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Quark and gluon content of the proton spin

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We examine the role of quarks and gluons in polarized deep-inelastic scattering in connection with the famous European Muon Collaboration experimental result. The gluon contribution generated by the axial anomaly is shown to have a well-defined physical meaning and its gauge-invariance properties are fully clarified. We also emphasize the role of the ghost contribution in the generalized Goldberger-Treiman relation and give a detailed calculation of the correction due to unequal quark masses and to π^0-η-η' mixing.

I. INTRODUCTION

The naive parton interpretation of the famous European Muon Collaboration (EMC) result [1] that the total contribution of the quarks to the proton spin $\Delta\Sigma = \sum_f \int dx[q_f^+(x) - q_f^-(x) + \bar{q}_f^+(x) - \bar{q}_f^-(x)]$ is compatible with zero is not only in contradiction with the naive quark model but also with all of our understanding of baryon spectroscopy. As emphasized by Lipkin [2] using the Wigner-Ekhardt theorem, the total spin of the sea quarks and gluons plus orbital momentum has to be very large (>6) in order to obtain this small value. It is then difficult to understand why the nucleon is the only stable state with $J = I = \frac{1}{2}$.

The EMC result has produced a stream of theoretical papers [3] with a broad spectrum of ideas from doubts in the experiment to questioning the applicability of perturbative QCD. In our opinion, however, this "spin crisis" can be turned into a spectacular success of QCD. It has been demonstrated [4] that the factorization theorem in QCD leads to the expression for the first moment of the flavor-singlet part of the structure function $g_1^p(x)$:

$$\Gamma_1^{\text{singl}} = \frac{\langle e^2 \rangle}{2} \left[\Delta\Sigma - \frac{\alpha_s}{2\pi} N_f \Delta g \right], \tag{1}$$

where the second term is due to a short-range interaction of photons with polarized gluons via the quark box diagram (see Fig. 1). For the first moment it reduces to the contribution of the triangle axial anomaly. It need not be a small correction in spite of α_s because $\Delta g \sim \alpha_s^{-1}$ due to evolution equations [5]: a zero-mass quark emitting a polarized gluon conserves its helicity and again emits gluons of the same polarization. The more gluons are emitted (the smaller x), the higher the gluon polarization (of course, this increase is always compensated by an orbital momentum [6]). So, the EMC effect could result from a compensation between $\Delta\Sigma$ and $\tilde{\Delta}g = (\alpha_s/2\pi)N_f \Delta g$.

This approach has been criticized recently in [7–10] (i) because of the nonlocality (k^2 dependence) of the box diagram together with a dependence of this contribution on the regularization procedure [8,9], and (ii) for the absence of a local gauge-invariant gluon spin operator. These criticisms were partly answered in Refs. [11–13], which we here want to elaborate upon and to present more clearly. This is done in Sec. II where we also show that the commonly used definitions of $\Delta\Sigma$ and Δg through the quark and gluon axial-vector currents J_ν^5 and K_ν are in fact gauge invariant although they have to be more accurately defined, due to a nonperturbative ghost contribution. The generalized Goldberger-Treiman (GGT) relation is also discussed here. In Sec. III we calculate the corrections to the GGT relation due to unequal quark masses and to mixing of the neutral pseudoscalar mesons. We demonstrate the cancellation of large isospin and SU(3)-breaking terms and observe that only a small correction ($\simeq 0.2$) remains. In Sec. IV we give a general discussion

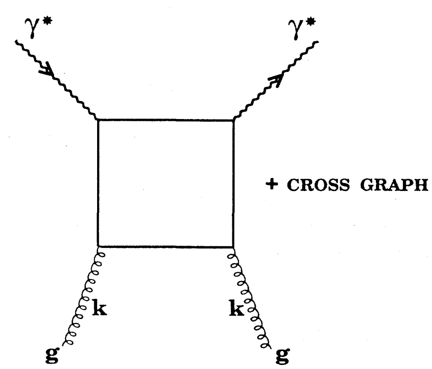

FIG. 1. The photon-gluon scattering graphs.

on the interpretation of the EMC result and our concluding remarks.

II. THE AXIAL ANOMALY CONTRIBUTION AND THE QUARK AND GLUON CONTENT OF THE PROTON SPIN

Consider first the ambiguity of the box contribution. In the QCD-modified parton picture there is no difficulty and no ambiguity in defining the first moment of the gluon spin distribution function (except the usual ambiguity in the renormalization scheme in higher orders of α_s). This picture is based on the factorization theorem which has the same theoretical foundation as the operator-product expansion (OPE) (i.e., proved in any order of perturbation theory) but many more applications. The object which generates the gluon distribution function there (polarized and unpolarized) is not a product of gauge-invariant operators but rather an ultraviolet regularized matrix element of a product of gluon fields $\langle p|A^a_\rho(0)A^a_\sigma(\xi)|p\rangle_{\text{reg}}$ convoluted with a contribution from a short-range, infrared regularized subprocess:

$$\int \sigma_{\rho\sigma}(\xi,q,\mu^2)\langle p|A^a_\rho(0)A^a_\sigma(\xi)|p\rangle_{\text{reg}} d^4\xi \ .$$

The gluon distribution functions are defined through the Taylor-series expansion of this matrix element in the limit $\xi\to 0$ and its gauge invariance is guaranteed by the gauge invariance of the subprocess which is on the mass shell, in the leading-twist approximation. So the matrix element of the gluon axial-vector current $K_\nu=\epsilon_{\nu\mu\sigma\rho}A^a_\mu F^a_{\sigma\rho}$ (or more precisely its projection onto the gauge vector n in the axial gauge $A^a_\nu n_\nu=0$) will appear as the first moment of the gluon spin distribution function [11]. It is gauge invariant in perturbative QCD (PQCD) since it contains only the transversal component with respect to n of the gluon field A. However, as we will see, beyond PQCD, it requires further specification.

