Superluminal Geometrodynamics of Braneworld Hyperdrive via Brane-Bulk Interaction

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Abstract. The idea of superluminal travel in the framework of General Relativity has firstly been introduced by Miguel Alcubierre. Instead of puncturing through a hole in spacetime as in a wormhole, this method of spacetime geometrical modification manipulates the geometrodynamics contraction and expansion of spacetime curvature so that the intended travelling object's hypersurface surfs on the surrounding spacetime hyperspace. In the general relativity framework, this feat requires exotic matter that violates the null energy condition. However, in the braneworld framework, the bulk space may naturally support the geometrodynamics requirement without violating the energy condition. In this paper, we will explore in more detail mathematically, how the concept has been derived in the framework of General Relativity, and how by embedding the extra-dimensional terms that represent bulk space of braneworld the travelling object's hypersurface may fly-through the bulk hyperspace. We begin by developing the metric to formulate the energy density expression that governs the spacetime of "flying-through-hyperspace" geometrodynamics in relation to the necessary parameters describing the spacetime hyper-warped bubble for the braneworld hyperdrive.

Keyword: General Relativity, Superluminal travel, Braneworld.

1. Introduction

To be a spacefaring civilization with current rocket propulsion technologies that is utilizing Newtonian orbital mechanics, humankind could only travel within the solar system and yet still acquire years of travel to surpass the outermost regions of the solar system. Thus, not to mention the sensible of humanity's dreams for interstellar or perhaps intra-galactic travel, if just by using Newtonian mechanics-base propulsion technologies, because even if such endeavour may approach up to 99% the speed of light [1] which is an almost impossible feat by itself, the journey will still take several years, even of tens to hundreds of thousands of years [2]. Thus, the speed of light, tremendously huge as it may seems, but nevertheless, if the astronomical scale of the universe is to be considered, light speed is still slow. Therefore, the idea of hyper-fast travel that could surpass many folds the speed of light is very much an interesting subject to study [3].

Surpassing the speed of light seems to violate the law of physics. This notion is true if Special Relativity (SR) is taken into consideration. SR says that any subject of travel that holds any amount of mass could never even achieve the light speed, not to mention exceeding it. This is due to the increase in relativistic mass asymptotically toward infinity as the subject approaches the speed of light, consequently requiring an infinite amount of energy. However, in General Relativity (GR) the concept of spacetime geometrodynamics [4] seems to show that the dynamics of spacetime itself does not constrain toward any limit. This notion has been proven at least in the cosmological scale level when we consider the signatures of the inflationary period of the universe [5] and the proven observable phenomenon of accelerating expansion of the universe [6]. GR has shown that the dynamics of stretching and folding of spacetime may exceed the speed of light by either short cutting the spacetime as in black hole and wormhole [7] or superluminally stretching the spacetime via *expansion* as during the inflationary period of post big bang or via *contraction* as during the theoretically possible scenario of the big crunch period, or the very dynamics scenario of spacetime expansion and contraction in the new cyclic universe conjecture by Anna Ijjas et. al. [8].

Thus, the idea of superluminal travel is not about finding a way of violating SR. Instead, it is a geometrodynamical possible scenario in various GR frameworks. Essentially there are two concepts of superluminal travel: “short cutting” or “surfing” on the spacetime. Short cutting concept is about reducing tremendously the distance between two different coordinates as in the wormhole theory which require the topological breaking of spacetime itself, either naturally (from subatomic quantum foam) [9] or induce (through entanglement) [10]. The surfing concept; involves embedding a bubble of warped spacetime hypersurface onto the surrounding hyperspace and using the transition region of warped spacetime properties between the bubble and the surrounding hyperspace that defines the bubble’s extrinsic curvature to govern its dynamic’s character of expansion and contraction of spacetime.

In this paper, we investigate the idea of spacetime “surfing” which was first introduced by Alqubierre [3] by further exploring the mathematical derivation underlying the subject in a rather elaborate detail. Moreover, we study the braneworld cosmological concept that can possibly influence the embedding mechanism of warp bubble. We will introduce a new method of warp drive using brane-bulk fluid [11] interaction that enhances the warp drive concept toward the hyperdrive concept. Braneworld [13] cosmology suggests that there exists 1-dimensional higher spatial extra-dimension than the 3-dimensional space, known as bulk, that may contain fluid-like substance described by Weyl tensor [7]. The 3-d universe is the 3-dimensional spatial brane floating in 4-dimensional spatial bulk. Thus, in deriving the warp bubble expression, the extrinsic curvature not only require to be dealt with warp bubble hypersurface with brane but also with bulk-brane interaction [14] in line with the idea that spacetime inflation is driven by geometrodynamics of the interaction.

2. The metric property of hyperspace warp bubble

The concept of spacetime “hyper-surf” is very much about manipulating spacetime locally, thus we may start by considering a *flat* spacetime metric that not only describes the hyperspace surrounding the “hyper-surf” spacetime region but also the “interior” of the hyper spacetime region that is by itself a flat hypersurface-brane with respect to the surrounding hyperspace-bulk region [14, 15, 16] that is

$$ds^2 = -dt^2 + \left(1 + Y_{,r}^2\right)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{1}$$

Imagine the “surfing” spacetime region as a warp bubble that moves along the x axis with a velocity of v,

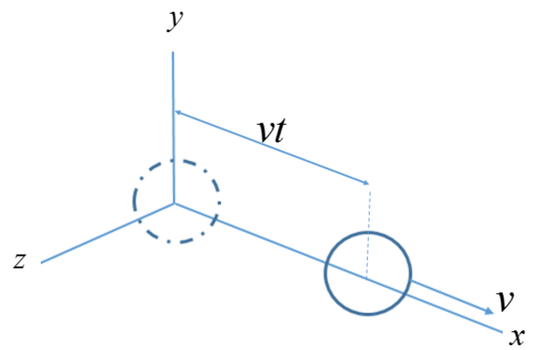

Figure 1. Spacetime warp bubble “surf” (moves) along x axis with velocity of v

so, we may rewrite the metric (1) as

$$ds^2 = -dt^2 + \left(dr\big|_x - vdt\right)^2 + Y_{,r}^2 dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \tag{2}$$

which preserved the essence of equation (1) whereby, if the bubble doesn’t move, the equation (2) will become (1) again. For representing the characteristic of the bubble where the center is located at $r = 0$ and the radius of the bubble can be assigned as R we may consider a function $f = f(r)$ where $f = 1$ always within the bubble’s radii $|r| = R$,

