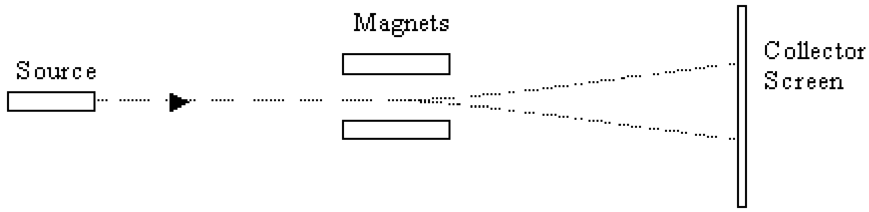
9.4 Spin and Polarization

The spin of a particle is quantized, so when we make a measurement at any specific angle we get only one of the two results UP or DOWN. This was shown by the famous Stern/Gerlach experiment, in which a beam of particles (atoms of silver) was passed through an oriented magnetic field, and it was found that the beam split into two beams, one deflected UP (relative to the direction of the magnetic field) and the other deflected DOWN, with about half of the particles in each.



This behavior implies that the state-vector for spin has just two components, v_{UP} and v_{DOWN}, for any given direction v. These components are weighted and the sum of the squares of the weights equals 1. (The overall state-vector for the particle can be decomposed into the product of a non-spin vector times the spin vector.) The observable "spin" then corresponds to three operators that are proportional to the Pauli spin matrices:

$$S_x = i\hbar \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad S_y = i\hbar \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad S_z = i\hbar \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

These operators satisfy the commutation relations

$$[S_x, S_y] = i\,\hbar\, S_z \qquad [S_x,S_z] = i\,\hbar\, S_y \qquad [S_y,S_z] = i\,\hbar\, S_x$$

as we would expect by the correspondence principle from ordinary (classical) spin. Not surprisingly, this non-commutation is closely related to the non-commutation of ordinary spatial rotations of a classical particle, in the sense that they're both related to the cross-product of orthogonal vectors. Given an orthogonal coordinate system [x,y,z] the angular momentum of a classical particle with momentum $[p_x, p_y, p_z]$ is (in component form)

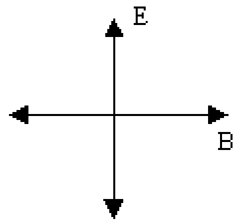
$$H_x = y\,p_z\ 0\ z\,p_y \qquad H_y = z\,p_x\ 0\ x\,p_z \qquad H_z = x\,p_y\ 0\ y\,p_x$$

Guided by the correspondence principle, we replace the classical components p_x, p_y, p_z with their quantum mechanical equivalents, the differential operators $-i\hbar$ d/dx, $-i\hbar$ d/dy,

$-i\hbar$ d/dz, leading to the S operators noted above.

Photons too have quantum spin (they are spin-1 particles), but since photons travel at the speed c, the "spin axis" of a photon is always parallel to its direction of motion, pointing either forward or backward. These two states correspond to left-handed and right-handed photons. Whenever a photon is absorbed by an object, an angular momentum of either +h/2s or -h/2s is imparted to the object. Each photon "in transit" may be considered to possess, in addition to its phase, a certain propensity to exhibit each of the two possible states of spin when it interacts with an object, and a beam of light can be characterized by the spin propensities (polarization) and phase relations of its constituent photons.

Polarization behaves in a way that is formally very similar to the spin of massive particles. In a sense, the Schrodinger wave of a photon corresponds to the electro-magnetic wave of light, and this wave is governed by Maxwell's equations, which tell us that the electric and magnetic fields oscillate transversely in the plane normal to the direction of motion (and perpendicular to each other). Thus a photon coming directly toward us "looks" something like this:



where E signifies the oscillating electric field and B the magnetic field. (This orientation is not necessarily fixed - it's possible for it to rotate like a windmill - but it's simplest to concentrate on "plane-polarized" photons.) The photon is said to be polarized in the direction of E.

A typical beam of ordinary light has photons with all different polarizations mixed together, but certain substances (such as calcite crystals or a sheet of Polaroid) allow photons to pass through only if their electric field is oscillating in one a particular direction. Therefore, when we pass a beam of light through a polarizing material, the light that passes through is "polarized", because all the photons have their electric fields aligned.

Since only photons with one particular alignment are allowed to pass, and since the incident beam has photons whose polarizations are distributed uniformly in all direction, one might expect to find that only a very small fraction of the photons would pass through a perfect polarizing substance. (In fact, the fraction of photons from a uniform distribution with polarizations *exactly* aligned with the polarizing axis of the substance should be vanishingly small.) However, we actually find that a sheet of Polaroid cuts the intensity of an ordinary light beam about in *half*. Just as in the Stern/Gerlach experiment with massive particles, the Polaroid sheet acts as a measurement for each photon, and gives one of two answers, as if the incoming photons were all polarized in one of just two directions, exactly parallel to the polarizing axis of the substance, or exactly perpendicular to it. This is analogous to the binary UP/DOWN results for spin-1/2 particles such as electrons.

If we place a second sheet of Polaroid behind the first, and orient its axis in the same direction, then we find that *all* the light which passes through the first sheet also passes through the second. If we rotate the second sheet it will start to cut down on the photons allowed through. When we get the second sheet axis at 90 degrees to the first, it will essentially block all the photons. In general, if the two sheets (i.e., measurements) are oriented at an angle of t relative to each other, then the intensity of the light passing all the way through is $I \cos(t)^2$, where I is the intensity of the original beam.

The thickness of the polarizing substance isn't crucial (assuming the polarization axis is perfectly uniform throughout the substance, because the first surface effectively "selects" the suitably aligned photons, which then pass freely through the rest of the substance. The light emerging from the other side is plane-polarized with half the intensity of the incident light. On the other hand to convert circularly polarized incident light into plane-polarized light *of the same intensity*, the traditional method is to use a "quarter-wave plate" thickness of a crystal substance such as mica. In this case we're not masking out the non-aligned components, but rather introducing a relative phase shift between them so as to force them into alignment. Of course, a particular thickness of plate only "works" this way for a particular frequency.

Incidentally, most people have personal "hands on" knowledge of polarized electromagnetic waves without even realizing it. The waves broadcast by a radio or television tower are naturally polarized, and if you've ever adjusted the orientation of "rabbit ears" and found that your reception is better at some orientations than at others (for a particular station) you've demonstrated the effects of electromagnetic wave polarization.

It may be worth noting that light polarization and photon spin, although intimately related, are not precisely synonymous. The photon's spin axis is always parallel to the direction of travel, whereas the polarization axis of a wave of light is perpendicular to the direction of travel. It happens that the polarization affects the behavior of photons in a formally similar way to the effect of spin on the behavior of massive particles. Polarization itself is often not regarded as a quantum phenomenon, and it takes on quantum behavior only because light is quantized into photons.

Regarding the parallel between Schrodinger's equations and Maxwell's equations, it's interesting to draw the further parallel between the real/imaginary complexity of the Schrodinger wave and the electric/magnetic complexity of light waves.

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