"Visualization in Curved Spacetimes: I. Visualization of Objects via Fourdimensional Ray-Tracing"

Hans-Peter Nollert
Computational Physics, Institut für Astronomie und Astrophysik, Universität Tübingen, Auf der Morgenstelle 10, 72076 Tübingen
nollert@tat.physik.uni-tuebingen.de

U. Kraus
Computational Physics, Institut für Astronomie und Astrophysik, Universität Tübingen, Auf der Morgenstelle 10, 72076 Tübingen

H. Ruder
Computational Physics, Institut für Astronomie und Astrophysik, Universität Tübingen, Auf der Morgenstelle 10, 72076 Tübingen

March 28, 2000

Abstract

A possibility for the visualization of arbitrary objects in curved spacetimes is the computation of an image as a realistic observer would see it. We present this approach first in the context of special relativity. In particular, we discuss the 'apparent geometry' of a moving object and the effects on the specific intensity that is seen by the observer. We examine the possibility of using polygon shading as an alternative technique which is simpler than a full ray-tracing approach. In general relativity, no such shortcut is possible, and we have to explicitly integrate the paths of all photons reaching the observer. The resulting image is therefore determined not only by the object itself, but also by the spacetime surrounding this object. In addition, the observer may now have to be described in a general relativistic context. We also discuss the possibility of including general relativistic ray-tracing in conventional ray-tracing software. As examples, we present pictures of a thin disc around a Kerr black hole and of Einstein rings. We also mention astrophysical consequences of the distortion of images as they occur for light curves of X-ray pulsars or spectra measured for accretion disks around compact objects. Examples for pictures and animations can be found on the world wide web, our home page is http://www.tat.physik.uni-tuebingen.de.

1 Introduction

Living in an essentially flat spacetime, we have not had the possibility to develop an intuition for the interpretation of perceptions, in particular of visual impressions, of objects in curved spacetimes. This is a major stumbling block for an intuitive grasp of the meaning of results in General Relativity. This is true for people who are not trained in General Relativity, but to some extent, it also concerns the hard-core relativist.

One possibility to train our intuition is using computers to model objects in curved spacetimes and to create images of them as we would see them if we either had sufficiently large telescopes, or could get close enough ourselves, to actually look at them in nature. Using this approach on simple objects, such as spheres, cubes, rings, etc., can help us train our intuition for interpreting such images. Using it on actual results of computations in numerical relativity will then enable us to interpret the results we have obtained, or they can help us in the diagnostics of the programs we develop.

2 Ray-Tracing Special Relativity

We will actually start with special relativity, i.e. the visualization of objects in flat, fourdimensional spacetime. We will see that there are many basic concepts as well as technical difficulties which can be discussed in this somewhat simpler setting, without obscuring them by the additional difficulties introduced by general relativity.

In fact, it is far from trivial to get the correct idea what an object moving at relativistic speed will actually look like to an observer. Einstein [1] himself does not seem to have realized the difference between measuring an object in a moving frame of reference, and looking at it. Gamov [2] actually gave a wrong description of what the world around us would look like if the speed of light were much lower than it really is.

If images are actually obtained by ray-tracing, then the conceptual difference to 'conventional' ray-tracing in Newtonian space is not really dramatic. Photons still travel on straight lines, but since they now travel at a finite velocity, we have to keep track of time while tracing the rays and looking at their intersections with objects in the scene. Of course, an appropriate Lorentz transformation must be used to obtain directions, specific intensities, etc. in the rest frame of the objects. This approach has been used to create images of several geometrical objects moving at relativistic speeds [3].

2.1 Geometrical appearance

Complete ray-tracing can give us a realistic image with all relevant effects included, but it is very expensive in terms of computational resources. If we are mainly interested in the geometry of the scene, we may settle for a simpler and much more efficient approach, i.e. polygon shading. In this technique, objects are described as a collection of luminous polygons, excluding exterior light sources. Efficient algorithms are available for projecting these polygons onto the screen, determining obstructions from view by intervening polygons, and for finding their shade for the resulting picture. Some of these functions may even be performed by specialized hardware, resulting in considerably higher speed for the whole procedure.

In a sense, we settle for a diagram, rather than a realistic image, of the scene. However, the threedimensional structure can be emphasized by using texture and non-isotropic emission characteristic on the surfaces. This can also recreate an impression similar to exterior light sources, especially a diffuse, ambient sort of lighting.

However, this approach is essentially static: Light rays are not followed along their paths, the scene does not move itself, but the observer may regard it from different perspectives and distances. How can the effect of a finite speed of light, the fact that time plays an important role now, be incorporated into such an approach?

We will see that it is indeed possible to transform the given geometry of some scene into another one in such a way that the effects of motion relative to the observer, together with the finite speed of light, can be included in this static approach. This will permit the use of polygon-shading at least for some visualization problems in special relativity.

2.1.1 Hyperbolic transformation, apparent positions

Imagine, say, a lattice, consisting of spheres and beams connecting them, passing over a camera. The camera shall take pictures at a very high shutter speed, such that the lattice moves very little during the time that the shutter is open for a given picture. We can therefore assume that the photons making up one picture have arrived at the camera at the same time. It is clear that photons coming from different points on the lattice had to be emitted at different instances in time, because they have a different distance to travel from their emission until reaching the camera. This is true both in the reference system of the lattice and in that of the observer. Using the Lorentz transformation and the traveling time for photons coming from different points on the lattice, we can then compute the points in space where the photons making up the picture have been emitted. The results for different speeds have been given in [4], they are shown in Fig. 1.

A thin rod traveling straight towards the observer, with its axis along the direction of travel, will appear elongated while it is approaching, and strongly contracted when it is receding. If it is aligned perpendicularly to the line of travel, it will assume the shape of a hyperbola. The shape of any other object can be transformed in the same way, of course, if we regard the lattice as a coordinate system for this object.

