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Axial Vector

Related terms:

Anomalies, Divergence, Pseudoscalar, Magnetic Moment, σ property

Representation Analysis of Magnetic Structures

Rafik Ballou, Bachir Ouladdiaf, in Neutron Scattering from Magnetic Materials, 2006

Axial vector matrix representation: Γ^{vect}.

The magnetic moment, being an axial vector, transforms as a polar vector under rotation but remains invariant under the inversion so that if α is a symmetry operation and I the inversion then $\Gamma^{\text{vect}}(\alpha I) = \Gamma^{\text{vect}}(\alpha)$. We show in Table 6 the transformation properties of the magnetic moments S^x, S^y and S^z, along x, y and z, under the 24 symmetry operators of the group O using the method described in Section 3.1. We deduce from the table that the nonzero traces of the axial representation are only the following:

Table 6. Transformation of the S^x, S^y, S^z moments under the symmetry operations of the group O

E	2_z	2_y	2_x	3^{+}_{xxx}	$3^{+}_{\bar{x}x\bar{x}}$	$3^{+}_{x\bar{x}\bar{x}}$	$3^{+}_{\bar{x}\bar{x}x}$	3^{-}_{xxx}	$3^{-}_{x\bar{x}\bar{x}}$	$3^{-}_{\bar{x}\bar{x}x}$	$3^{-}_{\bar{x}x}$
S_x	$-S_x$	$-S_x$	S_x	S_y	$-S_y$	$-S_y$	S_y	S_z	$-S_z$	$-S_z$	S_z
S_y	$-S_y$	S_y	$-S_y$	S_z	$-S_z$	S_z	$-S_z$	S_x	$-S_x$	S_x	$-S_x$
S_z	S_z	$-S_z$	$-S_z$	S_x	S_x	$-S_x$	$-S_x$	S_y	S_y	$-S_y$	$-S_y$
2_{xx0}	$2_{x\bar{x}0}$	4^{-}_{z}	4^{+}_{z}	4^{-}_{x}	2_{0yy}	$2_{0y\bar{y}}$	4^{+}_{x}	4^{+}_{y}	2_{x0x}	4^{-}_{y}	$2_{\bar{x}0}$
S_y	$-S_y$	$-S_y$	S_y	S_x	$-S_x$	$-S_x$	S_x	$-S_z$	S_z	S_z	$-S_z$
S_x	$-S_x$	S_x	$-S_x$	$-S_z$	S_z	$-S_z$	S_z	S_y	$-S_y$	S_y	$-S_y$
$-S_z$	$-S_z$	S_z	S_z	S_y	S_y	$-S_y$	$-S_y$	S_x	S_x	$-S_x$	S_x

$$\chi^{\text{vect}}(E) = \chi^{\text{vect}}(I) = 3, \quad \chi^{\text{vect}}(6C_4) = \chi^{\text{vect}}(6C_4) = 1,$$
$$\chi^{\text{vect}}(3C_2) = \chi^{\text{vect}}(3\sigma_h) = \chi^{\text{vect}}(6C_2^P) = \chi^{\text{vect}}(6\sigma_d) = -1. \tag{75}$$

We can then identify Γ^{vect} with Γ^{4g} from the character table of the group O_h. This result can also be obtained by reducing the representation Γ^{vect} over the Γ^{iw} of O_h.

The transformation-induced matrix representation $\Gamma_{32f}\Gamma^{vect} = \otimes\Gamma^{perm}_{32f}$ can then be written as the direct product $\Gamma_{32f} = \Gamma^{4g} \otimes \left(\Gamma^{1g} \otimes \Gamma^{5g} \otimes \Gamma^{2u} \otimes \Gamma^{4u}\right)$. This is decomposed into

$$\Gamma_{32f} = \Gamma^{2g} \oplus \Gamma^{3g} \oplus 2\Gamma^{4g} \oplus \Gamma^{5g} \oplus \Gamma^{1u} \oplus \Gamma^{3u} \oplus \Gamma^{4u} \oplus 2\Gamma^{5u} \tag{76}$$

using the following simpler decompositions: $\Gamma^{4g} \otimes \Gamma^{1g} = \Gamma^{4g}, \Gamma^{4g} \otimes \Gamma^{5g} = \Gamma^{2g} \otimes \Gamma^{3g} \otimes \Gamma^{4g} \otimes \Gamma^{5g}, \Gamma^{4g} \otimes \Gamma^{2u} = \Gamma^{5u}, \Gamma^{4g} \otimes \Gamma^{4u} = \Gamma^{1u} \otimes \Gamma^{3u} \otimes$. We easily check that the dimension of the direct sum of the matrix representations in the right-hand side of (76) is (1 + 2 + 2 • 3 + 3 + 1 + 2 + 3 + 2 • 3) = 24 = 3 • 8 as expected. Equation (76) indicates that the basis functions for the irreducible matrix representations not contained in the reduction of Γ_{32f}, that is, Γ^{1g} and Γ^{2u}, are necessary null.

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Anomalies

S.L. Adler, in Encyclopedia of Mathematical Physics, 2006

Neutral Pion Decay $\pi^0 \to \gamma\gamma$

As a result of the abelian chiral anomaly, the partially conserved axial-vector current (PCAC) equation relevant to neutral pion decay is modified to read

$$\partial^\mu \mathscr{F}^5_{3\mu}(x) = \left(f_\pi \mu_\pi^2/\sqrt{2}\right) \phi_\pi(x) + S\frac{\alpha_0}{4\pi} F^{\xi\sigma}(x) F^{\tau\rho}(x) \varepsilon_{\xi\sigma\tau\rho} \quad [6a]$$

with μ_π the pion mass, $f_\pi \simeq 131$ MeV the charged-pion decay constant, and S a constant determined by the constituent fermion charges and axial-vector couplings. Taking the matrix element of eqn [6a] between the vacuum state and a two-photon state, and using the fact that the left-hand side has a kinematic zero (the Sutherland–Veltman theorem), one sees that the $\pi^0 \to \gamma\gamma$ amplitude F is completely determined by the anomaly term, giving the formula

$$F = -(\alpha/\pi) 2S\sqrt{2}/f_\pi \quad [6b]$$

For a single set of fractionally charged quarks, the amplitude F is a factor of three too small to agree with experiment; for three fractionally charged quarks (or an equivalent Han–Nambu triplet), eqn [6b] gives the correct neutral pion decay rate. This calculation was one of the first pieces of evidence for the color degree of freedom of quarks.

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THE THEORY OF SYMMETRY

L.D. LANDAU, E.M. LIFSHITZ, in Quantum Mechanics (Third Edition), 1977

PROBLEMS

Problem 1. Find the selection rules for the matrix elements of the electric and magnetic dipole moments **d** and μ when symmetry ***O*** is present.

