Physical Quantities in Different Reference Systems According to Relativity

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The relativistic transformation of measurements from one observer S to another observer S′ is valid, of course, only when both observers are measuring the same physical quantity. What is the same quantity is, however, not a trivial problem in relativity. As a matter of fact, two different definitions have already been used in the literature, without realizing that they were not equivalent. Some of the pitfalls of such a confusion are discussed.

INTRODUCTION: AN APOLOGUE

A FEW days ago I called my friend, observer S′: "Hello, this is observer S speaking. I heard that you have been working on relativity recently. I just wondered whether you could help me in an experiment. Since you are moving away from me at constant velocity, wouldn't it be nice to check Einstein's theory once again? I know that you have very fine instruments aboard your spaceship, so why don't you check the formula $\lambda' = \lambda[(1+\beta)/(1-\beta)]^{\frac{1}{2}}$? I have just measured the color of my tie, a brilliant laser-green at exactly $\lambda = 5198.2$ Å. Why don't you measure λ'? . . . sure you can finish your other experiment first; I can certainly wait a couple of days . . . yes . . . yes. . . . That is precisely the same equipment I used . . . O.K., you'll call me back as soon as you get results? Fine . . . Good bye . . . bye. . . .

It was 3.15 a.m. on Monday when my phone rang. The voice from the other end was excited: "Sorry to wake you up so early, but . . . I just can't believe it . . . oh, by the way, this is S′ speaking . . . Einstein is wrong . . . is it you, S? Do you hear me? Einstein is wrong! No doubt about it. No, no, I am not kidding. I have never been so serious, S. Now, listen: you must be careful in giving the announcement. A letter to the *Phys. Rev. Letters* will do, but try to be somewhat vague, so that we can still change some details later on . . . yes, yes, let me tell you . . . I know it sounds incredible, but . . . I do find a redshift as expected, but the amount, gosh, . . . to make it short $[(1+\beta)/(1-\beta)]^{\frac{1}{2}}$ is one hundred times greater than given by relativity! Just to give you an idea of how much above experimental errors we are, let me tell you that I see a beautiful *yellow* tie. . . . Sure it will have great implication for the whole field of cosmology! How could one seriously believe in the Hubble redshift after our experiment? . . . Well, thank you . . . see you in Stockholm, then . . . bye–bye . . .

I called S′ after the press conference: "Just a pity—or good luck—you weren't here. Never seen such enthusiasm before! I was almost choked to death by fans trying to get a piece of my tie . . . well, anyhow, I was lucky enough to save a piece of my tie for the museum. . . . You never liked green ties? You prefer yellow ties? . . . Oh, come on, what do you mean we still have the yellow tie? . . . Oh, no! . . . oh, no! . . . Sure we are equivalent observers! . . . But this doesn't mean that, when I ask you to measure λ', you should look at *your own* tie!

WHAT IS THE SAME PHYSICAL QUANTITY FOR DIFFERENT OBSERVERS?

Special relativity gives us rules to compare results of an experiment performed by an observer S with results obtained by another observer S′, moving with constant velocity with respect to S. It is, of course, implied that both observers are experimenting upon the *same* physical system and that they are not being trapped into the trivial pitfall referred to in the apologue above.

Yet, the concept of sameness of a physical system for different observers is far from obvious. As a matter of fact, we can see that at least two different and contradictory definitions of sameness have already been used in the relativistic literature, with the more or less implicit assumption that they were one and the same definition. This caused many misunderstandings, which we discuss in this paper.

Relativity begins with the setting up of a one-to-one correspondence between points of the four-dimensional continuum of S and of S′. A point $x_\lambda \equiv (x,y,z,t)$ is said to be identical with the point $x_\lambda' \equiv (x',y',z',t')$ of the second observer when the four coordinates are connected by a Lorentz transformation. The next step is then to define a tensor quantity at the point (x,y,z,t) and take as same quantity for S′ the transformed tensor—transformed according to the rules of tensor calculus—at the point (x',y',z',t'). For example, a vector $\mathbf{v}_\mu$ at the point x_λ (z,y,z,t) will be the same event as the vector $\mathbf{v}_\mu'$ at the point x_λ' (x',y',z',t') when

