Electron time, mass and zitter

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Abstract: de Broglie's original idea that the electron has an internal clock has recently received experimental confirmation by measuring the period of the clock in an electron channeling experiment. This result has been explained by a new model of the electron, called the *zitter model* because it incorporates Schroedinger's qualitative *zitterbewegung* concept into a fully specified interacting particle model. The zitter electron is a lightlike charged particle with intrinsic spin that maintains it in a helical spacetime path, with curvature and frequency determined by the electron mass. Thus, electron mass is fully reduced to clock frequency in electron motion. This essay discusses details of the model and its implications.

To discover a natural time scale we look to the dynamics of elementary particles. Louis de Broglie was the first to do that when, in his 1924 doctoral thesis, he proposed that the electron possesses an internal clock. As two pillars of quantum mechanics, he accepted *Planck's Law*:

$$E = \hbar\omega \qquad (\textit{energy is frequency!}) \tag{1}$$

and *Einstein's Law*:

$$E = mc^2 \qquad (\textit{mass is energy!}). \tag{2}$$

Applying these laws jointly to the electron (with rest mass m_e) he proposed

$$\omega_B = \frac{m_e c^2}{\hbar} = 0.77634 \times 10^{21}\,\mathrm{s}^{-1} \qquad (\textit{mass is frequency!}) \tag{3}$$

for the frequency of the electron clock. The clock can then be modeled by a periodic function

$$\psi(\tau) = e^{i\omega_B \tau}\,, \tag{4}$$

where τ is proper time along the electron's spacetime history.

De Broglie went further to propose that a wave of the same frequency was associated with the motion of an electron [1]. As everyone knows, this *wave hypothesis* was immediately extended by Schroedinger to create his famous wave equation that has become a paradigm of quantum mechanics. Ironically, de Broglie's *clock hypothesis* has been ignored or forgotten in the physics literature since. Besides, how could one read time on a clock with a period of 10^{21} seconds?

Many decades passed before French experimental physicist Michel Gouanère resolved to search for the electron clock. I suppose it had to be a French experimentalist to take the clock hypothesis seriously, because de Broglie's views on quantum mechanics were dismissed or disparaged by most theoreticians, except for a small band of (mainly French) devotees.

As he related it to me, Gouanère discussed various experimental alternatives with his colleague M. Spighel until they seized on electron channeling as a feasible possibility. In a channeling experiment, electrons in a beam aligned close to a crystal axis are trapped in orbits spiraling around a single atomic row, so scattering is reduced and transmission through the crystal is greatly enhanced. Gouanère argued that if the electron clock is physically real, channeled electrons should interact resonantly with the crystal periodicity at some energy to produce a dip in transmission rate.

A prediction for the resonant energy is easily calculated. As de Broglie had already noted, the clock frequency observed in the laboratory ω_L at laboratory time t is related to the intrinsic clock frequency by $\omega_B \tau = \omega_L t$, so

$$\omega_L = \frac{\omega_B}{\gamma} = \frac{2\pi}{T_L}, \qquad \text{where} \qquad \gamma = (1 - \mathbf{v}^2 / c^2)^{1/2} \tag{4}$$

is the relativistic time dilation factor and **v** is the electron lab velocity parallel to the crystal axis. Therefore the distance traversed in a clock period is given by

$$d = T_L |\mathbf{v}| = \frac{2\pi h}{m_e c^2} \gamma \frac{m_e |\mathbf{v}|}{m_e} = \frac{hp}{(m_e c)^2}.$$

Along the <110> axis of a silicon crystal the atomic spacing is $d = 3.84$ Å, so the predicted resonance energy is

$$cp = m_e \gamma |\mathbf{v}| = \frac{d(m_e c^2)^2}{hc} = \frac{3.84\,\text{Å}(0.511034\,\text{MeV})^2}{0.01239852\,\text{MeV-Å}} = 80.87\,\text{MeV}. \tag{5}$$

This is easily within the accessible energy range for a channeling experiment.