Solution. The group ***O*** includes no reflections; the polar vector **d** and the axial vector μ are therefore transformed by the same irreducible representation, F_1. The decompositions of the direct products of F_1 with the other representations of the group ***O*** are

$$\begin{aligned} &F_1 \times A_1 = F_1, \quad F_1 \times A_2 = F_2, \quad F_1 \times E = F_1 + F_2, \\ &F_1 \times F_1 = A_1 + E_1 + F_1 + F_2, \quad F_1 \times F_2 = A_2 + E + F_1 + F_2 \end{aligned} \quad (1)$$

Hence the non-zero non-diagonal (with respect to energy) matrix elements are those for the transitions

$$F_1 \leftrightarrow A_1, E, F_1, F_2; \quad F_2 \leftrightarrow A_2, E, F_2 .$$

The symmetric and antisymmetric products of the irreducible representations of the group ***O*** are

$$\left[A_1^2\right]=\left[A_2^2\right]=A_1,\quad \left[E^2\right]=A_1+E,\quad \left[F_1^2\right]=\left[F_2^2\right]=A_1+E \tag{2}$$
$$\left\{E^2\right\}=A_2,\quad \left\{F_1^2\right\}=\left\{F_2^2\right\}=F_1.$$

The symmetric products do not contain F_1; hence there are no diagonal (with respect to energy) matrix elements of the vector **d** (which is invariant under time reversal). The magnetic moment, which changes sign under time reversal, has diagonal matrix elements for the states F_1 and F_2.

Problem 2. The same as Problem 1, but for symmetry D_{3d}.

Solution. The vectors **d** and μ have different transformation laws in the group D_{3d}:

$$d_x,d_y\sim E_u,\quad d_z\sim A_{2u},$$
$$\mu_x,\mu_y\sim E_g,\quad \mu_z\sim A_{2g};$$

here and in the Problems below, the symbol ~ stands for the words "is transformed by the representation". We have

$$\left.\begin{array}{l} E_u\times A_{1g}=E_u\times A_{2g}=E_u,\quad E_u\times A_{1u}=E_u\times A_{2u}=F_g,\\ E_u\times E_u=A_{1g}+A_{2g}+E_g,\quad E_u\times E_g=A_{1u}+A_{2u}+E_u.\end{array}\right\} \tag{3}$$

Hence the non-diagonal matrix elements of d_x, d_y are non-zero for the transitions $E_n\leftrightarrow A_{1g},A_{2g},E_g;E_g\leftrightarrow A_{1u},A_{2u}$. In the same way we find the selection rules

$$\begin{array}{ll} \text{for } d_z: & A_{1g}\leftrightarrow A_{2u};A_{2g}\leftrightarrow A_{1u};E_g\leftrightarrow E_u;\\ \text{for } \mu x,\mu y: & E_g\leftrightarrow A_{1g},A_{2g}E_g;E_u\leftrightarrow A_{1u},A_{2u},E_u;\\ \text{for } \mu_z: & A_{1g}\leftrightarrow A_{2g};A_{1u}\leftrightarrow A_{2u};E_g\leftrightarrow E_g;E_u\leftrightarrow E_u.\end{array}$$

The symmetric and antisymmetric products of the irreducible representations of the group ***D***$_{3d}$ are

$$\left.\begin{array}{l} \left[A_{1g^2}\right]=\left[A_{1u^2}\right]=\left[A_{2g^2}\right]=\left[A_{2u^2}\right]=A_{1g},\\ \left[E_{g^2}\right]=\left[E_{u^2}\right]=E_g+A_{1g},\quad \left\{E_g^2\right\}=\left\{E_u^2\right\}=A_{2g}.\end{array}\right\} \tag{4}$$

Hence we see that there are no diagonal (with respect to energy) matrix elements for any of the components **d**; for the vector μ, there are diagonal matrix elements of μ_z for transitions between states belonging to a degenerate level of the type E_g or E_u.

Problem 3. Find the selection rules for the matrix elements of the electric quadrupole moment tensor Q_{ik} when symmetry ***O*** is present.

Solution. The components of the tensor Q_{ik} (a symmetrical tensor with the sum Q_{ii} equal to zero) with respect to group ***O*** are transformed by the laws

$$Q_{xy},Q_{xz},Q_{yz}\sim F_2,\quad Q_{xx}+\in Q_{yy}+\in^2Q_{zz},Q_{xx}+\in^2Q_{yy}+\in Q_{zz}$$
$$\left(\in=e^{2\pi i/3}\right).$$

Decomposing the direct products of F_2 and E with all the representations of the group, we find the selection rules for the non-diagonal matrix elements:

$$\begin{array}{ll} \text{for } Q_{xy},Q_{xz},Q_{yz}: & F_1\leftrightarrow A_2,E,F_1,F_2;F_2\leftrightarrow A_1,E,F_1,F_2;\\ \text{for } Q_{xx},Q_{yy},Q_{zz}: & E\leftrightarrow A_1,A_2,E;F_1\leftrightarrow F_1,F_2;F_2\leftrightarrow F_2.\end{array}$$

The diagonal matrix elements exist (as we see from (2)) in the following states:

$$\begin{array}{ll} \text{for } Q_{xy},Q_{xz},Q_{yz}: & F_1,F_2,\\ \text{for } Q_{xx},Q_{yy},Q_{zz}: & E,F_1,F_2.\end{array}$$

Problem 4. The same as Problem 3, but for symmetry ***D***$_{3d}$.

Solution. The transformation laws of the components Q_{ik} with respect to the group ***D***$_{3d}$ are

$$Q_{zz} \sim A_{1g}; \quad Q_{xx} - Q_{yy}, Q_{xy} \sim E_g; \quad Q_{xz}, Q_{yz} \sim E_g.$$

Q_{zz} behaves as a scalar. Decomposing the direct products of E_g with all the representations of the group, we find the selection rules for the non-diagonal matrix elements of the remaining components Q_{ik}:

$$E_g \leftrightarrow A_{1g}, A_{2g}, E_g; \quad E_u \leftrightarrow A_{1u}, A_{2u}, E_u.$$

The diagonal elements are non-zero (as we see from (4)) only for the states E_g and E_u.

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MOTION IN A MAGNETIC FIELD

L.D. LANDAU, E.M. LIFSHITZ, in Quantum Mechanics (Third Edition), 1977

§111. Schrödinger's equation in a magnetic field

A particle that has a spin also has a certain "intrinsic" magnetic moment μ. The corresponding quantum-mechanical operator is proportional to the spin operator $\hat{\mathbf{s}}$, and can therefore be written as

$$\hat{\mu} = \mu \hat{\mathbf{s}}/s, \tag{111.1}$$

where *s* is the magnitude of the particle spin and μ a constant characterizing the particle. The eigenvalues of the magnetic moment component are $\mu_z = \mu\sigma/s$. Hence we see that the coefficient μ (which is usually called just the magnitude of the magnetic moment) is the maximum possible value of μ_z, reached when the spin component σ = *s*.