$$\mathbf{v}_\mu' = a_{\mu\nu}\mathbf{v}_\nu \quad x_\mu' = a_{\mu\nu}x_\nu, \tag{1}$$

where $a_{\mu\nu}$ is the matrix of the Lorentz transformation between S and S′. We do not need to discuss here the rather subtle epistemological problem of whether one first defines equal quantities for S and S′ and then finds that they are connected by Lorentz transformation, or whether one first defines Lorentz transformation and then through it one arrives at tensor quantities that are the same for the two observers. The question is irrelevant for the present discussion, since there seem to be no disagreement among physicists on the following statement: if observer S measures a certain tensor quantity, say $T_{\mu\nu}$ at the point x_λ, the same quantity for S′ is the tensor $T_{\mu\nu}'$ at the point x_λ', where the primed quantities are the quantities transformed according to the rules of tensor calculus. In other words, there is no difficulty in defining the concept of sameness on a local basis, i.e., at a certain point in space–time. We know what is the same point for different observers and what are same events at the same point.

The difficulty arises when S wants to define a nonlocal quantity.

To be specific, imagine that S wants to define the vector $A_\mu = B_\mu + C_\mu$, where B_μ is defined at the point x_λ and $\mathbf{C}_\mu$ at the point X_λ with $X_\lambda \neq x_\lambda$. Such a quantity occurs, for example, in the definition of the center of mass of a system of two particles. The quantity A_μ is now a function of two points x_λ and X_λ, i.e.,

$$A_\mu(x_\lambda, X_\lambda) = B_\mu(x_\lambda) + C_\mu(X_\lambda). \tag{2}$$

What is the same A quantity for the observer S′? We could define it as follows:

(a) The quantity $A_\mu(x_\lambda, X_\lambda)$ for S is the same as the quantity $A_\mu'(x_\lambda', X_\lambda')$ for S′ when all the primed quantities are obtained from the corresponding unprimed quantities through Lorentz transformation (tensor calculus).

It is easily seen that all the paraphernalia of tensor calculus in special relativity can be used for nonlocal as well as for local tensor quantities. Through definition (a) we know what it means to say that two observers are looking at the *same* physical object. It is the obvious, logical definition. It is nothing but the prescription given by all textbooks on relativity: always use covariant quantities! What is, then, the difficulty of defining the concept of sameness that we mentioned at the beginning of this paper? The fact is that the books do not always apply the rule (a) they have so clearly stated.

Let us go back to the quantity $A_\mu(x_\lambda, X_\lambda)$ and write it in the form $A_\mu(\mathbf{x},t,\mathbf{X},T)$. In general, $\mathbf{x} \neq \mathbf{X}$, $t \neq T$; but it is up to observer S to define whatever quantity he likes at whatever points he likes. For example, he is certainly free to define B_μ and C_μ at points

$$\mathbf{x} = \mathbf{X}, \quad t \neq T \tag{3}$$

or at points

$$\mathbf{x} \neq \mathbf{X}, \quad t = T. \tag{4}$$

In classical physics observer S very often uses definition (4) at a specific time in different space locations (for example, in the definition of total momentum, total angular momentum, etc.). On the contrary he is less interested in definition (3): who cares to measure, say, the average of the momentum of two different particles passing at the same point in space, one at four o'clock and the other at five o'clock?

We do not want to blame S for his preference for definition (4). We just want to remind him that he should then write

$$\left\{\begin{array}{c} A_\mu(\mathbf{x},t,\mathbf{X},T) = B_\mu(\mathbf{x},t) + C_\mu(\mathbf{X},T) \\ \text{with the supplementary condition: } t = T \end{array}\right\}. \tag{5}$$

If he writes, for short,

$$A_\mu(\mathbf{x},\mathbf{X},t) = B_\mu(\mathbf{x},t) + C_\mu(\mathbf{X},t), \tag{6}$$

he can mislead observer S′ into the following definition of same quantity:

(b) Observer S has defined a quantity A_μ by taking the sum of two vectors B_μ and C_μ at different spatial points at equal time. Therefore, I must define as same quantity for me, observer S′, the sum of the same two vectors at equal time t' for me. Since he looks at his own tie (time), I must look at my own tie (time)!

