simpler, and understandable components illustrates the scientific method in a way traditional instruction does not achieve.
(3) When quantitative reasoning or numerical calculations are required, students do not perceive this as mathematical drudgery, because they can see its application to some real physical phenomenon.
(4) The basic conceptual ideas important to an introductory non-calculus based physics course are not neglected using this approach. Students just encounter them in a slightly unorthodox sequence.

The disadvantages are:

(1) Teaching using the "top-down" approach is very difficult for physicists, requiring a great deal of self restraint to avoid presenting some background theory before carrying out an experiment or demonstration.
(2) The preparation of suitable lecture demonstrations is very time consuming, and instructors must usually prepare much supplementary written material.
(3) Some compromise in the breadth of material covered must be made to accommodate the demonstration-based approach of the course.

V. CONCLUSIONS

The "top-down" approach provides an alternative strategy for teaching introductory science literacy physics courses. The philosophy of reversing the traditional sequence of instruction, and working backwards from the complex to the simple seems to provide a framework for effective teaching of fundamental physical concepts, whilst still remaining interesting for students. Physics is above all, a subject based on observation, and our main point is to make the observations of our students provide the starting point of the lectures.

ACKNOWLEDGMENT

We would like to thank Dr. Graeme Putt for useful discussions relating to the preparation of this manuscript.

[1]Auckland University Paper 31.102, *The World of Physics*.
[2]A. B. Arons, *A Guide to Introductory Physics Teaching* (Wiley, New York, 1991).
[3]R. Baierlein, "Meeting Them on Their Own Ground," Phys. Today **48** (3), 87–88 (1995).
[4]D. G. Stork, "Teach Nonscience Majors 'Real World' Physics," Phys. Today **47** (5), 15 (1994).
[5]F. C. Michel, "Science Literacy at the College Level," Phys. Today **46** (1), 69–71 (1993).
[6]D. H. Frisch, "Teaching Physics to Poets and Vice Versa," Phys. Today **44** (2), 118 (1991).
[7]L. M. Lederman, "Physics for Poets," Phys. Today **43**(7), 15 (1990).
[8]L. C. McDermott and P. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity. Part I: Investigation of student understanding," Am. J. Phys. **60**, 994–1003 (1992).
[9]P. Shaffer and L. C. McDermott, "Research as a guide for curriculum development: An example from introductory electricity. Part II: Design of Instructional Strategies," Am. J. Phys. **60**, 1003–1013 (1992).
[10]P. G. Hewitt, "Concepts before Computation," Phys. Teach. **32**, 224 (1994).
[11]P. G. Hewitt, *Conceptual Physics* (Harper Collins, New York, 1993), 7th ed.
[12]A. Beiser, *Physics* (Addison-Wesley, Reading, MA, 1991), 5th ed.
[13]J. D. Cutnell and K. W. Johnson, *Physics* (Wiley, New York, 1992), 2nd ed.

Complex speeds and special relativity

Catherine Asaro[a]
Molecudyne Research, Box 1379, Laurel, Maryland 20725

(Received 1 September 1992; accepted 28 July 1995)

The quest to find faster-than-light particles has intrigued physicists for decades, though it has yet to turn up any real candidates. Even if a superluminal universe does exist, we have no way to reach it given that we must go through the speed of light, which to the best of our knowledge is impossible. In this paper, I show that by making speed complex, we can go around the speed of light in a manner analogous to the way a car faced with an infinitely tall road block might leave the road to go around that barrier. The treatment is a mathematical device; no known physical interpretation exists for the imaginary part of a complex speed. However, it can provide an entertaining problem in special relativity, one that may encourage students to think about the connections between equations and the physical universe. © *1996 American Association of Physics Teachers.*

I. INTRODUCTION

When I was teaching physics, I found that special relativity in particular provoked the interest of my students because it involved wonderfully strange effects that could be discussed without graduate level math.[1] They found it both fascinating and frustrating that the speed of light is a barrier which prevents anything in our subluminal (slower-than-light) universe from reaching superluminal (faster-than-light) speeds. Here I show that if we make speed a complex number, we can go *around* the speed of light the way a car faced with an insurmountable road block might leave the road to go around that barrier. In our present day understanding of the universe, no known physical interpretation exists for imaginary speed. However, the problem provides a useful exercise in special relativity, one that illustrates the value of learning how to interpret the results of equations in physical terms.

Section II gives an overview of the physics that students need in order to follow the discussions in this paper. Section III presents the extension of special relativity to complex speeds. Section IV gives a problem (relativistic "absorption") which illustrates how comparisons between different areas of physics have the potential to yield new insights.

II. SUPERLUMINAL PHYSICS

Although prerelativistic theorists considered the possibility of superluminal particles as early as the late 1800s, the paper that introduced many of the current ideas about the subject is the classic article by Bilaniuk, Deshpande, and Sudarshan on "meta" relativity.[2] Feinberg gave a quantum field theory of noninteracting superluminal particles and introduced the word tachyon, from the Greek word *tachys* meaning "swift."[3] Two words have been suggested for slower than light particles: "bradyon" from the Greek word for "slow," and "tardyon" from the obvious derivation.[4,5] Because of the similarity between the words tachyon and tardyon, I will use bradyon for subluminal particles. Following Bilaniuk and Sudarshan, I will use "luxon" for particles with luminal speed.[4]

Some scientists have hypothesized superluminal reference frames, relative to which tachyons behave like bradyons.[6–9] Fjelstad and his students show how perplex number theory provides insight into the differences among the various theories[10] (a perplex number satisfies $z=x+hy$, where $|h| = -1$). Others discuss the EPR paradox, which raises the question of whether or not distant particles can communicate at superluminal speeds.[11] Bibliographies of the literature on superluminal physics have been compiled,[12] and review articles have presented the subject in a manner suitable for undergraduates.[4,5] Results have been reported for various tachyon searches, though so far none of the projects has produced definitive evidence for superluminal particles.[13]

A. Properties

Special relativity predicts that no object can go from subluminal to superluminal speeds because to do so it would have to pass through the speed of light, where mass (and thus energy) becomes infinite relative to any slower observer. Time dilation also becomes infinite, which means slower observers would see time stop for the object; even if the transition from subluminal to superluminal speed somehow became possible, it would take an infinite amount of time for an observer to record it. Space contraction goes to zero at light speed; an observer measures the length of the object in the direction of travel as contracted to zero when it passes the observation point.[14] Although contraction to zero length does not necessarily forbid the transition, it is an odd enough prediction that even if the other two problems did not exist, it would still warrant questioning the likelihood of such a process. The theory does allow for massless particles, like photons, that travel at light speed, but if such particles ever slow down (or speed up), they cease to exist.[15]

However, despite Einstein's conviction that superluminal travel was impossible,[16] relativity does not rule out its existence. By assuming relativistic theory is valid in a superluminal universe, we can determine the behavior of tachyons.[2–12] Light speed is still an impassable barrier, but now it is the *slowest* possible speed. This suggests three classes of objects: bradyons (class I particles) that travel at subluminal speeds; luxons (class II) that have no mass and travel at light speed; and tachyons (class III) that travel at superluminal speeds.