The ratio $\mu/\hbar s$ gives the ratio of the intrinsic magnetic moment and the intrinsic angular momentum of the particle (when both are along the *z*-axis). For the ordinary (orbital) angular momentum, this ratio is $e/2mc$ (see *Fields*, §44). The coefficient of proportionality between the intrinsic magnetic moment and the spin of the particle is not the same. For an electron it is $-|e|/mc$, i.e. twice the usual value, as is found theoretically from Dirac's relativistic wave equation (see *RQT*, §33). The intrinsic magnetic moment of the electron (spin $\frac{1}{2}$) is consequently $-\mu_B$, where

$$\mu_B = |e|\,\hbar/2mc = 0 \cdot 927 \times 10^{-20} \mathrm{erg/gauss}. \tag{111.2}$$

This quantity is called the *Bohr magneton*.

The magnetic moment of heavy particles is customarily measured in *nuclear magnetons*, defined as $e\hbar/2m_pc$, with m_p the mass of the proton. The intrinsic magnetic moment of the proton is found by experiment to be 2·79 nuclear magnetons, the moment being parallel to the spin. The magnetic moment of the neutron is opposite to the spin, and is 1·91 nuclear magneton.

It should be noted that the quantities μ and **s** on the two sides of (111.1) are the same type of vector, as they should be: both are axial vectors. A similar equation for the electric dipole moment **d** (= constant × **s**) would contradict the symmetry under inversion of the coordinates: the relative sign of the two sides would be changed by inversion.†

In non-relativistic quantum mechanics, the magnetic field may be regarded as an external field only. The magnetic interaction between the particles is a relativistic effect, and a consistent relativistic theory is needed to take it into account.

In the classical theory, the Hamilton's function of a charged particle in an electromagnetic field is

$$H = \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 + e\phi,$$

where ϕ is the scalar and **A** the vector potential of the field, and **p** the generalized momentum of the particle; see *Fields*, §16. If the particle has no spin, the transition to quantum mechanics can be made in the usual manner: the generalized momentum must be replaced by the operator $\widehat{\mathbf{p}} = -i\hbar\nabla$, and we obtain the Hamiltonian†

$$\widehat{H} = \frac{1}{2m}\left(\widehat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)^2 + e\varphi. \tag{111.3}$$

If, on the other hand, the particle has a spin, this procedure does not suffice. This is because the intrinsic magnetic moment of the particle interacts directly with the magnetic field. In the classical Hamilton's function, this interaction does not appear, since the spin, which is a purely quantum effect, vanishes in the limit of classical mechanics. The correct expression for the Hamiltonian is obtained by including in (111.3) an extra term—$\widehat{\mu}$.**H** corresponding to the energy of the magnetic moment μ in the field **H**. Thus the Hamiltonian of a particle having a spin is‡

$$\widehat{H} = \frac{1}{2m}\left(\widehat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)^2 - \widehat{\mu}\cdot\mathbf{H} + e\varphi. \tag{111.4}$$

In expanding the square $\left(\widehat{\mathbf{p}} - e\mathbf{A}/c\right)^2$, we must bear in mind that $\widehat{\mathbf{p}}$ dose not in general commute with the vector **A**, which is a function of the coordinates. Hence we must write

$$\widehat{H} = \widehat{\mathbf{p}}^2/2m - (e/2mc)\left(\mathbf{A}\cdot\widehat{\mathbf{p}} + \widehat{\mathbf{p}}\cdot\mathbf{A}\right) + e^2\mathbf{A}^2/2mc^2 - (\mu/s) \tag{111.5}$$

According to the rule (16.4) for the commutation of the momentum operator with any function of the coordinates, we have

$$\widehat{\mathbf{p}}\cdot\mathbf{A} - \mathbf{A}\cdot\widehat{\mathbf{p}} = -i\hbar\,\mathrm{div}\mathbf{A}. \tag{111.6}$$

Thus $\widehat{\mathbf{p}}$ and **A** commute if div **A** ≡ 0. This holds, in particular, for a uniform field, if its vector potential is expressed in the form

$$\mathbf{A} = \tfrac{1}{2}\mathbf{H}\times\mathbf{r}. \tag{111.7}$$

The equation $i\hbar\partial\psi/\partial t = \widehat{H}\psi$ with the Hamiltonian (111.4) is a generalization of Schrödinger's equation to the case where a magnetic field is present. The wave functions on which the Hamiltonian acts in this equation are symmetrical spinors of rank 2*s*.

The wave functions of a particle in an electromagnetic field are not uniquely defined, because the choice of the field potentials is not unique: they are defined (see *Fields*, §18) only to within a gauge transformation

$$\mathbf{A} \to \mathbf{A} + \nabla f, \phi \to \phi - \frac{1}{c}\frac{\partial f}{\partial t}, \tag{111.8}$$

where *f* is an arbitrary function of the coordinates and the time. This transformation does not affect the values of the field strengths, and it is therefore clear that it cannot essentially alter the solutions of the wave equation; in particular, it must leave $|\Psi|^2$ unchanged, since it is easy to see that the original equation is restored if we make the changes (111.8) in the Hamiltonian and at the same time change the wave function according to

$$\Psi \to \Psi\exp\left(ief/\hbar c\right). \tag{111.9}$$

This non-uniqueness of the wave function does not affect any quantity having a physical significance (in whose definition the potentials do not appear explicitly).

In classical mechanics, the generalized momentum of a particle is related to its velocity by

$$m\mathbf{v} = \mathbf{p} - e\mathbf{A}/c.$$

In order to find the operator $\widehat{\mathbf{v}}$ in quantum mechanics, we have to commute the vector **r** with the Hamiltonian. A simple calculation gives the result

$$m\widehat{\mathbf{v}} = \widehat{\mathbf{p}} - e\mathbf{A}/c, \tag{111.10}$$

which is exactly analogous to the classical expression. For the operators of the velocity components we have the commutation rules

$$\left.\begin{aligned}\{\hat{v}_x, \hat{v}_y\} &= i\left(e\hbar/m^2c\right)H_z,\\ \{\hat{v}_y, \hat{v}_z\} &= i\left(e\hbar/m^2c\right)H_x,\\ \{\hat{v}_z, \hat{v}_x\} &= i\left(e\hbar/m^2c\right)H_y,\end{aligned}\right\} \tag{111.11}$$

which are easily verified directly. We see that, in a magnetic field, the operators of the three velocity components of a (charged) particle do not commute. This means that the particle cannot simultaneously have definite values of the velocity components in all three directions.

In motion in a magnetic field, the symmetry with respect to time reversal occurs only if the sign of the field **H** (and of the vector potential **A**) is changed. This means (see §§18 and 60) that Schrodinger's equation $\hat{H}\psi = E\psi$ must keep the same form when we take complex conjugates and change the sign of **H**. This is immediately evident for all terms in the Hamiltonian (111.4) except $-\hat{\mathbf{s}}.\,\mathbf{H}$. The term $-\hat{\mathbf{s}}.\,\mathbf{H}\psi$ in Schrodinger's equation becomes $\mathbf{s}*.\mathbf{H}\psi*$ under the transformation in question, and at first sight this destroys the required invariance, since the operator $\hat{\mathbf{s}}*$ is not the same as $-\hat{\mathbf{s}}$. It must be remembered, however, that the wave function is in reality a spinor $\psi^{\lambda\mu\cdots}$, and on time reversal a contravariant spinor must be replaced by a covariant one (see §60), so that in Schrodinger's equation the term $-\widehat{\mathbf{S}}.\,\mathbf{H}\psi^{\lambda\mu\cdots}$ is replaced by $-\widehat{\mathbf{S}}*.\,\mathbf{H}\psi_{\lambda\mu\ldots}$. It is easily seen by means of the definitions (57.4), (57.5) that the result of the action of the operator $\hat{\mathbf{s}}*$ on the components of the covariant spinor has the opposite sign to that of the operator $\hat{\mathbf{s}}$ on the components of the contravariant spinor. The operation of time reversal therefore leads to a Schrodinger's equation for the components $\psi_{\lambda\mu\ldots}$ which is of the same form as the original equation for the components $\psi^{\lambda\mu\cdots}$.