Observer S′ forgets that $t' \neq T'$, if $t = T$—as he would have easily found out, had he used the correct (covariant) definition (5)—and tries instead, by analogy with definition (6), to define a quantity

$$\mathfrak{A}_\mu'(\mathfrak{x}', \mathfrak{X}', t') = \mathfrak{B}_\mu'(\mathfrak{x}', t') + \mathfrak{C}_\mu'(\mathfrak{X}', t'), \qquad (7)$$

where we have used German letters for various quantities to indicate that they have absolutely nothing to do with the quantities (like A_μ', B_μ', etc.) obtained as transformed quantities according to definition (a).

As a matter of fact, definition (b) is so ambiguous that observer S′ is in trouble if we ask him to be more specific. What are the quantities $\mathfrak{x}'$ and $\mathfrak{X}'$? In most practical cases S has defined B_μ and C_μ at the points, $\mathbf{x}$ and $\mathbf{X}$, respectively, where two particles are present at time t. Observer S′ is often lucky to be able to avoid our question by simply saying: I'll take $\mathfrak{x}'$ and $\mathfrak{X}'$ as the points in space where I find the same two particles. At what time $\mathbf{t}'$? Again observer S′ is often lucky, since in most cases S has defined a conserved quantity A_μ, so that any time will do. In general, S′ would not be able to define t′ unambiguously, since in the original quantity there were two different time variables (t and T), and one cannot reduce a function of two variables into a function of one variable without introducing some arbitrary procedure.

We have shown so far that definition (a) and definition (b) of *same* physical quantity are not equivalent. We have also clearly indicated our preference for definition (a). However, we do not want to quarrel with physicists that prefer definition (b) or to be drawn into a philosophical discussion on the meaning of sameness. The only relevant question for physics is this: relativity gives rules to relate measurements made by S with measurements made by S′ only if definition (a) is adopted. Relativity then provides all the necessary tools (Lorentz transformation, tensor calculus) to establish the connection between the two sets of measurements.

As far as relativity is concerned, quantities like A_μ and $\mathfrak{A}_\mu'$ are different quantities, not necessarily related to one another. To ask the relation between $\mathfrak{A}_\mu'$ and A_μ, from the point of view of relativity, is like asking what is the relation between the measurement of the radius of the Earth made by an observer S and the measurement of the radius of Venus made by an observer S′. We can certainly take the ratio of the two measures; what is wrong is the tacit assumption that relativity has something to do with the problem just because the measurements were made by *two* observers. If there is any relation between the two measurements, it is certainly not a *relativistic* relation. To go back to the apologue in the Introduction, one should not try to explain a color difference on the basis of a relativistic redshift when a simpler explanation is available: one observer has bought a green tie and the other observer a yellow tie! Trivial? Not quite, if one considers how often physicists have made the same mistake. The examples are so numerous that to review them all one should have to write a book, not an article. We therefore discuss only few typical examples in the next section.

EXAMPLES

(1) The definition of length is always given according to definition (b). Only rarely is it pointed out clearly that the measurements of the two observers S and S′ *do not refer to the same set of events*.[1] The "ends" of the rod, being taken as contemporary for both observer at rest and moving observer, are in fact different points in the four-dimensional (absolute) space–time. Once this is clearly indicated, the accepted definition is not particularly harmful. Although it is a completely useless concept in physics,[2] it

[1] See the particularly clear discussion of this point in D. Bohm, *The Special Theory of Relativity* (W. A. Benjamin, Inc., New York, 1965), pp. 58–59 and pp. 64–65.

[2] Nobody will ever see the Lorentz contraction. To define it operationally, one has to assume an infinite velocity of light, contrary to relativity, i.e., in contradiction with the

will probably continue to remain in the books as an historical relic for the fascination of the layman.