They knew that funding for such an offbeat experiment would be impossible to secure, so they organized a research team and wrote a proposal to study “Kumakhov radiation” in the 54–110-MeV region on the linear accelerator at Saclay. It was not until the project was up and running that they informed other members of the team about what they really wanted to do. One day of accelerator time was diverted to the clock experiment, but publication was delayed for many years until analysis of the data was completed!

The experiment involved search for a transmission resonance in the channeled electron beam by scanning an energy window centered at the predicted resonance momentum (5). They found an 8% dip in transmission centered at $p_{\exp} = 81.1\,\text{MeV/c}$. The 0.28% difference between predicted and measured values fell within the estimated calibration error of ± 0.3%, though Gouanère confessed to me that he thought the experiment was more accurate than that.

Their results were published in *Annales de la Fondation Louis de Broglie* in 2005. Predictably, the impact was nil, as that journal attracts few readers. To get more visibility, Gouanère submitted a slightly modified account to *Physical Review Letters.* It was rejected in January 2007. The majority of reviewers regarded the reported results as physically implausible! Their response reminds me of Eddington’s ironic remark, “I won’t believe the experiment until it is confirmed by theory!” However, one reviewer suggested that the effect might be explained by Schroedinger’s *zitterbewegung*. Gouanère had never heard of zitterbewegung, so he Googled it and found an article of mine [2], which argues that zitterbewegung is fundamental for interpretation of the Dirac equation and *à fortiori* for interpretation of quantum mechanics.

We met in Paris in May 2007. Gouanère supplied me with more details about his experiment and convinced me that the results should be taken seriously. It happens that I had been sitting on a new model of electron zitterbewegung until experimental implications could be worked out. As Gouanère’s experiment offered a direct test of the model, I jumped at the chance to explain his data quantitatively. The results could not be more satisfactory:

- The *equations of motion* (given below) apply without modification, though some approximations are in order.
- The clock *interaction mechanism* is explained as a resonance of the periodic crystal lattice with a rotating electric dipole moment of the electron.
- A series of *resonant energies* is predicted and the puzzling factor of two difference between zitterbewegung and de Broglie frequencies is explained.
- The calculated *width* of the lowest resonance agrees with the data.
- The *discrepancy* between predicted and measured resonance energies is explained as an *apparent shift* in the maximum due to an unresolved doublet.
- Measurement of predicted *spin effects* will require higher resolution.

Details of the theoretical analysis are available in [3], and Gouanère's account of the experiment has finally been published in [4]. The main experimental uncertainty is due to the constrained conditions under which the experiment was performed. The observed resonance was not predicted nor, I believe, can it be explained by standard quantum mechanics. Though zitterbewegung is indeed inherent in the Dirac equation, that cannot explain the resonance without some theoretical modification as described below. Surely Gouanère's pioneering experiment should be refined and repeated to confirm results and test new predictions! Gouanère is currently lobbying for that.

The Zitter Model of the Electron

Schroedinger [5] coined the term *zitterbewegung* (trembling motion) to describe oscillations in free particle solutions of the Dirac equation. Its putative physical interpretation has been clearly described by Huang [6]:

> "The well-known *Zitterbewegung* may be looked upon as a circular motion about the direction of the electron spin with radius equal to the Compton wavelength $\times$ $1/2\pi$ of the electron. The intrinsic spin of the electron may be looked upon as the orbital angular momentum of this motion. The current produced by the *Zitterbewegung* is seen to give rise to the intrinsic magnetic moment of the electron."

Dirac himself concurred with this interpretation [7]. No doubt the weight of Dirac's authority accounts for its reiteration in textbooks on relativistic quantum mechanics and field theory to this day, despite the fact that it has not been subjected to a single experimental test. Indeed, the zitterbewegung concept has served as no more than metaphorical window dressing on abstract formalism, while its <u>staggering theoretical implications</u> remain unexamined!!