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SPIN

L.D. LANDAU, E.M. LIFSHITZ, in Quantum Mechanics: A Shorter Course of Theoretical Physics, 1974

§43. A particle in a magnetic field

A particle that has a spin also has a certain “intrinsic” magnetic moment μ. The corresponding quantum-mechanical operator is proportional to the operator $\hat{\mathbf{s}}$, and can therefore be written as

$$\hat{\mu} = \mu\hat{\mathbf{s}}/s, \tag{43.1}$$

where s is the magnitude of the particle spin and μ a constant characterising the particle. The eigenvalues of the magnetic moment component are $\mu_z = \mu\sigma/s$. Hence we see that the coefficient μ (which is usually called just the magnitude of the magnetic moment) is the maximum possible value of μ_z, reached when $\sigma = s$.

The ratio $\mu/\hbar s$ gives the ratio of the intrinsic magnetic moment and the intrinsic angular momentum of the particle (when both are along the z-axis). For the ordinary (orbital) angular momentum, this ratio is $e/2mc$ (see Mechanics and *Electrodynamics*, §66). The coefficient of proportionality between the intrinsic magnetic moment and the spin of the particle is not the same. For an electron it is $-|e|/mc$, i.e. twice the usual value; we shall see later that this value can be obtained theoretically from Dirac's relativistic wave equation. The intrinsic magnetic moment of the electron (spin $\frac{1}{2}$) is consequently $-\mu_B$, where

$$\mu_B = |e|\hbar/2mc = 0.927 \times 10^{-20}\,\mathrm{erg/gauss}. \tag{43.2}$$

This quantity is called the *Bohr magneton*.

The magnetic moment of heavy particles is customarily measured in *nuclear magnetons*, defined as $e\hbar/2m_pc$ with m_p the mass of the proton. The intrinsic magnetic moment of the proton is found by experiment to be 2.79 nuclear magnetons, the moment being parallel to the spin. The magnetic moment of the neutron is opposite to the spin, and is 1.91 nuclear magnetons.

It should be noted that the quantities μ and **s** on the two sides of (43.1) are the same type of vector, as they should be: both are axial vectors (both being given by vector products of two polar vectors). A similar equation for the electric dipole moment **d**(**d** = constant × **s**) would contradict the symmetry under inversion of coordinates: the relative sign of the two sides would be changed by the inversion.†

Let us ascertain the form of Schrödinger's equation for a particle moving in external electric and magnetic fields. In the classical theory, the Hamilton's function for a charged particle in an electromagnetic field has the form

$$H = \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 + e\Phi,$$

where Φ and **A** are the scalar and vector potentials of the field, and **p** is the generalised momentum of the particle (see *Mechanics and Electrodynamics*, §43). If the particle has no spin, the transition to quantum mechanics can be made in the usual manner; the generalised momentum must be replaced by the operator $\hat{\mathbf{p}} = -i\hbar\nabla$, and we obtain the Hamiltonian‡

$$\widehat{H} = \frac{1}{2m}\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)^2 + e\Phi. \tag{43.3}$$

If the particle has a spin, this procedure does not suffice. This is because the intrinsic magnetic moment of the particle interacts directly with the magnetic field. In the classical Hamilton's function this interaction does not appear, since the spin itself, which is a purely quantum effect, vanishes when we pass to the limit of classical mechanics. The correct expression for the Hamiltonian is obtained by adding to (43.3) a term $-\hat{\mu}.\mathbf{H}$, which corresponds to the energy of the magnetic moment μ in the field **H**†. Thus the Hamiltonian of a particle having a spin and in a magnetic field is

$$\widehat{H} = \frac{1}{2m}\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)^2 - \mu\,.\,\mathbf{H}. \tag{43.4}$$

The equation Ĥψ = *E*ψ for the eigenvalues of this operator is the required generalisation of Schrödinger's equation to the case of motion in a magnetic field. The wave function ψ in this equation is a spinor of rank 2*s*+1.

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Magnetic Point Groups and Space Groups

R. Lifshitz, in Encyclopedia of Condensed Matter Physics, 2005

Generalizations of Magnetic Groups

There are two natural generalizations of magnetic groups. One is to color groups with more than two colors, and the other is to spin groups where the spins are viewed as classical axial vectors free to rotate continuously in any direction.

An *n*-color point group G_C is a subgroup of $O(d) \times S_n$, where S_n is the permutation group of *n* colors. Elements of the color point group are pairs (*g*, *γ*) where *g* is a *d*-dimensional (proper or improper) rotation and *γ* is a permutation of the *n* colors. As before, for (*g*, *γ*) to be in the color point group of a finite object it must leave it invariant, and for (*g*, *γ*) to be in the color point group of a crystal it must leave it indistinguishable, which in the special case of a periodic crystal reduces to invariance to within a translation. To each element $(g, \gamma) \in G_C$ corresponds a phase function $\mathbf{\Phi}_g^\gamma(\boldsymbol{k})$, satisfying a generalized version of the group compatibility condition [6]. The color point group contains an important subgroup of elements of the form (*e*, *γ*) containing all the color permutations that leave the crystal indistinguishable without requiring any rotation *g*.

A spin point group G_S is a subgroup of $\mathrm{O}(d) \times \mathrm{SO}(d_s) \times 1'$, where $\mathrm{SO}(d_s)$ is the group of d_s-dimensional proper rotations operating on the spins, and $1'$ is the time inversion group as before. Note that the dimension of the spins need not be equal to the dimension of space (e.g., one may consider a planar arrangement of 3D spins). Also note that because the spins are axial vectors there is no loss of generality by restricting their rotations to being proper. Elements of the spin point group are pairs (g, γ), where g is a d-dimensional (proper or improper) rotation and γ is a spin-space rotation possibly followed by time inversion. Here as well, elements of the form (e, γ) play a central role in the theory, especially in determining the symmetry constraints imposed by the corresponding phase functions $\boldsymbol{\Phi}_e^\gamma(\boldsymbol{k})$ on the patterns of magnetic Bragg peaks, observed in elastic neutron diffraction experiments.