Significant differences exist between the behavior that special relativity predicts for tachyonic and bradyonic objects. Suppose we are traveling with a group of space ships that can go at any speed u as long as $|u|<c$, where c is the speed of light. Two observers are keeping track of us, one in reference frame S and the other in S'. The two frames share a common x axis, and S' moves at speed v relative to S, where $|v|<c$. Our speed u is measured relative to S. According to the Lorentz transformations, if the observer in S sees one of our ships travel a distance Δx in time Δt, then the distance and time intervals recorded in S' are given by $\Delta x'$ and $\Delta t'$, where[5]

$$\Delta x' = \gamma \Delta x\left(1-\frac{v}{u}\right), \tag{1}$$

$$\Delta t' = \gamma \Delta t\left(1-\frac{uv}{c^2}\right). \tag{2}$$

With the definition of a unitless speed $\beta=v/c$, the relativistic γ factor is

$$\gamma=\frac{1}{\sqrt{1-\beta^2}}. \tag{3}$$

Depending on the values of u and v, the distance interval $\Delta x'$ can have either the same or the opposite sign as Δx. However, $|uv|<c^2$ for all u and v, so $\Delta t'$ always has the same sign as Δt. In other words, whether an observer records a particular ship moving in the $+x$ or $-x$ direction depends on that observer's speed relative to the ship, but *every* observer agrees the ship goes forward in time.

Now consider what happens if the ships are superluminal. Assuming the Lorentz transformations are valid for $|u|>c$, we can show that time and space interchange character as compared to the subluminal universe. Because $|u|>|v|$, the space interval $\Delta x'$ must always have the same sign as Δx (I am assuming the observers are still subluminal). However, depending on the value of uv, the interval $\Delta t'$ may have either the same or the opposite sign as Δt. Thus all observers agree on the direction a ship travels in space but *not* on its direction in time. This is the causality paradox; if something can travel back in time, the effect of an event can be put before its cause. An observer could see you die before you were born!

Causality paradoxes are not the only problem we encounter in the superluminal universe. The Lorentz transformations predict that if we observe a particle moving "pastward," we will also measure its relativistic energy as negative. Here, I am using the energy E defined by

$$E=Mc^2, \tag{4}$$

where the mass of an object traveling with speed β is

$$M=m_0\gamma. \tag{5}$$

In the subluminal universe, m_0 refers to the mass of a particle as measured in the reference frame where it is at rest.[17] Using m_0 allows us to rewrite the energy as

$$E=m_0c^2+m_0(\gamma-1)c^2, \tag{6}$$

which is the sum of the rest energy and kinetic energy, both of which are inherently positive. Let E and Δt be, respectively, the energy and interval of time associated with a process in reference frame S, say a particle traveling with speed u through distance Δx. If E' and $\Delta t'$ are the corresponding energy and time intervals as observed in frame S' (which moves at speed v with respect to S), it is straightforward to show that[5]

$$\frac{E'}{E}=\frac{\Delta t'}{\Delta t}=\left(1-\frac{v^2}{c^2}\right)^{-1/2}\left(1-\frac{uv}{c^2}\right) \tag{7}$$

When $uv>c^2$, both E' and $\Delta t'$ change sign relative to their values in S.

Physicists object to particles with $E<0$ because no system would be stable against their emission.[3] Relativistic theory predicts that if one observer can record such an energy, then that state must be possible for all observers, which means negative energy states exist for all observers, contradicting the known stability of ordinary matter.[5]

We can resolve the time and energy problems using the *reinterpretation principle*.[2,4] According to reinterpretation, a tachyon with negative energy going into the past appears as an antitachyon with positive energy going into the future, traveling from its destination to its point of origin. Suppose particle P_a emits a tachyon with energy $E>0$ that travels until particle P_b absorbs it. Any observers who measure its energy as negative should also observe it going backward in time, which means they would record its absorption *before* its emission. Reinterpretation suggests the observers actually see the time-reversed process: P_b emits an antitachyon with positive energy E that travels into the future until P_a absorbs it. The laws of physics remain valid, though we may not all agree on the events we observe.

Reinterpretation can be seen as the temporal analogy of a much more familiar phenomenon. Consider what happens to cars traveling at sublight speeds. Person P_a is in a car that travels at speed v_a. Person P_b is in a car traveling in the same direction at speed v_b such that $v_a<v_b$. If we are standing still on the sidewalk, then when the two cars go by we observe both moving in the same direction relative to us. P_b sees P_a's car moving backward relative to his car, and P_a sees herself as stationary relative to her car. The various observers see different processes, but every process is consistent with the others; everyone agrees that P_a and P_b arrive at their destinations, but each person sees the arrivals occur in a different way.

Now consider superluminal speeds. For tachyons, it is the temporal rather than spatial direction that depends on the observer's reference frame. Say P_b is the pilot of a superluminal ship and P_a is a rocket scientist recording P_b's journey from an observatory on her home planet. P_b always observes himself moving forward in time because he is at rest relative to his ship. However, his speed is such that P_a observes him traveling pastward. Can P_a watch P_b go back in time and, say, affect his parent's history so that they die before he is born, creating a paradox? I would say no, for the following reason: P_b sees himself living in a timeline that *for him* always progresses futureward. In analogy with the spatial example given above, his observations must be consistent with those of all other observers. This suggests it is impossible for him to kill his parents; otherwise, that event would have already taken place in his timeline, which of course it has not. In other words, P_b's life as observed from other reference frames must be consistent with his own observations just as spatial observations must be consistent. This does not necessarily mean P_b can never appear in his own past, but rather that he cannot change that past after he has experienced it.

But what does P_a see when P_b goes pastward? Imagine that P_b travels to point x_1 in space and arrives there at time t_1, where position and time are measured in his frame of reference. At x_1, his ship takes on a speed such that the scientist P_a observes it traveling pastward. She records P_b moving into the past until he reaches point x_2 at time t_2 (as measured in his frame), after which he takes on a speed such that P_a once again records his motion as futureward. P_b then continues on to point x_3. Relativity predicts that P_a observes the ship at x_2' first, where the prime indicates the measurement is made in P_a's reference frame. However, P_b observes himself traveling forward in time continuously from point x_1 to point x_3. In other words, P_b records $t_1<t_2$ and P_a records $t_1' > t_2'$. So what does P_a actually see?

