still another argument that the basic formalism of quantum mechanics is all that is required to establish that orbital angular momentum components can have only integral eigenvalues. However, the formal solution (2) is based entirely on this formalism, and the restriction to integral quantum numbers for the eigenvalues of the orbital angular momentum operators (4) must be a consequence of the formal theory (2) as proved here. Buchdahl avoids altogether any use of the standard basis (2), and his arguments, therefore, do not reveal the manner in which orbital angular momentum fits into the standard formal theory.

Stress Effects due to Lorentz Contraction

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Relativistic stress effects caused by the Lorentz contraction described in a previous publication are related to implications of relativistic simultaneity. This relationship is discussed here, in some detail, in order to clarify certain pedagogically difficult aspects of the problem. Objections raised in a recent Note are also answered.

IN a recent letter,[1] Nawrocki raised what he considered to be valid arguments against the conclusions of a paper[2] by Beran and myself concerning relativistic stress effects. In the present article it is shown that all of his arguments are based on a single misconception concerning simultaneity. The concept of simultaneity, it will be recalled, is a crucial one and it was precisely this subtle concept that was involved in perhaps one of the most difficult steps in Einstein's discovery that time and space are related. It is, therefore, not surprising that Nawrocki and other students of relativity might find difficulty in understanding the problem under discussion. Perhaps one could say that the pedagogical value of this problem lies primarily in the fact that it illustrates in a somewhat dramatic way the physical relation between length contraction and simultaneity.

In order to clarify the thought experiment in question, let us consider it from the viewpoints of observers not only in the initial rest frame but also in other moving frames.

1. Observer in Initial Rest Frame

Two *identical* rockets, R-1 and R-2, are positioned along the x axis and aimed in the positive x direction with R-1 a distance d_0 in *front* of R-2. A thread or weak string is attached to their respective centers, and at a time $t=0$ they are fired *simultaneously*. The distance between the rockets, as measured in the initial frame, remains constant for all time because both rockets, by assumption, are identical and therefore have identical velocities at each instant of time due to the fact that they are fired simultaneously. On the other hand, the string tends to contract because of the Lorentz contraction and therefore it eventually breaks because of its finite strength. Further details of this thought experiment are found in reference 2.

2. Observer in Final Rest Frame

We next consider the viewpoint of an "observer" in the Lorentz frame, in which the rockets are at rest at the end of the experiment when they "run out of gas." In other words, let us consider the results of measurements *made with clocks and rods at rest with respect to an inertial frame moving with velocity v*, the final rocket velocity with respect to the initial frame. (Notice that in actuality the observer can be in *any* frame; it is the rest frame of the *measuring instruments* that is relevant. We nevertheless shall continue to use the word "observer" as a sort of shorthand notation.) As usual, the clocks are assumed to be synchronized by the Einstein convention using light signals so that x and t in the S or initial rest

[1] P. J. Nawrocki, Am. J. Phys. **30**, 771 (1962).
[2] E. Dewan and M. Beran, Am. J. Phys. **27**, 517 (1959).

frame will be related to x' and t' in the S' or final rest frame by the Lorentz transformation:

$$x'=(x-vt)/(1-\beta^2)^{\frac{1}{2}}, \quad t'=[t-(vx/c^2)]/(1-\beta^2)^{\frac{1}{2}},$$

where $\beta=v/c$.

The most important thing to notice here is the $(-vx/c^2)/(1-\beta^2)^{\frac{1}{2}}$ term in the time transformation. Its physical significance is that, although all the clocks in S' run at equal *rates* when measured in S, they are out of "phase" with one another. That is, from the point of view of S, they are synchronized with respect to *rate* but *not* with respect to dial reading. Grünbaum's article[3] clearly shows the relationship between this term and Einstein's synchronization convention. With reference to our thought experiment it implies that the S' observer measures time in a way that the two rockets will appear to him as starting off at *different times*. The assumption that the two rockets are identical implies that their trajectories can differ only by a spatial and temporal displacement. The final separation between the rockets when they come to rest in S' is therefore equal to their initial separation as measured in that frame plus the separation caused by the difference in starting times, Δt; where $\Delta t'=(vd_0/c^2)/(1-\beta^2)^{\frac{1}{2}}$. Thus the final separation, d_f', in S' is given by:

$$\begin{aligned} d_f' &= d_0(1-\beta^2)^{\frac{1}{2}}+v\Delta t' \\ &= d_0(1-\beta^2)^{\frac{1}{2}}+(v^2/c^2)[d_0/(1-\beta^2)^{\frac{1}{2}}] \\ &= d_0/(1-\beta^2)^{\frac{1}{2}}. \end{aligned}$$

