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The intrinsic magnetic moment of elementary particles

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In a purely nonrelativistic formulation, a Hamiltonian is obtained for the description of states of a charged particle with spin, interacting with an external electromagnetic field. For the specific cases of spin-$\frac{1}{2}$ and spin-1, the simplest choices for the Hamiltonian functions lead to a value for the (lowest order) intrinsic magnetic moment equal to the Bohr magneton in both cases. This is the same as is obtained in treatments that start from a relativistic formulation, as in the Dirac theory. The nonrelativistic and relativistic formulations both employ the "minimal coupling principle," but the resulting Hamiltonians with couplings are, for very general considerations, not completely determined. However, the corresponding ambiguity that occurs in the nonrelativistic theory is also present in the relativistic formulation. The simpler nonrelativistic theory is more transparent in exhibiting these features of the problem and dispels the notion that spin (and the associated magnetic moment) require relativistic formulations for a proper understanding.

I. INTRODUCTION

A very prevalent notion, propagated in countless textbooks, has to do with the origin or understanding of the magnitude of the fundamental intrinsic magnetic moment of the electron. Without radiative corrections, this quantity is given by the Bohr magneton

$$\mu_0 = e\hbar/2mc, \tag{1}$$

and the most common explanation heard for *why* it has this value is that "it comes out of the Dirac equation." Historically, the value for the electron's intrinsic moment was first determined experimentally from spectroscopic measurements, and it is true that the Dirac equation does imply the value (1) for the electron as well as its spin quantum number $s=\frac{1}{2}$. Basically because the Dirac equation is the result of a relativistic formulation, the notion developed that spin and the associated spin magnetic moment are "intrinsically relativistic phenomena." In fact, spin has nothing to do with relativity and considerations of relative motion. It has everything to do with spatial isotropy and rotational invariance. This should be fairly clear, since we speak of a particle's spin and spin states specifically through a discussion of different orientations of reference axes fixed in the rest frame of the particle. Then, if the intrinsic magnetic moment is a consequence of "motion" in the spin degree of freedom, it would seem more natural to seek an understanding of its characteristic value from a formulation that is nonrelativistic from the beginning instead of from a relativistic treatment that then considers the low energy limit.

This point has been made before,[1–5] but the idea does not seem to take hold and is periodically reintroduced into the literature. Some of the discussions[1,2] involve considerations of the Galilei group and its implications for the description of particle motion, making the analysis a little abstract, and the present paper takes a more elementary approach with an aim toward bringing out the possible ambiguities that may be present. We start with the basic Hamiltonian function (representing kinetic energy) for a free particle with no internal motion (that is, spin 0):

$$H_{\text{kin}} = \mathbf{p}^2/2m = p_j p_j/2m, \tag{2}$$

employing throughout this work the notation whereby the double appearance of an index in a single term implies summation over that index. We can introduce electromagnetic interactions by modifying the free-particle Schrödinger equation (2) with the replacements

$$p_j \to \pi_j \equiv p_j - (q/c)A_j,$$
$$E \to E - q\Phi, \tag{3}$$

in which A_j and Φ are components of the vector and scalar potentials associated with an electromagnetic field that couples to the charge q. The prescription (3), known as the "minimal coupling principle," is tied up with gauge invariance, and its application then yields the coupling terms associated with the interaction of the charge with the external field. If the particle has an intrinsic magnetic moment $\boldsymbol{\mu}$, we can then add an interaction potential energy term $-\boldsymbol{\mu}\cdot\text{curl}\,\mathbf{A}$, but nothing tells us what to use for the moment; moreover, insertion of the term in this manner is artificial. If, in fact, we were dealing with some kind of *composite* object such as a molecule, the hand insertion of a coupling term for the permanent moment would be appropriate, although in principle the moment term could be derived. The nucleon is another example of a composite particle in which the moment is known experimentally to high accuracy. Actually, the calculation of the neutron and proton moments in terms of quark intrinsic moments is a striking success of the quark model for nucleons, and the theory provides another test of the formula for the magnetic moment for spin-$\frac{1}{2}$ elementary particles. Further evidence on quark moments comes from analyses of mass differences in hadron multiplets in terms of (quark) magnetic hyperfine interactions.[6]