Extrapolating reinterpretation into a macroscopic realm suggests a possible interpretation of the above events. At point x_2' and time t_2', scientist P_a sees two ships create by pair production, one matter and the other antimatter (this requires large enough mass in the vicinity to ensure conservation laws are satisfied). The matter ship travels on to x_3'. However, the antimatter ship follows a reversed trajectory between x_2' and x_1'; it moves along a time-reversed path compared to what P_b himself observes when he travels from x_1 to x_2. Meanwhile, P_a sees a third ship approaching point x_1', a twin to the matter ship now traveling from x_2' to x_3' (the twin ships are not identical because they are at different points along the trajectory). At point x_1' and time t_1' the antimatter and matter ships meet and annihilate each other, producing an equivalent amount of energy, mass and charge as was used to create the antimatter and matter ships at x_2'. Although the different observers see dramatically different processes, in theory no physical laws are violated. The final result is the same in both reference frames: The ship arrives at its destination.

Of course, the macroscopic nature of this scenario raises problems. For one thing, the matter and antimatter ship must pair produce without annihilating; they must create together without occupying overlapping regions in space. The example also makes the rather odd prediction that P_a sees the antimatter ship gaining antifuel as it travels "backward" from point x_2' to x_1'. The reader may be able to supply additional oddities regarding the scenario, and perhaps think up possible solutions or explanations.

One might argue that the above scenario requires predetermination. Although this is more a philosophical than physical matter, my first inclination is to say consistency is a less severe constraint than predetermination. Actually, causality itself is a form of predetermination. In some ways a universe where objects move forward or backward in time, but not space, is analogous to a universe such as ours where objects move forward or backward in space, but not time. The equations for position, velocity and acceleration determine an object's behavior until a new force acts on it; if we know all of the forces involved, we can specify exactly how the object behaves at all times.

Another problem raised by superluminal travel is that relativity predicts that superluminal objects have imaginary mass. When $\beta>1$ is put into Eq. (5), the mass becomes

$$M=\frac{\mp i m_0}{\sqrt{\beta^2-1}}. \tag{8}$$

The $+im_0$ comes from the negative root of $\sqrt{-1}$ and is dropped in most treatments. For the time being, I will do likewise and deal only with $-im_0$. However, I will show later that the second root does play a useful role in the discussion.

Unfortunately, even if a universe with imaginary properties exists, we do not (yet) know how to interact with it. Superluminal reference frames provide one possible theoretical device for circumventing the imaginary nature of tachyons.[6–9] A tachyon with $\beta>1$ relative to subluminal frames acts like a bradyon with $1/\beta<1$ relative to superluminal frames, and tachyons obey the same physical laws in superluminal frames that bradyons obey in subluminal frames. Combined with reinterpretation, this leads to the intriguing suggestion that what we measure as antiparticles are the particles themselves, but traveling backward in time.

Another way to avoid the imaginary nature of tachyons is to postulate an imaginary mass m_0 for superluminal particles. In the superluminal universe we then have

$$m_0=i\mu \quad \text{when } |\beta|>1. \tag{9}$$

Here μ is real. An imaginary m_0 does not contradict known physics because the Lorentz equations predict that nothing can go slower than light speed in a superluminal universe, which means a tachyon has no rest frame. Substitution of Eq. (9) into Eq. (8) (using the $-im_0$ root) gives a real value: $M = \mu/\sqrt{\beta^2-1}$, which means $E=Mc^2$ is also real.

Time dilation and length contraction can be treated in a similar manner. Suppose a ship of proper length L_0 flies through a distance Δx in proper time T_0, where both L_0 and T_0 are measured in the reference frame of the ship (that is, the frame where the ship is at rest). Define T and L as the time and length measured by an observer in a reference frame traveling at speed β with respect to the ship. When $|\beta|>1$, it is useful to write L and T as

$$T=\frac{T_0}{\sqrt{1-\beta^2}}=\frac{-iT_0}{\sqrt{\beta^2-1}}, \tag{10}$$

$$L=L_0\sqrt{1-\beta^2}=iL_0\sqrt{\beta^2-1}. \tag{11}$$

If we write $T_0=i\tau$ and $L_0=-i\lambda$, where τ and λ are real, then T and L become real and positive at superluminal speeds.

The relativistic equations predict that at speeds near $|\beta|=1$, a decelerating faster-than-light ship experiences effects similar to an accelerating slower-than-light ship; an observer measures an increase in the magnitude of its mass, greater dilation in the passage of time on the ship, and more contraction in the magnitude of its length. Similarly, an accelerating superluminal ship is like a decelerating sublight ship: mass decreases, length increases, and time dilates less. At $|\beta|=\sqrt{2}$, the magnitude of the ship's mass equals m_0, the magnitude of its length equals L_0, and time passes at the same rate for both the object and the observer ($|T|=T_0$). In other words, M, L, and T have the same magnitudes at $\beta=\pm\sqrt{2}$ as they do in the ship's rest frame.

Nothing in the equations sets an upper limit on our speed in the superluminal universe. But as $|\beta|$ increases above $\sqrt{2}$, odd things happen; the observer keeping track of our superluminal progress records a *decrease* in the magnitude of our mass relative to m_0, an *increase* in the magnitude of our length relative to L_0, and an *increase* in how fast time passes for us relative to her reference frame. If we accelerate all the way to $|\beta|=\infty$, our disconcerted observer finds that for us $M=0$, $|L|=\infty$, and $T=0$. In other words, our ship has no mass and infinite length dilation, and an infinite amount of time passes for us while no time passes for the observer.[18] The term "transcendent" is used for objects with infinite speed.[2]

Bilaniuk and Sudarshan make the intriguing suggestion that the infinite $|L|$ for a transcendent particle is analogous to a particle at rest having infinite position uncertainty.[4] This follows from the Heisenberg uncertainty principle, which states that the product of the uncertainties in position and momentum (or in energy and time) must be finite, so if one variable has no uncertainty then the other must have infinite uncertainty. The momentum of a particle at rest equals zero, so its momentum uncertainty is zero and its position uncertainty is infinite. However, the interchange of space and time for superluminal particles suggests that if the spatial uncertainty is infinite for a transcendent tachyon, then it is the energy, rather than the momentum, which may be known exactly.

If our ship is traveling at a speed β, then its momentum is given by $p=Mv=Mc\beta$. What happens to its energy and momentum when it reaches transcendence? In general,

$$E=m_0c^2\gamma \quad \text{and} \quad p=m_0c\beta\gamma. \tag{12}$$

If $|\beta|>1$, then $|p|>|E|$. The limit $|\beta|\to\infty$ gives $E\to 0$ and $|p|\to\mu c$. So an observer records the object as having zero energy and finite momentum. This contrasts to the subluminal universe, where an object can have zero momentum but never zero energy. So bradyons have "zero-point energy" ($E=m_0c^2$) and tachyons have "infinite-point momentum" ($|p|=\mu c$). Just as $\beta=0$ is not invariant for bradyons, but depends on the observer's reference frame, so $|\beta|=\infty$ for tachyons depends on reference frame.