(2) The same remarks as for the length apply to the definition of volume $[V'=V(1-\beta^2)^{\frac{1}{2}}]$. Here, however, the definition is to be held responsible for many errors in the literature. Many authors, after writing

$$dV'=dV(1-\beta^2)^{\frac{1}{2}} \tag{8}$$

have considered expression (8) as the Jacobian in a volume integration,[3] without realizing that here again the two quantities dV' and dV *do not refer to the same set of events*. Example (3) below is a classical case of such a misunderstanding.[4]

(3) *Electromagnetic mass of the classical electron.* Here one computes the energy of the electrostatic field by a volume integration with the electron at rest. Thus one gets the energy $\mathcal{E}$ for observer S. A similar integration is then performed with the electron in motion, *using formula (8)*, thus forgetting that it refers to a different set of events. The quantity obtained in this way—i.e., using definition (b) of same quantity—must be written $\mathfrak{E}'$ in our notations and has no relation whatsoever with $\mathcal{E}$. However, at this point one thinks to have instead computed $\mathcal{E}'$ for which relativity predicts $\mathcal{E}'=\mathcal{E}(1-\beta^2)^{\frac{1}{2}}$. One finds instead $\mathfrak{E}'=(\frac{4}{3})\mathcal{E}/(1-\beta^2)^{\frac{1}{2}}$, and then one wonders where the extra $\frac{4}{3}$ factor comes from. Surely this must be due to some other mass of nonelectromagnetic origin, to some Poincaré's stresses, etc.

Details of the computation for this example, although elementary, are given in the Appendix for the reader's convenience.

It is a pity that the wrong point of view has found its way into most textbooks, from the old works of Laue and Pauli[5] to the recent, and otherwise excellent, books of Feynman.[6] The fact is even more surprising when one considers that the correct explanation has been given repeatedly over the past forty years. We give only two references, one old[7] and one recent,[8] in order to be able to add a few comments of historical interest, which we think appropriate for a paper in this JOURNAL.

Fermi, as far as we know, was the first to give the correct explanation. He did not state explicitly that the whole problem originated from a case of mistaken identity, as we have put it here. He used instead an equivalent, but more sophisticated, approach, discussing the concept of rigid body in connection with a variational principle. According to the reviewer of this work of Fermi [collected papers, Ref. 7]: ". . . This result, of which Fermi was particularly proud, was published by him, with minor alterations, in three different journals. . . ." No matter how many times Fermi published his result, he certainly did not succeed in eliminating Poincaré's stresses from physics. Even the same reviewer feels obliged to apologize for Fermi by stating that he ". . . evidently overlooked the explanation contained in M. v. Laue, Die Relativitätstheorie,[5] p. 218 and so he found for it an explanation of his own, essentially equivalent to the former. . . ." Needless to say, the explanation of Fermi is *not* essentially equivalent to the explanation of Laue, since Fermi is correct and Laue is wrong.

Nor does Rohrlich[8] state too explicitly that the case is one of mistaken identity, although he insists on covariance—and therefore on definition (a) of the same quantity. Rohrlich computations—essentially those reported in the Appendix—are so straightforward that they should have convinced everybody of the correctness of his conclusions. It is unfortunate that the same author had previously attempted another, ob-

theory that supposedly introduces that definition. See J. Terrell, Phys. Rev. **116**, 1041 (1959); V. F. Weisskopf, Phys. Today **13**, 24 (September 1960).

[3] See A. Gamba, Nuovo Cimento **37**, 1742 (1965) and the subsequent controversy [Nuovo Cimento **41B**, 72, 79, 81, 83, 84 (1966)].

[4] Of course, in the special case when the integrand (a vector) has vanishing divergence, the integral is independent of the integration volume. Thus in this particular case, definition (a) and definition (b) are equivalent. Obviously, one cannot conclude from this that a theory has *always* to be independent of the volume integration! This, however, is the point taken by F. R. Tangherlini [Am. J. Phys. **31**, 285 (1963)] in criticizing the work of Rohrlich (see Sec. 3). For details on this point, see Ref. 12, p. 130 ff.

[5] M. v. Laue, *Die Relativitätstheorie* (Frederick Vieweg und Sohn, Braunschweig, Germany, 1919), 3rd. ed.; W. Pauli, *Theory of Relativity* (Pergamon Press, Inc., New York, 1958).

[6] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley Publ. Co., Reading, Mass., 1964).

[7] E. Fermi, Nuovo Cimento **25**, 159 (1923), reprinted in E. Fermi, *Note e Memorie* (collected papers) (University of Chicago Press, Chicago, Ill., 1962), Vol. I.

