$ Taking new objects or entities into consideration always forces us to a criticism of our prejudices. If we require the phenomena to obey the *law* of (retarded) causality with respect to all the observers, then we cannot demand also the *description* "details" of the phenomena to be invariant: namely, we cannot demand in that case also the invariance of the "cause" and "effect" labels.[6,2]

physics, even if tachyons would not exist in our cosmos as asymptotically free objects. As already said, however, at least three or four different experimental sectors of physics seem to suggest the possible existence of faster-than-light motions; as our first aim, in the following we set forth some information (mainly bibliographical) about the experimental results obtained in a couple of those different physics sectors, with a mere mention of the others.

A) Neutrinos – First: A long series of experiments, started in 1971, seems to show that the square  $m_0^2$  of the mass  $m_0$  of muon-neutrinos, and more recently of electron-neutrinos too, is negative; which, if confirmed, would mean that (when using a naïve language, commonly adopted) such neutrinos possess an "imaginary mass" and are therefore tachyonic, or mainly tachyonic.[7,3] [In Extended Relativity, the dispersion relation for a free Superluminal object becomes  $\omega^2 - \mathbf{k}^2 = -\Omega^2$ , or  $E^2 - \mathbf{p}^2 = -m_o^2$ , and there is no need therefore of imaginary masses...].¶

**B)** Galactic Micro-quasars – Second: As to the apparent Superluminal expansions observed in the core of quasars[8] and, recently, in the so-called galactic microquasars[9], we shall not deal here with that problem, too far from the other topics of this paper: without mentioning that for those astronomical observations there exist orthodox interpretations, based on ref.[10], that are accepted by the majority of astrophysicists. For a theoretical discussion, see ref.[11]. Here, we may be interested in the fact that simple geometrical considerations in Minkowski space show that a single Superluminal light source would appear[11,3]: (i) initially, in the "optical boom" phase (analogous to the acoustic "boom" produced by a plane traveling with constant supersonic speed), as an intense source which suddenly comes into view; and that (ii) afterwards seem to split into TWO objects receding one from the other with speed V > 2c [all of this being similar to what is actually observed, according to refs.[9]].

C) Evanescent waves and "tunneling photons" – Third: Within quantum mechanics, it had been shown that the tunneling time does not depend on the width of the barrier, in the case of opaque barriers ("Hartman effect")[12]. This implies Superluminal and arbitrarily large (group) velocities V inside long enough barriers: see Fig.3 Experiments that may verify this prediction by, say, electrons are difficult. Luckily enough, however, the Schroedinger equation in the presence of a potential barrier is mathematically identical to the Helmholtz equation for an electromagnetic wave propagating, for instance, down a metallic waveguide along the x-axis: as shown, e.g., by R.Chiao et al.[13]; and a barrier height U bigger than the electron energy E corresponds (for a given wave frequency) to a waveguide of transverse size lower than a cut-off value. A segment of "undersized" guide —to go on with our example— does therefore behave as a barrier for the wave (photonic

<sup>&</sup>lt;sup>¶</sup>Let us put c = 1, whenever convenient, throughout this paper.

barrier)[13], as well as any other photonic band-gap filters. The wave assumes therein —like an electron inside a quantum barrier— an imaginary momentum or wave-number and gets, as a consequence, exponentially damped along x. In other words, it becomes an *evanescent* wave (going back to normal propagation, even if with reduced amplitude, when the narrowing ends and the guide returns to its initial transverse size). Thus, a tunneling experiment can be simulated[13] by having recourse to evanescent waves (for which the concept of group velocity can be properly extended[14]). The fact that evanescent waves travel with Superluminal speeds (cf.,e.g., Fig.4) has been actually *verified* in a series of famous experiments.



Figure 3: Behaviour of the average "penetration time" (in seconds) spent by a tunneling wavepacket, as a function of the penetration depth (in åangstroms) down a potential barrier (from Olkhovsky et al., ref.[12]). According to the predictions of quantum mechanics, the wavepacket speed inside the barrier increases in an unlimited way for opaque barriers; and the total tunneling time does *not* depend on the barrier width[12].

Namely, various experiments, performed since 1992 onwards by R.Chiao, P.G.Kwiat and A.Steinberg's group at Berkeley[15], by G.Nimtz et al. at Cologne[16], by A.Ranfagni and colleagues at Florence[17], and by others at Vienna, Orsay, Rennes[17], verified that "tunneling photons" travel with Superluminal group velocities. Let us add that also Extended Relativity had predicted[19] evanescent waves to be endowed with faster-than-*c* speeds; the whole matter appears to be therefore theoretically self-consistent. The debate in the current literature does not refer to the experimental results (which can be correctly reproduced by numerical elaborations[20,21] based on Maxwell equations only), but rather to the question whether they allow, or do not allow, sending signals or information with Superluminal speed[22,21,14].

In the abovementioned experiments one meets a substantial attenuation of the considered pulses during tunneling (or during propagation in an absorbing medium). However, by employing a "gain doublet", it has been recently reported the observation of



Figure 4: Simulation of tunneling by experiments with evanescent classical waves (see the text), which were predicted to be Superluminal also on the basis of Extended Relativity [3,4]. The figure shows one of the measurement results in refs. [15]; that is, the average beam speed while crossing the evanescent region ( = segment of undersized waveguide, or "barrier") as a function of its length. As theoretically predicted [19,12], such an average speed exceeds c for long enough "barriers".

undistorted pulses propagating with Superluminal group-velocity with a *small* change in amplitude.[23]



Figure 5: The very interesting experiment along a metallic waveguide with TWO barriers (undersized guide segments), i.e., with two evanescence regions [24]. See the text.