III. GOING AROUND THE TREE

Although differences exist between the various tachyon theories, they all agree that the singularity in the Lorentz transformations at $\beta=\pm1$ creates an infinitely high energy barrier between subluminal and superluminal speeds. Let us look now at a mathematical device for getting around the barrier at light speed.

The equations of special relativity are singular at $|\beta|=1$ because $\sqrt{1-\beta^2}$ appears in the denominator of the relativistic γ factor. This singularity is easily circumvented by allowing speed to become complex:

$$\beta=\beta_r+i\beta_i. \tag{13}$$

For the sake of simplicity, I will consider motion only in one dimension. Figure 1 shows the coordinates of an arbitrary complex speed β, where

$$\begin{aligned}\beta_+&=1+\beta=r_+\exp(i\theta_+),\\ \beta_-&=1-\beta=r_-\exp(i\theta_-).\end{aligned} \tag{14}$$

It is convenient to define such quantities because of the frequent occurrence of the term $1-\beta^2$ in the relativistic equations; with these definitions, $\beta_+\beta_-=1-\beta^2=r_+r_-\exp[i(\theta_++\theta_-)]$, which simplifies the mathematical manipulations. Here, $r_\pm$ and $\theta_\pm$ are the polar coordinates of $\beta_\pm$:

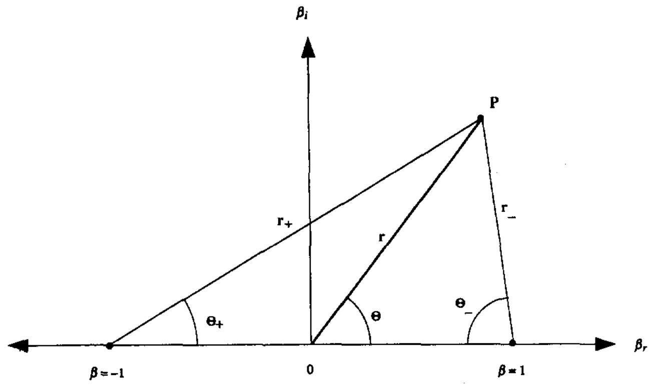

Fig. 1. Coordinates of point **P** in the complex speed plane. The horizontal axis specifies the real part of β and the vertical axis specifies its imaginary part, where β is the unitless complex speed. For points in the upper half-plane (as shown above), $\theta_+>0$ and $\theta_-<0$. For points in the lower half-plane, $\theta_->0$ and $\theta_+<0$. For the case illustrated, $\theta_+\cong 45°$ and $\theta_-\cong -80°$. The negative sign on θ_- can be motivated by drawing a vector from the origin to the point $\beta_-=1-\beta$. The angle made by that vector with the positive x axis is θ_-, which, by geometry, is the same angle shown as θ_- in the triangle.

$$r_\pm=\sqrt{(1\pm\beta_r)^2+\beta_i^2}, \quad \theta_\pm=\pm\tan^{-1}\left[\frac{\beta_i}{1\pm\beta_r}\right]. \tag{15}$$

For points in the upper half-plane (such as **P** shown in Fig. 1), $\theta_+>0$ and $\theta_-<0$. For points in the lower half-plane, $\theta_->0$ and $\theta_+<0$. In either case the mass and energy are

$$M=\frac{m_0}{\sqrt{r-r_+}}\exp[-i(\theta_-+\theta_+)/2], \tag{16}$$

$$E=\frac{m_0c^2}{\sqrt{r_-r_+}}\exp[-i(\theta_-+\theta_+)/2]. \tag{17}$$

Equation (17) is not single valued: $E(\theta_++\theta_-)\neq E(\theta_++\theta_-+2\pi)$. To make the energy a function we restrict its phase to an interval where it is single-valued. The function in such an interval is a *branch* of E, and the singular points $\beta=\pm1$ are branch points.[19] A *branch cut* originates at a branch point, cannot be crossed, and is drawn to define the interval of allowed values for $\theta_++\theta_-$ that make E single-valued. Figure 2 shows three out of the infinite number of ways to draw branch cuts for the energy given in Eq. (17). Each situation has a physical interpretation, which I will talk about next.

A. Bradyons

The branch cuts shown in Fig. 2(a) lie on the real β axis, one stretching from $-1\rightarrow-\infty$ and the other from $1\rightarrow\infty$. This configuration blocks the real axis for all $|\beta|>1$, so for real speeds only $|\beta|<1$ is allowed, which means both θ_+ and θ_- equal zero. The energy on the real axis is thus

$$E=\frac{m_0c^2}{\sqrt{1-\beta_r^2}}, \tag{18}$$

which is the usual form E takes in our subluminal universe. In the universe represented by Fig. 2(a), no object with real speed can ever have $|\beta|>1$. So this arrangement of branch

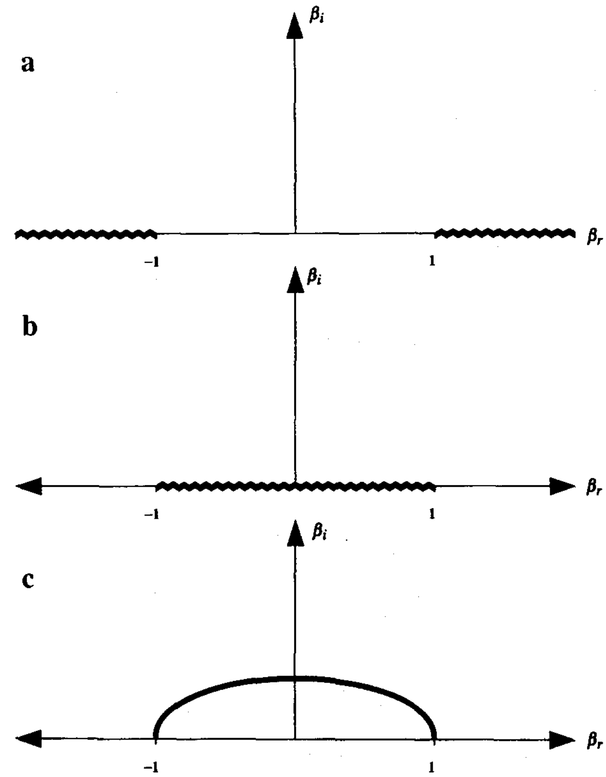

Fig. 2. Branch cuts in the complex β plane: situation corresponding to (a) bradyons (class I particles), (b) tachyons (class III particles), "massons" (class "0" particles). In each plot, the horizontal axis specifies the real part of β and the vertical axis specifies the imaginary part.

cuts corresponds, along the real axis, to the universe of bradyons.