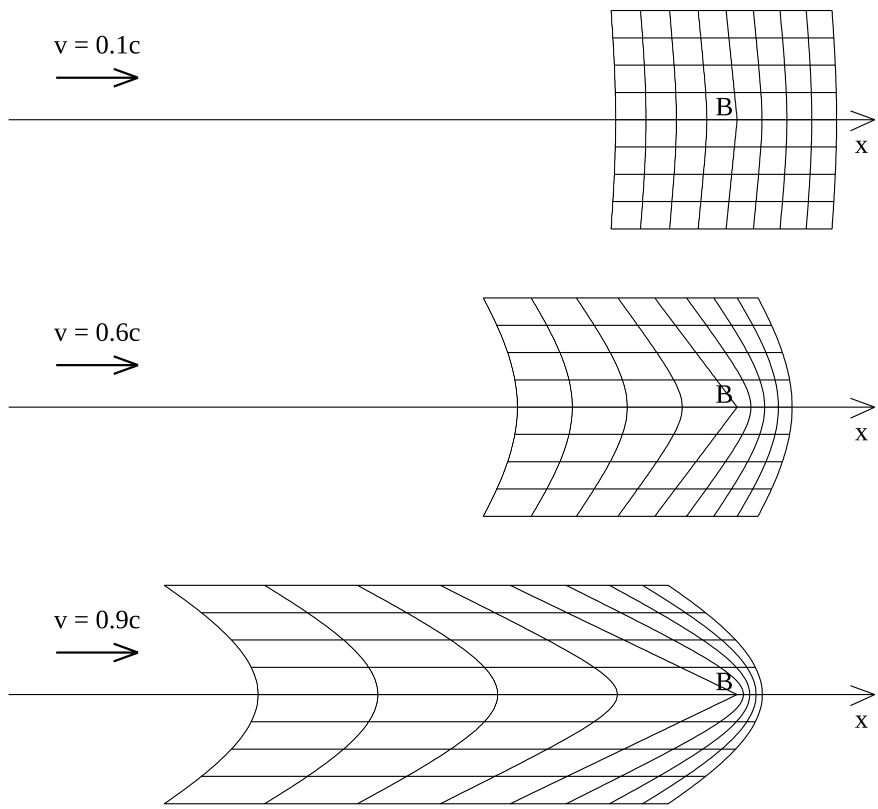

Figure 1: Apparent shape of a lattice passing over an observer B at different speeds.

The general formula for this transformation is

$$
\begin{aligned}
x &= \gamma\left(x' - \beta\sqrt{\rho'^2 + x'^2}\right) = \gamma\left(x' - \beta r'\right) \\
\rho &= \rho' \ ,
\end{aligned}
\tag{1}
$$

where $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$; x and $\rho = \sqrt{y^2+z^2}$ are measured in the observer's frame, x' and ρ' in the rest frame of the lattice. The coordinate systems are aligned such that $x = 0$ coincides with $x' = 0$ for $\rho' = 0$; the observer is positioned at the center of his reference frame.

Note that a ruler aligned vertically to the direction of travel, given by $(x' = \text{const}, \rho')$, will appear to the observer as having the shape of a hyperbola (x, ρ):

$$
\frac{(x - \gamma x')^2}{\beta^2\gamma^2 x'^2} - \frac{\rho^2}{x'^2} = 1 \ . \tag{2}
$$

Given the description of some object in its rest frame, all we have to do now is to transform the positions of its defining points according to (1). The transformed object can then be used as input for a conventional rendering program to create pictures as seen by our hypothetical camera. Animated sequences may be produced as well if the transformation (1) is redone every time the position of the object relative to the camera changes. Note that the transformation also depends on the direction of travel relative to the direction of observation.

Two examples obtained with this technique are shown in Figs. 2 and 3. The apparent rotation of the cube is discussed further in section 2.1.3. Note that the back side (green color) of the Brandenburg gate is already visible while the camera is still inside the gate, facing forward.

2.1.2 Meaning of the apparent shape

We should pause for a moment and ask the question what the meaning of this apparent shape of the lattice, or any other object, really is. Let us first summarize what it is *not*:

- It does not show the positions of the points as *measured* in the observer's rest frame.
- It is not the lattice as we would actually see it from the side, i.e. from the perspective chosen for Fig. 1.
- It is not what the observer at B would actually see: A rod aligned with the direction of movement is seen as a point, a rod aligned perpendicularly to it is seen as a line.
- It is not what an intelligent observer would reconstruct from the image he sees: An intelligent observer knows about special relativity and, given enough information, reconstructs the true shape of the object in its own rest frame.

The apparent shape is the collection of points (in the rest frame of the observer) where any one of those photons has been emitted which make up the picture seen by the observer at a given instance. Since the emission is a spacetime event, it is possible to give this apparent shape a physical, observable meaning by the following construction:

Suppose we could identify each photon that enters the camera at a given moment, e.g. by giving each photon a unique frequency. Suppose we have filled the space that the lattice traverses with detectors which are at rest with respect to the observer. These detectors store the information which photons have been emitted in their vicinity. For each photon in the picture, we find the detector which saw it being emitted, and make this detector raise a flag. All the detectors with raised flags then make up the 'apparent shape' of the lattice which is depicted in Fig. 1. The observer could then leave his fixed position and