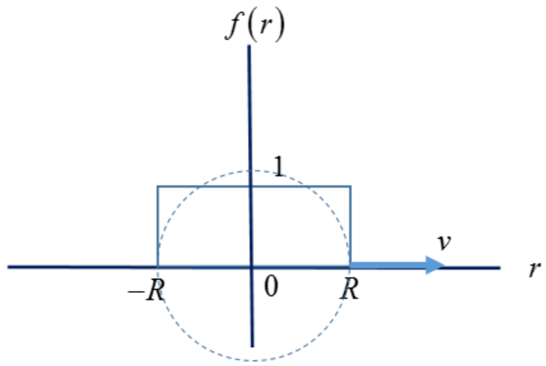

Figure 2. Function $f = f(r)$ characteristic with bubble radii

Thus, the essence of equation (2) is always preserved and we may rewrite equation (2) with the inclusion of the function $f = f(r)$ as

$$ds^2 = -dt^2 + \left(dr|_x - vf(r)dt\right)^2 + Y_{,r}^2 dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \quad . \tag{3}$$

If we consider condition at the center of the warp bubble where the spatial coordinate is $(vf(r)t, 0, 0)$ which then obviously shows equation (3) reduce to $ds^2 = -dt^2$. Since the different of proper time is defined as minus of the line element thus $d\tau^2 = -ds^2$ therefore $dt^2 = -ds^2 = d\tau^2$ which obviously implies the *null-time dilation* characteristic of the bubble as proper time is the same as coordinate time $\tau = t$. Inside the bubble, spacetime condition is as normal as in a stationary non relativistic condition. We may now expand this notion to find the geodesic and free fall characteristic of the warp bubble. By relationship of proper time and line element metric, $d\tau^2 = -ds^2 = -g_{\mu\nu}dx^\mu dx^\nu$ we may obtain the extremum [3] of the proper time as below to obtain Lagrangian terms in handling curvature invariants of the warp bubble [12]

$$\tau = \int \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}\, d\lambda \,, \tag{4}$$

which is the extremum expression of the proper time. We may obtain the Lagrangian from equation (4). But before expanding (4) to obtain the Lagrangian we may have to expand and rearrange metric equation (3). We also consider a flat spacetime inside the warped bubble where the extradimensional term that represents bulk space is negligible, that is $Y = 0$. Thus, we may write

$$\begin{aligned} ds^2 &= -dt^2 + \left(dr|_x - vf(r)dt\right)^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \,, \\ &= -\left(1 - v^2 f(r)^2\right)dt^2 - 2vf(r)drdt + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \,, \end{aligned} \tag{5}$$

so we identify the metric components which are

$$g_{00} = v^2 f(r)^2 - 1, \quad g_{01} = g_{10} = -vf(r), \quad g_{11} = 1, \quad g_{22} = r^2 \;, \quad g_{33} = r^2 \sin^2\theta \;, \tag{6}$$

by (4)

$$\left(\frac{d\tau}{d\lambda}\right)^2 = \frac{d\tau^2}{d\lambda^2} = \left(1 - v^2 f^2\right)\dot{t}^2 + 2vf\dot{r}\dot{t} - \dot{r}^2 - r^2\dot{\theta}^2 - r^2 \sin^2\theta\dot{\phi}^2 = -g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu \,. \tag{7}$$

From the pure definition of Lagrangian $L=\frac{1}{2}\dot{q}^2$ where $q=s\left(x(t)\right)$, as a coordinate position of a metric, thus

$$\dot{q}^2=\dot{s}^2=\left(-\dot{\tau}\right)^2=\frac{-d\tau^2}{d\lambda^2}=\frac{ds^2}{d\lambda^2}=2L \tag{8}$$

since $-ds^2=d\tau^2$; $ds^2=-d\tau^2$,

and from (7) and (8) $-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu=\left(1-v^2f^2\right)\dot{t}^2+2vf\dot{r}\dot{t}-\dot{r}^2-r^2\dot{\theta}^2-r^2\sin^2\theta\dot{\phi}^2=\frac{d\tau^2}{d\lambda^2}=\frac{-ds^2}{d\lambda^2}$ therefore $\left(\frac{ds}{d\lambda}\right)^2=\frac{ds^2}{d\lambda^2}=\frac{-d\tau^2}{d\lambda^2}=\dot{q}^2=g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu=-\left(1-v^2f^2\right)\dot{t}^2-2vf\dot{r}\dot{t}+\dot{r}^2+r^2\dot{\theta}^2+r^2\sin^2\theta\dot{\phi}^2=2L$,

thus, the Lagrangian is

$$L=\frac{1}{2}\left(-\left(1-v^2f^2\right)\dot{t}^2-2vf\dot{r}\dot{t}+\dot{r}^2+r^2\dot{\theta}^2+r^2\sin^2\theta\dot{\phi}^2\right), \tag{9}$$

and rearrange as,

$$L=\frac{1}{2}\left(-\dot{t}^2+r^2\dot{\theta}^2+r^2\sin^2\theta\dot{\phi}+\left(\dot{r}-vf\dot{t}\right)^2\right) . \tag{10}$$