However, Fig. 2(a) suggests many other possibilities. Suppose the speed of our space ship corresponds to **P** in Fig. 1:

$$\beta=r\exp(i\theta) \quad \text{at } \mathbf{P}. \tag{19}$$

If $\theta\neq0$, what will our observer measure for properties such as energy and momentum? At this point, she is ready to throw up her hands in frustration. How can she make observations if our speed, and thus many of our other attributes, has both a real and an imaginary part? However, being a persistent sort, she makes a go at the problem. To detect the energy (or any other complex quantity), it seems reasonable to expect she would measure either (A) its magnitude, or (B) its real part. For the time being, I will assume m_0 is real. However, in keeping with the ideas in the literature (and to make our observer's life more interesting), I will later consider the possibility that properties such as m_0, L_0, and T_0 are complex.

Using method A on the energy in Eq. (17) gives

$$|E|=\frac{m_0c^2}{\sqrt{r_+r_-}}, \tag{20}$$

whereas, with method B, a modulating factor appears:

$$E_r=\cos[(\theta_-+\theta_+)/2]\,\frac{m_0c^2}{\sqrt{r_-r_+}}, \tag{21}$$

where E_r is the real part of E. The two methods yield identical results for real subluminal speeds, which is essential if they are to agree with known physics. As $|\beta|$ (and thus β_+

and β_-) goes to ∞ in either Eq. (20) or (21), $E\to 0$. So, in both methods A and B, the energy behaves as expected at transcendent speeds. However, elsewhere the two equations give different results. The method A energy has no phase angle dependence; it varies only with the magnitude of β. The method B result depends on phase; as $|\theta_- + \theta_+|$ increases from $0\to\pi$, the magnitude of the cosine in Eq. (21) decreases, until at $|\theta_- + \theta_+| = \pi$ we get $E_r = 0$ as the observed energy. In other words, method B predicts the observed energy is zero at all real superluminal speeds (this will turn out to be important for other configurations where the branch cuts do not block the real superluminal axis).

The observer next considers the momentum p, where, in general,

$$p = \frac{m_0 c r}{\sqrt{r_+ r_-}} \exp[i(\theta - (\theta_+ + \theta_-)/2)]. \tag{22}$$

Method A gives

$$p = \frac{m_0 c r}{\sqrt{r_+ r_-}} \tag{23}$$

and method B yields

$$p_r = \cos[\theta - (\theta_- + \theta_+)/2] \frac{m_0 c r}{\sqrt{r_+ r_-}} \tag{24}$$

as the observed momentum, where p_r is the real part of p. As with the energy, method A predicts the momentum has no phase dependence and method B predicts it varies with phase. According to Eq. (24), the observed momentum equals zero whenever the speed is such that $\theta - (\theta_- + \theta_+)/2) = \pi/2$ (or an odd multiple of $\pi/2$). By plugging in angles, it can be shown that real superluminal values of β satisfy this condition.

Method A thus gives nonzero values for E and p at finite real superluminal speeds, whereas method B predicts the observed energy and momentum both vanish at such speeds. The branch cuts in Fig. 2(a) block that portion of complex speed space, but it becomes available if we move the cuts even slightly off the real axis. Method B would then appear to suggest that when our speed is real and superluminal, we cease to exist! Although the observer gleefully contemplates this possibility as an end to her observation problems, she quickly realizes she has determined no more than that method B predicts the *real* parts of E and p cease to exist for $|\beta|>1$, which is consistent with the fact that tachyons have yet to be detected in our universe. It does not by itself rule out the possibility that they "exist" with purely imaginary properties.

B. Luxons

Before considering superluminal particles I will look at luminal particles, or luxons, which travel at the speed of light and so exist on the boundary between subluminal and superluminal space. Relativity predicts that such particles exist only if they have $m_0=0$. The reason can be seen from Eqs. (3) and (5). A particle with $m_0=0$ traveling at light speed satisfies $M=0/0$, which is undefined and may yield a finite number. At subluminal (or superluminal) speeds such an object ceases to exist because $M=0$. So luxons are defined as massless particles moving at luminal speeds.

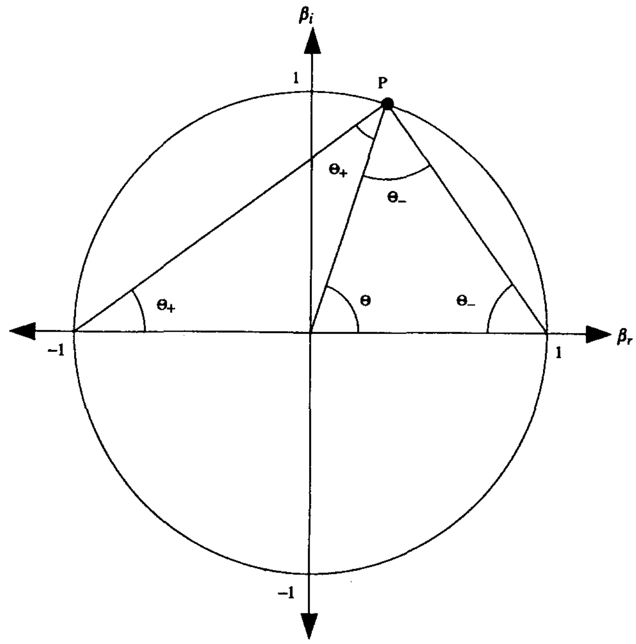

Fig. 3. The circle gives all values of β in the complex plane that have $|\beta|=1$. It crosses the real axis at the points $\beta=-1$ and $\beta=1$. At any point **P** on the circle, the magnitude of a particle's speed equals the speed of light. The angle $\theta_- \cong -50°$.

How do we extend the definition of a luxon to the complex speed plane? Consider all speeds with $|\beta| = \sqrt{\beta_r^2 + \beta_i^2} = 1$. As shown in Fig. 3, these correspond to a circle of radius one centered at the origin and intersecting the real axis at the branch points $\beta=\pm 1$. The fact that, on this circle, the relativistic equations encounter no singularities off the real axis suggests the possibility of luxons with mass. To investigate this idea, consider the mass as given by Eq. (16). With $\beta_r^2 + \beta_i^2 = 1$, it is straightforward to show $r_+ r_- = 2|\beta_i$. Using the fact that the magnitudes of the angles in a triangle add up to π then yields

$$M = \frac{m_0}{\sqrt{2|\beta_i|}} \exp\left[\pm i\left(\frac{\pi}{4} - \frac{|\theta|}{2}\right)\right], \tag{25}$$

where the plus sign is for points in the upper-half-plane ($\theta>0$) and the minus sign is for points in the lower-half-plane ($\theta<0$). Method A thus predicts the observer measures

$$|M| = \frac{m_0}{\sqrt{2|\beta_i|}} \tag{26}$$

and method B predicts she measures

$$M_r = \cos\left(\frac{\pi}{4} - \frac{|\theta|}{2}\right) \frac{m_0}{\sqrt{2|\beta_i|}}, \tag{27}$$

where M_r is the real part of M. In either method, if $m_0 \neq 0$ then the observed mass goes to ∞ as $\beta_i \to 0$. The only way to ensure M gives the observed result for particles with $\beta=\pm 1$ is to make $m_0=0$ for all speeds on the circle, which means $M=0$ everywhere except at $\beta=\pm 1$. This does not mean an object with mass can never have $|\beta|=1$, but rather that such an object must always have an imaginary component to its speed. Conversely, if an imaginary part is added to the speed of a massless luxon, γ becomes finite; the condition $m_0=0$

then requires $M=0$, which means the particle ceases to exist. For these reasons, I will continue to use the word luxon to mean only massless particles with $\beta=\pm 1$.