For elementary particles, we would like a formulation of the problem in which the intrinsic moment coupling term arises naturally from the application of the minimal coupling principle. To accomplish this a more general free-particle Hamiltonian is needed as a starting point, and it is not hard to establish the form that this function should take. This is done in the following two sections for the specific cases of spin-$\frac{1}{2}$ and spin-1, respectively. Basically, the procedure involves choosing a free-particle Hamiltonian that is a scalar (not pseudoscalar) function of the particle momentum vector $\mathbf{p}$ and the spin angular momentum pseudovector $\mathbf{s}$. The function must be *equivalent to* the elementary form (2) in the absence of an external electromagnetic field and it must be consistent with the classical limit for the general problem. These guidelines are the same as those employed in a relativistic formulation except in that case Lorentz invariance is also imposed. As we shall see, once a more general free-particle Hamiltonian is found (for the individual cases of spin s), both the charge- and moment-coupling terms fall out automatically, as in the relativistic theory. We then see that for such particles, the charge and mass can be considered fundamental but the associated magnetic moment is not and its interaction term need not be inserted artificially into the Schrödinger equation.

Concerning the classical limit, perhaps it should be emphasized here that this provides no guideline for the exact expression for the moment-coupling term. The classical force on a permanent moment in a magnetic field B is given by $\mathbf{F}_m = \boldsymbol{\mu}\cdot\nabla\mathbf{B}$, and this is small compared with the "deflecting" force $\mathbf{F}_q = (q/c)\mathbf{v}\times\mathbf{B}$ on the charge for a certain condition on the magnetic field. In terms of a spatial variation scale length L_B, the field gradient is of order B/L_B, and if the magnetic moment is of order $q\hbar/mc$ (as we expect), the ratio (F_m/F_q) of the two forces is of order $\lambda\!\!\!^{-}/L_B$, where $\lambda\!\!\!^{-}$ is the particle de Broglie wavelength. That is, in the classical limit $\lambda\!\!\!^{-}/L_B \ll 1$, the moment term is unimportant and classical mechanics cannot provide a guide to its exact form.

In an attempt to provide a largely self-contained discussion, the mathematical manipulations in this paper will be indicated explicitly, although some identities employed are also found in standard references. For example, derivations of basic results involving the Pauli- and other spin-matrices will be given in some detail. However, for the basic properties of these matrices the reader should consult textbooks on quantum mechanics.

II. SPIN-$\frac{1}{2}$ CASE

The aim in the formulation outlined here is to find a new generalized free-particle Hamiltonian function H_0 which, on application of the minimal coupling prescription (3), will yield a complete Hamiltonian H' that includes a kinetic energy term, charge coupling terms, plus an intrinsic moment coupling term:

$$H' = H_{\text{kin}} + H_q + H_m. \tag{4}$$

The new function H_0 must be such that, in the absence of external fields, it is equivalent to the form (2). It is not hard to infer the form of this function. Now describing both the spatial and spin motion of the particle, for spin-$\frac{1}{2}$, the wave function is represented by a two-component column. The Hamiltonian operator is then a 2×2 matrix, formed from factors involving the Pauli spin matrices. These matrices are related to the particle spin operator by

$$\mathbf{s} = \tfrac{1}{2}\boldsymbol{\sigma}, \tag{5}$$

and their properties can be summarized in a single relation:

$$\sigma_j\sigma_k = \delta_{jk}I + i\epsilon_{jkl}\sigma_l. \tag{6}$$

Here, I is the 2×2 identity matrix and ϵ_{jkl} is the Levi-Civita symbol; in the term $\sigma_j\sigma_k$ in (6), matrix multiplication is implied.