Let us emphasize that some of the most interesting experiments of this series seem to be the ones with two "barriers" (e.g., with two gratings in an optical fiber, or with two segments of undersized waveguide separated by a piece of normal-sized waveguide: Fig.5). For suitable frequency bands —i.e., for "tunneling" far from resonances—, it was found that the total crossing time does not depend on the length of the intermediate (normal) guide: namely, that the beam speed along it is infinite[24,25]. This agrees with what predicted by Quantum Mechanics for the non-resonant tunneling through two successive opaque barriers (the tunneling phase time, which depends on the entering energy, has been shown to be *independent* of the distance between the two barriers[26]); something that has been accepted and generalized in Aharonov et al.[26] Such a prediction has been verified a second time, taking advantage of the circumstance that quite interesting evanescence regions can be constructed in the most varied manners, like by means of different photonic band-gap materials or gratings (it being possible to use from multilayer dielectric mirrors, or semiconductors, to photonic crystals...). And indeed a very recent confirmation came —as already mentioned— from an experiment having recourse to two gratings in an optical fiber.[25]

We cannot skip a further, indeed delicate, topic, since the last experimental contribution to it [23] aroused large interest. Even if in Extended Relativity all the ordinary causal paradoxes seem to be solvable [3,6], nevertheless one has to bear in mind that (whenever it is met an object,  $\mathcal{O}$ , traveling with Superluminal speed) one may have to deal with negative contributions to the tunneling times [27,12]: and this should not be regarded as unphysical. In fact, whenever an "object" (particle, electromagnetic pulse,...)  $\mathcal{O}$  overcomes the infinite speed [3,6] with respect to a certain observer, it will afterwards appear to the same observer as the "anti-object"  $\overline{\mathcal{O}}$  traveling in the opposite space direction [3,6]. For instance, when going on from the lab to a frame  $\mathcal{F}$  moving in the same direction as the particles or waves entering the barrier region, the object  $\mathcal{O}$  penetrating through the final part of the barrier (with almost infinite speed[12,21,26,27], like in Figs.3) will appear in the frame  $\mathcal{F}$  as an anti-object  $\mathcal{O}$  crossing that portion of the barrier in the opposite space-direction [3.6]. In the new frame  $\mathcal{F}$ , therefore, such anti-object  $\overline{\mathcal{O}}$  would yield a negative contribution to the tunneling time: which could even result, in total, to be negative. For any clarifications, see refs. [28]. What we want to stress here is that the appearance of such negative times is predicted by Relativity itself, on the basis of the ordinary postulates [3,6,21,28]. (In the case of a non-polarized beam, the wave anti-packet coincides with the initial wave packet; if a photon is however endowed with helicity  $\lambda = +1$ , the anti-photon will bear the opposite helicity  $\lambda = -1$ ). From the theoretical point of view, besides refs. [3,6,12,21,27,28], see refs. [29]. On the (quite interesting) experimental side, see papers [30], the first one having already been mentioned above.

Let us *add* here that, via quantum interference effects it is possible to obtain dielectrics with refraction indices very rapidly varying as a function of frequency, also in three-level atomic systems, with almost complete absence of light absorption (i.e., with quantum induced transparency)[31]. The group velocity of a light pulse propagating in such a medium can decrease to very low values, either positive or negatives, with *no* pulse distortion. It is known that experiments have been performed both in atomic samples at room temperature, and in Bose-Einstein condensates, which showed the possibility of reducing the speed of light to a few meters per second. Similar, but negative group velocities, implying a propagation with Superluminal speeds thousands of time higher than the previously mentioned ones, have been recently predicted also in the presence of such an "electromagnetically induced transparency", for light moving in a rubidium condensate.[32] Finally, let us recall that faster-than-*c* propagation of light pulses can be (and was, in same cases) observed also by taking advantage of anomalous dispersion near an absorbing line, or nonlinear and linear gain lines —as already seen—, or nondispersive dielectric media, or inverted two-level media, as well as of some parametric processes in nonlinear optics (cf., e.g., G.Kurizki et al.[30]).

**D)** Superluminal Localized Solutions (SLS) to the wave equations. The "X-shaped waves" – The fourth sector (to leave aside the others) is not less important. It came into fashion again, when it was rediscovered that any wave equation —to fix the ideas, let us think of the electromagnetic case— admit also solutions as much sub-luminal as Super-luminal (besides the ordinary waves endowed with speed c/n). Actually, starting with the pioneering work by H.Bateman, it had slowly become known that all homogeneous wave equations (in a general sense: scalar, electromagnetic, spinorial,...) admit wavelet-type solutions with sub-luminal group velocities[33]. Subsequently, also Superluminal solutions started to be written down, in refs.[34] and, independently, in refs.[35] (in one case just by the mere application of a Superluminal Lorentz "transformation"[3,36]).

An important feature of some new solutions of these (which attracted much attention for possible applications) is that they propagate as localized, non-dispersive pulses: namely, as "undistorted progressive waves", according to the Courant and Hilbert's[33] terminology. It is easy to realize the practical importance, for instance, of a radio transmission carried out by localized beams, independently of their being sub- or Super-luminal. But non-dispersive wave packets can be of use even in theoretical physics for a reasonable representation of elementary particles [37], and so on. Within Extended Relativity since 1980 it had been found [38] that — whilst the simplest subluminal object conceivable is a small sphere, or a point as its limit— the simplest Superluminal objects results by contrast to be (see refs. [38], and Fig.6 and Fig.7 of this present paper) an "X-shaped" wave, or a double cone as its limit, which moreover travels without deforming —i.e., rigidly in a homogeneous medium[3]. It is not without meaning that the most interesting localized solutions happened to be just the Superluminal ones, and with a shape of that Even more, since from Maxwell equations under simple hypotheses one goes on kind. to the usual *scalar* wave equation for each electric or magnetic field component, one can expect the same solutions to exist also in the field of acoustic waves, and of seismic waves (and of gravitational waves too). Actually, such beams (as suitable superpositions of Bessel beams[39]) were mathematically constructed for the first time, by Lu et al. [40], in acoustics: and were then called "X-waves" or rather X-shaped waves.