A real ambiguity exists, however, in defining the first moment of the quark distribution. Actually, the first moment of the box-diagram contribution (see Fig. 1) off the mass shell ($k^2\neq 0$ and $m_q\neq 0$) has the form [9]

$$\Gamma_1^{\text{box}}=-\frac{\alpha_s}{2\pi}N_f\left[1+\frac{2m_q^2/k^2}{\sqrt{1+4m_q^2/k^2}}\times\ln\left[\frac{\sqrt{1+4m_q^2/k^2}-1}{\sqrt{1+4m_q^2/k^2}+1}\right]\right] . \qquad (2)$$

There are two gauge-invariant local (k^2-independent) limits of this quantity, which can be considered as contributions of parton subprocesses,

$$\Gamma_1^{\text{box}}\to\begin{cases}-\dfrac{\alpha_s}{2\pi}N_f, & m_q^2/k^2\to 0\ ,\\ 0, & k^2/m_q^2\to 0\ .\end{cases}$$

They correspond to the cross section of the $\gamma^* g\to q\bar{q}$ subprocess for either $m_q=0$ or $m_q\neq 0$ on the mass shell $k^2\to 0$, giving two different definitions of the quark contribution $\Delta\Sigma$ and different evolution equations. In the first case, the anomaly part [first term in Eq. (2)] is subtracted from the quark distribution function and included into the subprocess. It gives Eq. (1) for Γ_1^{singl} and the evolution equations

$$\dot{\Delta\Sigma}=0 \ \text{ and } \ \dot{\Delta\Sigma}-\dot{\tilde{\Delta g}}=\gamma(\Delta\Sigma-\tilde{\Delta g}) \qquad (3)$$

and γ is an anomalous dimension and the overdot denotes $d/d\ln Q^2$. In the second case, the whole contribution in Eq. (2) is in the quark distribution function, i.e., $\Gamma_1^{\text{singl}}\sim\Delta\Sigma'=\Delta\Sigma-\tilde{\Delta g}$ and the evolution equations become $\dot{\Delta\Sigma}'=\gamma\Delta\Sigma'$ and $\dot{\Delta\Sigma}'+\dot{\tilde{\Delta g}}=0$.

Which of these definitions is better? We believe that for light quarks the first one is better for the following reasons.

(i) When the mass m_q is small the cancellation of the anomaly by the mass term in Eq. (2) occurs only in a small part ($|k^2|\ll m_q^2$) of the whole integration region $|k|^2\leq Q^2$. However, in this small part, other effects (in particular nonperturbative) are expected to be more important than the cancellation.

(ii) Equations (3) are just the analytic continuation of the evolution equation for higher moments $m\geq 2$. This is a necessary condition for the existence of soft x-dependent distribution functions $\Delta\Sigma(x)$ and $\Delta g(x)$ which are measured experimentally.

(iii) Since $\Delta\Sigma$ is independent of Q^2 one has a closer connection between low- (naive quark model) and high-energy (parton) pictures of the proton.

(iv) Because of a possible cancellation of the two terms in Eq. (1), the contradiction of the EMC result and the quark model need not be so severe.

Now turn to the gauge-invariance problem. There is no such problem for higher moments $m\geq 2$ because there are two towers of local gauge-invariant operators which are built from quark and gluon fields q and A_ν. So, being defined as an analytic continuation of moments of x-dependent spin distribution functions to $m=1$, $\Delta\Sigma$ and $\tilde{\Delta g}$ also have to be gauge invariant. The formal continuation of the local operators gives, however, as was discussed earlier

$$\Delta\Sigma=\tfrac{1}{2}\langle p|J^5_\nu-\tilde{K}_\nu|p\rangle n_\nu \ \text{ and } \ \tilde{\Delta g}=-\tfrac{1}{2}\langle p|\tilde{K}_\nu|p\rangle n_\nu\ , \qquad (4)$$

where n_ν is a lightlike vector satisfying $n^2=0, np=1$ and where $J^5_\nu=\sum_f\bar{q}_f\gamma_\nu\gamma_5 q_f, \tilde{K}_\nu=N_f(\alpha_s/2\pi)K_\nu$ are proportional to the quark and gluon spin operators (strictly speaking, K_ν is not the gluon spin; nevertheless, $K_\nu n_\nu$ is its projection). The subtraction of $\tilde{K}_\nu$ in the definition of $\Delta\Sigma$ in Eq. (4) is done in accordance with Eq. (3) to make $\Delta\Sigma$ independent of Q^2 as a consequence of the Adler-Bardeen relation

$$\partial_\nu J^5_\nu=\partial_\nu\tilde{K}_\nu=N_f\frac{\alpha_s}{2\pi}F^a_{\rho\sigma}\tilde{F}^a_{\rho\sigma}\ . \qquad (5)$$

The problem with gauge invariance arises since the operator K_ν in Eq. (4) is apparently gauge variant at least with respect to large gauge transformations. Of course in the projection $K_\nu n_\nu$ remains only the variance of the large (homotopically nontrivial) gauge transformations which are absent in perturbation theory. However, beyond perturbation theory Eq. (4) can be wrong due to nonperturbative contributions. So the resolution of the problem lies in a term nonanalytic in m in the matrix elements of Eq. (4) due to a nonperturbative contribution [14]. The most important part of this contribution is tightly connected with the well-known U(1) problem in QCD [15,16].