Let me state the literal implications of the zitterbewegung concept explicitly:

(1) The Dirac equation provides a statistical description of electron behavior with an underlying particle substructure.

(2) The electron is a point charge moving at the speed of light in circular motion with angular momentum of magnitude $\hbar/2$ observed as electron spin.

(3) The phase of the Dirac wave function is a measure of angular displacement in the circular motion. (Thus, electron spin and phase are inseparably related!)

(4) The circular motion generates the observed magnetic moment of the electron.

(5) The circular motion also generates an electric dipole field fluctuating with zitterbewegung frequency on the order of 10^{21} Hertz.

In [2] and elsewhere, I have argued for adopting these five statements (in one form or another) as principles for a *zitterbewegung interpretation of quantum mechanics*. Taken literally, these principles cry out for a specific model of the particle substructure that is amenable to quantitative experimental test and compatible with established successes of quantum mechanics. I have pursued that goal for decades, with clear progress only recently [3], after overcoming a long-standing misconception about electron spin. The resulting model has already passed the empirical test of Gouanère's electron clock experiment. Now let me present details of the model's design with emphasis on the role of time.

My research objective has been to create a particle model of the electron that incorporates the essential features of *spin* and *zitterbewegung* in the Dirac equation that are listed above. Since the model entails extension of Schroedinger's original concept and "zitterbewegung" is such a mouthful, I use the term **zitter** to refer to the extended concept. Accordingly, I refer to the new particle model as th*e zitter model of the electron*, or simply the *zitter electron*.

As Einstein advised, the model should be "as simple as possible — but not simpler!" Since simplicity and transparency of a physical theory depends heavily on the mathematical formalism employed, I present the model in the coordinate-free language of *geometric algebra* [8, 9] that played an essential role in its development [3]. Readers are not expected to be conversant with this language, but I believe it will seem sufficiently familiar to convey a clear impression of the structure of the model, which is my purpose. For readers who are not satisfied with that, I have attached an appendix providing translations of key expressions into standard tensor language.

We model the electron as a point particle in spacetime with a *lightlike history* $z = z(\tau)$, so its *velocity*

$$u = \frac{dz}{d\tau} \equiv \dot{z} \qquad \text{is a null vector:} \qquad u^2 = \dot{z}^2 = 0\,. \tag{6}$$

As proper time cannot be defined on a lightlike curve, a physical definition of the time parameter τ must be determined by other features of the model. We shall see that an intrinsic definition of electron time derives from the assumption that the electron has an intrinsic angular momentum or spin. The *spin* $S = S(\tau)$ is a bivector quantity (see the appendix for its tensor form). For the sake of internal consistency in the model, it turns out the spin must be a null bivector, as expressed by:

$$S^2 = 0\,, \qquad Su = S \cdot u = 0\,. \tag{7}$$

The particle is charged so it interacts with any external electromagnetic field $F = F(z)$.

Particle dynamics is determined by a system of coupled equations of motion for velocity u, momentum p, and spin S:

$$\dot{u} = \frac{1}{r} + \frac{q}{m_e} F \cdot u\,. \tag{8}$$

$$\dot{p} = qF \cdot u + \nabla\Phi\,, \tag{9}$$

$$\dot{S} = u \wedge p + \frac{q}{m_e} F \times S\,, \tag{10}$$

where a *zitter radius vector* r is defined by

$$\frac{1}{r} = \left(\frac{2}{\hbar}\right)^2 p \cdot S\,, \tag{11}$$

and a *spin-zitter potential* is defined by

$$\Phi = \Phi(\tau, z) = \frac{q}{m_e} S \cdot F\,. \tag{12}$$

Note the units with $c = 1$ and the two coupling constants: charge q and charge to mass ratio q / m_e, where m_e is the electron rest mass.