It is more important for us that the X-shaped waves have been indeed produced in experiments both with acoustic and with electromagnetic waves; that is, X-beams were produced which, in their medium, travel undistorted with a speed larger than sound, in the first case, and than light, in the second case. In acoustics, the first experiment was



Figure 6: An intrinsically spherical (or pointlike, at the limit) object appears in the vacuum as an ellipsoid contracted along the motion direction when endowed with a speed v < c. By contrast, if endowed with a speed V > c (even if the *c*-speed barrier cannot be crossed, neither from the left nor from the right), it would appear[37] no longer as a particle, but rather as an "X-shaped" wave[37] traveling rigidly (namely, as occupying the region delimited by a double cone and a two-sheeted hyperboloid —or as a double cone, at the limit–, moving Superluminally and without distortion in the vacuum, or in a homogeneous medium).

performed by Lu et al. themselves[41] in 1992. In the electromagnetic case, certainly more intriguing, Superluminal localized X-shaped solutions were first mathematically constructed by us (cf., e.g., our Fig.8 taken from refs.[42]), and later on experimentally produced by Saari et al.[43] in 1997 by visible light (Fig.9), and more recently by Mugnai, Ranfagni and Ruggeri at Florence by microwaves[44], and, again in optics but exploting this time the response of a nonlinear medium, by Di Trapani et al.[44]. Further experimental activity is in progress; while in the theoretical sector the activity has been not less intense, in order to build up —for example— new analogous solutions with finite total energy or more suitable for high frequencies (and with adjustable bandwidth), on the one hand[45], and localized solutions Superluminally propagating even along normal waveguides, on the other hand[46]: and we are going to contribute, below, just to questions of this type.

Before going on, let us eventually mention the problem of producing an X-shaped Superluminal wave like the one in Fig.7, but truncated —of course– in space and in time (by the use of a finite antenna, radiating for a finite time: a *dynamic* antenna, in general):



Figure 7: Here we show the intersections of an "X-shaped wave" [37] with planes orthogonal to its motion line, according to Extended Relativity "[2-4]. The examination of this figure suggests how to construct a simple dynamic antenna for generating such localized Superluminal waves (such an antenna was in fact adopted, independently, by Lu et al.[39] for the production of such non-dispersive beams).



Figure 8: Theoretical prediction of the Superluminal localized "X-shaped" waves for the electromagnetic case (from Lu, Greenleaf and Recami[40], and Recami[40]).

in such a situation, the wave will keep its localization and Superluminality only along a

certain "depth of field", decaying abruptly [40,42] afterwards. Actually, such localized Superluminal beams appear to keep their good properties only as long as they are fed by the waves arriving (with speed c) from the antenna: Taking account of the time needed for fostering such Superluminal pulses (i.e., for the arrival of the feeding speed-c waves coming from the antenna). one concludes that these localized Superluminal beams are probably unable to transmit *information* faster than c. However, they don't seem to have anything to do with the illusory "scissors effect", since they carry energy-momentum Superluminally along their field depth [for instance, they can get two (tiny) detectors at a distance L to click after a time smaller than L/c. As we mentioned above, the existence of all these X-shaped Superluminal (or "Super-sonic") waves, together with the Superluminality of evanescent waves, are compatible with (Extended) Relativity: a theory based, let us repeat, on the ordinary postulates of SR and that consequently cannot be in contrast with any of its fundamental principles. And in fact all the previous results may be obtained from the Maxwell equations (or the wave equation) only. It is worthwhile mentioning, moreover, that one of the first applications of the X-waves (which takes advantage of their propagation without deformation) is in progress in the field of medicine, and precisely of ultrasound scanners [47].

# 3. SLSs to the Maxwell equations with arbitrary frequencies or finite total-energy: An outline.

As we recalled above, since many years it has been known that localized (nondispersive) solutions exist to the (homogeneous, and even non-homogeneous) wave equation[33-36], endowed with subluminal or Superluminal[42] velocities. Particular attention has been paid to the Superluminal localized solutions, which seem to propagate not only in vacuum but also in media with boundaries[48], like normal-sized metallic waveguides[14] and, possibly, optical fibers. Such SLS have been experimentally produced.

However, all the analytical SLSs known to us, with one exception [49], have a broadband frequency spectrum starting from zero, they being appropriate for low frequency

We can become convinced about the possibility of realizing it, by imaging the simple ideal case of a negligibly sized Superluminal source S endowed with speed V > c in vacuum and emitting electromagnetic waves W (each one traveling with the invariant speed c). The electromagnetic waves will result to be internally tangent to an enveloping cone C having S as its vertex, and as its axis the propagation line x of the source[3]. This is analogous to what happens for a plane that moves in the air with constant supersonic speed. The waves W interfere negatively inside the cone C, and constructively only on its surface. We can place a plane detector orthogonally to x, and record magnitude and direction of the W waves that hit on it, as (cylindrically symmetric) functions of position and of time. It will be enough, then, to replace the plane detector with a plane antenna which *emits*—instead of recording— exactly the same (axially symmetric) space-time pattern of waves W, for constructing a cone-shaped electromagnetic wave C that will propagate with the Superluminal speed V (of course, without a source any longer at its vertex): even if each wave W travels with the invariant speed c. For further details, see the first of refs.[42].