Actually, in any covariant gauge,

$$\begin{aligned} \langle p'|J_\nu^5|p\rangle &= \bar{u}(p')[\gamma_\nu\gamma_5 G_1(q^2)+q_\nu\gamma_5 G_2(q^2)]u(p)\;, \\ \langle p'|\tilde{K}_\nu|p\rangle &= \bar{u}(p')[\gamma_\nu\gamma_5 \tilde{G}_1(q^2)+q_\nu\gamma_5 \tilde{G}_2(q^2)]u(p)\;. \end{aligned} \tag{6}$$

The same expressions can be written also in the form

$$\begin{aligned} \langle p'|J_\nu^5|p\rangle &= 2M_N s_\nu G_1(q^2)+q_\nu(sq)G_2(q^2)\;, \\ \langle p'|\tilde{K}_\nu|p\rangle &= 2M_N s_\nu \tilde{G}_1(q^2)+q_\nu(sq)\tilde{G}_2(q^2)\;, \end{aligned} \tag{6'}$$

where M_N is the proton mass, s_ν its spin four-vector, and the contributions parallel to it are, in terms of quark and gluon distributions,

$$G_1(0)=\Delta\Sigma-\tilde{\Delta g} \quad \text{and} \quad \tilde{G}_1(0)=-\tilde{\Delta g}\;.$$

Because of the gauge invariance of $\partial_\nu\tilde{K}_\nu$ and the absence of a zero-mass axial pole state in PQCD both $q^2G_2, q^2\tilde{G}_2\to 0$ when $q^2\to 0$. So both $G_1(0)$ and $\tilde{G}_1(0)$ are gauge invariant if one disregards nonperturbative effects. However, this is not correct in general. The current K_ν has to couple with a zero-mass ghost pole whose mixing with Nambu-Goldstone η_0' supplies it with an additional mass $\Delta m_{\eta'}^2=\lambda^4/f_{\eta'}^2$ [17], where λ^2 is the $\tilde{K}_\nu$-ghost coupling and $f_{\eta'}$ is the η'-decay parameter $\langle 0|J_\nu^5|\eta'\rangle = f_{\eta'}q_\nu$. This pole contributes to $\langle p'|\tilde{K}_\nu|p\rangle$ through the diagrams shown in Fig. 2 and gives

$$q_\mu\tilde{G}_2(q^2)=\frac{q_\mu\lambda^2}{q^2}\sqrt{N_f}\left[\frac{\Delta m_{\eta'}g_{\eta'}NN}{m_{\eta'}^2-q^2}-g_{QNN}\right]\;, \tag{7}$$

where g_{QNN} is the ghost nucleon coupling constant $(\partial_\nu\tilde{K}_\nu\equiv Q)$. Then

$$\lim_{q^2\to 0} q^2\tilde{G}_2(q^2)=G=\sqrt{N_f}f_{\eta'}\left[\frac{\Delta m_{\eta'}^2}{m_{\eta'}^2}g_{\eta'NN}-\Delta m_{\eta'}g_{QNN}\right]\;. \tag{8}$$

This ghost contribution is nonanalytic in m (the moment number) and nonperturbative in nature. Its physical meaning is in a periodic dependence of the QCD potential $[\sim\mathrm{Tr}(H^2)]$ on a collective variable $X=\int d^3x\, K_0(x,t)$, which changes by a winding number under a homotopically nontrivial gauge transformation [18] $X\to X+n$. The reason of the pole is the same as that of a gapless excitation in an ideal conductor.

An important comment is now in order. Notice that the ghost pole can contribute only to the form factor $\tilde{G}_2$ and does not contribute to $\tilde{G}_1$. This is because the effective ghost nucleon vertex (and η'-ghost vertex) has to contain a derivative $\bar{N}\partial_\nu G_\nu\gamma_5 N$, where G_ν is the ghost field. It is the consequence of the no-ghost radiation condition in any physical process [19]. Therefore the dependence of the gauge parameter drops out from the ghost

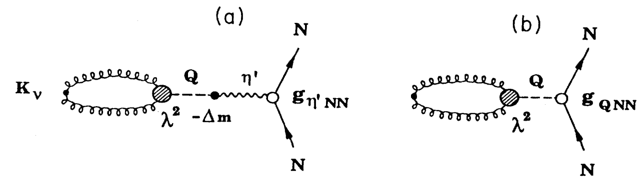

FIG. 2. The two different ghost contributions to $\bar{N}N$: (a) coupling through the physical η', (b) direct coupling.

propagator (see Fig. 2) $(g_{\mu\nu}+aq_\mu q_\nu/q^2)\,[-q^2(a+1)]^{-1}$, where a is a gauge parameter, and similarly from the $\tilde{K}_\nu G_\nu$ vertex. That is the reason why the expression (7) is in fact gauge invariant. This means, however, that $\tilde{G}_1$ has to be gauge invariant also, due to the gauge invariance of $2M_N\tilde{G}_1+q^2\tilde{G}_2$. So, the whole matrix element of K_ν, for symmetric states where $q^2=0$, has to be gauge invariant.

Turn now to the renormalization properties of the G's. The quark current J_ν^5 is known to be renormalized multiplicatively with an anomalous dimension γ, i.e., $\dot{J}_\nu^5=\gamma J_\nu^5$ whereas $J_\nu^5-\tilde{K}_\nu$ is not renormalized; therefore, $\dot{\tilde{K}}_\nu=\gamma J_\nu^5$. Using Eq. (6) one finds

$$\dot{G}_{1,2}=\gamma G_{1,2} \quad \text{and} \quad \dot{\tilde{G}}_{1,2}=\gamma G_{1,2}\;.$$