We may parameterize the geodesic using proper time itself taking $\lambda=\tau$ thus the dot indicates the derivative with respect to τ . By Euler Lagrange equation $\frac{\partial}{\partial\tau}\left(\frac{\partial L}{\partial\dot{x}^\mu}\right)-\frac{\partial L}{\partial x^\mu}=0$ and from (10) we have 4 equations as follow

$$\frac{\partial}{\partial\tau}\left(-\dot{t}-vf\left(\dot{r}-vf\dot{t}\right)\right)+\left(\dot{r}-vf\dot{t}\right)\dot{t}\frac{d\left(vf\right)}{dt}=0 \ , \tag{11}$$

$$\frac{\partial}{\partial\tau}\left(\dot{r}-vf\dot{t}\right)+\left(\dot{r}-vf\dot{t}\right)\dot{t}\frac{\partial\left(vf\right)}{\partial r\big|_x}=0 \ , \tag{12}$$

$$\frac{\partial\dot{\theta}}{\partial\tau}+\left(\dot{r}-vf\dot{t}\right)\dot{t}\frac{\partial\left(vf\right)}{\partial\theta}=0 \ , \tag{13}$$

$$\frac{\partial\dot{\phi}}{\partial\tau}+\left(\dot{r}-vf\dot{t}\right)\dot{t}\frac{\partial\left(vf\right)}{\partial\phi}=0 \ , \tag{14}$$

where obviously the solution shall be

$$\dot{x}^\mu=\left(\dot{x}^0,\dot{x}^1,\dot{x}^2,\dot{x}^3\right)=\left(\dot{t},\dot{r},\dot{\theta},\dot{\phi}\right)=\left(1,vf\dot{t},0,0\right), \tag{15}$$

which are 4 velocities for Eulerian observers that follow timelike geodesics. Since $\dot{x}=v\left(t\right)f\left(r\left(t\right)\right)\dot{t}$, as at $r=0$, $f=1$ and moreover $\lambda=\tau$ so that $\dot{t}=dt/d\lambda=dt/d\tau=1$ (since $t=\tau$ in the warp bubble), thus the spatial velocity of the warp bubble is at v which is not constrained by the speed limit of light. This notion of timelike characteristic in the warp bubble can also be proven by considering the following and referring to the metric equations (6), thus by $\dot{x}_\mu=g_{\mu\nu}\dot{x}^\nu$

for $\dot{x}_0 = g_{0\nu}\dot{x}^\nu$, since $g_{02} = g_{03} = 0$, $\frac{\partial t}{\partial \tau} = 1$, $\frac{\partial x}{\partial \tau} = \frac{\partial x}{\partial t} = v$ and $f = 1$

$$\dot{x}_0 = \left(v^2 f^2 - 1\right)\frac{\partial t}{\partial \tau} + \left(-vf\right)\frac{\partial x}{\partial \tau} = -1 \tag{16}$$

for $\dot{x}_1 = g_{1\nu}\dot{x}^\nu$, since $g_{12} = g_{13} = 0$, $\frac{\partial t}{\partial \tau} = 1$, $\frac{\partial x}{\partial \tau} = \frac{\partial x}{\partial t} = v$ and $f = 1$

$$\dot{x}_1 = \left(-vf\right)\frac{\partial t}{\partial \tau} + \frac{\partial x}{\partial \tau} = 0 , \tag{17}$$

The rest, that is $\dot{x}_2 = \dot{x}_3 = 0$ obviously, which therefore

$$\dot{x}_\mu = \left(-1, 0, 0, 0\right) \tag{18}$$

is defined as covector that is normal to the hypersurface [3] given by $dt = 0$, and by equation (15) $\dot{x}_\mu \dot{x}^\mu = -1$ confirming the time-like characteristic of the warp bubble. This ensure that the vehicle inside the bubble can be at stationary or moving at sub-light speed that does not violate the speed of light.

Defining $\beta = -vf\left(r\right)$ we may rewrite equation (5) as below

$$ds^2 = -\left(1 - \beta^2\right)dt^2 + 2\beta dr dt + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 . \tag{19}$$

In tensorial form

$$\beta^2 = \beta^i \beta_i \ ,$$
$$dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 = \delta_{ij} dx^i dx^j ,$$
$$\beta dx = \beta_i dx^i , \tag{20}$$

where $i, j = 1, 2, 3$, which by equation (5) obviously shows that $\beta = -v$ is only in the direction of the bubble's motion. Otherwise it is zero. Therefore, for this case where the bubble is "surfing" along the x axis only $\beta_1 = \beta_{r|x} = -v$ while others $\beta_2, \beta_3 = \beta_{r|y}, \beta_{r|z} = 0$. Now equation (19) in general tensorial form, can be rewritten as

$$ds^2 = -\left(1 - \beta^i \beta_i\right)dt^2 + 2\beta_i dx^i dt + \delta_{ij} dx^i dx^j \ . \tag{21}$$

For even more generalized term to suit the metric tensor g_{00} of equation (21) the term $\alpha = \sqrt{\alpha^i \alpha_i} = 1$ is introduced, thus precisely the equation (21) should be written as

$$ds^2 = -\left(\alpha^2 - \beta^i \beta_i\right)dt^2 + 2\beta_i dx^i dt + \delta_{ij} dx^i dx^j \tag{22}$$

which is the metric as firstly proposed by Alqubierre [3] describing the spacetime metric of warp bubble hypersurface.

3. Shape function

The function $f = f(r)$ is considered as the shape function of the bubble where the conditions required are as in Figure 2 where the value is $f(r)\big|_{|r|<R} = 1$ inside the bubble and $f(r)\big|_{|r|>R} = 0$ outside the bubble. Alqubierre [3] has shown that the function can be written as

$$f(r) = \frac{\tanh(\sigma(r+R)) - \tanh(\sigma(r-R))}{2\tanh(\sigma R)}, \tag{23}$$

σ is a factor of the bubble's thickness. We may investigate this function in four conditions of warp bubble radii which are as follows:

3.1 The radii is at the positive bubble radius (toward the direction of travel) $r = R$

At the bubble radius, transition between $f = 1$ and $f = 0$ is supposed to be abrupt. However equation (23) will ensure some delay in the transition but controllable. This transition from 1 to 0 also represents the spacetime contraction in front of the warp bubble in the direction of travel. As $r = R$, equation (23) can be shown to reduce to

$$\begin{aligned} f(r)\big|_{r=R} &= \frac{\tanh(\sigma(2R))}{2\tanh(\sigma R)} \\ &= \frac{\tanh(\sigma(R)) + \tanh(\sigma(R))}{2\tanh(\sigma R)\left(1+\tanh^2(\sigma(R))\right)}, \end{aligned}$$

thus

$$f(r)\big|_{r=R} = \frac{1}{1+\tanh^2(\sigma R)} \quad . \tag{24}$$

The equation (24) implies that the function's transition is dependent on the value of the bubble thickness factor. As the bubble thickness factor reduces toward zero the transition will occur abruptly from $f(r)\big|_{|r|<R} = 1$ toward $f(r)\big|_{|r|>R} = 0$.