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Neutron Scattering - Magnetic and Quantum Phenomena

V. Ovidiu Garlea, Bryan C. Chakoumakos, in Experimental Methods in the Physical Sciences, 2015

4.3.3.1 Shubnikov Space Groups

To describe the invariance of magnetic structure, a new "spin-reversal" operator (aka antisymmetry, antiidentity, or time-inversion) that defines the current loop type symmetry of an axial vector is used. The antiidentity operation was introduced by Heesch [101], but magnetic symmetry is usually termed Shubnikov symmetry, after the crystallographer Shubnikov who rediscovered the antisymmetry concept as a way to expand the classical symmetry groups. This antisymmetry operator, identified by $1'$, can be combined with any conventional symmetry operator h to form a new "primed" operator h'. The effect of such symmetry operators on a magnetic moment described as axial vectors associated with a current loop is exemplified in Figure 9. It is important to emphasize that the magnetic moment $\boldsymbol{\mu}_j$ is transformed only by the rotational part of the operator $g = \{h|\mathbf{t}\}$. The resulting moment $\boldsymbol{\mu}_j'$ can be mathematically expressed as:

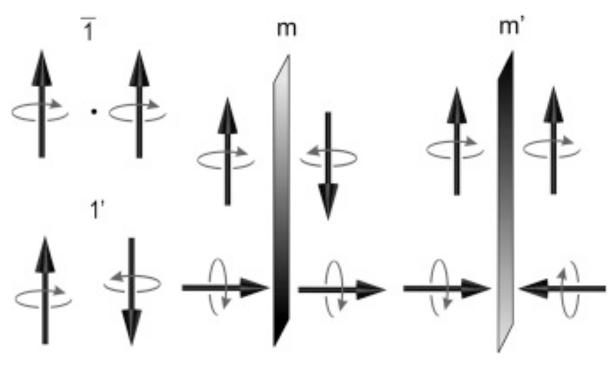

Figure 9. Transformations of magnetic moments described as axial vectors associated with a current loop, under the action of inversion, spin-reversal, mirror, and antimirror operators.

$$\boldsymbol{\mu}_j' = g\boldsymbol{\mu}_j = \det(h)\,\delta h\boldsymbol{\mu}_j \tag{15}$$

where the determinant det(h) describes the current loop type symmetry, while the term δ takes the value 1 for unprimed symmetry elements and −1 for the primed ones. The position in the zeroth cell of the transformed moment changes according to the equation:

$$\mathbf{r}_j' = g\mathbf{r}_j = \{h|\mathbf{t}\}\,\mathbf{r}_j = h\mathbf{r}_j + \mathbf{t} = \mathbf{r}_i + \mathbf{a}_{gj} \tag{16}$$

The vector $\mathbf{a}_{gj}$ is called the "returning vector" because it links the transformed position $\mathbf{r}'_j$ outside the zeroth cell to a symmetry equivalent $\mathbf{r}_i$ inside the zeroth cell. For the operator g to be a magnetic symmetry operator, it should leave the moment invariant such as $\boldsymbol{\mu}'_j = \boldsymbol{\mu}_i$. Following the above formulas, notice that the moment is not reversed by the inversion operation $\overline{1}$ (see Figure 9). Furthermore, the mirror operator m leaves the moment invariant only if it aligns perpendicular to the plane of the mirror, whereas the m' leaves invariant only the moment lying within the mirror.

By adding the spin reversal operator to any of the standard rotational operators the number of crystallographic point groups increases from 32 to 122. If one defines the time-reversal group as formed by two elements $\mathbf{\Theta} = \{1,1'\}$, a magnetic group $\mathbf{M}$ can be obtained as a subgroup of the direct product of $\mathbf{R}$ with the crystallographic group $\mathbf{G}$: $\mathbf{M} \subset \mathbf{G} \otimes \mathbf{\Theta}$. The magnetic point groups can be classified into three types. The first type is made by those identical to the 32 crystallographic point groups, not involving the $1'$ operation, termed "single-color" or "colorless." Note that the group nomenclature uses the analogy between the concept of spin-reversal and color change, however, as noted above, color and spin differ in the way the regular group operations act upon them.

The second type, named "gray" groups, consist of the 32 groups containing symmetry elements h in both pure and prime forms, from the construction $\mathbf{M} = \mathbf{G} \cup \mathbf{G}1'$. The presence of a spin-reversal operator in each such group precludes nonzero magnetic moments, and consequently they are also known as "paramagnetic" groups. The third type involves unprimed elements of the subgroup $\mathbf{H}$ of index 2 of $\mathbf{G}$ ($\mathbf{H}$ is a so-called "halving subgroup" of $\mathbf{G}$), and the remaining operators $\mathbf{G}\backslash\mathbf{H}$ (read $\mathbf{G}$ "not" $\mathbf{H}$) that are being primed, from the construction $\mathbf{M} = \mathbf{H} \cup (\mathbf{G}\backslash\mathbf{H})1'$. These resulting 58 magnetic groups are called "black-white" or Heesch groups. A lucid presentation on how to expand the crystallographic point groups to include antisymmetry operations is given by Boisen [102]. Because $\mathbf{H}$ is a halving subgroup, $\mathbf{M} = \mathbf{H} \cup (\mathbf{G}\backslash\mathbf{H})1'$ has the same number of symmetry elements as $\mathbf{G}$, and exactly half are antisymmetry operations.

It is quite evident that not all of the colorless and black-white magnetic point groups defined above can be realized in a magnetically ordered system. In that sense, a point group is called admissible if all its operators leave at least one spin component invariant. There are 31 admissible magnetic point groups that are listed together with their admissible moment direction in Table 2 [85,97].

Table 2. Magnetic Point Groups and Corresponding Moment Directions That Enable magnetic Order

Admissible Magnetic Point Groups				Admissible Moment Directions
1	$\overline{1}$			Any direction
$2'$	$2'/m'$	$m'm2'$		Perpendicular to the twofold axis, and to the unprimed plane for $m'm2'$
m'				Any direction within the plane
m				Perpendicular to the plane
$m'm'm$				Perpendicular to the unprimed plane
$2'2'2$				Along the unprimed axis
2	$2/m$	$m'm'2$		Along the twofold axis
4	$\overline{4}$	$4/m$	$42'2'$	Along the fourfold axis

$4m'm'$	$\bar{4}2m'$		$4/mm'm'$		Along the fourfold axis
3	$\bar{3}$	$32'$	$3m'$	$\bar{3}m'$	Along the threefold axis
6	$\bar{6}$	$6/m$	$62'2'$		Along the sixfold axis
$6m'm'$	$\bar{6}m'2'$		$6/mm'm'$		Along the sixfold axis

Reproduced from Ref. [97].

To derive the magnetic lattices, the concept of translation group is generalized to the case of Shubnikov symmetry by considering the antitranslation operation **t′** = **t**1′. Note that this concept replaces in a way the propagation vector formalism defined in the previous section, causing a limitation of the Shubnikov symmetry to the commensurate structures with **k** = (0,0,0) and **τ**/2. The single color magnetic lattices coincide with the 14 conventional Bravais lattices, while the “paramagnetic” lattices do not need to be considered because in that case the crystal is not magnetically ordered. The derivation of black-white translation groups can be done in a similar way as done for the magnetic point groups, by using the subgroups of index 2, $\mathbf{H_L}$ of translation group **T**: $\mathbf{M_L} = \mathbf{H_L} + (\mathbf{T} - \mathbf{H_L})1'$. These result in 34 black-white Bravais lattices which are listed in Refs [85,98].