The dynamical equations admit an integral of the motion

$$m \equiv p \cdot u = m_e + \Phi\,, \tag{13}$$

which defines *mass m* as a dynamical quantity. Thus, the potential Φ determines a mass shift due to interaction. Since the momentum p is a timelike vector ($p^2 > 0$), it is necessarily non-collinear with the lightlike velocity u, and some momentum is contained in S. Of course, the relation between p and S is determined by the dynamics.

Self-consistency of the three dynamical equations has been assured by deriving them from a Lagrangian [3]. Therein lies an important lesson: Equations like (9) and (10) have been proposed and studied by many authors, most notably Corben [10]. However, in analogy with Dirac theory, Corben assumed that the particle velocity u, like the Dirac current, is a timelike vector, and also that the spin is a spacelike bivector with magnitude $|S| = \hbar / 2$. Having argued that the Dirac current should be regarded as the average of a lightlike charged current over a zitter period [2], I struggled unsuccessfully for years to define a lightlike particle model with spacelike spin. It was not until I looked at a long-forgotten paper by Weyssenhoff [11] that I understood a lightlike particle *must* have lightlike spin. However, Weyssenhoff treated only the free particle case, and it was not obvious how to generalize it to an interacting particle. It was only with a Lagrangian formulation that I was able to convincingly generate the self-consistent system of equations (6) to (12). And, by the way, that made it obvious why a model with lightlike velocity and spacelike spin is impossible.

The Zitter Electron Clock

Now we are prepared to examine the clock mechanism in the zitter model. In the particle equation of motion (8), the last term on the right is the usual Lorentz force, while the first term describes an intrinsic curvature with *radius vector* $r = r(\tau)$. From (11) it follows that $r \cdot u = r \cdot p = 0$ and a *radius of curvature* for the electron history can be defined by

$$\lambda = |r| = \frac{\hbar}{2m} = \omega^{-1} \tag{14}$$

A *center of curvature* $x = x(\tau)$ can be defined by

$$x(\tau) = r(\tau) + z(\tau)\,. \tag{15}$$

It is a timelike curve with velocity $v = \dot{x}$, so its arc length defines a *proper time* τ, which we can choose as time parameter of our model. This time parameter is extrinsically related to spacetime geometry, so it provides a link to the intrinsic geometry of electron motion.

For a free particle ($F = 0$), the electron equations of motion can be integrated exactly. As illustrated in Fig. 1, the solution $z = z(\tau)$ is a *lightlike helix* centered on a straight timelike line collinear with the momentum p. The general solution can pictured as a deformation of this helix by interaction with an external electromagnetic field. More specifically, it can be described as follows:

The velocity $v = v(\tau)$ defines an *instantaneous rest frame* for the electron. In this frame the orbit of the electron can be pictured as circulating at the speed of light with radius vector $r(\tau)$ about a fixed point $x(\tau)$. According to (14), the *zitter radius and frequency* are inversely related by

$$\lambda\omega = \lambda_e \omega_e = c = 1\,, \tag{16}$$

where subscripts indicate free particle values. From (13) it follows that the frequencies are related by

$$\omega = \omega_e + \frac{2}{\hbar}\Phi\,. \tag{17}$$

This shows how zitter frequency and radius vary with interaction. When mass is increased by interaction, the frequency increases and the radius decreases to maintain the speed of light.