Figure 9: Scheme of the experiment by Saari et al., who announced (PRL of 24 Nov.1997) the production in optics of the beams depicted in Fig.8: In this figure one can see what shown by the experiment, i.e., that the Superluminal "X-shaped" waves which run after and catch up with the plane waves (the latter regularly traveling with speed c). An analogous experiment has been performed with microwaves at Florence by Mugnai, Ranfagni and Ruggeri (PRL of 22 May 2000).

regions only. This fact can be a problem, because it is difficult or even impossible to define a carrier frequency for those solutions (as well as to use them in high frequency applications). It is therefore interesting to obtain exact SLSs to the wave equation with spectra localized at higher (arbitrary) frequencies and with an adjustable bandwidth (in other words, with a well defined carrier frequency). In previous work of ours[49] we showed how to shift the spectrum to higher frequencies, without dealing however with its bandwidth. In the second part of this paper we are going to propose analytical and exact SLSs in vacuum, whose spectra can be localized inside any range of frequency with adjustable bandwidths, and therefore with the possibility of choosing a well defined carrier frequency. In this way, we can construct (without any approximation) radio, microwave, optical, etc., localized Superluminal waves.

We shall also show how, taking advantage of our methodology, we can obtain the first analytical approximations to SLSs in *dispersive* media (i.e., in media with a frequency dependent refractive index). One of the interesting points of our approach is that all results are obtained from simple mathematical operations on the classic "X-wave" function.

Before going on, let us set forth an infinite family of generalizations of the classic X-shaped wave; and briefly mention the problem of the construction of *finite* total-energy SLSs (which is being considered also in refs.[45]). However, We shall have to skip here other questions, like that of obtaining SLSs which propagate along waveguides (a question that is being considered in refs.[46]).

### 3.1 – Standard low-frequency SLSs, and generalizations

Let us start from the axially symmetric solution (Bessel beam) to the wave equation in vacuum, in cylindrical co-ordinates:

$$\psi(\rho, z, t) = J_0(k_\rho \rho) e^{+ik_z z} e^{-i\omega t}$$
(1)

with the conditions

$$k_{\rho}^{2} = \frac{n^{2}\omega^{2}}{c^{2}} - k_{z}^{2} ; \qquad k_{\rho}^{2} \ge 0 , \qquad (2)$$

where  $J_0$  is the zeroth-order ordinary Bessel function; quantities  $k_z \equiv k_{\parallel}$  and  $k_{\rho} \equiv k_{\perp}$  (often called simply k, with ambiguous notations) are the axial and the transverse wavenumber magnitude, respectively; while  $\omega$  is the angular frequency; c the light speed in vacuum; and, in the vacuum case, n = 1.

It is essential to stress right now that the dispersion relation (2), with positive (but not constant, a priori)  $k_{\rho}^2$  and real  $k_z$ , while enforcing the consideration of the truly propagating waves only (with exclusion of the evanescent ones), does allow for both subluminal and Superluminal solutions; the latter being the ones of interest here for us. [Conditions (2) correspond in the  $(\omega, k_z)$  plane to confining ourselves to the region delimited by the straight lines  $\omega = \pm ck_z$ ].

In this Sec.3 we shall consider, for simplicity, the vacuum case. A general, axially symmetric superposition of Bessel beams (with  $\Phi'$  as spectral weight-function) will therefore be:

$$\Psi(\rho, z, t) = \int_0^\infty \mathrm{d}k_\rho \, \int_0^\infty \mathrm{d}\omega \, \int_{-\omega/c}^{+\omega/c} \mathrm{d}k_z \, \psi(\rho, z, t) \,\delta\left(k_\rho - \sqrt{\frac{\omega^2}{c^2} - k_z^2}\right) \,\Phi'(\omega, k_z; k_\rho) \,, \quad (3)$$

where we put n = 1 for simplicity's sake.

Let us recall also that each Bessel beam is associated with an ("axicone") angle  $\theta$ , linked to its speed V by the known relations[38]:

$$\tan \theta = \sqrt{V^2 - 1}; \quad \sin \theta = \frac{\sqrt{V^2 - 1}}{V}; \quad \cos \theta = \frac{c}{V} , \qquad (4)$$

where V > c, and more precisely  $V \to c$  when  $\theta \to 0$  while  $V \to \infty$  when  $\theta \to \pi/2$ .

In the first one of refs.[45], we have however shown that instead of eq.(3) one may consider the more easily integrable Bessel beam superposition [with  $V \ge 1$ ]

$$\Psi(\rho,\zeta,\eta) = \int_0^\infty \mathrm{d}k_\rho \int_0^\infty \mathrm{d}\alpha \int_0^\infty \mathrm{d}\beta \ J_0(k_\rho\rho) \ \mathrm{e}^{i\alpha\zeta} \ \mathrm{e}^{-i\beta\eta} \times \\ \times \delta\left(k_\rho - \sqrt{(\alpha^2 + \beta^2)(V^2 - 1) + 2(V^2 + 1)\alpha\beta}\right) \ \Phi(\alpha,\beta;k_\rho)$$
(3)

where  $(\alpha, \beta)$ , which replace the parameters  $(\omega, k_z)$ , are

$$\alpha \equiv \frac{1}{2V}(\omega + Vk_z); \qquad \beta \equiv \frac{1}{2V}(\omega - Vk_z) , \qquad (5)$$

in terms of the new ("V-cone") variables:

$$\begin{cases} \zeta \equiv z - Vt \\ \eta \equiv z + Vt \end{cases}$$
(6)

The base functions  $\psi(\rho, z, t)$  are to be therefore rewritten as

$$\psi(\rho,\zeta,\eta) = J_0(k_\rho\rho) \exp i[\alpha\zeta - \beta\eta].$$

Let us now derive the classic "X-shaped solution", together with some new[50] generalizations of it.