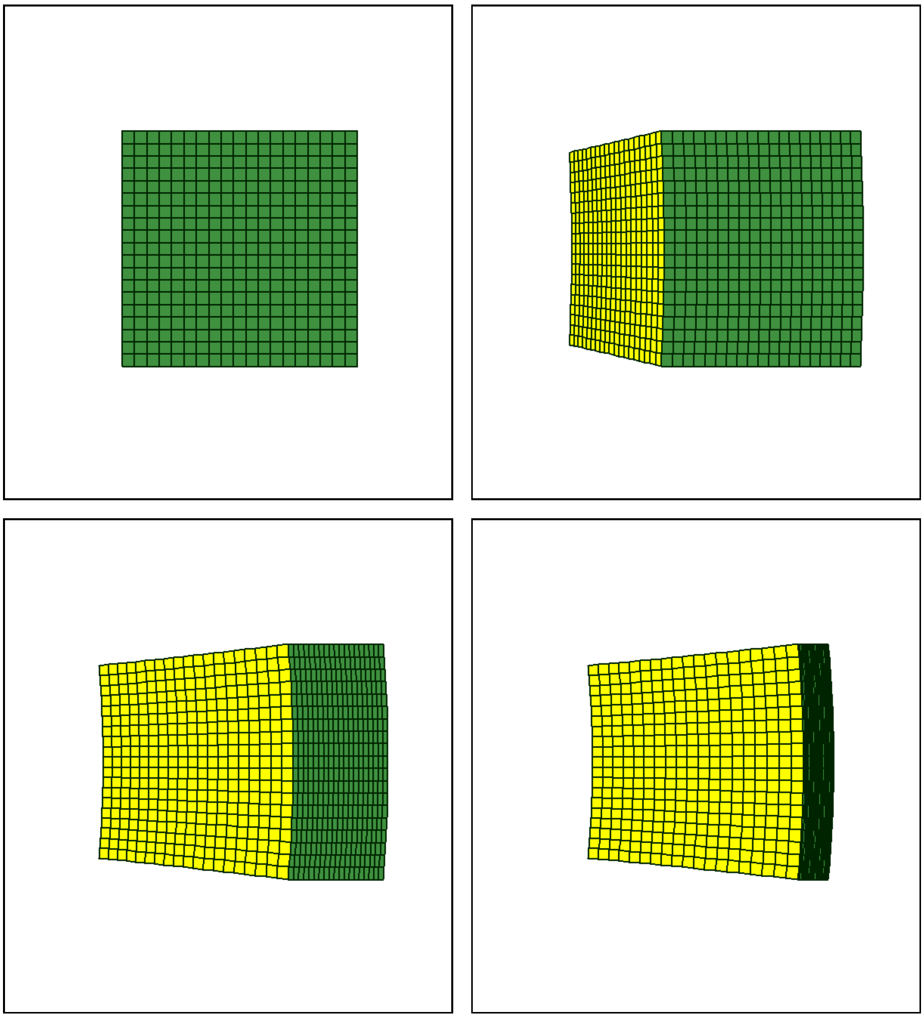

Figure 2: Pictures of a cube passing a camera non-centrally, taken when the cube appears at its closest distance to the camera. The side of the cube facing the camera is colored green, the back side (with respect to the direction of motion) is colored yellow. The velocity (from left to right and top to bottom) is $v \sim 0$, $v = 0.5c$, $v = 0.9c$, and $v = 0.99c$.

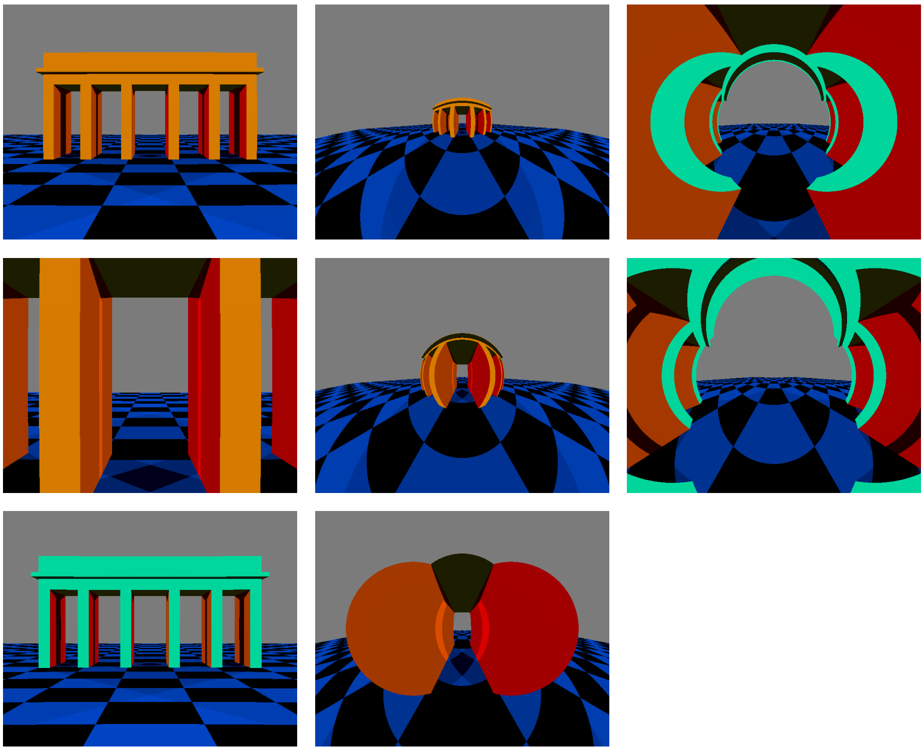

Figure 3: Passing through the Brandenburg gate (Berlin, Germany). The different sides of the gate have been given distinctive colors in order to identify them more easily. On the left, the camera is moving at a non-relativistic speed; the bottom picture shows a look backwards towards the back side of the gate. In the middle and on the right, the camera moves at $v = 0.99c$.

view this (static) arrangement of detectors from any position and any direction he chooses.

2.1.3 Projection and field of view

The process of projecting the threedimensional scene onto a twodimensional image can cause deceptions as well. In the scene (Fig. 2) where a cube passes the observer non-centrally at some distance, the impression is that the cube is not stretched or contracted, but mostly rotated. However, when we regard the cube as a part of the lattice in Fig. 1, we realize that it cannot be rotated. Rather, it appears sheared along the direction of travel. For the setup sketched in Fig. 4, Fig. 5 shows how the apparent shape of the cube (left), when projected on the image plane, gives the same picture as a cube which is not distorted, but merely rotated (right). The cube in Figs. 4 and 5 is assumed to be small relative to its distance from the camera. The moment of observation is chosen such that the apparent image of the cube is seen in a direction perpendicular to its direction of motion, but the same general argument applies for an arbitrary combination of directions.