The Adler-Bardeen relation Eq. (5) leads to the equality

$$2M_N[G_1(q^2)-\tilde{G}_1(q^2)]=q^2[\tilde{G}_2(q^2)-G_2(q^2)]\;,$$

both sides of which are now seen to be Q^2 independent. In the limit $q^2\to 0$ it gives

$$G_1(0)-\tilde{G}_1(0)=\Delta\Sigma=\frac{G}{2M_N}=\frac{\sqrt{N_f}f_{\eta'}g_{\eta_0'}NN}{2M_N}\;, \tag{9}$$

where $g_{\eta_0'NN}=g_{\eta'NN}-\Delta m_{\eta'}g_{QNN}$ is the η_0'-nucleon interaction coupling constant. Equation (9) is *not* the formula proposed in Ref. [16] due to the presence of g_{QNN} and to the fact that one should not neglect Okubo-Zweig-Iizuka (OZI) rule violations in the η' coupling. It also differs *formally* from the formula derived in Ref. [15] but this could be just due to a difference in the definition of $g_{\eta_0'NN}$. The source of this difference is the inclusion in the same one-particle-irreducible (1PI) vertex Γ_{QNN} in Ref. [15] of two terms. The first term is a truly 1PI multimeson contribution from the continuum spectrum, which we call $\tilde{G}_1=-\tilde{\Delta g}$ and the second term, which is reducible with respect to the ghost, is the contribution containing the direct ghost nucleon interaction [see Fig. 2(b)]. They have different kinematic structures in $\langle p'|\tilde{K}_\nu|p\rangle$ but the same structure in $\langle p'|\partial_\nu\tilde{K}_\nu|p\rangle$. So the term $\tilde{G}_1$, in Ref. [15], is in the right-hand side (RHS) of Eq. (9) and, in fact, included in $g_{\eta_0'NN}$ whose definition and consequent numerical values and renormalization properties are different from ours. Although we did not succeed yet to find a formal way to separate these two contributions, we believe these two terms are different in nature and this difference can be easily seen in comparing QCD with QED where we have the axial anomaly but no ghost pole due to the Abelian character of the gauge group. Indeed, in QED $q^2\tilde{G}_2 \to 0$ in the symmetric limit and if one disregards the first term $\tilde{G}_1$, one obtains $\langle e|\partial_\mu K_\mu|e\rangle = 0$, which is certainly not correct. So, in our opinion the $\tilde{G}_1$ term has to be taken into account separately from the ghost contribution. One way to demonstrate this would be to construct an effective gauge-invariant Lagrangian with the ghost contribution, which is not a trivial task. Phenomenological considerations (see Sec. IV) make it plausible to assume g_{QNN} small, so we should recover our previous formula (Ref. [16]), whereas the relation derived in Ref. [15] probably yields a large g_{QNN} by including finite ghost contributions to $\tilde{G}_1$. Interesting enough, a small g_{QNN} (according to our definition) brings back agreement between Eq. (9) and the final proposal of Shore and Veneziano [15] for the quark component of the proton spin. Thanks to this observation, the discussion and results described in the next section will be quite independent of the above controversy.

In conclusion, we must stress once more that quark and gluon spin first moments defined through a hard subprocess in the QCD-improved parton model are not exactly represented by matrix elements of the currents $J_\nu^5 - \tilde{K}_\nu$ and $\tilde{K}_\nu$. The distinction is due to the nonperturbative ghost contribution. The same contribution determines the total quark spin fraction via the GGT relation (9).

III. FLAVOR SYMMETRY BREAKING IN THE GENERALIZED GOLDBERGER-TREIMAN RELATIONS AND IN THE PSEUDOSCALAR NONET

The discussion in this section is rather independent of the exact form and interpretation of the flavor singlet Goldberger-Treiman relation [Eq. (9)], which still is under dispute in the literature [15,16,20,21]. In our discussion we keep our form of the relation as the first-order approximation.

When considering corrections from finite quark masses the Adler-Bardeen relation takes the form

$$\partial_\nu J_\nu^5 = \partial_\nu \tilde{K}_\nu + \sum_i 2m_i \bar{q}_i i\gamma_5 q_i \ , \qquad (10)$$

which, together with the well-known relations for the triplet and octet, leads to the following relations in the triplet, octet, and singlet channels, respectively (for $N_f = 3$ and $q^2 = 0$):

$$2M_N g_A^3 = 2(m_u \nu_u - m_d \nu_d) \ , \qquad (11a)$$

$$2M_N g_A^8 = 2(m_u \nu_u + m_d \nu_d - 2m_s \nu_s) \ , \qquad (11b)$$

$$2M_N \Delta\Sigma = 2(m_u \nu_u + m_d \nu_d + m_s \nu_s) + G \ , \qquad (11c)$$

where

$$\nu_i = \langle N|\bar{q}_i\gamma_5 q_i|N\rangle / \bar{N} i\gamma_5 N \ ,$$

and $G = \lim_{q^2\to 0} q^2\tilde{G}_2$ is the residue of the ghost pole contribution [see Eq. (8)].

Flavor symmetry breaking enters into those equations in two ways: (i) nondegenerate m_q's; (ii) π^0-η-η' mixing in the coupling constants.

The effect of (i) is explicitly seen from Eqs. (11a)–(11c) and we can solve for $\Delta\Sigma$ by eliminating the $m_q\nu_q$'s from the RHS of Eq. (11c) using the other two. First, one can use Eq. (11b) to eliminate ν_s and second, due to the identity

$$3(m_u\nu_u + m_d\nu_d) \equiv \tfrac{3}{2}[(m_u+m_d)(\nu_u+\nu_d) + (m_u - m_d)\nu_3]$$

together with $\nu_3 = \nu_u - \nu_d = \sqrt{2} f_\pi g_{\pi NN}/(m_u + m_d)$ from Eq. (11a), neglecting the term $(m_u - m_d)(\nu_u + \nu_d)$ (which leads to higher-order isospin corrections), one can reexpress Eq. (11c) as

$$\Delta\Sigma = G/2M_N \pm \frac{3}{2}\frac{m_u - m_d}{m_u + m_d} g_A^3 - \frac{1}{2} g_A^8 \ , \qquad (12)$$

where the second terms "violates isospin" and where the plus or minus sign is for proton or neutron, respectively. The third term "violates $SU(3)_F$." Both terms are large, reducing $\Delta\Sigma$ by more than 50%.