C. Tachyons

Figure 2(b) shows an alternate way to make γ single-valued. Instead of two branch cuts, we have only one going from $-1 \rightarrow +1$. Now no points can be taken on the real axis where $|\beta|<1$, so particles with real speed must *always* be superluminal. Figure 2(b) thus corresponds, on the real axis, to a universe of tachyons.

For purely real speeds, Eq. (16) gives $M=im_0/\sqrt{r_+r_-}$ if $\beta>1$ and $M=-im_0/\sqrt{r_+r_-}$ if $\beta<-1$ (in the interval $0\leq\theta\leq\pi$ that defines the upper half-plane). This is consistent with the discussion following Eq. (8), which notes that strictly speaking $\gamma=\mp i/\sqrt{\beta_2-1}$. Method A predicts that $M=m_0/\sqrt{r_+r_-}$ is observed at real superluminal speeds and method B gives $M_r=0$ as the observed mass. Although so far we have assumed m_0 is real, the results of method A are reminiscent of those obtained from theories that make proper mass imaginary for tachyons. One way to provide an explanation as to *why* method A would allow us to observe the magnitude of a complex quantity is to extend the idea of imaginary mass to the complex plane;

$$m_0=\mu \exp[i(\theta_+ + \theta_-)/2], \tag{28}$$

where μ is real. Putting Eq. (28) into Eq. (17) then gives

$$E=\frac{\mu c^2}{\sqrt{r_-r_+}}. \tag{29}$$

Along the real superluminal axis this is the same result as that obtained with tachyon theories where $m_0=i\mu$. However, for points along the real axis, Eq. (28) gives $m_0=-i\mu$ if $\beta>1$ and $m_0=i\mu$ if $\beta<-1$. The predictions of Eq. (28) are physically pleasing because they suggest that as speed changes, m_0 rotates smoothly through the complex plane rather than jumping discontinuously from a real quantity in our universe to an imaginary quantity in the superluminal universe.

The γ factor also appears in the time and length relations given in Eqs. (10) and (11), suggesting the definitions

$$T_0=\tau \exp[i(\theta_+ + \theta_-)/2],$$
$$L_0=\lambda \exp[-i(\theta_+ + \theta_-)/2], \tag{30}$$

to make T and L real.

In either Fig. 2(a) or 2(b) it is impossible to draw a curve that lies on both subluminal and superluminal sections of the real axis. In other words, if our ships are traveling at real sublight speeds we cannot get into a superluminal universe with real speed, and vice versa; if we leave the road to go around the road block, we cannot return to the road after we pass the barrier.

However, there are other ways to draw the branch cuts.

D. Massons

If we think of complex β space as curved into a sphere, then Figs. 2(a) and 2(b) are actually different configurations of the same branch cut. We can envision this by sticking two pins in a ball so that they are separated by a few degrees of arc. The surface of the ball corresponds to β space and the pins to $\beta=\pm 1$. A rubber band stretched between the pins represents the branch cuts. If we pull the band around the ball so that it stretches the long way from pin to pin, we have the situation in Fig. 2(a). The curving of complex β space allows ∞ and $-\infty$ to "meet" on the far side of the ball, so that the two cuts are actually one. To obtain Fig. 2(b), we move the band so that it stretches the short distance between the pins.

However, with a bit of glue we can attach the rubber band to the ball in ways other than the two discussed above. In fact, an infinite number of configurations exist. The only requirement is that we anchor the branch cut at $\beta=\pm 1$. Figure 2(c) shows a third possibility. Now all real speeds are available except $\beta=\pm 1$, which means our ships can start at real subluminal speeds and accelerate (via complex space) to real superluminal speeds. This suggests the possibility of a "class 0" particle that can go at any speed, real or complex, *except* $\beta=\pm 1$. Class 0 would contain all particles with mass, so I will call them massons (to contrast with luxons, which have no mass). For such particles, the relativistic equations vary smoothly with the phase $(\theta_- + \theta_+)$.

No known method exists for observing imaginary speeds, so even if we were detecting particles with such speeds we would not know. We continually make leaps in understanding which revolutionize our view of the universe, as witnessed by quantum mechanics and relativity. In the past such leaps of understanding have been accompanied by a reformulation of the equations that describe physical phenomena; Newton's equations, for example, gave way to those of Schrödinger and Einstein. The validity of a theory is determined by how well it predicts observed results. The important point for the student to remember is that the equations of physics are not "absolute truths," but rather they represent models that describe the universe to the best of our current knowledge. A lack of evidence is not the same as proof of impossibility; theories may well turn out to be incomplete when pushed to describe new results previously inaccessible to experiment.[20]

IV. RELATIVISTIC "ABSORPTION"

Precedent does exists for the imaginary part of a complex function having a physical meaning, as in the damped dispersion equations for n, the refractive index of light. The real part of n gives the speed of light in a material and its imaginary part provides a measure of how much the material absorbs light. Perhaps an imaginary speed might also manifest as a physical property we could measure in the real universe. This section presents a problem that students and relativity buffs might find intriguing, one based on an analogy between the complex forms of the dispersion and relativity equations. The analogy is purely speculative; it has no known physical basis. What it illustrates is how comparisons between different areas of physics have the potential to yield new insights.

Consider a molecule in an isotropic dielectric medium subject to an electromagnetic field $E=E_0 \sin \omega t$. There are N electrons per unit volume, m is the electron mass and e is charge. To a first approximation, the system can be described as a mechanical oscillator driven by a sinusoidal force with frequency ω. The refractive index for a rarified medium, $n(\omega)$, is then determined from[21]

$$n^2(\omega)=1+\frac{Ne^2}{\epsilon_0 m}\left[\frac{1}{\omega_0^2-\omega^2}\right]. \tag{31}$$

Here ω_0 is the resonance frequency of the oscillator (for simplicity, I will consider a system with only one resonance). With the definition of a unitless dispersion frequency $\beta_d=\omega/\omega_0$, and a "dispersion" γ_d such that

$$\gamma_d=\sqrt{\frac{\omega_0^2\epsilon_0 m}{Ne^2}(n^2-1)}, \tag{32}$$

Eq. (31) can be written as

$$\gamma_d=\frac{1}{\sqrt{1-\beta_d^2}} \tag{33}$$

Thus γ_d has the same form as the relativistic γ, with ω and ω_0 playing analogous roles in dispersion to v and c in special relativity.