[8] F. Rohrlich, Am. J. Phys. **28**, 639 (1960).

scure, explanation of the same problem.[9] This has probably led some to the mistaken conclusion that they were one and the same explanation, somewhat detracting from the merit of Rohrlich's clear presentation.

Taking into account the fact that on this very popular question a great number of papers are continuously published, with suggested explanations that range from general relativity[10] to self-induction effects of the charged electron,[11] one can probably understand why the correct explanation has not yet found its way into most textbooks.[12]

(4) In a recent paper Van Dam and Wigner[13] discuss, among other things, the following problem. Consider two charged particles of equal mass, moving along the x axis, separating from each other. In the reference system in which the center of mass of the two particles is at rest, the total angular momentum $M_{\mu\nu}$ is obviously zero. Then they compute the total angular momentum for a moving observer S′—using definition (b), i.e., they compute $\mathfrak{M}_{\mu\nu}'$—and they find to their, but not to our, surprise, that it does not vanish, even asymptotically. Therefore they ask the question: where is the extra term coming from? They do not attribute the effect to Poincaré's stresses (!), but to an equivalent interaction angular momentum of the field.

The solution should, by now, be obvious to our readers. To ask where is the missing term is but an implicit admission that $\mathfrak{M}_{\mu\nu}'$ and $M_{\mu\nu}$ refer to the same physical quantity, whereas this is not true.

Finally, we would like to point out that the problem of the interaction angular momentum is but a different aspect of the old problem of the right-angle lever, which was given a wrong interpretation [definition (b)] by Laue, and has been recently solved [definition (a)] by Arzeliès.[14]

[9] J. M. Jauch and F. Rohrlich, *Theory of Photons and Electrons* (Addison-Wesley Publ. Co., Reading, Mass., 1955), p. 410.

[10] R. Penney, Phys. Rev. **137**, B1385 (1965).

[11] J. W. Zink, Am. J. Phys. **34**, 211 (1966).

[12] A notable exception being F. Rohrlich's *Classical Charged Particles* (Addison-Wesley Publ. Co., Reading, Mass., 1965).

[13] H. Van Dam and E. P. Wigner, Phys. Rev. **142**, 838 (1966).

[14] H. Arzeliès, Nuovo Cimento **35**, 783 (1965).

APPENDIX: ELECTROMAGNETIC MASS OF THE CLASSICAL ELECTRON

Imagine the (classical) electron as a sphere of radius R with a uniform surface-charge density. In the rest system, assume that the whole mass m of the electron is due to the electrostatic energy of the field. Then

$$mc^2=\mathcal{E}=\int_{\substack{\text{over the whole volume}\\ \text{at time } t=\text{constant}}}\frac{E^2}{8\pi}dV$$

$$=\int_R^\infty \frac{e^2}{8\pi r^4}4\pi r^2 dr=\frac{e^2}{2R} \qquad \text{(A1)}$$

Similarly for the momentum $\mathbf{p}$

$$\mathbf{P}=\int_{\substack{\text{over the whole volume}\\ \text{at time } t=\text{constant}}}\frac{\mathbf{E}\times\mathbf{H}}{4\pi c}dV=0, \qquad \text{(A2)}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields, respectively.

An observer S′ is in motion with respect to the electron with constant velocity $\boldsymbol{\beta}\equiv[(u/c),0,0]$; then for S′

$$E_x'=E_x \quad E_y'=E_y/(1-\beta^2)^{\frac12} \quad E_z'=E_z/(1-\beta^2)^{\frac12} \qquad \text{(A3)}$$
$$H_x'=0 \quad H_y'=-\beta E_z/(1-\beta^2)^{\frac12} \quad H_z'=\beta E_y/(1-\beta^2)^{\frac12}.$$

Assume that S′ is now measuring the energy and the momentum of the electron according to the two definition (a) and (b) of sameness given in the text. Let us begin with the usual (wrong) definition (b). Then