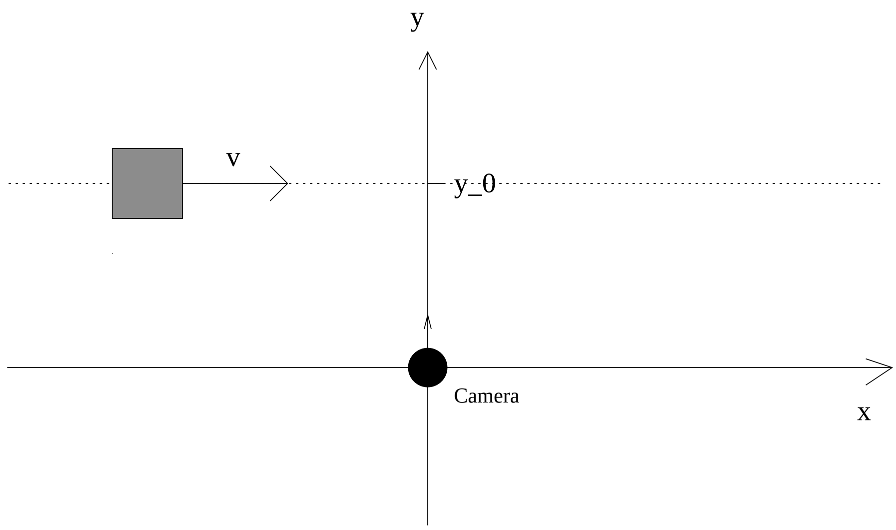

Figure 4: A cube passing a camera.

Finally, the field of view we use can have a profound influence: With a fish-eye lens, we will obtain effects which look very similar to the ones we have seen here. In order to exclude artificial effects caused by using an inappropriate perspective or camera size, one should always record a scene at non-relativistic speeds in order to use it as a standard against which the relativistic scene can

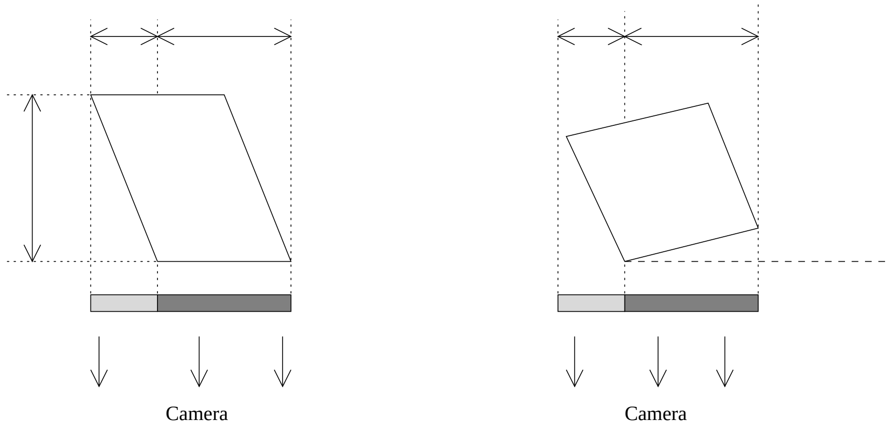

Figure 5: Projection effect for the cube passing a camera.

be judged.

2.2 Lighting

In the last section, we have discussed the geometry of the pictures that we obtain. In order to see anything, we need light. In order to obtain the pictures of the last section, we assumed that all objects in the scene are self-luminous, with an isotropic emission. All effects of objects being illuminated by other parts of the scene, or of the spectral shift and the intensity change of the emitted radiation due to the Doppler effect, have been neglected. For realistic (and more impressive) images, however, we will have to take into account the effects of exterior light sources as well as the Doppler effect on the radiation emitted or reflected by the scene.

2.2.1 Spectral shift

The relativistic Doppler effect shifts the frequency of emitted radiation for an approaching or receding object according to:

$$\nu/\nu_0 = \frac{1-\beta\cos\theta}{\sqrt{1-\beta^2}} \tag{3}$$

where θ is the angle between the direction of movement and the direction of observation. If the object is moving directly towards or away from the observer ($\theta = \pi$, 0), then

$$\nu/\nu_0 = \sqrt{\frac{1\pm\beta}{1\mp\beta}} \quad = \quad 2.000 \quad (0.5000) \qquad \beta = 0.60 \quad \text{app.} \quad \text{(rec.)}$$

$$
\begin{aligned}
&= 6.245 \quad (0.1601) \qquad \beta = 0.95 \quad \text{app. (rec.)} \qquad (4)\\
&= 14.11 \quad (0.0709) \qquad \beta = 0.99 \quad \text{app. (rec.)}
\end{aligned}
$$

If the object is emitting monochromatically somewhere in the visible range of the spectrum, then its image will be shifted towards the ultraviolet when it is approaching, and towards the infrared when it is receding. Even at a mildly relativistic speed, it may completely disappear from the visible range of the spectrum. In order to keep the object visible for the whole time, it has to have a continuous spectrum (e.g. a Planck spectrum) with considerable intensity in the ultraviolet and in the infrared.

2.2.2 Intensity shift

Along with the spectral shift due to the relativistic Doppler effect comes a change in the specific intensity. This change can easily be computed since

$$\frac{I_\nu}{\nu^3} = \text{const.} \tag{5}$$

is an invariant scalar along the path of any photon.

Using (5), in order to compare the brightness of an object when it is approaching vs. when it is receding, we have to keep in mind that we compare intensities at different *observed* frequencies, i.e. at those determined by (3). If we keep the observed frequency fixed, we may assume a spectrum which is flat over the relevant range, or we take into account how the (emitted) intensity depends on the frequency. Using the values of (4), we see that even for a mildly relativistic speed of $v/c = 0.6$, the ratio of the intensity of the approaching to that of the receding object is 64! This will make it impossible to display the intensity change realistically without losing almost all the resolution in brightness that a computer screen offers. The situation becomes worse for a higher speed, of course.