However, classical dispersion theory neglects absorption. Energy losses due to such processes cause the oscillating atom to behave like a damped oscillator. The damping force is $f=mG(dR/dt)$, where G is a constant characteristic of the damping and dR/dt is the time derivative of the displacement R experienced by the electron cloud. When absorption is included, the index of refraction can be written[22]

$$n^2(\omega)=1+\frac{Ne^2}{\epsilon_0 m}\left[\frac{1}{(\omega_0^2-\omega^2)-iG\omega}\right]. \tag{34}$$

Combining Eqs. (34) and (32) yields

$$\gamma_d=\frac{1}{\sqrt{1-\beta_d^2-iG\beta_d/\omega_0}}. \tag{35}$$

The complex form of the relativistic γ is found by substituting $\beta=\beta_r+i\beta_i$ into Eq. (3);

$$\gamma=\frac{1}{\sqrt{1-\beta_r^2+\beta_i^2-2i\beta_r\beta_i}}. \tag{36}$$

In dispersion theory, the resonance occurs at $\omega=\pm\omega_0$, which can be rewritten as $\beta_d=\pm 1$ (only the $+\omega_0$ root has known physical meaning). When absorption is ignored, $\gamma_d\rightarrow\infty$ at the resonance frequency. With the inclusion of absorption, the real part of γ_d goes to zero at $\beta_d=\pm 1$.

Similarly, the relativistic $\gamma\rightarrow\infty$ at $\beta=\pm 1$. However, when we make β complex, the real part of γ does not necessarily go to zero at the points where the purely real γ is singular. The location of the zeros is determined by the hyperbola

$$\beta_r^2-\beta_i^2=1. \tag{37}$$

Speed can be either positive or negative, so both the positive and negative branches of the equation have physical meaning. If we take as the relativistic analog of the dispersion resonance the points where the real part of the complex γ equals zero (which includes as the points where the purely real $\gamma\rightarrow\infty$), then the analogy suggests that an object with β on the hyperbola in Eq. (37) is "absorbed" out of real space into a purely imaginary universe. This is consistent with the fact that we measure $m_0=0$ for luminal particles. As $\beta_i\rightarrow 0$, the relativistic resonance goes to ± 1, which is consistent with known physics.

The relativistic model has two variables, β_r and β_i, whereas the dispersion model has only one, β_d. Comparison of γ and γ_d suggests the following analogies:

dispersion	relativity
$1-\beta_r^2+\beta_i^2$ $\leftrightarrow$	$1-\beta_d^2$
$2\beta_r\beta_i$ $\leftrightarrow$	$G\beta_d/\omega_0$

We can try out various scenarios depending on what we do with the extra degree of freedom introduced by the additional relativistic variable. Comparing v/c with ω/ω_0 suggests the real part of the relativistic β compares with the dispersion β_d. The reader may enjoy investigating the implications of other choices.

Note that the comparison $\beta_r\leftrightarrow\beta_d$ does not require $\beta_i=0$; rather, it indicates that a constant in the dispersion model compares with a variable in the relativistic model. The comparison $\beta_i\leftrightarrow G/(2\omega_0)$, which comes from applying $\beta_r\leftrightarrow\beta_d$ to the second analogy relationship, suggests that the imaginary part of the complex speed "damps" the mass (or energy) of an object in a manner analogous to the way absorption damps the oscillation of an electron cloud subject to an external field.

When a molecule has more than one resonance, a sum over ω_{0j} appears in Eq. (34) (the j index runs over all resonance frequencies). This raises the intriguing suggestion that $\pm c$ (or the associated hyperbola) may be only the first of many singularities in the allowed speeds of an object.

Gascon has introduced real transformation formulas for the relativistic equations that contain a discrete spectrum of singularities. The first speed is invariant with respect to subluminal space, but the remaining speeds are not.[23]

ACKNOWLEDGMENTS

This work was partially supported by the National Science Foundation through a grant for the Institute for Theoretical Atomic and Molecular Physics at Harvard University and Smithsonian Astrophysical Observatory. The hospitality of G. R. Dierkson at the Max Planck Institut für Astrophysik in Garching, Germany, and of Ron Polidan and the Laboratory for Astronomy and Solar Physics at Goddard Space Flight Center are gratefully acknowledged. The author also thanks the referees for their comments.

[a]e-mail: asaro@stars.gsfc.nasa.gov.

[1]Several good resources for special relativity are E. F. Taylor and J. A. Wheeler, *Space-time Physics: Introduction to Special Relativity* (Freeman, New York, 1992), 2nd ed.; R. T. Weidner and R. L. Sells, *Elementary Modern Physics* (Allyn and Bacon, Boston, 1980), 3rd ed., pp. 16–85; A. P. French, *Special Relativity, The M.I.T. Introductory Series* (Norton, New York, 1968). Another classic is Y. P. Terletskii, *Paradoxes in the Theory of Relativity* (Plenum, New York, 1968). For a treatment that does not use many equations, see E. Chaisson, *Relatively Speaking: Relativity, Black Holes, and the Fate of the Universe* (Norton, New York, 1988), pp. 45–105.

[2]O. M. P. Bilaniuk, V. K. Deshpande, and E. C. G. Sudarshan, "'Meta' Relativity," Am. J. Phys. **30**, 718–723 (1962). For articles on pre-relativistic theories, see Ref. 1 of this paper.

[3]G. Feinberg, "Possibility of Faster-than-Light Particles," Phys. Rev. **159**, 1089–1105 (1967).

[4]O. M. Bilaniuk and E. C. G. Sudarshan, "Particles beyond the Light Barrier," Phys. Today 43–51, May (1969).

[5]M. N. Kreisler, "Are there Faster-than-Light Particles?" Am. Sci. **61**, 201–208 (1973).

[6]L. Parker, "Faster-than-Light Inertial Frames and Tachyons," Phys. Rev. **188**(5), 2287–2292 (1969).

[7]E. Recami and R. Mignani, "Generalized Lorentz transformations in Four Dimensions and Superluminal Objects," Nuovo Cimento **14** A, 169–189 (1973).

[8]A. F. Antippa, "A One-Dimensional Causal Theory of Tachyons," Il Nuovo Cimento **10** A(3), 389–406 (1972).