(1) *with definition* (*b*), observer S′ replaces dV with dV' in the integrals, according to formula (8), and integrates at constant t'. He finds

$$\mathfrak{P}_x'=\int_{\substack{\text{over the whole volume}\\ \text{at time } t'=\text{constant}}}\frac{(\mathbf{E}'\times\mathbf{H}')_x}{4\pi c}dV'$$

$$=\int\frac{E_y'H_z'-E_z'H_y'}{4\pi c}dV'$$

$$=\frac{\beta}{c(1-\beta^2)^{\frac12}}\int\frac{E_y^2+E_z^2}{4\pi}dV=\frac{4}{3}\frac{mu}{(1-\beta^2)^{\frac12}}, \qquad \text{(A4)}$$

since for symmetry

$$\int E_x^2 dV=\int E_y^2 dV=\int E_z^2 dV=\frac{1}{3}\int E^2 dV. \qquad \text{(A5)}$$

Similarly, he finds

$$\mathfrak{P}_y' = \mathfrak{P}_z' = 0$$

$$\mathfrak{E}' = \int \frac{E'^2 + H'^2}{8\pi} dV' = \left(1 + \frac{\beta^2}{3}\right) \frac{\mathcal{E}}{(1-\beta^2)^{\frac{1}{2}}}. \quad \text{(A6)}$$

The reader may note that one gets an extra factor $\frac{4}{3}$ in (A4), but an extra factor $[1 + (\beta^2/3)]$ in (A6). Most authors stop at formula (A4) and then, applying relativity, imply that the same factor also appears in the calculation of $\mathfrak{E}'$. The reduction of the latter factor to $\frac{4}{3}$ requires a more elaborate analysis (using Poincaré's stresses, of course). We do not enter into these irrelevant details in view of the fact that this approach, after all, is incorrect.

The correct solution is obtained

(2) *with definition* (*a*). In this case observer S′ writes (A1) and (A2) in a covariant form

$$p_\mu = \int T_{\mu\nu} dS_\nu \quad \text{(A7)}$$

with

$$T_{\mu\nu} = \frac{1}{4\pi} \begin{vmatrix} E_x^2 + H_x^2 - W & E_x E_y + H_x H_y & E_x E_z + H_x H_z & i(E_y H_z - E_z H_y) \\ E_x E_y + H_x H_y & E_y^2 + H_y^2 - W & E_y E_z + H_y H_z & i(E_z H_x - E_x H_z) \\ E_x E_z + H_x H_z & E_y E_z + H_y H_z & E_z^2 + H_z^2 - W & i(E_x H_y - E_y H_z) \\ i(E_y H_z - E_z H_y) & i(E_z H_x - E_x H_z) & i(E_x H_y - E_y H_z) & W \end{vmatrix} \quad \text{(A8)}$$

and

$$W = \tfrac{1}{2}(E^2 + H^2). \quad \text{(A9)}$$

Observer S′ realizes that observer S made the particular choice

$$dS_\nu = (0,0,0,dV) \quad \text{(A10)}$$

with

$$T_{\mu\nu} = \frac{1}{4\pi} \begin{vmatrix} E_x^2 - W & E_x E_y & E_x E_z & 0 \\ E_x E_y & E^2 - W & E_y E_z & 0 \\ E_x E_z & E_y E_z & E_x^2 - W & 0 \\ 0 & 0 & 0 & W \end{vmatrix} \quad \text{(A11)}$$

and an integration at constant t. This does not mean that he (observer S′) should then integrate at constant t'. Instead, observer S′ writes, according to relativity

$$p_\mu' = \int T_{\mu\nu}' dS_\nu', \quad \text{(A12)}$$

where

$$dS_1' = \frac{i\beta dV}{(1-\beta^2)^{\frac{1}{2}}}, \quad dS_2' = dS_3' = 0, \quad dS_4' = \frac{dV}{(1-\beta^2)^{\frac{1}{2}}} \quad \text{(A13)}$$

and $T_{\mu\nu}'$ is obtained from formula (A8) by substituting primed quantities as given by (A3). Accordingly he gets

$$p_1' = \int T_{11}' dS_1' + \int T_{14}' dS_4' = \frac{i\beta}{4\pi(1-\beta^2)^{\frac{1}{2}}} \int \left[E_x^2 - \tfrac{1}{2}\left(E_x^2 + \{E_y^2 + E_z^2\} \frac{1+\beta^2}{1-\beta^2} \right) + \frac{E_y^2 + E_z^2}{1-\beta^2} \right] dV$$