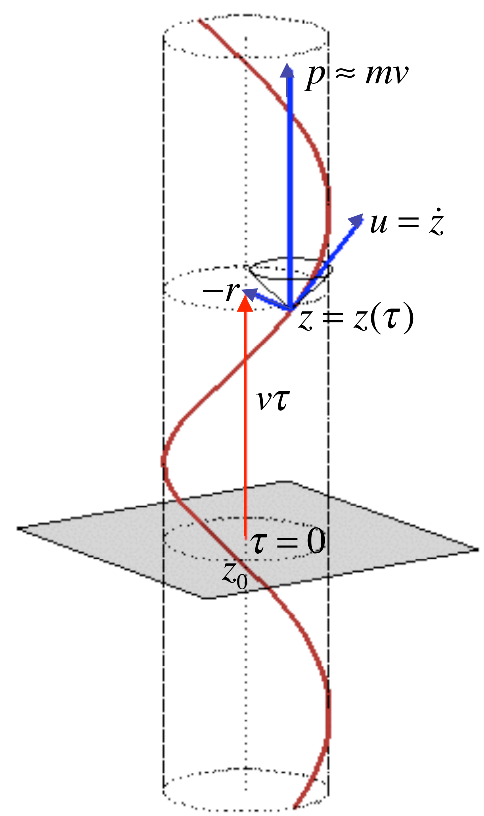

Fig.1

Intrinsic electron time and length scales are determined by free particle values. The zitter frequency is about one *Zetta* (10^{21}) Hertz; precisely, twice the de Broglie frequency in (3:

$$\omega_e = \frac{2m_e c^2}{\hbar} = 1.5527 \text{ ZHz}\,, \tag{18}$$

Accordingly, the fundamental unit of electron time T_e is about 4 zepto (10^{-21}) seconds;

$$T_e = \frac{2\pi}{\omega_e} = 4.0466 \text{ zs}\,. \tag{19}$$

The fundamental unit of length is the *zitter* (or *spin*) *radius*

$$\lambda_e = \frac{c}{\omega_e} = \frac{\hbar}{2m_e c} = 1.93079 \times 10^{-3} \text{ Å} = \frac{\lambda_C}{4\pi}\,, \tag{20}$$

where λ_C is the electron Compton wavelength.

Angular displacement of the zitter circulation,

$$\Delta\varphi = \int \omega\, d\tau \tag{21}$$

provides an intrinsic measure of electron clock time, independent of physical units. The angle φ is the analog of phase in the Dirac wave function.

Reading the electron clock requires analysis of the spin-zitter interaction. The physical structure of the interaction is most evident in relation to the instantaneous rest system of the electron. As fully explained in [3], geometric algebra enables a rest system decomposition of spin and electromagnetic field into the simple complex forms:

$$S = m_e\mathbf{r}_e + i\mathbf{s}, \qquad\qquad F = \mathbf{E} + i\mathbf{B}. \tag{22}$$

The vectors **E** and **B** are, of course, electric and magnetic fields in the rest frame. The *spin vector* **s** is actually an invariant defined as the average of the spin bivector S over a zitter period; thus

$$\overline{S} = i\mathbf{s}, \qquad \text{with magnitude} \qquad |\overline{S}| = |\mathbf{s}| = \frac{\hbar}{2}, \tag{23}$$

in exact agreement with Dirac theory. The zitter radius vector $\mathbf{r}_e$ has fixed magnitude $|\mathbf{r}_e| = \lambda_e$. Now the spin-zitter interaction can be expressed in the perspicuous form

$$\Phi = \frac{q}{m_e} S \cdot F = \frac{q}{m_e}\langle (m_e\mathbf{r}_e + i\mathbf{s})(\mathbf{E} + i\mathbf{B})\rangle = q\mathbf{r}_e \cdot \mathbf{E} - \frac{q}{m_e}\mathbf{s} \cdot \mathbf{B}. \tag{24}$$

The last term in (24) is the familiar *Zeemann interaction* with the correct gyromagnetic ratio (g = 2) as predicted by Dirac theory. Inserted into the momentum equation (9), we see that it gives a Stern-Gerlach force when the magnetic field is inhomogeneous.

The other term in (24) is an electric dipole interaction with *dipole moment* $\mathbf{d} = -q\mathbf{r}_e$ rotating around the spin axis **s** with the zitter frequency. This term will average to zero over a zitter period unless the field **E** is also oscillating with a comparable frequency so it interacts resonantly with the dipole. That is how the zitter model explains the interaction between electron clock and crystal lattice in the channeling experiment.