[9]L. Marchildon, A. F. Antippa, and A. E. Everett, "Superluminal coordinate transformations: the two dimensional case," Can. J. Phys. **61**, 256–263 (1983). See also the references given in Marchildon for an overview of the work done in this area.

[10]P. Fjelstad, "Extending special relativity via the perplex numbers," Am. J. Phys. **54**, 416–422 (1986); A. Ronveaux, "About 'Perplex Numbers,' " Am. J. Phys. **55**, 392 (1987).

[11]R. Y. Chiao, P. G. Kwiat, and A. M. Steinberg. "Faster than Light?," Sci. Am. 52–60, August (1993). For a very readable discussion of the EPR paradox, see L. Hammer, "Useful Speeds," Mindsparks No. 5 **2** (2), 26–27 (1994).

[12]V. F. Perepelitsa, *Philosophical Problems of the Hypothesis of Superluminal Speeds*, edited by Yu. B. Molchanov (Nauka, Moscow, 1986); L. M. Feldman, "Short Bibliography on Faster-than-Light Particles (Tachyons)," Am. J. Phys. **42**, 179–181 (1974).

[13]For a very readable review of earlier experimental work, see G. Feinberg, "Particles That Go Faster than Light," Sci. Am. **222**, 68–77, February (1970). An experiment suitable for undergraduates is given in B. A. Logan and A. Ljubicic, "Background Counting Rates and Possible Tachyonic Decays," Am. J. Phys. **44**, 789–790 (1976). More recent work includes R. W. Clay, "A Search for Tachyons in Cosmic Ray Showers," Aust. J. Phys. **41**, 93–99 (1988); A. Meszaros, "Indirect Experimental Evidence against the Tachyons," Astrophys. Space Sci. **123**, 490–410 (1986); H. F. Masjed, "Search for Tachyons Associated with Extensive Air Showers in the Ground Level Cosmic Radiation," 19th Int. Cosmic Ray Conf. **8**, 298–301 (1985); V. D. Ashitkov, T. M. Kirina, A. P. Klimakov, R. P. Kokoulin, and A. A. Petrukhin, "Results of a Search for Monopoles and Tachyons in Horizontal Cosmic Ray Flux," *ibid.* 222–225; F. Ashton *et al.*, "Preliminary Results of a Search for Faster than Light Objects in Cosmic Rays at Sea Level," 15th Int. Cosmic Ray Conf. **7**, 370–375 (1977). Intriguing speculations on superluminal rocket propulsion appear in C. Powell, "Propulsion by Tachyon Beams," 25th Joint Propulsion Conf. 3114–3118 (1989) and G. L. Bennett, "Boundary Conditions on Faster-than-Light Transportation Systems," 29th Joint Propulsion Conference June (1993).

[14]The contraction appears as a rotation for parts of an object that subtend an infinitesimal solid angle as seen by the observer. This rotation is only an approximation for infinitesimal angles, as can been seen by imagining a relativistic train traveling along rails which are at rest relative to the observer. The train cannot rotate while the rails remain fixed. See J. Terrell, "Invisibility of the Lorentz Contraction," Phys. Rev. **116**, 1041–1045 (1959).

[15]E. Hecht, *Optics, Second Edition* (Addison-Wesley, Reading, MA, 1987), p. 544.

[16]A. Einstein, "Zur Elektrodynamik bewegter Körper," Annal. Phys. **17**, 891–921 (1905).

[17]Historically, m_0 has been called the rest mass. Bilaniuk and Sudarshan point out that the word is somewhat of a misnomer for tachyons given that a superluminal particle has no rest frame (Ref. 4.) They suggest using the term "proper mass."

[18]This raises the question of just *how* the observer makes any measurements when our entire existence goes by in time $T=0$ for her. We will assume she can take data infinitely fast. Physics books often show helpful pictures of an observer comfortably seated in a chair while a rocket or relativistic train obligingly passes by in front of the chair, somehow being clearly visible despite its phenomenal speed.

[19]R. V. Churchill, J. W. Brown, and R. F. Verhey, *Complex Variables and Applications, Third Edition* (McGraw Hill, New York, 1974). The definition of a branch cut is given on p. 64. A discussion of a case similar to that of the relativistic γ function appears on pp. 89–92. For a different approach to the use of complex numbers in special relativity, see V. S. Olkhovsky and E. Recami, "About Lorentz Transformations and Tachyons," Lett. Nuovo Cimento **1**, 165–168 (1971).

[20]We can hypothesize candidates for complex particles (keeping in mind that since no evidence exists to support such hypotheses, we are playing a sort of physics game). Several possibilities come to mind. As mentioned in the text, antiparticles might be pastward-traveling tachyons as observed in our sublight universe. Neutrinos, which cannot be detected directly, are other candidates for complex and/or superluminal particles, as is the "dark matter" in the universe. In a lighter vein, making speed complex nicely solves a problem that has long plagued science fiction writers, which is how to "design" fictional engines for faster-than-light spaceships. The idea inspired my story "Light and Shadow" (C. Asaro, *Analog, Science Fiction and Science Fact*, April 1994) and parts of my novel *Primary Inversion* [C. Asaro (Tor Books, March 1995)]. Relativity buffs interested in more on treatments of the subject using general relativity might enjoy the science fact article "Faster-than-Light" by Dr. Robert Forward in the February 1994 issue of *Analog*.

[21]See Ref. 16, pp. 56–63, or M. Born and E. Wolf, *Principles of Optics, Fifth Edition* (Pergamon, Oxford, 1985), pp. 90–98 (section 2.3.4).

[22]The derivation appears in Ref. 21, section 3.5.1, p. 57, and problem 3.28 on p. 77 (solution on p. 632). In this paper I use the Born and Wolf convention $E=E_0\exp(-i\omega t)$ for the applied field rather than $E_0\exp(i\omega t)$ because it is more consistent with the form of the relativistic equations. However, both definitions of E contain the same physics.

[23]F. Gascon, "Special Relativity with a Discrete Spectrum of Singular Velocities," Il Nuovo Cimento **41**, 317–326 (1977).

UNEXPLAINED PHENOMENA

Another demand of strong adherents to a belief is that their critics provide alternative explanations of the items of "evidence" they have presented. This presumption is based on a fallacious understanding of the character of scientific judgment. One does not attempt to fit every piece of alleged information into the general assessment of an idea. As Hudson Hoagland once said (in another connection, i.e., psychic phenomena and UFOs)

> There will [always] be cases which remain unexplained because of lack of data, lack of repeatability, false reporting, wishful thinking, deluded observers, rumors, lies, frauds. ...Unexplained cases are simply unexplained. They can never constitute evidence for any hypothesis. ...The basic difficulty...is that it is impossible for science ever to prove a universal negative.

Irving H. Klotz, *Diamond Dealers and Feather Merchants* (Birkhauser, Boston, 1986), pp. 112–113.