Let us start by choosing the spectrum [with a > 0]:

$$\Phi(\alpha,\beta) = \delta(\beta - \beta') e^{-a\alpha} , \qquad (7)$$

a > 0 and  $\beta' \ge 0$  being constants (related to the transverse and longitudinal localization of the pulse).

In the simple case when  $\beta' = 0$ , eq.(3') can be easily integrated over  $\beta$  and  $k_{\rho}$  by having recourse to identity (6.611.1) of ref.[51], yielding

$$X \equiv \Psi_{\rm X}(\rho,\zeta) = \int_0^\infty \mathrm{d}\alpha \, J_0(\rho\alpha\sqrt{V^2 - 1}) \,\mathrm{e}^{-\alpha(a - i\zeta)} = = \left[(a - i\zeta)^2 + \rho^2(V^2 - 1)\right]^{-1/2} \,, \tag{8}$$

which is exactly the classic X-shaped solution proposed by Lu & Greenleaf[39] in acoustics, and later on by others[42] in electromagnetism, once relations (4) are taken into account. See Fig.10. It may be useful to recall the obvious circumstance that any solution that depends on z (and t) only through the variable  $\zeta \equiv z - Vt$  will be rigidly moving with speed V.

Many other SLSs can be easily constructed; for instance, by inserting into the weight function (7) the extra factor  $\alpha^m$ , namely  $\Phi(\alpha, \beta) = \alpha^m \delta(\beta) \exp[-a\alpha]$ , while it is still  $\beta' = 0$ . Then an infinite family of new SLSs is obtained (for  $m \ge 0$ ), by using this time identity (6.621.4) of the same ref.[51]:

$$\Psi_{\mathbf{X},m}(\rho,\zeta) = (-i)^m \frac{\mathrm{d}^m}{\mathrm{d}\zeta^m} \left[ (a-i\zeta)^2 + \rho^2 (V^2 - 1) \right]^{-1/2}$$
(9)

which generalize[50] the classic X-shaped solution, corresponding to m = 0: namely,  $\Psi_{\rm X} \equiv \Psi_{{\rm X},0}$ . Notice that all the derivatives of the latter with respect to  $\zeta$  lead to new SLSs, all of them being X-shaped.

In the particular case m = 1, one gets the SLS

$$\Psi_{\mathbf{X},1}(\rho,\zeta) = \frac{-i(a-i\zeta)}{\left[(a-i\zeta)^2 + \rho^2(V^2-1)\right]^{3/2}}$$
(10)

which is the first derivative of the X-shaped wave, and is depicted in Fig.11. One should notice that, by increasing m, the pulse becames more and more localized around its vertex. All such pulses travel, however, without deforming.



Figure 10: In Fig.10 it is represented (in arbitrary units) the square magnitude of the "classic", X-shaped Superluminal Localized Solution (SLS) to the wave equation[41,42], with V = 5c and a = 0.1 m: cf. eqs.(8) and (6). An infinite family of SLSs however exists, which generalize the classic X-shaped solution; the Fig.11, below, depicts the first of them (its first derivative) with the same parameters: see the text and eq.(10). The successive solutions in such a family are more and more localized around their vertex. Quantity  $\rho$  is the distance in meters from the propagation axis z, while quantity  $\zeta$  is the "V-cone" variable (still in meters)  $\zeta \equiv z - Vt$ , with  $V \geq c$ . Since all these solutions depend on z (and t) only via the variable  $\zeta$ , they propagate "rigidly", i.e., without distortion (and are called "localized", or non-dispersive, for such a reason). Here we are assuming propagation in the vacuum (or in a homogeneous medium).

Solution (8) is suited for low frequencies only, since its frequency spectrum (exponentially decreasing) starts from zero. One can see this for instance by writing eq.(7) in the  $(\omega, k_z)$  plane: By eqs.(5) one obtains



Figure 11: See the caption of the previous Figure 10.

$$\Phi(\omega, k_z) = \delta\left(\frac{\omega - Vk_z}{2V} - \beta'\right) \exp\left[-a\frac{\omega + Vk_z}{2V}\right]$$

and can observe that  $\beta' = 0$  in the delta implies  $\omega = Vk_z$ . So that the spectrum becomes  $\Phi = \exp[-a\omega/V]$ , which starts from zero and has a width given by  $\Delta \omega = V/a$ .

By contrast[50], when the factor  $\alpha^m$  is present, the frequency spectrum of the solutions can be "bumped" in correspondence with any value  $\omega_M$  of the angular frequency, provided that m is large [or a/V is small]: in fact,  $\omega_M$  results to be  $\omega_M = mV/a$ . The spectrum, then, is shifted towards higher frequencies (and decays only beyond the value  $\omega_M$ ).

#### 3.2 – An example of finite total-energy solution

The previous solutions travel without any deformation, but possess infinite totalenergy (as well as the plane waves). The finite total-energy solutions, by contrast, do (slightly, in general) get deformed while propagating. To show this by an example, let us go on to a more general choice for the weight-function: namely, by inserting into eq.(3') for  $\beta \geq \beta_0$  the spectrum

$$\Phi = e^{-a\alpha} e^{-b(\beta - \beta_0)} \qquad \text{for } \beta \ge \beta_0 . \tag{7'}$$