2.2.3 The influence of aberration

The direction of a plane wave emitted by a moving object is tilted towards the (forward) direction of motion. Therefore, radiation emitted (or reflected) by an object will be focussed towards the line of motion in the forward direction and spread away from it in the backward direction. This beaming results in a change of intensity in a given direction. This change, however, is already covered by (5). Therefore, all we need to know is the relativistic Doppler shift for a light ray reaching the observer from a given direction.

Conversely, if a light source is at rest with respect to the observer, then a moving object will see it closer to the forward direction of motion than it appears in the observer's frame. Within our approach of "hyperbolic transformation (1) + polygon shading", we can take this change of direction into account if we restrict ourselves to point sources at infinity: We just have to move each light source to the position it would have in the object's rest frame. The beaming of light emitted (or reflected) by the object, on the other hand, has to be included separately, even for an object which emits radiation isotropically in its rest frame. The invariant intensity (5) can be used for this purpose.

2.2.4 Obstructions and Shadows

Usually, parts of a scene are obstructed from view by other parts. In addition, one part of a scene might block light from an exterior light source from reaching another part of the scene. The 'transformation approach' we described in the last section will correctly include obstruction, but not shadows: shadows can only be treated correctly by a full ray-tracing approach. This is also true, of course, of light that is reflected by one object and illuminates another.

3 Ray-Tracing General Relativity

In special relativity, light rays can still be considered straight lines in flat spacetime. This is different in general relativity, and therefore, the paths of all photons reaching an observer will have to be integrated explicitly, using the geodesic equation. The resulting image is therefore determined not only by the object itself, but also by the spacetime surrounding this object.

A general relativist will probably be quite happy with the possibility of visualizing relativistic effects of the results of numerical calculations. A more astrophysically minded person might ask, however, if it would not be better to build telescopes powerful enough to actually look at an object like an X-ray pulsar in nature. In Table 1 we give the necessary aperture of an ideal telescope (one whose resolution is limited by diffraction) needed to resolve a ten kilometer object at various typical distances. It is obvious that such a telescope cannot be realized for objects which are outside our own solar system.

Also, current technology does not allow us to take a closer look by traveling to objects outside our own solar system, and this is likely to remain the same for at least several thousand years to come. Therefore, the computer is the only telescope, and the only spaceship, that will allow us to have a good look at, say, the X-ray pulsar Her X-1.

3.1 Requirements

The requirements for a universal general relativistic visualization code are the following:

We want to be able to work in any metric. In particular, this means a metric without any symmetry, which may also be time dependent. If the metric is the result of a numerical calculation, it may be given in terms of numerical data on a (possibly irregular) lattice.

Distance		Telescope aperture
20 000 km	Australia	1 mm
400 000 km	Earth – Moon	20 mm
80 million km	Earth – Mars	4 m
4 billion km	Earth – Neptune	200 m
4×10^{13} km = 4 ly.	Nearest star	2 000 km
400 light years	Cosmic neighborhood	200 000 km
12 000 light years	X-ray pulsar Her X-1	6 million km

Table 1: Telescope apertures necessary for a diffraction limited resolution of a ten kilometer object for some typical cosmic distances.

We want a resolution which corresponds to that of a standard computer monitor, i.e. about 1000^2 pixels. Since realistic, astrophysical systems will generally carry information about specific intensity, we should have about 3×8 bit color resolution. Luckily, the null geodesics of photons do not depend on the energy of the photon, so we don't have to compute several paths for one pixel. On the other hand, spectral changes due to gravitational redshift and Doppler shift have to be taken into account explicit For an animated sequence of, say, 24 frames per second and a duration of 1 minute, more than 10^9 light rays have to be integrated and intersections with objects in the scene have to be checked. To our knowledge, presently no such fully universal code exists. If it did, it would require too much CPU time to generate sequences in an acceptable time. Therefore, compromises based on symmetries have to be made and adaptive techniques have to be used in order to reduce the computational resources that are required. Since individual light rays don't influence each other, parallelization is also a promising possibility.

3.2 Geodesic equation

Photons follow null geodesics in the given spacetime:

$$\ddot{x}^{\mu} + \Gamma^{\mu}{}_{\nu\lambda}\dot{x}^{\nu}\dot{x}^{\lambda} = 0 \ . \tag{6}$$

The immediate consequence is that there are no simple 'tricks' such as the transformation (1) in special relativity any more. It will be necessary to explicitly follow the path of each photon. Therefore, algorithms like polygon shading will not be applicable any more, and a full ray-tracing, or at least ray-casting, approach is required.

A related problem concerns the definition and description of objects in the scene, rather than the light rays. As long as we are dealing with objects which are the result of some calculation, such as the surface of a neutron star, the description of it is coordinate invariant if the underlying problem is formulated correctly. However, in order to demonstrate the effect of a certain geometry, we may want to visualize simple objects, like spheres, cubes, etc. These objects have to be described in terms of some coordinates. However, an object which satisfies the equation for a sphere in one set of coordinates may not do so in another. It is thus necessary to construct a coordinate invariant description of the properties of the objects. In addition, something like a cube may even be impossible to construct in an arbitrary spacetime.

3.3 Camera

In a general relativistic framework, we should also consider the influence that the spacetime may have on the camera. In order to avoid unnecessary complications, we will assume the simplest possible camera, i.e. a pinhole camera. There are two basic possibilities for the location of the camera:

1) The camera is located in the asymptotically flat part of the spacetime.

For actual observations, this is clearly the most realistic possibility. It has the advantage that we do not need a general relativistic description of the camera. However, the angles which distinguish the different light rays making up the picture vanish in the asymptotic limit. This technical problem may be solved by using a large, but finite distance, or — more elegantly — by using other quantities, such as the impact parameter, to characterize light rays.

2) The camera is located near the source of the gravitational field.

This possibility is potentially more interesting, but now we need a fully relativistic description of the camera. In general, we can assume the camera to be small with respect to the length scale of the spacetime we are picturing. It will then fit into its own locally inertial frame, and in the case of acceleration, we can assume that it accelerates 'as a whole'. However, we will need to determine a local tetrad corresponding to the motion of the camera through the spacetime. All angles have to be measured with respect to this tetrad.