In terms of the pseudoscalar operator $\boldsymbol{\sigma}\cdot\mathbf{p}$ formed from an axial and polar vector, we construct the scalar operator

$$H_0 = \frac{1}{2m}(\boldsymbol{\sigma}\cdot\mathbf{p})(\boldsymbol{\sigma}\cdot\mathbf{p}). \tag{7}$$

This expression has the desired form as a generalization of (2) for the description of intrinsic spin and magnetic moment effects. It contains the spin operator $\boldsymbol{\sigma}$ and it is quadratic in the momentum. The multiplying factor is determined from the equivalent form that H_0 reduces to when there are no external fields, and this limit is the first thing to check. As we shall immediately see, the only difference between (2) and (7) is that the latter yields a term associated with coupling of the intrinsic moment with an external magnetic field. Moreover, this coupling has the appropriate form and it gives the *value* for the intrinsic moment. First, as to the limit when there are no external fields, we note that the momentum operators commute with one another and with the components σ_j, but the latter do not commute with each other. In the product of dot products in (7), we can then take half the sum with relabelled and interchanged dummy indices and apply the identity (6). We then find that

$$H_0 \underset{\text{no fields}}{\longrightarrow} H_{\text{kin}}(\mathbf{p})I, \tag{8}$$

which is the property required for the function, that is, that it reduces to (2) in this limit. The Hamiltonian H' is then found from the form (7) by the minimal coupling prescription (3):

$$H' = H_0(\mathbf{p} \to \boldsymbol{\pi}) + q\Phi I. \tag{9}$$

This simple procedure, starting from the fundamental Hamiltonian (7), provides the correct complete Hamiltonian that includes the effects of coupling to both the charge and intrinsic moment.

It is an elementary exercise to rewrite the Hamiltonian to a form that demonstrates its features explicitly. The $\boldsymbol{\sigma}$ operator commutes with $\boldsymbol{\pi}$, the $\mathbf{p}$'s commute, but the components of $\boldsymbol{\pi}$ do not commute ($\boldsymbol{\pi} \times \boldsymbol{\pi} \neq 0$). Then

$$H' = \frac{1}{2m} \sigma_j \sigma_k \pi_j \pi_k + q\Phi I \tag{10}$$

$$= \frac{1}{4m} (\sigma_j \sigma_k \pi_j \pi_k + \sigma_k \sigma_j \pi_k \pi_j) + q\Phi I, \tag{11}$$

and the first term can be rewritten with the help of the identity (6) as an anticommutator ($\sigma_j \sigma_k + \sigma_k \sigma_j = 2\delta_{jk} I$):

$$\sigma_j \sigma_k \pi_j \pi_k = 2\pi_j \pi_j I - \sigma_k \sigma_j \pi_j \pi_k. \tag{12}$$

With this substitution the Hamiltonian (11) becomes

$$H' = \frac{1}{2m} \pi_j \pi_j I + \frac{1}{4m} \sigma_k \sigma_j (\pi_k \pi_j - \pi_j \pi_k) + q\Phi I, \tag{13}$$

and in the second term here we can write $\sigma_k \sigma_j = i\epsilon_{kjl} \sigma_l$ and identify vector products:

$$i\epsilon_{kjl} \sigma_l (\pi_k \pi_j - \pi_j \pi_k) = 2i\sigma_l (\boldsymbol{\pi} \times \boldsymbol{\pi})_l. \tag{14}$$

The Hamiltonian is now in the fundamental and transparent form

$$H' = \frac{1}{2m} \boldsymbol{\pi}^2 I + \frac{i}{2m} \boldsymbol{\sigma} \cdot (\boldsymbol{\pi} \times \boldsymbol{\pi}) + q\Phi I. \tag{15}$$

Here, since $\boldsymbol{\pi} = -i\hbar \nabla - (q/c)\mathbf{A}$, the operator $\boldsymbol{\pi} \times \boldsymbol{\pi}$ reduces to

$$(iq\hbar/c) \text{curl } \mathbf{A} = (iq\hbar/c)\mathbf{B}, \tag{16}$$

in terms of the external magnetic field $\mathbf{B}$. All of the interaction terms in (4) can then be identified.

In addition to the kinetic energy term (8), there is the usual charge-interaction Hamiltonian

$$H_q = [-(q/mc)(\mathbf{p} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{p}) + (q^2/2mc^2)\mathbf{A}^2 + q\Phi]I. \tag{17}$$

However, now we have the important moment-interaction term:

$$H_m = -\boldsymbol{\mu} \cdot \mathbf{B}, \tag{18}$$

in which the magnetic moment is given by

$$\boldsymbol{\mu} = \frac{q\hbar}{2mc} \boldsymbol{\sigma}, \tag{19}$$

indicating the Bohr magneton (1) for its maximum magnitude.