3.2 The radii is at the center of the bubble $r = 0$

The center of the bubble is where any physical object, a test particle or even a vehicle, stays stationary. As $r = 0$ equation (23) can be shown to reduce to 1 as the following

$$f(r)\big|_{r=0} = \frac{\tanh(\sigma(R)) - \tanh(\sigma(-R))}{2\tanh(\sigma R)} \quad \text{and since } \tanh(\sigma(-R)) = -\tanh(\sigma(R)),$$

$$f(r)\big|_{r=0} = \frac{\tanh(\sigma(R)) + \tanh(\sigma(R))}{2\tanh(\sigma R)} = 1 \quad . \tag{25}$$

3.3 The radii is at the negative bubble radius (toward the opposite of the direction of travel) $r = -R$

Similar to the case in *3.1* the transition from 0 to 1 at the rear of the bubble radius is supposed to be abrupt. However, the equation (23) will ensure controllable delay in the transition. This transition from 0 to 1 also represents the spacetime expansion at the rear of the warp bubble as it moves away toward the frontal spacetime contraction in the direction of travel. At $r = -R$, equation (23) can be shown to reduce similarly as in the case of *3.1*

$$f(r)\Big|_{r=-R}=\frac{-\tanh\left(\sigma(-2R)\right)}{2\tanh(\sigma R)} \quad \text{and since} \quad \tanh\left(\sigma(-2R)\right)=-\tanh\left(\sigma(2R)\right),$$

$$=\frac{\tanh\left(\sigma(2R)\right)}{2\tanh(\sigma R)}=\frac{\tanh\left(\sigma(R)\right)+\tanh\left(\sigma(R)\right)}{2\tanh(\sigma R)\left(1+\tanh^2\left(\sigma(R)\right)\right)} \quad ,$$

therefore

$$f(r)\Big|_{r=-R}=\frac{1}{1+\tanh^2(\sigma R)} \quad . \tag{26}$$

3.4 The radii is at the exterior region of the bubble $|r|>R$

The exterior region will exhibit flatness of spacetime after the shape function transitions at the rear spacetime expansion $(f=0\to f=1)$ and at the frontal spacetime contraction $(f=1\to f=0)$. From equation (23) as $r<-R$, $\left(r|_{r<-R}+R\right)<0$ and $\left(r|_{r<-R}-R\right)<-2R$ thus

$$f(r)\Big|_{r<0}=\frac{\tanh\left(\sigma\left(\left(r|_{r<-R}+R\right)<0\right)\right)-\tanh\left(\sigma\left(\left(r|_{r<-R}-R\right)<-2R\right)\right)}{2\tanh(\sigma R)},$$

since $\tanh\left(\sigma\left(\left(r|_{r<-R}+R\right)<0\right)\right)\approx-1$ and $\tanh\left(\sigma\left(\left(r|_{r<-R}-R\right)<-2R\right)\right)\approx-1$ thus $f(r)\Big|_{r<-R}\approx 0$

as $r>R$, $\left(r|_{r>R}+R\right)>2R$ and $\left(r|_{r>R}-R\right)>0$ thus

$$f(r)\Big|_{r>R}=\frac{\tanh\left(\sigma\left(\left(r|_{r>R}+R\right)>2R\right)\right)-\tanh\left(\sigma\left(\left(r|_{r>R}-R\right)>0\right)\right)}{2\tanh(\sigma R)},$$

since $\tanh\left(\sigma\left(\left(r|_{r>R}+R\right)>2R\right)\right)\approx 1$ and $\tanh\left(\sigma\left(r|_{r>R}-R\right)>0\right)\approx 1$ thus $f(r)\Big|_{r>R}\approx 0$

thus

$$f(r)\Big|_{|r|>R}\approx 0 \quad . \tag{27}$$

By Alqubierre warp bubble shape function equation (23) sharp transition of shape function as in Figure 2 becoming rather gradual as depicted in the following figure

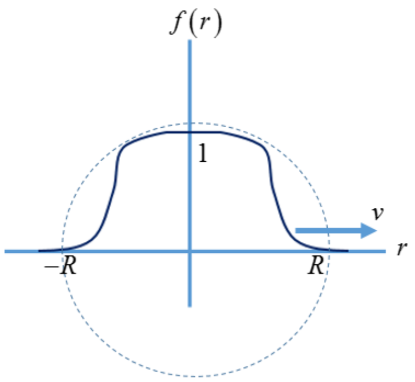

Figure 3. Function $f=f(r)$ characteristic with bubble radii with gradual transition

The transition region between the warp bubble interior and the surrounding spacetime is actually the governing factor for the dynamic characteristic of the warp bubble.

4. Dynamic of expansion and contraction

The transition region is best described by deriving the extrinsic curvature. The relationship between the extrinsic curvature of the warp bubble and the surrounding spacetime is depicted below.

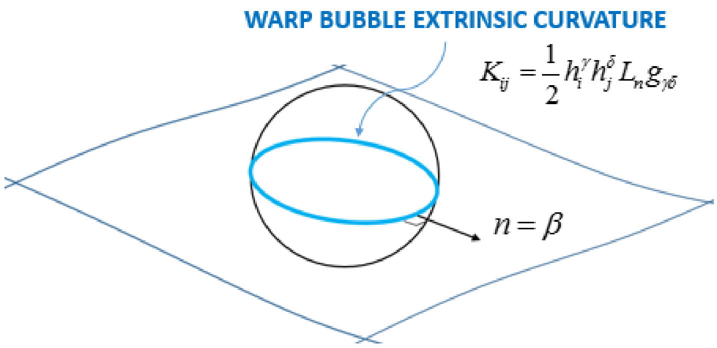

Figure 4. The extrinsic curvature between warp bubble and the surrounding spacetime