In direct correlation with the magnetic point groups and translational groups, one can obtain a total of 1651 Shubnikov space groups that consist of 230 single-color, 230 paramagnetic, and 1191 black-white groups. The latter type of magnetic space groups can in turn be grouped in two categories: 674 Shubnikov groups of the “first kind” where the subgroup of translation is the same as that of the space group, and 517 of “second kind” where the translation subgroup contains antitranslations leading to primitive magnetic unit cells larger than the primitive crystal cells. Such magnetic lattices correspond to those defined by the propagation vector **k** = **τ**/2.

Representations for space-group symmetry operations can be given by the following 4 × 4 matrix,

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} & t_1 \\ h_{21} & h_{22} & h_{23} & t_2 \\ h_{31} & h_{32} & h_{33} & t_3 \\ 0 & 0 & 0 & \pm 1 \end{bmatrix}$$

where

$$R = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

is the point-group operation, and the vector **t** embodies the location and translation of the space-group operation. The ±1 in the (4,4) entry denotes either a regular operation (+1) or an antisymmetry operation (−1). The meaning of the vector **t** is not always obvious by inspection, yet a simple recipe for constructing and interpreting space group symmetry operations is given in Ref. [103]. The various shorthand notations for space-group symmetry operations, and those adopted in the International Tables, are given by Litvin and Kopský (2011).

There are two notations used in the literature for describing magnetic space groups following Belov–Neronova–Smirnova (BNS) [84] and Opechowski–Guccione (OG) [85]. Both notations are identical for the major part of magnetic space groups except for the second kind black-white magnetic space groups. A list of all magnetic space groups using the OG notation has been compiled by Litvin [87], followed by a reinterpretation in terms of the BNS notation by Grimmer [88]. Other excellent resources are the ISOTROPY Software Suite [104] and Bilbao Crystallographic Server [105–107] that host databases and programs related to crystallographic and magnetic symmetry.

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Advances in Atomic, Molecular, and Optical Physics

Andrei Derevianko, Sergey G. Porsev, in Advances In Atomic, Molecular, and Optical Physics, 2011

5 Hyperfine-Induced Vector Light Shift in the $^3P_0$ State

The second-order light shift involves two interactions with the laser field. The product of two interactions $(D\cdot\mathscr{E})(D\cdot\mathscr{E}^*)$ may be recoupled into the scalar, vector (axial), and tensor components of the dynamic polarizability (these are irreducible tensors of rank 0, 1, and 2 acting in the electronic space). Because of the angular selection rules, for the $J = 0$ clock states, only the scalar polarizability is of relevance and it was the focus of our discussion in Section 3. The hyperfine interaction, nevertheless, removes the spherical symmetry of the atoms and leads to residual vector, $\alpha^A_{\gamma F}(\omega)$, and tensor, $\alpha^T_{\gamma F}(\omega)$ a.c., polarizabilities. These may affect the performance of the clock: the vector light shift may cause a small Stark-shift dependence on the polarization of the trapping light.

To determine the effect of the HFI on the a.c. polarizability, we carry out an analysis in the third-order perturbation theory. We apply the Floquet formalism (Section 3.1) with respect to a combined operator

$$V = V_{\mathrm{HFI}} + V_{E1}(t).$$

The third-order energy shift of the atomic energy level reads

$$E_a^{(3)} = \sum_{b,c\neq a}\frac{V_{ab}V_{bc}V_{ca}}{\left(E_b^{(0)}-E_a^{(0)}\right)\left(E_c^{(0)}-E_a^{(0)}\right)} - V_{aa}\sum_{b\neq a}\frac{V_{ab}V_{ba}}{\left(E_b^{(0)}-E_a^{(0}\right.}$$

where matrix elements are evaluated with respect to the dressed basis and inner products involve time-averaging. The relevant terms (involving two E1 laser-atom interactions and one HFI coupling) are

$$\begin{aligned}E_a^{(3)} = & \sum_{b,c\neq a}\frac{(V_{\mathrm{HFI}})_{ab}(V_{E1})_{bc}(V_{E1})_{ca}}{\left(E_b^{(0)}-E_a^{(0)}\right)\left(E_c^{(0)}-E_a^{(0)}\right)} + \sum_{b,c\neq a}\frac{(V_{E1})_{ab}(V}{\left(E_b^{(0)}-E_a^{(0)}\right.} \\ & + \sum_{b,c\neq a}\frac{(V_{E1})_{ab}(V_{E1})_{bc}(V_{\mathrm{HFI}})_{ca}}{\left(E_b^{(0)}-E_a^{(0)}\right)\left(E_c^{(0)}-E_a^{(0)}\right)} - (V_{\mathrm{HFI}})_{aa}\sum_{b\neq a}\frac{(}{(}\end{aligned}$$

Notice that we work in the dressed atom picture, i.e., the states *a*, *b*, *c* are products of atomic and photonic states. Also $(V_{E1})_{aa} = 0$ because of the parity/angular/photon number selection rules leading to a simplification of the last term. Explicitly, after the time averaging, Equation (13), (now *a*, *b*, *c* are the "bare" atomic states and the matrix elements are computed using the traditional inner products)

$$
\begin{aligned}
E_a^{(3)}(\omega) &= T_a(\omega) + C_a(\omega) + B_a(\omega) + O_a(\omega), \\
T_a(\omega) &= \sum_{b,c\neq a} \frac{(V_{\mathrm{HFI}})_{ab} v_{bc}^{(+)} v_{ca}^{(-)}}{\left(E_b^{(0)} - E_a^{(0)}\right)\left(E_c^{(0)} - \omega - E_a^{(0)}\right)} \\
&\quad + \sum_{b,c\neq a} \frac{(V_{\mathrm{HFI}})_{ab} v_{bc}^{(-)} v_{ca}^{(+)}}{\left(E_b^{(0)} - E_a^{(0)}\right)\left(E_c^{(0)} + \omega - E_a^{(0)}\right)}, \\
C_a(\omega) &= \sum_{b,c\neq a} \frac{v_{ab}^{(+)} (V_{\mathrm{HFI}})_{bc} v_{ca}^{(-)}}{\left(E_b^{(0)} - \omega - E_a^{(0)}\right)\left(E_c^{(0)} - \omega - E_a^{(0)}\right)} \\
&\quad + \sum_{b,c\neq a} \frac{v_{ab}^{(-)} (V_{\mathrm{HFI}})_{bc} v_{ca}^{(+)}}{\left(E_b^{(0)} + \omega - E_a^{(0)}\right)\left(E_c^{(0)} + \omega - E_a^{(0)}\right)}, \\
B_a(\omega) &= [T_a(\omega)]^*, \\
O_a(\omega) &= -(V_{\mathrm{HFI}})_{aa} \sum_{b\neq a} \frac{v_{ab}^{(+)} v_{ba}^{(-)}}{\left(E_b^{(0)} - \omega - E_a^{(0)}\right)^2} \\
&\quad - (V_{\mathrm{HFI}})_{aa} \sum_{b\neq a} \frac{v_{ab}^{(-)} v_{ba}^{(+)}}{\left(E_b^{(0)} + \omega - E_a^{(0)}\right)^2}.
\end{aligned}
$$

If we represent these contributions diagrammatically, then $T_a(\omega)$, $C_a(\omega)$, and $B_a(\omega)$ can be treated as the top, center, and bottom diagrams, respectively. The naming convention reflects the position of the HFI in the diagram. $O_a(\omega)$ combines other corrective terms; for the case at hand, the $O_a(\omega)$ term is irrelevant since the expectation value $(V_{\mathrm{HFI}})_{aa} = 0$ for $J = 0$ states.