Hence S' sees[4] the string break, *not* because of a Lorentz contraction, but because of a physical separation of the rockets! Notice that this argument is quite independent of the amount of acceleration, or its functional dependence on time. In our previous paper, we made the assumption that the acceleration was not "too large" simply to avoid the complication due to finite propagation times. There is no other restriction on the acceleration. One should also notice that the ratio of rocket length to separation distance is the same for both S and S' at the end of the experiment but that during the experiment it is *not* constant in *time*.

[3] A. Grünbaum, Am. J. Phys. **23**, 450 (1955).

[4] In this paper, the word "see," when referring to a particular observer, is used in the sense of physical measurement rather than in Terrell's sense.

3. Observer in Frame Instantaneously at Rest with Respect to *R*-2 during Acceleration

An observer riding R-2 during the acceleration would have to switch frames at each instant. Each of these changes would necessitate a re-synchronization of the phases of all his clocks. This re-synchronization does not "automatically" come about by the effect of the change of rates. In other words, the $(vx)/c^2(1-\beta^2)^{\frac{1}{2}}$ term in the time transformation implies that each change of instantaneous Lorentz frame involves an entire re-synchronization of clocks in a manner which depends on clock position. That is, as v increases, the rate of change of clock phase with respect to distance (or the "gradient" of the phase in the x direction) also increases. This implies, as one can easily show, that the R-2 observer would regard the *rate* of R-1's fuel consumption as being faster than that of R-2 and that R-1's acceleration is greater than that of R-2.[5] In other words, the fuel pumps of the rockets which meter the fuel at a "constant rate" can be regarded as clocks in some sense. If we take into account the effects of the *observer's* clock synchronization, then it can be shown that the *rate* of fuel consumption in R-1 as measured by R-2 would seem *larger* than that of R-2. The measured acceleration would also appear larger in a way consistent with the difference of fuel consumption; hence, there would be an increase in the spatial separation which would break the string when the latter reached its elastic limit.

This experiment can also be regarded from the point of view of the principle of equivalence. A uniform gravitational field in the direction opposite to the acceleration would allow one to consider R-2 as stationary. The increase of fuel consumption in R-1 would now be considered as being due to the gravitational "violet shift" because R-1 would be at a higher gravitational potential than R-2. It is interesting to notice that a gravitational field which could transform away the acceleration of an extended body (such as an elevator) cannot be uniform in the above sense because, as we have seen, such a field would

[5] The "rocket clock" is a standard clock, and the constantly readjusted clocks are coordinate clocks [see C. Møller, *The Theory of Relativity* (Clarendon Press, Oxford, 1955), pp. 33, 226].

tend to set up stresses similar to those in the rocket experiment.

4. Remarks on Nawrocki's Arguments

We have seen that one consistently reaches the same conclusions from the point of view of all observers, provided one takes into account time synchronization effects. With this in mind, we now turn our attention to statements made in Nawrocki's letter.

(a) At the start of his discussion, Nawrocki claims that our conclusions imply that two fundamental methods of measuring length (i.e., by measuring rods or by light signals and clocks) give contradictory results. However, we have just seen that the distance between the rockets *cannot* be considered to be a "proper length" since it is *time-dependent* in all frames with the exception of S with respect to which it is moving. The measurement of this distance by rods and by light signals will be consistent in any given frame, but in no frame will it be a "rest length."

(b) Next, he misquotes Evett and Wangsness as follows: ". . . the distance between corresponding points of the two rockets remains constant [for all Galilean observers]." The additional words in the square brackets are due only to Nawrocki's confusion and have no connection with the intentions of those authors. This resolves the mystery of why they "inexplicably" agree with our conclusion.

(c) Finally, he concludes his note with three arguments attempting to show that the ratio of the distance between the rockets to the length of one rocket must remain constant in time in any instantaneous rest frame of the midpoint between the rockets. These three arguments are now considered in chronological order.