We then obtain [for  $\beta_0 \ll 1$ ] the Superluminal "Modified Power Spectrum Pulse" [52]

$$\Psi_{\rm SMPS}(\rho,\zeta,\eta) = e^{b\beta_0} X \int_{\beta_0}^{\infty} d\beta \ e^{-(b+i\eta-Y)\beta} = X \ \frac{\exp[(Y-i\eta)\beta_0]}{b-(Y-i\eta)}$$
(11)

in which the integration over  $\beta$  runs now from  $\beta_0$  (no longer from zero) to infinity; and

$$Y \equiv \frac{V^2 + 1}{V^2 - 1} \left( (a - i\zeta) - X^{-1} \right) \,.$$

Our SMPS pulses, solutions (11), do not propagate rigidly any longer (as can be inferred also from the circumstance that they depend on  $\eta$ , besides on  $\zeta$ ). They are still Superluminal, however, as can be expected from the fact that they are superpositions of beams with constant  $\theta$  [and therefore with constant V: cf. eqs.(4)]. Actually, on integrating eq.(11) with  $\rho = 0$ , one gets  $\Psi_{\text{SMPS}}(\rho = 0, \zeta, \eta) = e^{-i\beta_0\eta} [(a - i\zeta)(b + i\eta)]^{-1}$ , and it is possible to verify, e.g., that the maximum amplitude of the (easily evaluated) real part of  $\Psi_{\text{SMPS}}(0, \zeta, \eta)$  goes on corresponding to z = Vt.



Figure 12: Representation of our Superluminal Modified Power Spectrum (SMPS) pulses, eq.(11). These beams possess *finite* total energy, and therefore get deformed while traveling. This Figure 12 depicts the shape of the pulse, for t = 0, with V = 5c, a = 0.001 m, b = 100 m, and  $\beta_0 = 1/(100 \text{ m})$ . In Fig.13, below, it is shown the same pulse after a 50 meters propagation.

It is worthwhile to emphasize that our solutions (11) possess a *finite total energy*.\*\* This is easily verified. In particular, their amplitude decreases with time: see Fig.12 and Fig.13.

We shall show elsewhere [45] how to construct, e.g., the finite-energy Superluminal generalization (SSP) of the so-called (luminal) "splash pulse" (SP) which was introduced

<sup>\*\*</sup>One should recall that the first finite energy solution, the MFXW, different from but analogous to our one, appeared in ref.[53].



Figure 13: See the caption of the previous Figure 12.

in refs.[52]. Our SSP will appear to be the finite-energy version of the classic X-shaped solution.

# 4. New localized Superluminal exact solutions to the Maxwell equations, with arbitrary frequencies and adjustable bandwidths.

Let us finally come to the main topic (of the second part) of this paper. From eq.(2), when putting  $ck_z \equiv n\omega \cos\theta$ , one can write  $ck_\rho \equiv n\omega \sin\theta$ . In the new coordinates  $(\omega, \theta)$ , solution (1) becomes

$$\psi(\rho,\zeta) = J_0(\frac{n\omega}{c}\rho\sin\theta) \exp\left[+i\frac{n\omega}{c}\zeta\cos\theta\right]$$
(12)

where we used the relations  $\zeta \equiv z - Vt$  and  $V = c/(n \cos \theta)$ , and we confine ourselves for the moment to a homogeneous medium with n = constant. Equation (12) contains two free parameters:  $\omega$  and  $\theta$ . This equation (which represents the "Bessel beam") tells us once more that such a beam is transversally localized in energy, and propagates without dispersion. Considering a superposition of waves with different frequencies and  $\theta$  (and therefore V) constant,<sup>††</sup> one can get the SLSs in the alternative form

$$\Psi(\rho,\zeta) = \int_0^\infty S(\omega) J_0(\frac{n\omega}{c}\rho\sin\theta) \exp\left[+i\frac{n\omega}{c}\zeta\cos\theta\right] d\omega .$$
 (13)

We have to find out a spectrum  $S(\omega)$  which preserves the integrability of eq.(13) for any

<sup>&</sup>lt;sup>††</sup>Superpositions of beams with different values of  $\theta$  (and therefore of V) will be considered elsewhere.

frequency range: In order to be able to shift our spectrum towards the desired frequency, let us locate it around a central frequency,  $\omega_c$ , with an (arbitrary) bandwidth  $\Delta \omega$ . Then, let us choose the spectrum

$$S(\omega) = \left(\frac{\omega}{V}\right)^m e^{-a\omega} \tag{14}$$

where V is the wave velocity, while m and a are free parameters. [For m = 0, it is  $S(\omega) = \exp[-a\omega]$ , and one gets the (standard) X-wave spectrum]. For  $m \neq 0$ , some mathematical manipulations yield the relations

$$m^{-1} = (\Delta \omega_{\pm}/\omega_{\rm c}) - \ln[1 + (\Delta \omega_{\pm}/\omega_{\rm c})]$$
(14a)

$$\omega_{\rm c} = \frac{m}{a} \ . \tag{14b}$$

where, because of the non-symmetric character of spectrum (14), we had to call  $\Delta\omega_+$ (> 0) and  $\Delta\omega_-$  (< 0) the bandwidth to the right and to the left of  $\omega_c$ , respectively; so that  $\Delta\omega = \Delta\omega_+ - \Delta\omega_-$ . [However, already for small values of m (typically, for  $m \ge 10$ ), one has  $\Delta\omega_+ \approx -\Delta\omega_-$ ]. Once defined  $\omega_c$  and  $\Delta\omega$ , one can determine m from the first equation. Then, using the second one, a is found. Figure 14 illustrates the behavior of relation (14a): One can observe that the smaller  $\Delta\omega/\omega_c$  is, the higher m must be; parameter m plays the important role of controlling the spectrum bandwidth.



Figure 14: Behavior of the derivative number, m, as a function of the normalized bandwidth frequency,  $\Delta \omega_{\pm}/\omega_{\rm c}$ . Given a central frequency,  $\omega_{\rm c}$ , and a bandwidth,  $\Delta \omega_{\pm}$ , one finds the exact value of m by substituting these values into eq.(14a).

From the X-wave spectrum, it is known that a is related to the (negative) slope of the spectrum. On the contrary, quantity m has the effect of rising the spectrum. In this way, one parameter compensates for the other, producing the localization of the spectrum

inside a certain frequency range. At the same time, this fact also explains (because of relation (14b)) why an increase of both m and a is necessary to keep the same  $\omega_c$ . This can be seen from Fig.15.