Basic features of the zitter model can now be summarized as follows:

- The *spacetime history* of electron is a lightlike helix.
- Electron *mass* (≈ zitter frequency) is a measure of helical curvature.
- Electron *phase* (≈ zitter angle) is a measure of helical rotation.
- Electron *spin* is a measure of helical orientation.
- Electron zitter generates a *static magnetic dipole* and *rotating electric dipole!*

All this fits together neatly into simple picture of the working mechanism in the electron clock:

Electron motion is governed by the spin *S,* which confines its lightlike history to the surface of a timelike *zitter tube* aligned with the momentum p. The momentum determines an intrinsic decomposition of the spin (22) into a spatial part i**s** specifying the tube cross section and a temporal part m**r** specifying the *temporal pitch* of the helix. As this constitutes the essential core of the zitter model, let's call it the **spin-zitter mechanism**.

Now we can picture interaction with an external electromagnetic field as deforming the zitter tube, and that opens possibilities for generalizations to other particles and interactions as suggested below.

Zitter Universality

When Einstein was once asked why he ignored the exciting discoveries of new ”elementary particles,” he replied,

> *“You know, it would be sufficient to really understand the electron!”*

Indeed, electron theory is the workshop where quantum mechanics was designed built and tested. So what is fundamentally true for the electron is probably true for all fermions and inherited by bosons that are composite states of fermions. Let me speculate on implications of this dictum for *universality* of the spin-zitter mechanism. Three major research issues come to the fore.

The first issue is confirmation of the zitter model,
so we can be sure that zitter is a fundamental property of the electron. So far its only test has been the channeling clock experiment, and that bears repeating before we can be sure of the result. However, reasons for overlooking zitter for so long are obvious. Since the zitter frequency is so short its effects will average to zero under most circumstances and be observable only under resonant conditions. Actually, zitter resonance may be quite common, for one can argue [3] that it explains many unique features of quantum mechanics. Even so, subtle deviations from standard quantum mechanics are to be expected from the fluctuating zitter dipole.

The second issue is compatibility with quantum mechanics.
As spin and temporal zitter are already present in the Dirac equation, the complete spin-zitter mechanism can be incorporated by a simple modification that replaces the timelike Dirac current by a lightlike current [3]. The Dirac current can then be recovered by averaging over a zitter period. It seems likely that the resulting *zitter–Dirac equation* is sufficiently similar to the original Dirac equation to reproduce empirically confirmed results, such as the spectrum of the hydrogen atom. Its main difference is existence of a zitter electric dipole, in perfect correspondence with the zitter particle model. Differences in predictions between zitter–Dirac and zitter particle models remain to be determined. The particle model has the advantage of clarity in physical interpretation and simplicity in application. The zitter–Dirac equation has the advantage that all the standard techniques of quantum mechanics and quantum field theory can be applied to it.

Finally, it is worth noting that the zitter concept has implications for interpretation of the nonrelativistic Schroedinger equation: Its derivation from the Dirac equation shows that the spatial part of the spin in (22) is dropped while the temporal part is retained in the wave function phase factor, though the contribution of the rest mass is factored out. Thus, one concludes that the phase of the Schroedinger wave function describes zitter phase shifts.