III. SPIN-1 CASE

Like the case of spin-$\frac{1}{2}$, the spin-1 problem has a long history[7] going back to the 1930s and the introduction of the "vector meson hypothesis." Again, treatments started from a relativistic formulation, but here a nonrelativistic approach is taken from the beginning. We try to insert the spin operator into some free-particle Hamiltonian that we construct, imposing the requirement that the function be a scalar and be equivalent to the ordinary Hamiltonian (2) when there are no external fields. For spin-1 the particle has three spin substrates with $m_s = 1, 0, -1$, but we can label these with an index running from 1 to 3. This notation is convenient, even though the index representing the Cartesian axes also runs from 1 to 3. The latin letters $j,k,l,m,n,\ldots$ will be employed for both kinds of indices in the formulation below. When two indices appear together as isolated subscripts or singly to designate the component of a wave function, as in Eq. (22) below, those indices will always represent spin substrates, even when they are attached to Cartesian vector components [Eqs. (26) and (27)].

For spin-1 the most convenient matrix representation of the operators for the three spatial components s_j is

$$(s_j)_{kl} = i\epsilon_{kjl}; \tag{20}$$

this specific representation is simple and convenient for mathematical manipulations. As in the case of general spin, the three components satisfy the commutation relations

$$s_j s_m - s_m s_j = i\epsilon_{jml} s_l. \tag{21}$$

The wave function is a three-component column and the Schrödinger equation now involves a Hamiltonian matrix operator:

$$H_{kl} \psi_l = E \psi_l. \tag{22}$$

As an analog of the spin-$\frac{1}{2}$ case where the operator (7) was introduced, consider the expression

$$S \equiv (\mathbf{s} \cdot \mathbf{p})(\mathbf{s} \cdot \mathbf{p}), \tag{23}$$

which in matrix form becomes, employing (20),

$$S_{kl} = -\epsilon_{kjm} \epsilon_{mnl} p_j p_n. \tag{24}$$

But $\epsilon_{kjm} = -\epsilon_{mjk}$, and the identity

$$\epsilon_{mjk} \epsilon_{mnl} = \delta_{jn} \delta_{kl} - \delta_{jl} \delta_{kn} \tag{25}$$

can be applied, giving

$$S_{kl} = \mathbf{p}^2 \delta_{kl} - p_k p_l. \tag{26}$$

We are then led to choose

$$H_{kl} = (S_{kl} + p_k p_l)/2m \tag{27}$$

as our generalized free-particle Hamiltonian so that a pure kinetic energy term results in the field-free limit. Applying the minimal coupling prescription, we take

$$H'_{kl} = H_{kl}(\mathbf{p} \to \boldsymbol{\pi}) + q\Phi \delta_{kl} \tag{28}$$

for our general Hamiltonian including effects of coupling to an external electromagnetic field. This expression can now be rearranged and put into a form so that the various couplings (including the moment interaction) are exhibited. We find

$$H'_{kl} = (\boldsymbol{\pi}^2/2m) \delta_{kl} + q\Phi \delta_{kl} + M_{kl}, \tag{29}$$

where the first two terms give the charge coupling contributions and

$$M_{kl} = (\pi_k \pi_l - \pi_l \pi_k)/2m \tag{30}$$

is the term associated with the interaction of the intrinsic moment with the external magnetic field. As in the spin-$\frac{1}{2}$ case, we find that M_{kl} reduces to

$$M_{kl}=(iq\hbar/2mc)(\partial_k A_l-\partial_l A_k), \tag{31}$$

which can then be expressed in terms of the matrix element $(\mathbf{s}\cdot\mathbf{B})_{kl}$. This dot product can be written as

$$i\epsilon_{kml}\epsilon_{mjn}\partial_j A_n=-i(\partial_k A_l-\partial_l A_k), \tag{32}$$

and we have

$$M_{kl}=-(\boldsymbol{\mu}\cdot\mathbf{B})_{kl}, \tag{33}$$

with

$$\boldsymbol{\mu}=\frac{q\hbar}{2mc}\,\mathbf{s} \tag{34}$$

as the intrinsic moment.