We carry out the angular reduction of these diagrams. We find that the magnetic-dipole HFI does not bring in neither the scalar nor the tensor contribution: there is only the vector component of the a.c. polarizability. In principle, the tensor contribution to $J = 0$ polarizability might appear because of the electric-quadrupole moment of the nucleus; the strength of this interaction is typically two orders of magnitude smaller than that of the magnetic HFI and we neglect this effect. The final result simplified for the $J = 0$ states reads

$$\delta E_a = -\left(\tfrac{1}{2}\mathscr{E}\right)^2 \mathscr{A} \alpha_{\gamma F}^{A}(\omega) \frac{M_F}{2I},$$

M_F being the projection of F (i.e., the projection of the nuclear spin I for $J = 0$). The degree of the circular polarization is defined in terms of $\mathscr{A} = \sin 2\theta$ for an electromagnetic wave $\mathscr{E} = \mathscr{E}\mathbf{e}_x \cos\theta \cos(\omega t - kz) + \mathscr{E}\mathbf{e}_y \sin\theta \sin(\omega t - kz)$. The shift is expressed in terms of the vector polarizability

$$\alpha_{\gamma F}^{A}(\omega) = -\sqrt{\tfrac{2}{27}} \left\{ C_{1,1}^{(1)}(\gamma J, \omega) + 2 T_{1,1}^{(1)}(\gamma J, \omega) \right\},$$

where the dynamic reduced sums are expressed in terms of the reduced matrix elements of the dipole operator and the HFI coupling

$$
\begin{aligned}
T_{J',J''}^{(K)}(\gamma J, \omega) &= \mu_I \sum_{\gamma'} \sum_{\gamma'' \neq \gamma} \langle \gamma J || \mathscr{T}^{(1)} || \gamma'' J'' \rangle \langle \gamma'' J'' || D || \gamma' J' \rangle \langle \\
&\quad \times \left(\tfrac{1}{E - E''} \tfrac{1}{E - E' + \omega} + (-1)^K (\omega \to -\omega) \right), \\
C_{J',J''}^{(K)}(\gamma J, \omega) &= \mu_I \sum_{\gamma' \gamma''} \langle \gamma J || D || \gamma' J' \rangle \langle \gamma' J' || \mathscr{T}^{(1)} || \gamma'' J'' \rangle \langle \gamma'' J \\
&\quad \times \left(\tfrac{1}{E - E' + \omega} \tfrac{1}{E - E'' + \omega} + (-1)^K (\omega \to -\omega) \right).
\end{aligned}
$$

In these formulas, E is the energy of the state of interest. Notation $(-1)^K$ $(\omega \to -\omega)$ means that the preceding term is multiplied by $(-1)^K$ and ω is replaced by $-\omega$. For $J = 0$, the selection rules require $J' = J'' = 1$ for both reduced sums.

Analyzing these expressions numerically in the CI + MBPT approach, we find that the vector polarizability of the 6 $^3P_0$ state of Yb is much larger than that for the ground state, as in the case of Sr (Katori et al., 2003). For Sr, Katori et al. (2003) estimated the vector polarizability by adding HFS correction to the energy levels of intermediate states. Our analysis is more complete and we find that the dominant effect is not because of corrections to the energy levels, but it is rather because of perturbation of the 6^3P_0 state by the HFS operator. The resulting values of $\alpha^{A}_{6^3P_0}(\omega^*)$ are −0.10 a.u. for ^{171}Yb and 0.075 a.u. for ^{173}Yb. Recently, the value $\alpha^{A}_{6^3P_0}(\omega^*)$ for ^{171}Yb was experimentally found to be −0.08 (Lemke et al., 2009) in very good agreement with the theoretical result. In practice, this translates to requiring $\mathscr{A} < 10^{-6}$ at laser intensities of 10 kW/cm^2 for keeping the induced clock shifts below the mHz level.

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Axial Vector Nuclear Sum Rules and Exchange Effects

MAGDA ERICSON, in Nuclear, Particle and Many Body Physics, 1972

Publisher Summary

The nuclear pionic vertex and its relation to the π-nuclear scattering length are discussed in this chapter using dispersion techniques. This chapter explains the information on mesonic effects in the pionic vertex and in the axial vector current matrix element, which can be extracted using these techniques. The result thus obtained is that the sum of pionic vertices between the ground state and all excited nuclear states is connected to an integral of the total 7r-nuclear cross section in the (3,3) resonance energy and above. It describes that the exchange current effects are related in a model-independent way to shadow phenomenon in the cross section. The Goldberger-Treiman relation extends the result presented in the chapter to Gamow-Teller matrix elements, which is also called nuclear Adler-Weisberger sum rule. Estimates are given for the renormalization of the axial coupling constant in some nuclei. The chapter additionally presents the many-body effects of the pionic vertex.

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SYMMETRIES: SOME RECENT THEORETICAL WORK*

Ernest M. Henley, in Few Particle Problems, 1972

E Second Class Axial Currents

The normal strangeness non-changing weak current, j_μ of Eq. (2) has definite properties under isospin rotations and thus under G-parity conjugation, where

$$\mathrm{G} = \mathrm{C}\ \exp(\mathrm{i}\pi\,\mathrm{I}_2)$$

For the vector and axial vector currents of Eq. (3)

$$\begin{aligned} \mathrm{G}\,\mathrm{v}_\mu \mathrm{G}^{-1} &= \mathrm{v}_\mu \\ \mathrm{G}\,\mathrm{a}_\mu \mathrm{G}^{-1} &= \text{-}\mathrm{a}_\mu . \end{aligned} \tag{9}$$

Such currents are referred to as first class currents. Weinberg[55]) suggested the possible existence of second class currents with the opposite G-parity transformation properties.