3.4 Realizing relativistic ray-tracing with conventional ray-tracing programs

There are two major modifications that have to be applied to conventional ray-tracing programs in order to handle relativistic spacetimes:

- In addition to the three space coordinate, we have to keep track of the time coordinate as a photon travels through spacetime.
- Light rays are now geodesics of the spacetime, rather than just straight lines.

The first modification is rather straightforward. In fact, for stationary spacetimes and stationary scenes, it may even be omitted. The second modification is much more demanding: Ray-tracing codes need efficient algorithms to determine intersections between light rays and objects in the scene. These algorithms rely heavily on the light rays being straight lines. Changing this is much more involved than the integration of the photon path itself, and it will considerably reduce the efficiency of the code.

It is therefore desirable to leave the intersection algorithms as they are, and approximate the light rays by segments of straight lines [7]. Fig. 6 demonstrates that even a strongly bent light ray can be approximated adaptively with only a few segments. These segments are then passed to the intersection algorithm.

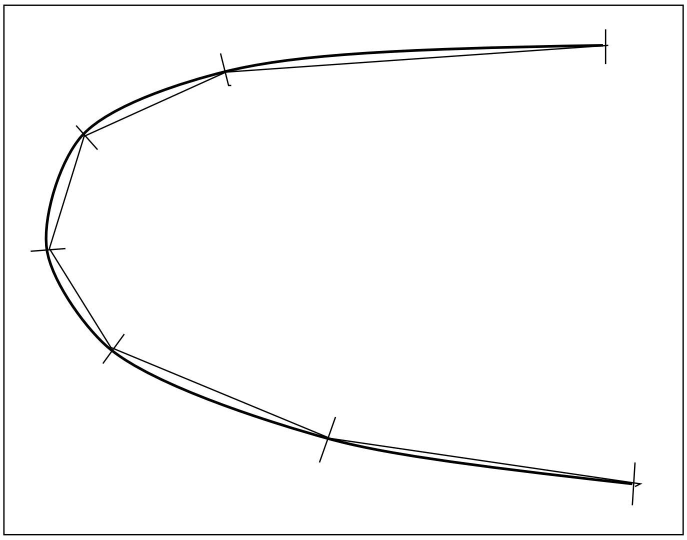

Figure 6: Approximating the path of a photon by straight line segments.

Figure 7 shows how relativistic visualization can be incorporated into a conventional ray-tracing program in such a way that the physicist using this software has to supply only information about the spacetime he is studying, without having to worry about the integration of the photon paths, the intersection with objects, or the rendering of the image.

The ray-tracer calls an interface which performs the integration of the light rays and adaptively converts them into straight line segments. Given the last position in terms of coordinates x_n, it returns the next position x_{n+1} such that the ray-tracer may assume that light travels in a straight line between x_n and x_{n+1}. In order to integrate the light ray (which should typically be done with a much smaller step size than the conversion to straight line segments), the interface calls a subroutine supplied by the user, passing the position and direction of the photon to the subroutine, and expecting the second derivative in return. This is just the information which can be provided using the geodesic equation (6).

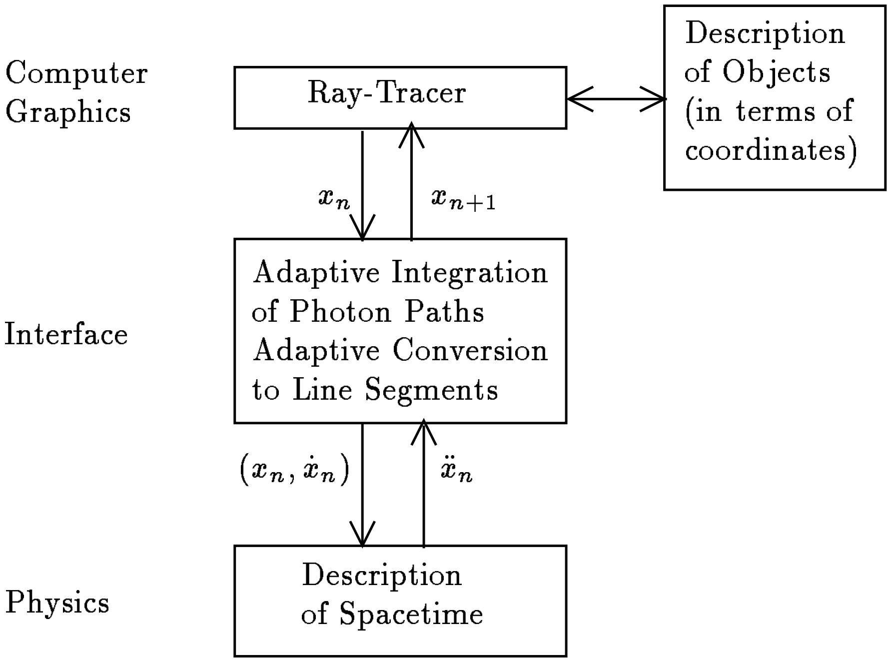

Figure 7: Including relativistic ray-tracing in a 'conventional' ray-tracing program.

3.5 Astrophysical Examples

Figure **??** in [9] shows a picture of the surface of a realistic, rotating neutron star, as seen by an asymptotic observer. Due to the rotation, the surface becomes oblate and is not spherically symmetric any more. However, its internal structure is still rotationally symmetric with respect to the rotation axis of the star. In the picture, however, the surface looks different on either side of the rotation axis. This effect is especially pronounced near the equator of the star. The reason is the different bending of light rays: Due to the dragging of inertial frames near the star, photons are deflected differently depending on whether they are traveling with or against the star's rotation.