Figure 15: Normalized spectra for  $\omega_c = 23.56 \times 10^{14}$  Hz and different bandwidths. The solid line corresponds to m = 27, and the dotted line to m = 41. See the text. Both spectra have the same  $\omega_c$ : To get this result, one can observe that (taking the narrower spectrum as a reference) both m and a have to increase. This figure shows moreover the important role of m in controlling the spectrum bandwidth.

One can verify that, on inserting into eq.(13) the exponential spectrum  $S(\omega) = \exp[-a\omega]$ , one gets once more the ordinary X-wave, namely

$$X = V \left[ \sqrt{(aV - i\zeta)^2 + \rho^2 (n_0^2 V^2 / c^2 - 1)} \right]^{-1} ,$$

where now we explicitly wrote down the refraction index  $n_{\rm o}$  of the considered (dispersionless) medium. To exemplify the use of our solution (13), let us rewrite it with the spectrum (14):

$$\Psi(\rho,\zeta) = V \int_0^\infty (\frac{\omega}{V})^m J_0\left(\frac{\omega}{V}\rho\sqrt{n_0^2 \frac{V^2}{c^2} - 1}\right) e^{-(aV - i\zeta)\omega/V} d(\frac{\omega}{V})$$
(15)

which shows that the use of a spectrum like (14) allows shifting the proposed solution towards any frequency while confining it within the desired frequency range.

It is not difficult to show (or verify) that eq.(15) can be written in the interesting forms

$$\Psi(\rho,\zeta) = (-1)^m \frac{\partial^m X}{\partial (aV - i\zeta)^m} , \qquad (16)$$

and, by use of identity (6.621) of ref.[51],

$$\Psi(\rho,\zeta) = \frac{\Gamma(m+1) X^{m+1}}{V^m} F\left(\frac{m+1}{2}; -\frac{m}{2}; 1; \left(n_0^2 \frac{V^2}{c^2} - 1\right) \rho^2 \frac{X^2}{V^2}\right),$$
(16')

where F is the Gauss hypergeometric function. [Equation (16') can be useful in the cases of large values of m].

Let us call attention to equations (16) and (16'): to our knowledge,<sup>‡‡</sup> no analytical expression has been previously met for X-*type* waves, apt at being localized in the neighborhood of any chosen frequency with an adjustable bandwidth. Figure 16 shows an example of X-type wave for microwave frequencies: it has shape and properties similar to the classic X-wave's.



Figure 16: The real part of an X-shaped beam for microwave frequencies in a dispersionless medium. It corresponds to  $\omega_c = 6 \times 10^9$  GHz and  $\Delta \omega = 0.9 \omega_c$ , while the values of m and a, calculated by using eqs.(14a)) and (14b), are m = 10 and  $a = 1.6667 \times 10^{-9}$ . The resulting wave possesses both a longitudinal and a transverse localization.

### 5. Superluminal localized waves in dispersive media.

<sup>&</sup>lt;sup>‡‡</sup>It can be noticed that  $\partial X/\partial (aV - i\zeta) = (iV)^{-1}\partial X/\partial t$ . Time derivatives of the X-wave have been actually considered by J.Fagerholm et al.[50]: however the properties of the generating spectrum (like its ability in frequency shifting and bandwidth controlling) have not been studied in previous work.

Let us now go on to dealing with *dispersive* media, in which case the refraction index depends on the wave frequency:  $n = n(\omega)$ ; so that we shall have[49]

$$\begin{cases} ck_z \equiv n(\omega) \,\omega \,\cos\theta \\ ck_\rho \equiv n(\omega) \,\omega \,\sin\theta . \end{cases}$$

Looking for a localized wave that does not suffer dispersion, one can exploit the fact that  $\theta$  determines the wave velocity: Namely, one can choose a particular frequency dependence of  $\theta$  to *compensate* for the (geometrical) dispersion due to the variation of n with the frequency.[49] When a dispersionless pulse is desired, the constraint  $k_z = d + \omega b$  must be satisfied. On using the first one of the last two equations, we infer such a constraint to be forwarded by the following relationship between  $\theta$  and  $\omega$ :

$$\cos(\theta(\omega)) = \frac{(d+b\,\omega)c}{\omega\,n(\omega)} \,, \tag{17}$$

with d and b arbitrary constants (b being related to the wave velocity: b = 1/V). If we choose, for convenience, d = 0, eq.(13) becomes

$$\Psi(\rho,\zeta) = \int_0^\infty S(\omega) J_0\left(\rho\omega\sqrt{\frac{n^2(\omega)}{c^2} - b^2}\right) e^{+i\omega b\zeta} d\omega .$$
(18)

Let us stress that eq.(18) constitutes an analytical (integral) formula which represents a wave propagating without dispersion in a *dispersive* medium; it is well suited for any desired frequency range, and a priori can find applications in acoustics, microwave physics, optics, etc.