In terms of the charge and mass, the magnitude of the intrinsic magnetic moment is then *the same* as in the case of spin-$\frac{1}{2}$. This is quite a remarkable result, and the cases of spin-$\frac{3}{2}$ and spin-2 have also been examined[8–10] as well as the case of general half-integer spin[8] (always starting from a relativistic formulation) with, again, the same result. This prompted Belinfante[10] to suggest that the magnitude of the magnetic moment is given by the same formula independent of spin. The result is usually expressed in terms of the *gyromagnetic ratio* g, defined as a factor in a general relation between magnetic moment and spin:

$$\boldsymbol{\mu}=\frac{gq\hbar}{2mc}\,\mathbf{s}\ \text{(general spin)}. \tag{35}$$

In terms of g, Belinfante's conjecture would correspond to a general formula

$$g=1/s; \tag{36}$$

however, this general expression has never been derived. It is a surprising result, since (classical) intuition would seem to suggest an increase in the intrinsic moment as s increases.

IV. EXTENDED DISCUSSION

In the previous two sections we have seen how a Hamiltonian can be obtained for charged particles with spin, including the effects of coupling to an external electromagnetic field, in which the intrinsic magnetic moment is inferred. It is a "minimalist" approach, imposing simplicity of mathematical form, applying restrictions of gauge invariance and consistency with the classical limit for particle motion, the need for the Hamiltonian to be a scalar and rotational invariant, and employing the mechanical application of the minimal coupling principle. Higher-order effects such as radiative corrections are ignored, of course, so that the procedure yields only the zeroth-order moment. No considerations of Lorentz invariance are introduced; the formulation is completely nonrelativistic with the $1/c$ in the moment formula coming from an application of minimal coupling. The moment and its coupling term occur naturally in the formulation, rather than having to be inserted "by hand." The particle's mass and charge are then to be considered fundamental but not its magnetic moment. The correct result is obtained for the electron, the muon, and for quarks, so that for spin-$\frac{1}{2}$ the Hamiltonian (7) should be regarded as a fundamental form. For spin-1 particles, taking the same basic approach, the corresponding Hamiltonian (29) is obtained, with the result that the formula for the magnitude of the intrinsic moment is the same as in the spin-$\frac{1}{2}$ case. The only known spin-1 elementary particle with finite mass and charge is the $W^{\pm}$ vector boson that mediates the charged-current weak interactions, but there are no direct measurements of the magnetic moment.

But can other Hamiltonians be chosen that yield different moments, and are such ambiguities present in the relativistic formulations? For the specific case of spin-$\frac{1}{2}$, suppose, simplifying the notation, we call the operator (7) Π and the first term on the right of (15) K. That is, specifically,

$$\Pi=\frac{1}{2m}\,(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})(\boldsymbol{\sigma}\cdot\boldsymbol{\pi});\quad K=\frac{1}{2m}\,\boldsymbol{\pi}^2 I. \tag{37}$$

We might consider in a more general Hamiltonian a linear combination of Π and K. The coefficients in the linear combination would not be independent, since the Hamiltonian must reduce to its free-particle form when there are no external fields. We would then have

$$H''=(1-\rho)\Pi+\rho K+q\Phi I, \tag{38}$$

with ρ as some parameter having any positive or negative value. This Hamiltonian satisfies the requirements of gauge invariance and the classical limit, and we find that on rewriting the first term in (37), the only difference from (15) is that the second term therein is now multiplied by a factor $1-\rho$. This same factor would then be applied to the result for the inferred moment. In fact, we might even add an additional term to the Hamiltonian (37) proportional to $\boldsymbol{\sigma}\cdot\mathbf{B}$ which would also be consistent with gauge invariance and the classical limit. However, this term has the same form as the moment coupling term (multiplied by $1-\rho$) and could be lumped within that term. That is, we could write our generalized Hamiltonian in the form

$$H'''=\Pi+\Sigma+q\Phi I, \tag{39}$$

where

$$\Sigma=\eta(q\hbar/2mc)(\boldsymbol{\sigma}\cdot\mathbf{B}). \tag{40}$$

In the definition of Σ, factors are inserted to make the arbitrary parameter η dimensionless (and presumably of order unity—or zero). With (40) chosen as a general Hamiltonian, the fundamental intrinsic moment would be given by (19) multiplied by $1+\eta$, that is, it would be determined by the value of the arbitrary parameter η. In this very general formulation the intrinsic moment can have any value, rather than being a consequence of the basic theory. The correct value—apparently for all fundamental spin-$\frac{1}{2}$ particles—corresponds to the choice $\eta=0$ in H''' or $\rho=0$ in H''. That is, maximum simplicity in the mathematical formulation yields the correct fundamental theory.