But, we must also take into account π^0-η-η' mixing, which for Eq. (11c) enters into G [see Fig. 2(b)]. This contribution is again determined by the diagrams in Fig. 2; however, instead of one η' pole we have to put the singlet combination of η', η, π^0 poles. For $q^2 \to 0$ this reduces, as is well known, to the following substitution in the first term of Eq. (8):

$$\frac{g_{\eta' NN}}{m_{\eta'}^2} \to \frac{g_{\eta' NN}}{m_{\eta'}^2}\cos\theta_3 \pm \frac{g_{\pi NN}}{m_\pi^2}\theta_2 - \frac{g_{\eta NN}}{m_\eta^2}\sin\theta_3 \ , \qquad (13)$$

where θ_2 and θ_3 are the π^0-η' and η-η' mixing angles, respectively. Inserting conventional values for these angles and for the couplings $g_{\eta NN}$ and $g_{\eta' NN}$, the second "isospin-violating" terms of Eqs. (12) and (13) cancel each other and similarly the $SU(3)_F$-violating third terms almost cancel each other. As a result the numerical value of $\Delta\Sigma$ changes very little. As we shall see this cancellation is exact to first order in the limit of large anomaly mass $\Delta m_{\eta'}^2$.

This cancellation of large correction terms looks accidental, but obviously there must be a physical reason for it which we now clarify. Using different techniques other authors [19,22] have reached similar, but less general conclusions. In short we write Eqs. (11a)–(11c) in matrix form:

$$2M_N\alpha \begin{bmatrix} g_A^3/\sqrt{2} \\ g_A^8/\sqrt{6} \\ \Delta\Sigma/\sqrt{3} \end{bmatrix} = \left[\alpha\mathcal{O} \begin{bmatrix} 2m_u & 0 & 0 \\ 0 & 2m_d & 0 \\ 0 & 0 & 2m_s \end{bmatrix} \mathcal{O}^{-1} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta m_{\eta'}^2 \end{bmatrix} \right] \begin{bmatrix} \nu_3 \\ \nu_8 \\ \nu_1 \end{bmatrix} , \qquad (11')$$

where

$$\mathcal{O}=\begin{bmatrix}1/\sqrt{2} & -1/\sqrt{2} & 0\\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6}\\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3}\end{bmatrix}, \quad \begin{bmatrix}\nu_3\\ \nu_8\\ \nu_1\end{bmatrix}=\mathcal{O}\begin{bmatrix}\nu_u\\ \nu_d\\ \nu_s\end{bmatrix}$$

and

$$\overline{\Delta\Sigma}=\Delta\Sigma-(G-\Delta m_{\eta'}^2\nu_1/\alpha)/2M_N$$

and in which $\mathcal{O}$ is the orthogonal matrix which transforms from the ideal $(u\bar{u},d\bar{d},s\bar{s})$ frame to the (triplet, octet, singlet) frame. In this construction α can have an arbitrary value, which we shall fix below. Now, one observes that the matrix in the square brackets of Eq. (11′) is of the same form as the 0^{-+} squared mass matrix with quark mass terms and an anomaly term $(\Delta m_{\eta'}^2)$. If one can choose $\alpha\simeq m_\pi^2/(m_u+m_d)$ this matrix will be equal to the true pseudoscalar mass matrix. As we shall see, then the difference between $\Delta\Sigma$ and $\overline{\Delta\Sigma}$ reduces to the contribution from g_{QNN} only, i.e., to that from the second term in Eq. (8).

So, let

$$M_{0^{-+}}^2=\alpha\mathcal{O}\begin{bmatrix}2m_u & 0 & 0\\ 0 & 2m_d & 0\\ 0 & 0 & 2m_s\end{bmatrix}\mathcal{O}^{-1}+\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \Delta m_{\eta'}^2\end{bmatrix}$$
$$=\alpha\begin{bmatrix}m_u+m_d & (m_u-m_d)/\sqrt{3} & (m_u-m_d)\sqrt{2/3}\\ (\text{sym.}) & (m_u+m_d+4m_s)/3 & (m_u+m_d-2m_s)\sqrt{2}/3\\ (\text{sym.}) & (\text{sym.}) & 2(m_u+m_d+m_s)/3+\Delta m_{\eta'}^2/\alpha\end{bmatrix} \quad (14)$$

be the mass matrix. The physical π^0,η,η' squared masses are the eigenvalues of this matrix which is diagonalized by another orthogonal matrix, the mixing matrix Ω:

$$\Omega=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta_3 & -\sin\theta_3\\ 0 & \sin\theta_3 & \cos\theta_3\end{bmatrix}\begin{bmatrix}1 & -\theta_1 & \theta_2\\ \theta_1 & 1 & 0\\ -\theta_2 & 0 & 1\end{bmatrix}, \quad (15)$$

where θ_1 is the π^0-η mixing angle and θ_2,θ_3 are as in Eq. (13) above, the π^0-η' and η-η' mixing angles, respectively.

Now Ω diagonalizes the sum of the quark mass term (Q) and the anomaly term (A):

$$\begin{bmatrix}m_{\pi^0}^2 & 0 & 0\\ 0 & m_\eta^2 & 0\\ 0 & 0 & m_{\eta'}^2\end{bmatrix}=Q+A \quad (16)$$

where

$$Q=2\alpha\Omega\mathcal{O}\begin{bmatrix}m_u & 0 & 0\\ 0 & m_d & 0\\ 0 & 0 & m_s\end{bmatrix}\mathcal{O}^{-1}\Omega^{-1}, \quad (17)$$

$$A=\Omega\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \Delta m_{\eta'}^2\end{bmatrix}\Omega^{-1}, \quad (18)$$