Beside the flat spacetime base metric that represents the locality, the "surfing" concept is also about embedding a bubble of warped spacetime hypersurface onto the surrounding hyperspace. The transition region of warped spacetime properties between the bubble and the surrounding hyperspace can be elaborated further by the extrinsic curvature. Consider the definition of an extrinsic curvature that is the Lie derivative of the projection tensor $h_{ij} = h_i^{\gamma} h_j^{\delta} g_{\gamma\delta}$ which is the rate of change of the projection tensor along the normal vector field n_i [13]

$$K_{ij} = \frac{1}{2} h_i^{\gamma} h_j^{\delta} L_n g_{\gamma\delta} . \tag{28}$$

The normal vector of concern here is actually the vector representing the direction of travel of the warp bubble $n = \sqrt{n_i n^i} = \sqrt{\beta_i \beta^i} = \beta$. Thus we may rewrite (28) as follows

$$K_{ij} = \frac{1}{2} h_i^{\gamma} h_j^{\delta} L_{\beta} g_{\gamma\delta} , \tag{29}$$

which can then be expanded as the covariant derivatives of the warp bubble direction of travel vector β_i and reduce toward the expression derived elaborately as the following

$$K_{ij} = \frac{1}{2}\left(\partial_i \beta_j + \partial_j \beta_i + \beta^1 \frac{\partial g_{ij}}{\partial x} \right) \tag{30}$$

thus, reduced to

$$K_{ij} = \frac{1}{2}\left(\partial_i \beta_j + \partial_j \beta_i \right), \tag{31}$$

which is essentially an equation of motion of the warp bubble. The matrix of the extrinsic curvature is

$$K = \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{pmatrix}, \tag{32}$$

where its trace describes the spacetime dynamic's characteristic χ indicating the warp bubble surrounding space of either expansion or contraction

$$\begin{aligned}\chi &= -\alpha TrK \ , \\ &= -\alpha\left(K_{11}+K_{22}+K_{33}\right), \\ &= -\alpha\left(\partial_r\beta_r - vf\left(r\right)\frac{\partial^2 Y}{\partial r^2}Y_{,r} + \partial_\theta\beta_\theta + \partial_\phi\beta_\phi\right)\end{aligned} \tag{33}$$

but since $\beta_\theta = \beta_\phi = 0$,

thus, the dynamics characteristic term is reduced to

$$\chi = -\alpha\left(\partial_r\beta_r - v\frac{\partial^2 Y}{\partial r^2}Y_{,r}\right) . \tag{34}$$

where $v\frac{\partial^2 Y}{\partial r^2}Y_{,r} = v_{bulk}$, is the velocity of the bulk. Therefore, the dynamics characteristic can also be expressed as $\chi = \alpha\left(v_{bulk} - \partial_r\beta_r\right)$. The dynamics characteristic term indicates the warp bubble surrounding space of either expansion or contraction and also its proportionality with velocity. Expanding $\partial_r\beta_r$

$$\begin{aligned}\partial_r\beta_r &= -\frac{\partial vf\left(r\right)}{\partial r} \\ &= -\left(v\left(t\right)\frac{\partial f\left(r\right)}{\partial x} + f\left(r\right)\frac{\partial v\left(t\right)}{\partial x}\right) = -v\left(t\right)\frac{\partial f\left(r\right)}{\partial r}\end{aligned} \tag{35}$$

since

$$v = v\left(t\right) = \frac{dx_b\left(t\right)}{dt} \ , \ \text{thus} \ \frac{\partial v\left(t\right)}{\partial x} = \frac{\partial\left(dx_b/dt\right)}{\partial x} = 0 \tag{36}$$

It represents the velocity of the warped bubble with respect to its surrounding spacetime (in the brane) without considering the bulk space factor. Thus, it can also be considered as the on-brane velocity $v = v_{brane}$. In equation (36) and the following equations, x_b is the position of the moving warp bubble with respect to its point of origin.

$$r = r\left(x,t\right) = \left[\left(x - x_b\left(t\right)\right)^2 + y_b^2 + z_b^2\right]^{\frac{1}{2}} \ , \ x_b = x_b\left(t\right) \ \text{thus} \ dx_b = \frac{dx_b\left(t\right)}{dt}dt \, , \quad x_b = \frac{dx_b\left(t\right)}{dt}t \tag{37}$$

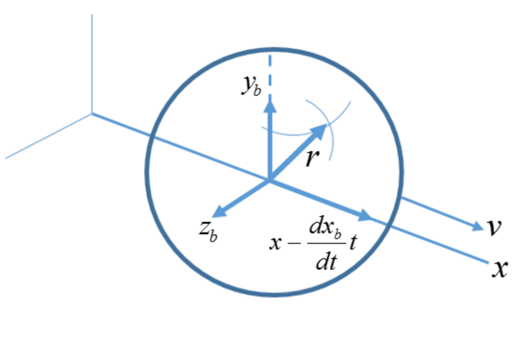

Figure 5. Warp bubble dynamics parameters

From (35) and (37), since $\partial_{r|x}\beta_{r|x} = -v(t)\dfrac{\partial f(r)}{\partial x} = -v_{brane}\dfrac{\partial f(r)}{\partial r}\dfrac{\partial r(x,t)}{\partial x}$, considering $v = v_{brane}$, it can be shown that $\partial_{r|x}\beta_{r|x} = -v_{brane}\dfrac{(x - x_b(t))}{r}\dfrac{\partial f(r)}{\partial r}$. Thus, from equation (34), the spacetime dynamic characteristic

$$\chi = -\alpha\left(\partial_{r|x}\beta_{r|x} - v_{bulk}\right) = \alpha\left(v_{brane}\frac{(x - x_b(t))}{r}\frac{\partial f(r)}{\partial r} + v_{bulk}\right). \tag{38}$$

let $\alpha = 1$ for simplicity and hyperdrive velocity is defined as $v_{HD} = v_{brane} + v_{bulk}$