In fact, the same type of considerations of a generalized Hamiltonian have to appear in the Dirac theory. Here, in the free-particle Dirac equation (with the usual notation in which $\not{p}=\gamma^{\mu}p_{\mu}$, where γ^{μ} is a Dirac matrix)

$$(\not{p}-mc)\psi=0, \tag{41}$$

we make the replacement $\not{p}\rightarrow\not{\pi}$. However, at this point the remark made by Pauli[11] in his *Handbuch* article should be noted. Pauli considered the possibility of an additional term in the Dirac equation with coupling, of the form, say,

$$M\psi=\delta(q\hbar/mc^2)F_{\mu\nu}\gamma^{\mu}\gamma^{\nu}\psi, \tag{42}$$

in which δ is some arbitrary constant and $F_{\mu\nu}$ is the electromagnetic field tensor; the factor $q\hbar/mc^2$ is chosen to make δ dimensionless. The modified Dirac equation is then

$$(\not{\pi}-mc)\psi+M\psi=0. \tag{43}$$

The introduction of this term is consistent with Lorentz invariance, gauge invariance, and the classical limit. Its inclusion modifies, in particular, the moment interaction effects (and the inferred moment) as well as the general applications of the Dirac theory. The standard Dirac theory corresponds to the straight application of the minimal coupling principle to the basic form for the free-particle Dirac equation and to the choice $\delta=0$. That is, the simplest form is taken—the analog of the choice $\eta=0$ in the nonrelativistic formulation—and the result is, of course, a correct relativistic theory. That is, in both the nonrelativistic and relativistic formulations the correct theories result from adopting the value zero for the corresponding arbitrary constants. Regarding the magnetic moment and its effects, however, as is emphasized in this paper, perhaps the nonrelativistic theory should serve as a guide to the relativistic theory rather than the reverse. This could have been done in the 1925–1928 era, since the electron's intrinsic moment was known experimentally.

Finally, some mention should be made of classical theories of particles with intrinsic moments. This part of the subject also has a long history, and the literature can, again, be misleading. Relativistic theories of classical charged particles are described in the standard textbooks[12] and, unlike the quantum mechanical theories, no prescription is indicated for the intrinsic magnetic moment of a particle of charge q and mass m. Actually, a superficial study of some of the classical treatments might lead to the notion that a "natural" value of a particle's gyromagnetic ratio g might be 2 (independent of spin), rather than the result (36) that quantum mechanical treatments seem to give. Really, however, classical electromagnetism has nothing to say about what the intrinsic moment should be and g has to be considered as an undetermined parameter with no prescription for its value.

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[12]See J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975); W. H. Panofsky and M. Phillips, *Classical Electricity and Magnetism*, 2nd ed. (Addison-Wesley, Reading, MA, 1962); R. H. Good, Jr. and T. J. Nelson, *Classical Theory of Electric and Magnetic Fields* (Academic, San Diego 1971); F. Rohrlich, *Classical Charged Particles* (Addison-Wesley, Reading, MA, 1965). These textbooks also contain references to early and later fundamental papers on the subject. Of the four listed here, the one by Good and Nelson may have the best overall discussion.

A C-AVERAGE AT MIT

In a way, the C felt good. It was an honest C.

"Uh, this could be a problem, sir," I said. "I just talked to Professor Beretta, and he said that I got a C in Thermo, too. I don't know how I did in Fluids, but I doubt it was an A."

"If you have a C in my class and a C in Thermo, the trend would indicate you'll probably have a C in Fluids, too. The subject matter's similar," he said.

Everything's a trend to this guy—points on a graph with a line in between them.

I asked, "What would 3 C's mean for my future at MIT?"

"Well," he answered, ...

Pepper White, *The Idea Factory—Learning to Think at MIT* (Penguin Books, New York, 1991), p. 76.