but both Q and A have off-diagonal terms of same order as Eq. (14). The mixing angles θ_i are determined by the cancellation of $Q_{ij}+A_{ij}=0$ (for $i\neq j$). To first order in the small isospin mixing angles θ_1 and θ_2 one finds simple algebraic relations for these (denoting $a=\Delta m_{\eta'}^2/\alpha$):

$$\theta_1=-\theta_2\frac{2m_s+a}{2\sqrt{2}m_s}=-\theta_2\frac{m_{\eta'}^2+m_\eta^2-2m_\pi^2}{2\sqrt{2}(m_K^2-m_\pi^2)}=-0.017, \quad (19a)$$

$$\theta_2=\left[\frac{3}{2}\right]^{1/2}\frac{m_d-m_u}{a}$$
$$=-\left[\frac{3}{2}\right]^{1/2}\frac{m_{K^+}^2-m_{K^0}^2+m_{\pi^0}^2-m_{\pi^+}^2}{m_{\eta'}^2+m_\eta^2-2m_K^2}=+0.009, \quad (19b)$$

$$\theta_3=\frac{1}{2}\arctan\frac{-4\sqrt{2}m_s}{3a-2m_s}$$
$$=\frac{1}{2}\arctan\frac{-4\sqrt{2}(m_K^2-m_\pi^2)}{3m_{\eta'}^2+3m_\eta^2+2m_\pi^2-8m_K^2}=-0.31, \quad (19c)$$

where we also give the results when expressed in terms of squared pseudoscalar masses and with their numerical values. The latter agree very well with the computer diagonalizations of Ref. [23]. Of these, the formula for θ_3 is well known, but θ_1 and θ_2 have not appeared in the literature, although the scheme for the mass matrix is well known (see, e.g., Ref. [23]). Phenomenological consequences of the isospin mixing have been discussed by many authors [24–29]. These estimates of θ_1 and θ_2 are also in reasonable agreement with a phenomenological analysis using data from the decays $\eta\to 3\pi^0$, $\eta'\to 3\pi^0$, $\eta'\to\eta 2\pi^0$, and $\psi'\to\psi\pi^0$ from which it is found, within a model [29], $\langle\pi^0|H|\eta\rangle=-0.0059$ GeV^2 and $\langle\pi^0|H|\eta'\rangle=-0.0055$ GeV^2 with 10% errors. These correspond to $\theta_1=-0.021$ and $\theta_2=+0.006$ with the same type of error.

In the limit of the large anomaly term $[m_{\eta'}\to\infty$ whereby also $\eta\to\eta_8$ and $m_\eta^2=(4m_K^2-m_\pi^2)/3]$ the expression for θ_1 reduces to the well-known formula of U-

spin invariance [30] giving

$$\theta_1 \to \theta_1^{U\ \mathrm{spin}} = -\frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s} = \frac{m_{K^+}^2 - m_{K^0}^2 + m_{\pi^0}^2 - m_{\pi^+}^2}{\sqrt{3}(m_{\eta_8}^2 - m_\pi^2)} = -0.011 \ . \quad (20)$$

In the same limit (keeping $m_s \gg m_u, m_d$), $\theta_3 \to -2\sqrt{2}m_s/a \simeq -2\sqrt{2}m_K^2/(3m_{\eta'}^2)$ and one sees that the $\sin\theta_3$ term of Eq. (13) also cancels to first order the $\frac{1}{2}g_A^8$ term of Eq. (12), i.e., in this limit, the $SU(3)_F$ part cancels to first order just as the isospin part. This can be seen explicitly from

$$-\frac{\sqrt{3}}{2M_N}(f_\eta g_{\eta NN})\sin\theta_3 \frac{\Delta m_{\eta'}^2}{m_\eta^2} \to \frac{\sqrt{3}}{2M_N}\left[2M_N g_A^8 \frac{1}{\sqrt{6}}\right]\frac{2\sqrt{2}(m_K^2 - m_\pi^2)}{3m_{\eta'}^2}\frac{\Delta m_{\eta'}^2}{m_\eta^2} \simeq g_A^8 \frac{2(m_K^2 - m_\pi^2)}{3m_\eta^2}$$

$$= g_A^8 \frac{2(m_K^2 - m_\pi^2)}{(4m_K^2 - m_\eta^2)} \simeq \frac{1}{2}g_A^8 \ .$$

Turning back to our original problem and Eqs. (11a)–(11c) or (11′) we see that these equations are diagonalized by the same Ω:

$$2M_N \begin{bmatrix} g_A^3/\sqrt{2} \\ g_A^8/\sqrt{6} \\ \Delta\Sigma/\sqrt{3} \end{bmatrix} = \Omega^{-1} \begin{bmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\eta^2 & 0 \\ 0 & 0 & m_{\eta'}^2 \end{bmatrix} \frac{1}{\alpha} \begin{bmatrix} \nu_\pi \\ \nu_\eta \\ \nu_{\eta'} \end{bmatrix} , \quad (21)$$

where the ν's on the RHS transformed by the same Ω, are the pseudoscalar densities, which in the pole approximation can be replaced by only one physical pole. In order to obtain the Goldberger-Treiman relation for g_A^3 with mixing it has to satisfy

$$\Omega \frac{1}{\alpha} \begin{bmatrix} \nu_3 \\ \nu_8 \\ \nu_1 \end{bmatrix} = \frac{1}{\alpha} \begin{bmatrix} \nu_\pi \\ \nu_\eta \\ \nu_{\eta'} \end{bmatrix} \simeq \begin{bmatrix} f_\pi g_{\pi NN}/m_\pi^2 \\ f_\eta g_{\eta NN}/m_\eta^2 \\ f_{\eta'} g_{\eta' NN}/m_{\eta'}^2 \end{bmatrix} , \quad (22)$$

where $g_{\pi NN}$, etc., are the physical coupling constants after mixing is taken into account.