$$= \frac{i\beta}{8\pi(1-\beta^2)^{\frac{1}{2}}} \int E^2 dV = \frac{i\beta p_4}{(1-\beta^2)^{\frac{1}{2}}} \quad \text{(A14)}$$

and similarly

$$p_2'=p_3'=0, \tag{A15}$$

and

$$p_4'=\int T_{41}'dS_1'+\int T_{44}'dS_4'=\frac{1}{4\pi(1-\beta^2)^{\frac{1}{2}}}\left\{-\int\frac{(E_y^2+E_z^2)\beta^2}{1-\beta^2}dV+\int\frac{1}{2}\left\{E_x^2+(E_y^2+E_z^2)\frac{1+\beta^2}{1-\beta^2}\right\}dV\right\}$$

$$=\frac{1}{8\pi(1-\beta^2)^{\frac{1}{2}}}\int E^2dV=\frac{p_4}{(1-\beta^2)^{\frac{1}{2}}} \tag{A16}$$

in perfect agreement with relativity.

The Leibniz–Clarke Controversy: Absolute versus Relative Space and Time

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There is an increasing current interest in the philosophy and history of physics. This paper discusses an interesting and important (but relatively little known among physicists) controversy in the history of theories of space and time, the Leibniz–Clarke correspondence. Leibniz represents the relativist view of space and time. Clarke, a disciple of Newton, upholds the absolute space and time of Newtonian mechanics.

INTRODUCTION

THE position of modern physics is that absolute space and time are meaningless concepts. This is based on the wholehearted acceptance by the physics community of an empirical, operationalist approach. However, the philosophy community is not as absolutely certain as are the physicists, and they feel that there is still ample room for speculative thought on the ultimate nature of space and time. This difference in viewpoint reflects a general intellectual situation in which each discipline largely goes its own way independently of other disciplines. Awareness of this problem has resulted in attempts at interdisciplinary approaches and, in particular, in physics has led to heightened interest in the philosophy and history of physics.

In the much less specialized and compartmentalized Newtonian age, physicists were philosophers and philosophers were physicists. It was in this age that there occurred a physical–philosophical correspondence of great importance to the history of speculative thought on theories of space and time. This was the correspondence between Leibniz and Clarke on absolute versus relative space and time.

Clarke[1] was a disciple of Newton and engaged in a famous interchange of philosophical letters with Leibniz[2] at the beginning of the 18th century. Alexander describes the correspondence as follows: "The exchange of papers between Leibniz and Clarke is the most frequently cited of

[1] Dr. Samuel Clarke (1675–1729) was a theologian, philosopher, and scientist. He was the foremost disciple of Newton. His translation into Latin of Rohault's *Physics* (1697) was very popular and in it he added extensive footnotes relating to Newton's physics. He also translated Newton's *Opticks* into Latin (1706). In 1704 and 1705 he delivered two sets of Boyle lectures. In the first he attempted to prove the existence of God by mathematical methods and in the second he tried to show that moral laws are on as firm a footing as mathematical propositions. In 1717 he published *A Collection of Papers which passed between the late Learned Mr. Leibniz, and Dr. Clarke, in the Years 1715 and 1716. Relating to the Principles of Natural Philosophy and Religion.* This latter work is the subject of this paper.

[2] Gottfried Wilhelm Leibniz, Freiherr von (1646–1716) is one of the major philosophers who have influenced Western thought. He shares with Newton the honor of being the discoverer of the calculus. Leibniz's philosophical system is based on a set of ultimate entities called monads. The grouping of the monads to form a universe is governed by the principle of contradiction and the principle of sufficient reason. The unfolding of events in the universe of monads follows a pre-established harmony whose author is God. Some of Leibniz's important works are as follows: *The Monadology* (1714), *Principles of Nature and of Grace* (1714), *On the Ultimate Origination of Things* (1697), *The Theodicy* (1710), *Correspondence With Clarke* (1715–1716), *New Essays on the Human Understanding* (1702–1703), and the *Correspondence with Arnauld* (1686–1687).