In a more speculative direction, the zitter–Dirac equation has surprising implications for electroweak theory [3, 12]. Zitter–Dirac employs only two of the four components in a Dirac spinor, so assignment of physical meaning to the residual two components is up for grabs. There are two important considerations to guide the choice. First, the generator of electromagnetic gauge transformations in the Dirac equation is a spacetime bivector, so it is tied to spacetime geometry. Second, the maximal symmetry group of the full Dirac equation is precisely the SU(2)×U(1) group of electroweak theory. Consequently, we have a fully geometric embedding of electroweak theory if we identify the residual part of the Dirac wave function with the electron neutrino, so the entire wave function represents a lepton doublet. Incorporating electroweak interactions is then straightforward [3, 12]. However, two significant differences appear. First, the zitter–Dirac equation is of Majorana type rather than Weyl type. Second, only left-handed components are needed for the electron, because the charge current is lightlike. It seems that right-handed components are needed in standard theory only to make the charge current timelike. Evidently, this version of electroweak theory deserves serious consideration if the spin-zitter mechanism is taken seriously.

If the spin-zitter mechanism is a universal property of elementary fermions, it must be augmented by some mechanism that determines the spectrum of rest masses (zitter frequencies). It seems doubtful that the Higgs mechanism can serve this purpose, but that is an open question. Perhaps a clue can be found in the following.

The third issue is compatibility with general relativity.
Einstein was never satisfied with the energy-momentum source of his gravitational field equation, evidently because of its non-geometric character. The *spin-zitter hypothesis* may go a long way to correct this deficiency, as it reduces rest masses to geometric frequencies in the curvature of lightlike particle paths. The obvious question is then "How does this relate to gravitational mass and sources of the gravitational field?" Without proposing a definitive answer, let us consider some possibilities.

Perhaps rest masses originate from gravitational interactions. Rosquist [13] points out that the standard argument for irrelevance of gravitational interactions at the Compton scale is seriously flawed, because it is based on Newtonian concepts. He argues instead from the Kerr-Newman solution to the Einstein-Maxwell equations, because that is the closest thing in General Relativity to a model of the electron with charge and spin. Therefrom he concludes that gravitational and electromagnetic interactions are comparable at a scale set by the spin radius (20). This provocative conclusion challenges us to extend the zitter electron model to include gravitational-electromagnetic self interaction in conformity with principles of General Relativity — a tall order that will not be easily filled.

The spin-zitter hypothesis has implications for gravitational fields as well as sources. It tells us that there is no mass without spin. Hence the gravitational field equations must be generalized to include spin. That has been done in *Gauge Theory Gravity* [9, 12], which presumes sources described by the Dirac equation and shows that spin gives rise to torsion in the gravitational field. It will be worthwhile to revisit that theory to ascertain modification due to null spin bivectors as required by the spin-zitter hypothesis.

It seems that commentary on the nature of time is not complete these days without some bold extrapolation to cosmic consequences. The Dark Matter problem is a popular target! So let me throw out a *timely* solution: We know that the cumulative gravitational force from stars in a galaxy, for example, is insufficient to account for the angular momentum of the galaxy. Perhaps the discrepancy is not due to missing matter, but missing angular momentum in the gravitational field, as might be supplied by a torsion component. Indeed, the spin-zitter hypothesis requires that masses of particle sources are accompanied by spins. Perhaps gravitational torsion due to spin, like gravitational force due to mass, is negligible at the atomic level but accumulates to an observable effect at the galactic level. Enough said!

Now is the time to terminate speculation before the reputation of the spin-zitter hypothesis is seriously compromised!

Appendix

Here are some correspondences between expressions in geometric algebra and their component forms in standard tensor calculus:

$$p \cdot u = p^{\mu} u_{\mu}, \qquad u \wedge p \Leftrightarrow u_{\mu} p_{\eta} - u_{\nu} p_{\mu}, \qquad S \cdot F = F_{\alpha\beta} S^{\beta\alpha},$$

$$F \cdot u \Leftrightarrow F^{\mu\nu} u_{\nu}, \qquad F \times S \Leftrightarrow \frac{1}{2}(F_{\mu\alpha} S^{\alpha}{}_{\eta} - F_{\eta\alpha} S^{\alpha}{}_{\mu}), \qquad \nabla S \cdot F \Leftrightarrow S^{\beta\alpha} \partial_{\mu} F_{\alpha\beta}.$$

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