Now we can show that in the same one-pole approximation the difference of $\Delta\Sigma$ and $\overline{\Delta\Sigma}$ in the third row of Eq. (21) reduces to the contribution from g_{QNN} only [Fig. 2(b)]. For this, it is enough to observe that, from Eq. (22), assuming that $f_\pi = f_\eta = f_{\eta'} = f$, one gets

$$\frac{1}{\alpha}\nu_1 = f\left[\frac{g_{\eta' NN}}{m_{\eta'}^2}\cos\theta_3 - \frac{g_{\eta NN}}{m_\eta^2}\sin\theta_3 \pm \frac{g_{\pi NN}}{m_\pi^2}\theta_2\right] .$$

So this relation which fixes α also leads to the cancellation of the first term of G [see Eq. (8) with the substitution (13)] which appears in the definition of $\overline{\Delta\Sigma}$. Thus the three Goldberger-Treiman relations, with $SU(3)_F$ breaking taken into account, can be written as

$$\Omega \begin{bmatrix} g_A^3/\sqrt{2} \\ g_A^8/\sqrt{6} \\ \Delta\Sigma/\sqrt{3} + f\Delta m_{\eta'} g_{QNN}/2M_N \end{bmatrix} = \frac{f}{2M_N}\begin{bmatrix} g_{\pi NN} \\ g_{\eta NN} \\ g_{\eta' NN} \end{bmatrix} , \quad (23)$$

where all quantities in the RHS include the π^0-η-η' mixing. The transition from the exact $SU(3)_F$ to broken $SU(3)_F$ is seen to be a very smooth one, provided Ω is $\simeq \mathbb{1}$ as is physically the case.

Note that, by separating off the quark and the anomaly terms, one obtains similar cancellations as in Eq. (16) for off-diagonal terms of $\langle Q \rangle$ and $\langle A \rangle$ in Eqs. (17) and (18). Some of these terms are enhanced by large ratios such as $m_{\eta'}^2/m_\pi^2$ coming from the ν's.

Now, using the third row of this equation one can obtain

$$\Delta\Sigma = \frac{\sqrt{3}f(g_{\eta' NN}/\cos\theta_3 - \Delta m_{\eta'} g_{QNN})}{2M_N} - \frac{g_A^8}{\sqrt{2}}\tan\theta_3 \pm \left[\frac{2}{3}\right]^{2/3} g_A^3(\theta_2 - \theta_1\tan\theta_3) , \quad (24)$$

or up to first order in the mixing angles

$$\Delta\Sigma = \frac{\sqrt{3}f(g_{\eta' NN} - \Delta m_{\eta'} g_{QNN})}{2M_N} - \frac{g_A^8}{\sqrt{2}}\theta_3 \pm \left[\frac{2}{3}\right]^{2/3} g_A^3\theta_2 \ . \quad (24')$$

Several comments are now in order. First, all large symmetry-breaking corrections are canceled and only a small one proportional to the θ_j's survives. The transition from exact $SU(3)_F$ to broken $SU(3)_F$ is a smooth one provided one simultaneously takes into account the nondegenerate m_q and the π^0-η-η' mixing. Such cancellations within another context were already discussed long ago [24,25] and recently also for the pion mass [31]. For the pion mass the cancellation can easily be seen in our framework directly from the pseudoscalar mass matrix. Only if one insists in separating, in a pure $SU(3)_F$ reference frame, a purely singlet gluonic quantity from the quark mass contributions, one does get large cancellations of terms of order $(m_u - m_d)/(m_u + m_d)$ and $\theta_2 m_{\eta'}^2/m_\pi^2$. Physically this means only that the small gluonic admixture in the pion gives a large contribution in the singlet channel because of the smallness of m_π^2 compared to $m_{\eta'}^2$. This is, however, a very natural contribution and the transition to exact $SU(3)_F$ is a very smooth one, due to the cancellation with quark mass terms. On the other hand, if one neglects the mixing keeping only the nondegenerate m_q's one gets a large unphysical isospin and $SU(3)_F$-breaking terms and one can even get a gluonic contribution of the wrong sign [32].

Second, in the limit of $m_u, m_d \ll \Delta m_{\eta'}, m_s$, $\theta_1, \theta_2 \to 0$ but $\theta_3 \neq 0$, Eq. (24′) reproduces the result of Veneziano [15] (up to the difference in LHS discussed in Sec. II) only if the first-order correction from θ_3 is also neglected. (Recall that $g_{\eta'_0 NN} = g_{\eta' NN} - \Delta m_{\eta'} g_{QNN}$.) The source of the difference could be in a different one-meson approximation. Instead of the Nambu-Goldstone-boson poles of Veneziano we have used the physical meson poles.

Third, there is a difference in $\Delta\Sigma$ for the proton and neutron in Eq. (24′). However, it is so small that it can produce the breaking of the Bjorken sum rule, only in the fourth digit.

IV. DISCUSSION AND CONCLUDING REMARKS

In this paper we have tried to elucidate that our choice of including the axial anomaly into the parton subprocess rather than into the quark spin distribution function is more of a physical than a mathematical problem. The main physical argument is that the anomaly is connected with the spin asymmetry for $q\bar{q}$ production in polarized photon-gluon scattering when the gluon is on the mass shell and the mass of the quarks is neglected. It is natural that the contribution of such a process is proportional to the difference $\Delta g(x)$ of the number of gluons polarized in opposite directions, with respect to the proton spin.

One of the main results of this paper is a clear understanding of how Δg is related to the matrix element of the gauge-invariant axial gluon current $\tilde{K}_\nu$ in QCD. These quantities are proportional to each other in perturbative QCD. However, beyond perturbative theory $\langle \tilde{K}_\nu \rangle$ contains an additional contribution from the zero-mass ghost pole relevant to the resolution of the U(1) problem in QCD. It is precisely this pole which helps resolve the contradiction between the QCD modified parton model and the OPE. One important result of this paper is the proof that the forward matrix element of $\tilde{K}_\nu$ is, in fact, gauge invariant. Thus, the main objection against the picture where gluons contribute to the EMC result, i.e., that the separation of the quark and gluon contributions is gauge dependent, is no longer valid.