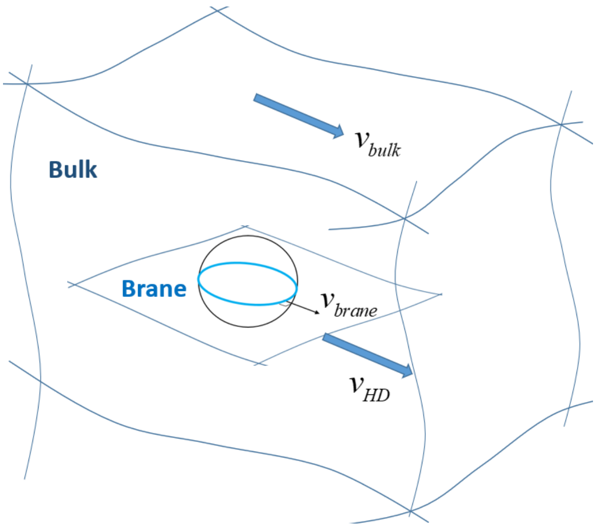

Figure 6. The extrinsic curvature between hyperdrive warp bubble and the surrounding spacetime

The hyperdrive velocity is an accumulation of warped bubble on-brane velocity pushed by the underlying bulk velocity as depicted in Figure 6. Analogically as if wind that represent bulk, push the kite surfing surfer that represents the warp bubble. Thus, from equation (38), the overall dynamics characteristics indicator expression is $\chi = (v_{HD} - v_{bulk})\dfrac{(x - x_b(t))}{r}\dfrac{\partial f(r)}{\partial r} + v_{bulk}$. The extrinsic curvature of the hyperdrive warped bubble with brane and bulk interaction is also depicted in Figure 6.

From this expression we may study the warp bubble regional characteristics that consist of *interior-exterior* and *transition* regions. In the *interior region* of the warp bubble, $f = 1$, while in *the exterior region* of the warp bubble, $f = 0$. Consider only the dynamics of warp bubble with respect to the surrounding space that is the brane only, thus $v_{bulk} = 0$. The warp bubble interior and exterior region spacetime dynamics characteristics can be depicted as

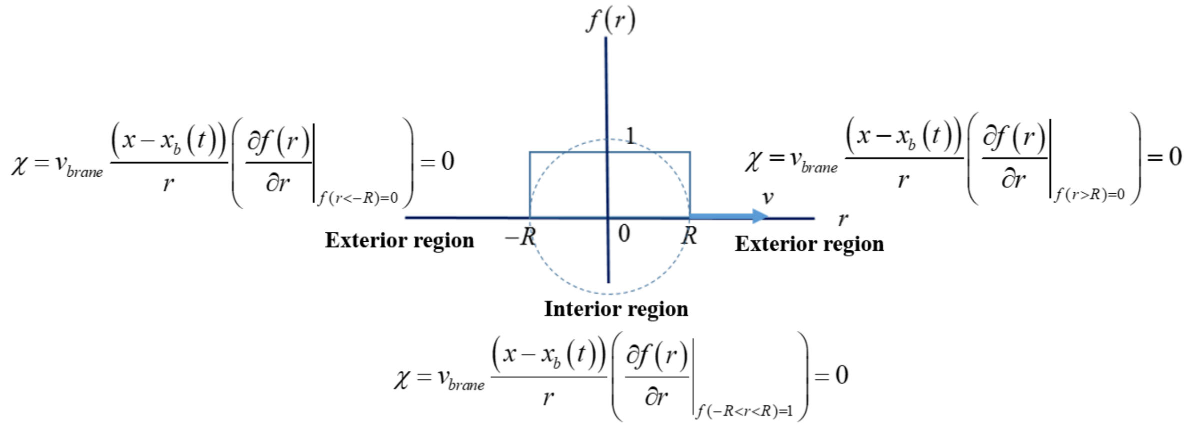

Figure 7. Spacetime dynamic characteristics in the warp bubble interior and exterior regions

Thus, the expression that represent interior-exterior region is

$$\chi = v_{brane}\frac{(x-x_b(t))}{r}\left(\frac{\partial f(r)}{\partial r}\bigg|_{f(-R<r<R)=1}\right) = v_{brane}\frac{(x-x_b(t))}{r}\left(\frac{\partial f(r)}{\partial r}\bigg|_{f(|r|>R)=0}\right) = 0\ . \tag{39}$$

This indicates the disconnection between warp bubble regions of interior and exterior while preserving flat non-dynamics character.

At the *transition* where $f(r): 0 \Leftrightarrow 1$, $\theta \neq 0$, these regions constitute two thickening walls with thickness $\xi = 1/\sigma$ along the direction of the warp bubble travel.

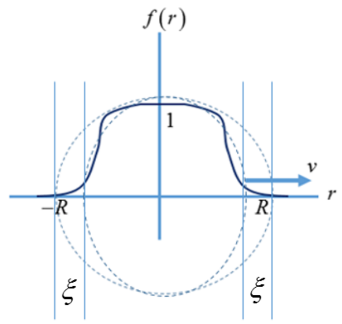

Figure 8. Spacetime dynamic's characteristics at the warp bubble's transition regions

The transition region will show the characteristics of spacetime expansion and contraction. At the front of the bubble, in the direction of travel

$$\chi = v_{brane}\frac{(x-x_b(t))}{r}\left(\frac{\partial f(r)}{\partial r}\bigg|_{(f=1)\to(f=0)}\right) < 0\,, \tag{40}$$

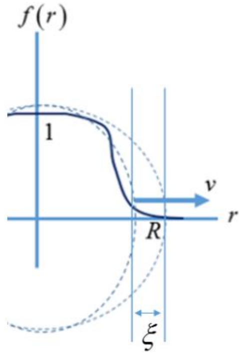

Figure 9. Transition region at the front of the bubble

since, $\left.\dfrac{\partial f(r)}{\partial r}\right|_{(f=1)\to(f=0)}<0$, thus $x-x_b(t)>0$, therefore $x_b(t)<x$, which represents *space contraction*. In the transition region at the rear of the bubble, opposite to the direction of travel (as opposed to contraction condition ($\theta<0$)), thus $\theta>0$ that is equation (38) is positive in value.

$$\chi=v_{brane}\frac{(x-x_b(t))}{r}\left(\left.\frac{\partial f(r)}{\partial(r)}\right|_{(f=1)\to(f=0)}\right)>0, \tag{41}$$

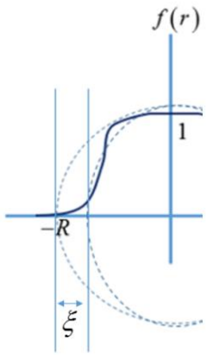

Figure 10. Transition region at the rear of the bubble

since also that $\dfrac{\partial f(r)}{\partial(r)}=\left.\dfrac{\partial f(-r)}{\partial(-r)}\right|_{(f=1)\to(f=0)}<0$, which implies that $\dfrac{\partial f}{\partial r}<0$ as always (of either during contraction or expansion), and $\theta>0$ (as opposed to contraction condition), thus $x-x_b(t)<0$, therefore $x_b(t)>x$, which represents *space expansion*.