Are these second class currents responsible for the CP- and T-violations observed in the kaon system? Indeed, either first or second class currents may be regular or irregular under time reversal transformations. In the second case the resulting interaction Hamiltonian will be odd under time-reversal transformations. As discussed in section B, the beta decay time reversal tests carried out so far do not rule out a sizeable irregular second class current because they occur between mirror nuclei.[33]) Furthermore, since decay rates do not detect T-odd parts of an interaction Hamiltonian, the measurements of δ also do not bear on this point. However, possible tests of the presence[of]/irregular second class currents have been suggested by Holstein and others.[56])

Although both the conserved vector current theory and experiments bearing on it rule out second class vector currents, there were until recently few experiments which bore on the existence of second class axial vector currents. The careful measurements of Alburger and Wilkinson[57]) and of Wilkinson and other collaborators[57]) of the ratios of ft values for Gamow-Teller mirror beta decays showed evidence for 10–20% deviations of this ratio from unity. Although we just heard from Professor Wilkinson that the evidence for these deviations in even-even nuclei (except for A=8) has largely disappeared over the past year, the experiments attracted widespread theoretical interest.

In terms of an impulse approximation treatment of beta-decay, the simplest interpretation of the deviation of δ

$$\delta = (\mathrm{ft})^{+}/(\mathrm{ft})^{-} - 1 \tag{10}$$

from 0 is the existence of a second class axial current. The free nucleon matrix element of this current can be written as

$$< \mathrm{p}(\mathrm{p'})\left|\mathrm{a}_\mu\right|\mathrm{n}(\mathrm{p}) >= \overline{\mathrm{u}}_\mathrm{p}(\mathrm{p'})[\mathrm{g_a}\gamma_\mu + \mathrm{f_a q}_\mu - \mathrm{ih_a}\sigma_{\mu\nu}\mathrm{q}^\nu]\gamma_5\mathrm{u_n}(\mathrm{p}) \tag{11a}$$

$$= \overline{\mathrm{u}}_\mathrm{p}(\mathrm{p'})[\mathrm{g}'_\mathrm{a}\gamma_\mu + \mathrm{f_a q}_\mu - \mathrm{h}'_\mathrm{a}\mathrm{p}_\mu]\gamma_5\mathrm{u_n}(\mathrm{p}) \tag{11b}$$

with q = p'-p, P = p+p', $h'_a = h_a$, $g'_a \approx g_a$. The terms proportional to h_a and h'_a are second class ones.

In Eqs. (11) f, g, and h are form factors which can depend on q^2 as well as p'^2, and p^2 when the nucleons are not on their mass shells. Although the two forms (11a) and (11b) are equivalent for nucleons on the mass shell (neglecting electromagnetic mass differences), they lead to different results when extrapolated to off-mass shell matrix elements such as those which occur in beta decay.[58])

The impulse approximation is useful for obtaining an understanding of nuclear phenomena in terms of the basic weak interaction. However, one can also treat nuclei as “elementary” particles with form factors, and this formulation is more appropriate for general symmetry arguments.[59]) Comparisons of the two methods have recently been made;[60]) they lead to essentially equivalent results for allowed beta decays, for instance. Any differences should be ascribed to off-mass shell extrapolations and to meson exchange effects.

The importance of meson exchange effects in the second class current problem was stressed by Lipkin[61]) and by Delorme and Rho.[62]) Lipkin, in particular, pointed out that the ω-$\pi^\pm$ exchange graph shown in Fig. 4 could give rise to an energy-independent δ unlike the impulse approximation treatment of Eqs. (11) which yields an asymmetry proportional to the beta decay energy releases.

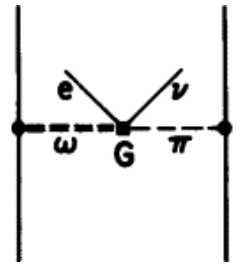

Fig. 4. The ω-π exchange current contribution to beta decay.

In a recent analysis of the second class current problem for mirror beta decay asymmetries, Kubodera, Delorme, and Rho[63]) include an arbitrary fraction of both forms of Eqs. (11) as well as exchange current contributions. For the asymmetry δ they find an energy dependent term, $b(W_0^+ + W_0^-)$, which is proportional to the sums of the contributions from Eqs. (11a) and (11b), an energy dependent term, $aK(W_0^+ + W_0^-)$, which is determined by the ω-π exchange current, and an energy-independent part aJ which comes only from Fig. 4

$$\delta(\mathrm{A}) = \mathrm{aJ(A)}+[\mathrm{ak(A)\text{-}b}](\mathrm{w_0}^+ + \mathrm{w_0}^-) \tag{12}$$

In Eq. (12), $W_0^\pm$ is the energy release in the positron/electron β-decay. The exchange current terms J, K depend on nuclear structure factors whereas a and b are constants. As pointed out by Professor Wilkinson, the almost complete absence of energy dependence of δ required by the nuclear mass A=8 data[64]) must be considered accidental if the above theoretical description is correct. Energy independence of δ would not be expected for other nuclei; this prediction should be tested. In order to fit the large mass eight beta decay asymmetry δ, the authors find that a large (unmeasured) beta decay rate $\omega \rightarrow \pi + \mathrm{e} + \nu$, proportional to G^2 of Fig. 4, is required. By fixing the two adjustable parameters a and b in Eq. (12) from the A=8 $(ft)^\pm$ asymmetry measurements, Kubodera, Delorme, and Rho predict asymmetries in other nuclei which are generally smaller than those observed.

Of course, an important question is whether the beta decay mirror asymmetries found by Wilkinson, Alburger, and collaborators can be explained by other means than the assumption of second class currents. For instance, electromagnetic corrections to beta decay matrix elements can give effects which mimic second class currents. However, such corrections would not be expected to give rise to asymmetries δ of the order of 10% but rather 1% or less. But there are other origins of charge asymmetries. The major one of these is probably the binding energy differences between proton in e^+ and neutron in e^- decays;[65]) other possibilities are effects due to a charge asymmetry of nuclear forces, and different nuclear deformations in the mirror nuclei.[65]) The estimates made to-date do not appear to be able to account for δ in some odd-A nuclei and for A=8. On the other hand, the recent results summarized by D. H. Wilkinson for the mirror ft-values in even nuclei (excluding A=8) suggest that these corrections be examined further. More accurate tests of the ratio of the beta decay rates $\Sigma^+ \rightarrow \Lambda + \mathrm{e}^+ + \nu / \Sigma^- \rightarrow \Lambda + \mathrm{e}^- + \overline{\nu}$ would also be helpful.

Measurements of the asymmetry δ in mirror beta decays is not a unique indicator of second class currents. Equally or more difficult tests have been suggested not only in beta decays, but also in muon capture and in ν scatterings from nuclei.[56,66]) When some of these tests have been carried out, we may be able to reach a more definite conclusion about the existence or non-existence of second class currents.

I have tried to summarize recent theoretical work on some symmetries, and have attempted to outline some remaining problems. They are legion. For instance, despite considerable effort we still do not understand parity violation in nuclei, or for that matter the basic PV nuclear force. Nor do we know the cause of the time reversal violation seen in the kaon decays. The definite existence of a charge asymmetry of the nuclear force has yet to be established. Lastly, although exotic currents can be, and have been, called upon to explain asymmetries observed in nature, the correct interpretations appear to be more subtle. Despite these shortcomings, the research on symmetries has improved our understanding of basic phenomena.

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