Let us mention how it is possible to realize relation (17) for *optical* frequencies, by following a procedure similar to the one illustrated in Figure 17. The wave vector for each spectral component suffers a different deviation in passing through the chosen device (axicone, hologram, and so on[41-43,54]): and such a deviation, associated with the dispersion due to the medium, can be such that the phase velocity remains the same for every frequency. This corresponds to no dispersion for the beam. We may consider a nearly gaussian spectrum, like that in eq.(14), and assume the refractive index (for the frequency range of interest) to depend linearly on  $\omega$ , that is,  $n(\omega) = n_0 + \omega \delta$ , with  $\delta$  a free parameter: a linear dependence that is actually realizable for frequencies far from the resonances. In this case, on using spectrum (14), a relation similar to eq.(22) is found, which can be evaluated via a Taylor expansion, that (if  $\delta$  is small enough) can be truncated at its first derivative. The expression of such a first derivative can be integrated (by using identity 6.621.4 of ref.[51]); and eventually we find that our eq.(18) admits the approximate solution

$$\Psi(\rho,\zeta) = (-1)^m \frac{\partial^m X}{\partial (aV - i\zeta)^m} + (-1)^{m+4} \frac{V^3 n_0}{c^2 \left(n_0^2 \frac{V^2}{c^2} - 1\right)} \frac{\partial^{m+2}}{\partial (aV - i\zeta)^{m+2}} \left[ (aV - i\zeta) X \right] \delta^{(19)}$$

which could be written also in a way similar to eq.(16'). It is interesting to notice that also the SLS, eq.(19), for a dispersive medium has been obtained by simple mathematical operations (derivatives) acting to the standard "X-wave".



Figure 17: Sketch of a generic device (axicone, hologram, etc.) suited to properly deviating the wave vector of each spectral component.

## 6. – Optical applications

Let us make two practical examples, both for optical frequencies: in particular, with optical fibers. The bulk of the dispersive medium be fused Silica (SiO<sub>2</sub>). Far from the medium resonances (which is our case), the refractive index can be approximated by the well-known Sellmeier equation [55]

$$n^{2}(\omega) = 1 + \sum_{j=1}^{N} \frac{B_{j} \omega_{j}^{2}}{\omega_{j}^{2} - \omega^{2}} , \qquad (20)$$

where  $\omega_j$  are the resonance frequencies,  $B_j$  the strength of the *j*th resonance, and N the total number of the material resonances that appear in the frequency range of interest. For typical frequencies of "long-haul transmission" in optics, it is appropriate to choose N = 3, which yields the values[55]  $B_1 = 0.6961663$ ;  $B_2 = 0.4079426$ ;  $B_3 = 0.8974794$ ;  $\lambda_1 = 0.0684043 \ \mu\text{m}$ ;  $\lambda_2 = 0.1162414 \ \mu\text{m}$ ; and  $\lambda_3 = 9.896161 \ \mu\text{m}$ . Figure 18 illustrates the relation between n and  $\omega$ , and specifies the range we are going to adopt

here. Our two examples consist in localizing the spectrum around the angular frequency  $\omega_c = 23.56 \times 10^{14}$  Hz (which corresponds to the wavelength  $\lambda_c = 0.8 \ \mu$ m, with the two different bandwidth  $\Delta \omega_1 = 0.55 \omega_c$  and  $\Delta \omega_2 = 0.4 \omega_c$ . The values of a and m corresponding to these two situations are  $a = 1.14592 \times 10^{-14}$ , m = 27, and  $a = 1.90986 \times 10^{-14}$ , m = 41, respectively. On looking at the two "windows" in Fig.18, one notices that the Silica refraction index does not suffer strong variations, and that a linear approximation to  $n = n(\omega)$  is quite satisfactory. Moreover, for both the situations and their respective  $n_0$  values, the value of  $\delta$  results to be very small (which means that in these cases the mentioned truncation of the Taylor expansion is certainly acceptable).



Figure 18: Variation of the refractive index  $n(\omega)$  with frequency for fused Silica. The solid line is its behaviour, according to Sellmeir's formulae. The open circles and squares are the linear approximations for m = 41 and m = 27, respectively. See the text.

The beam intensity profiles for the two bandwidths are shown in Fig.19 and Fig.20. In the first figure, one can see a pattern similar to that of an ordinary X-wave (cf. Fig.3a); but now the pulse is much more localized spatially and temporally (typically, it is a femtosecond pulse). The second figure shows some small differences, with respect to the first one, in the spatial oscillations inside the wave envelope[56]: This can be due to the circumstance that the carrier wavelength, for certain values of the bandwidth, becomes shorter than the width of the spatial envelope; so that one meets a well defined carrier frequency. Let us repeat that both these waves are transversally and longitudinally localized, being moreover free from dispersion (since the dependence of  $\Psi$  on z (and t) is through the variable  $\zeta = z - Vt$  only) just as it happens for a classic X-shaped wave.

### 7. – Conclusions

In the first part of this article (after a brief theoretical introduction based on Special



Figure 19: The real part of an X-shaped beam for optical frequencies in a dispersive medium, with m = 27. It refers to the larger window in Fig.16.



Figure 20: The real part of an X-shaped beam for optical frequencies in a dispersive medium, with m = 41. It refers to the inner window in Fig.16.

Relativity) we have mentioned the various experimental sectors of physics in which Superluminal motions seem to appear. In particular, a bird's-eye view has been presented of the experiments with evanescent waves (and/or tunneling photons), and with the "localized Superluminal solutions" (SLS) to the wave equation, like the so-called X-shaped beams; we have added to such an "experimental", very short review a number of references, sufficient in some cases to provide the interested readers with reasonable bibliographical information.

In the second part of this paper, we have first worked out analytical Superluminal

localized solutions to the wave equation for arbitrary frequencies and with adjustable bandwidth in vacuum. The same methodology has been then used to obtain new, analytical expressions representing X-type waves (with arbitrary frequencies and adjustable bandwidth) which propagate *in dispersive media*. Such expressions have been obtained, on one hand, by adopting the appropriate spectrum (which made possible to us both choosing the carrier frequency rather freely, and controlling the spectral bandwidth), and, on the other hand, by having recourse to simple mathematics. Finally, we have illustrated some examples of our approach with applications in optics, considering fused Silica as the dispersive medium.

The case of the very interesting (and more orthodox, in a sense) *subluminal* Localized Waves, solutions to the wave equations, will be dealt with in another paper.

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