Figure 8 shows a thin disc around a Kerr black hole. This arrangement can be regarded as a schematic representation of an accretion disc around a massive, rotating black hole, as they occur in active galactic nuclei. The disc is assumed to be rigid and to have negligible mass. The checkerboard pattern on the disc is not defined in a coordinate-independent way; rather, each patch covers a given range of ϕ in Boyer-Lindquist coordinates. For comparison, both the mass and the angular momentum of the black hole are set to zero in the upper left picture. The mass is nonzero in the upper right picture, the disc appears distorted and the Einstein ring, consisting of the indirect images, appears. In the lower left picture, the angular momentum becomes nonzero as well. The additional distortion due to frame dragging is clearly visible. The Einstein ring, however, disappears: Due to frame dragging, photons hit the disc again before being able to complete an orbit around the black hole. The picture on the lower right shows a close-up view of the central region.

A well-known consequence of gravitational lensing is the so-called Einstein ring: Due to symmetry, the image of an object right behind a gravitational lens will have the form of a ring in the image plane. In Fig. 9 we present the 'true' Einstein ring: Imagine a giant billboard at the end of the universe with a portrait of a famous physicist painted on it. While viewing this portrait, a black hole passes between us and the billboard.

While it is unlikely that this 'true' Einstein ring will ever be observed, images of galaxies distorted by gravitational lensing have actually been seen [10]. Figure 10 shows an image of the galaxy cluster Abell 2218 taken with the Hubble Space Telescope. Due to gravitational lensing, this galaxy cluster provides a powerful "zoom lens" for galaxies that are so far away they may not normally be observable with even the largest available telescopes. In particular, several hundred arclets can be identified. These are distorted images of a very distant galaxy population extending 5-10 times farther than the lensing cluster. In addition, Abell 2218 has a total of seven multiple images generated by gravitational lensing.

3.6 Astrophysical Application: Light curves of X-ray pulsars

Another consequence of relativistic light deflection is not directly related to visualization, but we want to point it out here because it is of great significance for astrophysics: The change of light curves of X-ray pulsars or of accretion discs around black holes.

X-ray pulsars consist of binary systems where one component is a magnetic neutron star. Matter is accreted from the companion star, it is eventually funneled, by the strong magnetic field, towards the magnetic poles where it forms two hot spots. Light deflection increases the fraction of the rotation period of the star where each of these hot spots remains visible, resulting in a reduced modulation of the light curve. Conversely, the interpretation of light curves of X-ray pulsars without taking light deflection into account results in improbably large hot spots (up to 60° half opening angle), which are needed to reduce the modulation [11]. We have shown that the same analysis, with light deflection included, yields hot spots with reasonable sizes (around 10° half opening angle) [12].

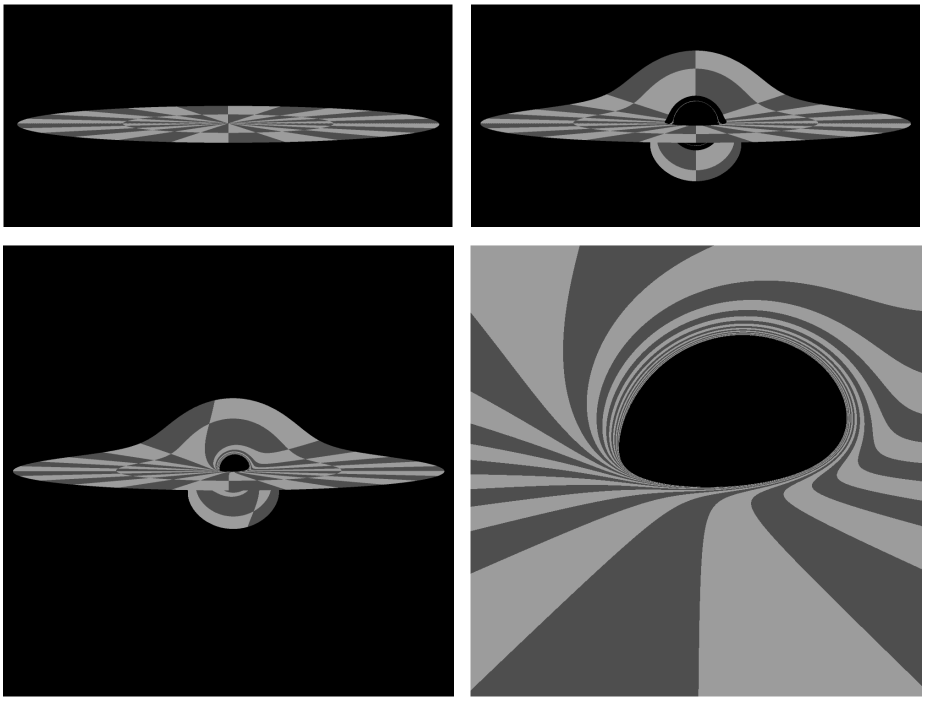

Figure 8: A thin disc around a black hole. Upper left: $M = 0$, $J = 0$. Upper right: $M \neq 0$, $J = 0$. Lower left: $M \neq 0$, $J \neq 0$. Lower right: close-up of the central region.

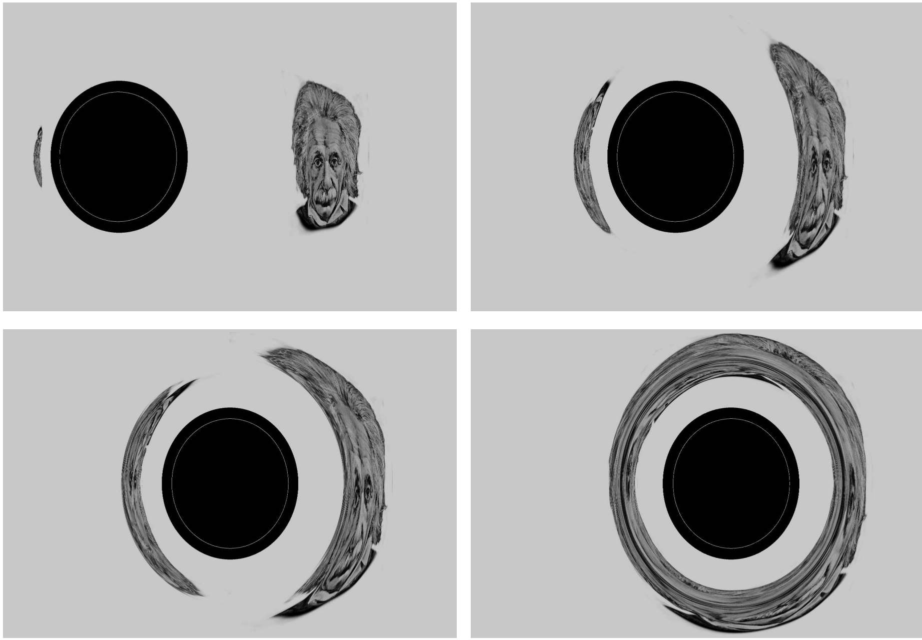

Figure 9: The 'true' Einstein ring.