5. Velocity expression

By the Einstein field equation, the relationship between the warp bubble velocity and the energy density can be acquired. Expanding the Einstein curvature tensor $G_{\mu\nu}$ in terms of Ricci tensor, rearranging in terms of the energy momentum tensor and multiplying both sides of the equation with timelike vector t^μ as below

$$\frac{1}{8\pi}\left(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}\right)=T_{\mu\nu}\ , \qquad T_{\mu\nu}t^\mu t^\nu=\frac{1}{8\pi}\left(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}\right)t^\mu t^\nu . \tag{42}$$

$-T_{\mu\nu} = \rho_{warp}$ can be shown that the energy momentum tensor $T_{\mu\nu}$ in equation (42) will result to the following as proven by Lobo and Visser [13] but require a little modification since the warp bubble direction of travel is in the x direction (Fig. 5)

$$T_{\mu\nu}t^{\mu}t^{\nu} = -\frac{v_{HD}^2}{32\pi}\left(\left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2\right), \tag{43}$$

which is the energy density of the warp bubble that violates the energy condition $T_{\mu\nu}t^{\mu}t^{\nu} < 0$ that is

$$T_{\mu\nu}t^{\mu}t^{\nu} = -\rho_{H\_warp} \ . \tag{44}$$

The velocity term emerging in equation (43) from Lobo and Visser calculation is regarded as the overall velocity of the hyperdrive warp bubble v_{HD}. It is known that the shape function $f = f(r)$, thus $\partial f = \frac{\partial f(r)}{\partial r}\partial(r)$, therefore $\frac{\partial f}{\partial y} = \frac{\partial f(r)}{\partial r}\frac{\partial r}{\partial y}$ and $\frac{\partial f}{\partial z} = \frac{\partial f(r)}{\partial r}\frac{\partial r}{\partial z}$ from (37) as $y_b = y$ and $z_b = z$

$$\frac{\partial f}{\partial y} = \frac{\partial f(r)}{\partial r}\frac{\partial\left[\left(x - x_b(t)\right)^2 + y^2 + z^2\right]^{\frac{1}{2}}}{\partial\left(\left(x - x_b(t)\right)^2 + y^2 + z^2\right)}\frac{\partial\left(x - x_b(t)\right)^2 + y^2 + z^2}{\partial y} = \frac{y}{r}\frac{\partial f(r)}{\partial r} \tag{45}$$

$$\frac{\partial f}{\partial z} = \frac{\partial f(r)}{\partial r}\frac{\partial\left[\left(x - x_b(t)\right)^2 + y^2 + z^2\right]^{\frac{1}{2}}}{\partial\left(\left(x - x_b(t)\right)^2 + y^2 + z^2\right)}\frac{\partial\left(x - x_b(t)\right)^2 + y^2 + z^2}{\partial z} = \frac{z}{r}\frac{\partial f(r)}{\partial r} \tag{46}$$

by (43) to (46) the expression of the warp bubble energy density is

$$\rho_{H\_warp} = \frac{v_{HD}^2}{32\pi}\left(\frac{y^2 + z^2}{r^2}\right)\left(\frac{\partial f(r)}{\partial r}\right)^2 \ . \tag{47}$$

The derivative $\partial f(r)/\partial r \approx \sigma$ which is actually the slope of the transition region in the condition where $f(r): 0 \Leftrightarrow 1$ and $\theta \neq 0$ which is also the thickness factor. The thickness of the bubble is $\xi = 1/\sigma$ (Fig. 7) thus from (47) the warp bubble velocity expression can be written as

$$v_{HD} = v_{brane} + v_{bulk} \approx \xi r\sqrt{\frac{32\pi\rho_{warp}}{y^2 + z^2}} = \frac{\xi r}{A}\left(32\pi\rho_{H\_warp}\right)^{\frac{1}{2}} \ . \tag{48}$$

This equation shows that the thickness of the warp bubble ξ and the hyperdrive energy density ρ_{H_warp} are the two most important factors that determine the hyperdrive warp bubble speed v_{HD}. The hyperdrive energy density consists of brane density and bulk density terms as

$$\rho_{H\_warp} = \rho_{brane} + \rho_{bulk} . \tag{49}$$

From equations (47), (48) and (49) the expression of the energy densities can be shown as

$$\rho_{brane} = \frac{v^2}{32\pi}\left(\frac{A}{r}\frac{\partial f(r)}{\partial r}\right)^2, \tag{50}$$

which is the brane energy density representing energy density that is required for the warp bubble dynamics with respect to the surrounding space, and

$$\rho_{bulk} = \frac{\left(v_{bulk}^2 + 2vv_{bulk}\right)}{32\pi}\left(\frac{A}{r}\frac{\partial f(r)}{\partial r}\right)^2, \tag{51}$$

which is the bulk energy density representing energy density that is required for the warp bubble dynamics with respect to bulk space.

6. Discussion

Figure 6 shows that the warp bubble is pushed by the underlying bulk space of the braneworld model. The expression of brane velocity has been developed by analysing the metric property of hyperspace warp drive by obtaining the extremum expression of proper time from equation (4). By finding the Lagrangian of the equations (9) and (10), we derived the warp bubble velocity where proper time is exactly the same with relativistic time, which indicates that the velocity is not constrained by the speed of light. This can also prove timelike characteristic of the warp bubble interior, thus ensuring that the vehicle inside the bubble remains stationary, whereas the bubble itself may move at a velocity with no special relativistic constrain. This leads to the notion that the inertial reference frame is preserved inside warp bubble. A more generalized metric tensor has been derived as shown in equation (22) which is similar to the Alqubierre spacetime metric expression. But for this case, the extradimensional term that represents bulk space is ready to be embedded for the calculation of the warp bubble extrinsic curvature after the shape function is properly defined as shown in Figure 3.