The same ghost contribution plays also the main role in obtaining the generalization of the Goldberger-Treiman relation to the flavor-singlet channel. Another new result of this paper is a detailed calculation of the corrections from finite quark masses and from η'-η-π^0 mixing. Although both corrections are larger, they cancel each other to first order in the expression for $\Delta\Sigma$ and only a modest correction (~ 0.2) remains. As a side product of this analysis, we also found new simple algebraic expressions for the π^0-η and π^0-η' mixing angles [Eqs. (19a) and (19b)] which include the effect of the anomaly pole contribution to the singlet mass. After this work was completed, there appeared another paper [33] which also discusses this problem, although less generally, and within another framework, but with partly similar conclusions.

Concerning the evaluation of $\Delta\Sigma$ by means of the GGT relation [see Eq. (24)], unfortunately neither $g_{\eta' NN}$ nor g_{QNN} are well enough known experimentally. There are two sources of information on the value of $g_{\eta' NN}$. The first comes from NN scattering phase shift analysis within the framework of the one-boson-exchange potential (OBEP) model [34]. It gives $g_{\eta' NN} \simeq 7.3$ with presumably a large error. The second comes from $\eta' \to 2\gamma$ decay calculated through the baryon triangle loop contribution [35] and it gives $g_{\eta' NN} = 6.3 \pm 0.4$. Both estimates are close to the SU(6) value [36] $g_{\eta' NN} = 6.5$. However in the OBEP analysis of NN scattering, no ghost pole exchange was taken into account. This contribution would generate a contact NN interaction and for small squared momentum transfer $|t| \ll m^2_{\eta'}$, it would change the $g_{\eta' NN}$ obtained to $\sqrt{g^2_{\eta' NN} - m^2_{\eta'} g^2_{QNN}}$. If the second term is large the two methods would give two fairly different values. So it seems reasonable to assume that $m_{\eta'} g_{QNN} \ll g_{\eta' NN} \simeq 6.3$ (the sign is unknown and assumed to be positive). Using this together with the values $g^8_A = 0.68$, $g^3_A = 1.254$, and $f = f_\pi = 132$ MeV, one obtains, from Eq. (24′),

$$\Delta\Sigma = 0.92 \pm 0.06 \ . \tag{25}$$

Of course, we are aware of the fact that both sources are uncertain and therefore this value of $\Delta\Sigma$ is questionable. In order to obtain some new information on both couplings constants, new experiments are necessary. One of the experiments would be the energy behavior of $A_{LL}(s,t)$, the double helicity asymmetry near the forward direction, in proton-deuteron (or deuteron-deuteron) elastic scattering in the Regge region ($s \gg t$). The ghost exchange in this region will correspond to a $j=0$ fixed singularity which is not Reggeized, in contrast with η', η exchanges. Assuming the Pomeron-nucleon vertex does not contain a spin-flip part and since ρ and π trajectories do not contribute for an isoscalar target (and/or beam), the main contribution comes from the ghost-η and η' exchanges. Because of the fact that the intercepts of the η and η' trajectories are below $j=0$, the ghost exchange must dominate and we would expect a non-Regge behavior of A_{LL} if g_{QNN} is large enough. However, a significant energy interval is necessary to check this.

The same ghost contribution to $\langle \tilde{K}_\nu \rangle$ has also helped us in understanding [37] the true meaning of the result of Mandula on a lattice simulation for this matrix element [38]. The smallness of the number he obtained implies, in fact, not necessarily that the gluon contribution is small but rather that it is canceled by the ghost contribution. In fact two different matrix elements involving $\tilde{K}_\nu$ were computed [39], namely $\langle s', 0 | \tilde{\mathbf{K}} \cdot \mathbf{s} | 0, s \rangle$ and

$$\lim_{p \to 0} \frac{s \cdot p}{p^2} \langle s', 0 | \partial_\nu \tilde{K}_\nu | \mathbf{p}, s \rangle$$

which lead to the following bounds:

$$|\Delta\tilde{g} - \tfrac{1}{3} G/2M_N| < \tfrac{1}{20} \quad \text{and} \quad |\Delta\tilde{g} - G/2M_N| < \tfrac{5}{20} \ . \tag{26}$$

respectively.

If one takes these results at face value, firstly it is clear that the ghost contribution G is not negligible compared to $\Delta\tilde{g}$ and secondly, due to the Adler-Bardeen relation, i.e., by using Eq. (9), the second bound reads

$$|\Delta\Sigma-\Delta\tilde{g}|<0.25 ,$$

which is in agreement with the EMC experimental result. However, by combining the two bounds of Eq. (26) one gets

$$|\Delta\Sigma|<0.45 \quad \text{and} \quad |\Delta\tilde{g}|<0.20 . \qquad (26')$$

This means that both $\Delta\Sigma$ and $\Delta\tilde{g}$ are small, $\Delta\Sigma$ being less than half our previous estimate Eq. (25). Nevertheless it is not clear at all that the lattice size was large enough to take into account long-range instantons which build the contribution to G. So more accurate computations are certainly required in order to get a more truthful conclusion.

Finally if one tries to use a reliable parametrization of $\Delta g(x)$ convoluted with $\sigma(\gamma^* g\to q\bar{q})$ to fit the x dependence of the EMC data, one finds that either the gluon contribution lies mainly in a region below the existing data ($x<0.01$) [9,40] or one needs some contribution from the strange quark [13]. However, this last possibility of a substantial Δs seems to be excluded from a very recent analysis [41] of the Chicago-Columbia-Fermilab Rochester (CCFR) data which has measured accurately the strange-quark content of the nucleon.

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