4 Discussion

This approach of simulating 'realistic' images has, of course, advantages as well as drawbacks:

+ The resulting pictures are generally pretty and impressive.

+ They represent what an actual observer would see.

+ Therefore, they may help in the interpretation of actual measurements, such as light curves of X-ray pulsars.

+ Any physical system can be visualized, since the simulation corresponds to the process of just looking at something.

On the other hand:

– It may be very expensive to carry out, making compromises necessary.

– It may be hard to predict which conclusions somebody will draw from a given image.

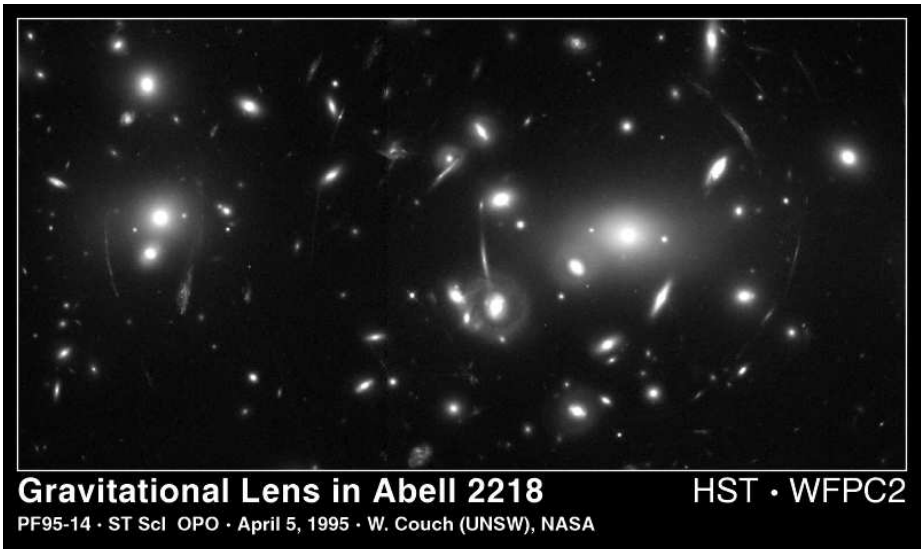

Figure 10: Gravitational lensing by the galaxy cluster Abell 2218. Image courtesy of W. Couch (University of New South Wales), R. Ellis (Cambridge University), and NASA.

– It may be hard to interpret because many effects are superimposed: The object itself, the metric surrounding it, projection effects, etc. (cf. the picture of the rotating neutron star).

– Drastic changes in specific intensity can make a realistic visualization impossible.

– Existing structure may be insufficient for visualization (e.g. the surface of a neutron star), thus artificial structure may have to be introduced.

– Invisible properties (magnetic fields, etc.), even though they can be included in the visualization, don't quite fit the concept of producing 'realistic' images.

Acknowledgement(s)

This work was partially supported by the Deutsche Forschungsgemeinschaft (DFG). A. Geyer [13] has computed the pictures for Fig. 8, C. Zahn [14] the pictures for Fig. 9. J. Kern supplied the pictures for Fig. 2, R. Rau and M.

Eichel those for Fig. 3. We also wish to thank R. Rau for many stimulating discussions.

References

[1] Einstein, A. (1905): Zur Elektrodynamik bewegter Körper. Ann. D. Phys. **17**, 891

[2] Gamov (1940): Mr. Tompkins in Wonderland.

[3] Hsiung, P.-K. and Dunn, R.H.P. (1989): Visualizing Relativistic Effects in Spacetime. Proceedings of Supercomputing '89, 597–605

[4] Ruder, H. and Ruder, M. (1993): Die Spezielle Relativitätstheorie. (Vieweg, Braunschweig)

[5] Viergutz, S.U. (1993): Image generation in Kerr geometry. I. Analytical investigation on the stationary emitter-observer probblem. Astron. Astrophys. **272**, 355

[6] Nemiroff, R.J. (1993): Visual distortions near a neutron star and black hole. Am. J. of Physics **61**, 619

[7] Gröller, E. (1995): Nonlinear ray tracing: visualizing strange worlds. The Visual Computer **11**, 263–274

[8] Quien, N., Wehrse, R., and Kindl, C. (1995): Licht auf Abwegen. Spektrum der Wissenschaft **Mai 1995**, 56–67

[9] Herold, H. (1995): 152th WE-Heraeus Seminar

[10] Fort, B. and Mellier, Y. (1994): Arc(let)s in clusters of galaxies. The Astron. Astrophys. Rev. **5**, 293–292

[11] Leahy, D.A. (1991): Modelling observed X-ray pulsar profiles. MNRAS **251**, 203–212

[12] Riffert, H., Nollert, H.-P., Kraus, U., and Ruder, H. (1993): Fitting Pulse Profiles of X-Ray Pulsars: The Effects of Relativistic Light Deflection. Ap. J. **406**, 185–189

[13] Geyer, A. (1992): Abstrahlung einer Akkretionsscheibe um ein rotierendes Schwarzes Loch. Diplomarbeit, Tübingen

[14] Zahn, C. (1991): Vierdimensionales Ray-Tracing in einer gekrümmten Raumzeit. Diplomarbeit, Stuttgart