The shape function is defined by using tangent hyperbolic expression that neatly separate 3 regions with respect to the warp bubble radii as depicted in Figure 3 where it represents analogically as spacetime mold that is ready for more detail analysis using warp bubble brane velocity terms derived from the extrinsic curvature expression and expressed in term of dynamics characteristic (38). Figure 4 shows the extrinsic curvature of the warp bubble hypersurface interacting with the surrounding space.

The Lie derivative of the projection tensor of equation (28) represents the extrinsic curvature which can be expanded as a 3 by 3 matrix. Considering the warp bubble as perfect spherical symmetric, the trace of the matrix represents the dynamics characteristic that indicates the warp bubble surroundings either expanding or contracting which is later used to describe the warp bubble shape function characteristics in greater detail. Since the bulk extradimensional term $Y(r)$ is included in the dynamic characteristic expression (34), the characteristic expression can be expressed in terms of bulk and brane velocities (38). The sum of these two velocities is the hyperdrive velocity v_{HD}. The shape function based on the tangent hyperbolic function at 3 different radii conditions of equations (25), (26) and (27) was analysed by using the dynamic characteristic χ of equation (38) and the result has shown relationship with the warp bubble velocity and the slope of the transition region which is also representing the bubble thickness ξ. These have shown how the warp bubble is propelled through the contraction and expansion of hyperspace that is the spacetime surrounding the warp bubble hypersurface. This also leads to the notion that the motion inertial reference frame is also preserved in the exterior of the warp bubble. Therefore, the reference frame in the interior of the warp bubble and the exterior of the warp bubble is completely separated.

Finally, using the Einstein field equation (42), it has been shown that the energy density acquires exotic matter property of negative energy at the slope of the transition region which also indicates that beside the hyperdrive energy density ρ_{H_warp}, which consist of brane and bulk components, the thickness of the warp bubble boundary ξ also plays an important factor in determining the velocity of the warp bubble, which by itself is not constrained by any relativistic limitations (48). As the energy density has the brane and bulk components, the velocity of the warp bubble also consists of the brane and bulk components and the figure 6 shows figuratively *kite surfing-like dynamics* of embedded hypersurface onto a hyperspace of braneworld. This shows the underlying extra-dimensional of hyperspace influence on the warp bubble geometrodynamics on the brane which is the 3-dimensional space universe. In another words, the braneworld "floats" in 4 spatial dimensional bulk of hyperspace.

7. Conclusion

Superluminal capability is fundamentally based on the concept of separating inertial reference frames between the warp-bubble and its surrounding spacetime. The transition region of the warp bubble is fundamentally manipulated on how the shape-function characteristic of the warp bubble interacts with the surrounding spacetime. The trace of the extrinsic curvature matrix provides its relationship with the warp bubble velocity and the slope of the transition region which also represents the bubble thickness factor. The velocity of a hyperdrive warp bubble is as superposition of the velocity of the warp bubble on the brane and the velocity of the underlying bulk. Therefore, the hyperdrive velocity is larger than the warp drive velocity that is occurring only on the brane whereas the hyperdrive velocity is the addition of the underlying bulk velocity to the warp drive velocity on the brane. Similar mathematical characteristics of superposition also occur for the hyperdrive energy density which resulted from the addition of brane energy density and bulk energy density. Therefore, the hyperdrive requires much more energy than that of a warp drive. A hyperdrive that utilizes the underlying higher dimensional spatial energy of bulk space is analogous to a *kite surfer* that uses *wind* as the bulk force, which adds up to the dynamics of surfing, which is originally propelled by the *surfing wave motion* analogous to the brane.

8. Reference

[1] David K 2017 Relativistic Light Sails *arXiv:1704.04310v3* (Department of Astronomy Columbia University)

[2] Coffey and Jeffrey 2013 How big is the Milky Way? *Universe Today*.

[3] Alcubierre M 1994 The warp drive: hyperfast travel within general relativity *Class.Quant.Grav.11:L73-L77*

[4] Wheeler J A 1963 *Geometrodynamics* New York Academic Press.

[5] Sumanta, Chakraborty, Tanmoy Paul and Soumitra Sen Gupta 2018 Inflation driven by Einstein-Gauss-Bonnet gravity. *Phys. Rev.* D **98***, 083539*

[6] Riess et al. 1998 Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *The Astronomical Journal*. 116(3): 1009-1038.

[7] Anuar A, W. A. Tajuddin 2018 The effects of spatial dynamics on a wormhole throat, *Modern Physics Letters A*, Vol. 33, No. 6, 18500360

[8] Anna Ijjas, Paul J. Steinhardt 2021 Entropy, Black holes, and the New Cyclic Universe, *arXiv:2108.07101v1 [gr-qc]*

[9] Wheeler J A 1955 Geons *Physical Review 97 (2) :511*

[10] Maldacena and Susskind L 2013 Cool horizons for entangled black holes, *Fortsch. Phys. 61(9): 781-811 arXiv:1306.0533.*

[11] Antoniadis, S Cotsakis, Klaoudatou 2021 Regular braneworlds with nonlinear bulk-fluids, *The European Physical Journal C, Vol **81**; 771*.

[12] Mattingly, B et.,al. 2021 Curvature Invariant for the Alcubierre and Natario Warp Drives *Particle **2020** 3(3), 642-659.*

[13] Philip D. Mannheim 2005 *Brane-Localize Gravity*. World Scientific Publishing.

[14] Bisabr, Y., Ahmadi, F., 2020 Deflation of Vacuum Energy During Inflation Due to Brane-Bulk Interaction, *Journal of Cosmology and Astroparticle Physics, Vol 2020*

[15] Zheltukhin, A. A. 2014 Branes as solutions of gauge theories in gravitational field, *The European Physical Journal C, Vol **74**; 3048*.

[16] Marteens, R. 2010.Brane-World Gravity, *Living Reviews in Relativity, **13**, Article 5.*