(1) The first argument is based on the statement, "all measured distances in the rest frame are obviously unaffected by the Lorentz–Fitzgerald contraction." As we have just seen, however, the distance between the rockets *is* affected by the acceleration when measured in any instantaneous frame moving, in some sense, with the rockets. Although S blames the breaking of the string on the Lorentz contraction, S' blames it on a relative velocity between the rockets. Thus we see that Nawrocki's error is due to his neglect of the $(vx)/c^2(1-\beta^2)^{\frac{1}{2}}$ term in the Lorentz transformation.

(2) In his second argument, Nawrocki states that acceleration, although important in the twin paradox, plays no decisive role in the rocket problem. The reason he gives is that we are concerned only with a length "dx" and not with "$3\sim1$ space time quantities." It has been shown above, however, that one of the main points of the problem is to demonstrate the connection between time synchronization and length contraction and that a spatial property, as seen by one observer, can involve temporal properties as seen by another observer. Hence, his statement is false and we see that the acceleration does indeed play an important role just as in the twin paradox.

(3) In his last argument, which is a slight modification of the preceeding one, Nawrocki attempts to consider the problem from the point of view of several Lorentz observers. Again he neglects synchronization effects and arrives at a contradiction. As usual, the answer to his argument is that he forgot about relativistic simultaneity. The breaking of the string is due to the Lorentz contraction in the initial frame, to the relative velocity (caused by lack of simultaneity) in the final frame, and to a mixture of both effects in other frames.

Thus we conclude, as we stated in the beginning, that Nawrocki's arguments are based on a single misunderstanding about the role of simultaneity in relativity.

As an exercise, the reader may test his grasp of the above concepts by means of the following problem. (See Figs. 1 and 2.) Consider a pole vaulter who lives in "Tompkin's Wonderland"[6] where the velocity of light is so small that if he points the pole in the direction of his movement it contracts by a large amount. Now, suppose that one day he runs with his contracted pole through the front door of a barn which is so small that, were it not for the contraction, the pole

[6] In *Mr. Tompkins in Wonderland*, one *ignores* the important fact that the Lorentz contraction is invisible [as was shown by Terrell, Phys. Rev. 116, 1041 (1959)]. When Gamow wrote this famous book, physicists were not aware of this invisibility; hence, for the sake of convenience, we *define* "Tompkins' Wonderland" as a place where one sees what is physically measurable, as opposed to what is seen by means of the usual optical equipment such as eyes and cameras.

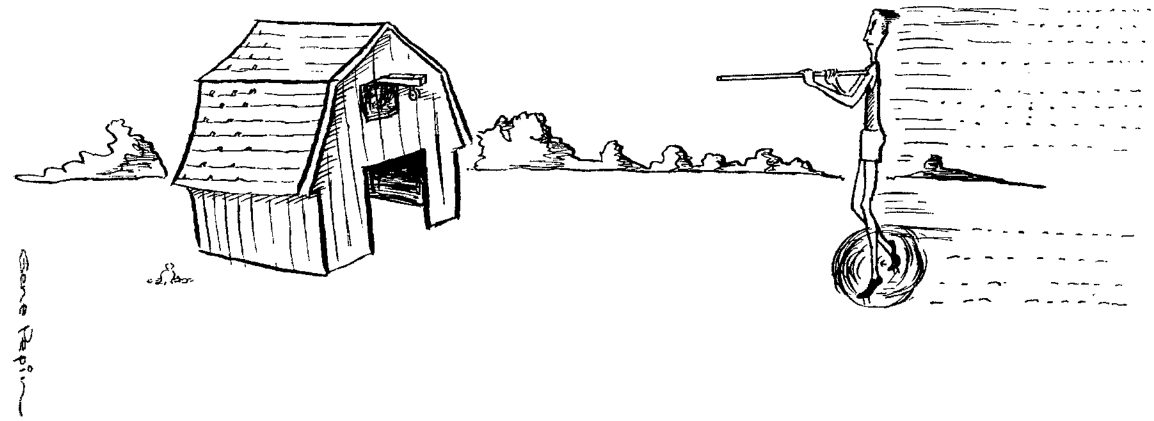

FIG. 1. Pole vaulter running toward open front door of barn with contracted pole.[6]

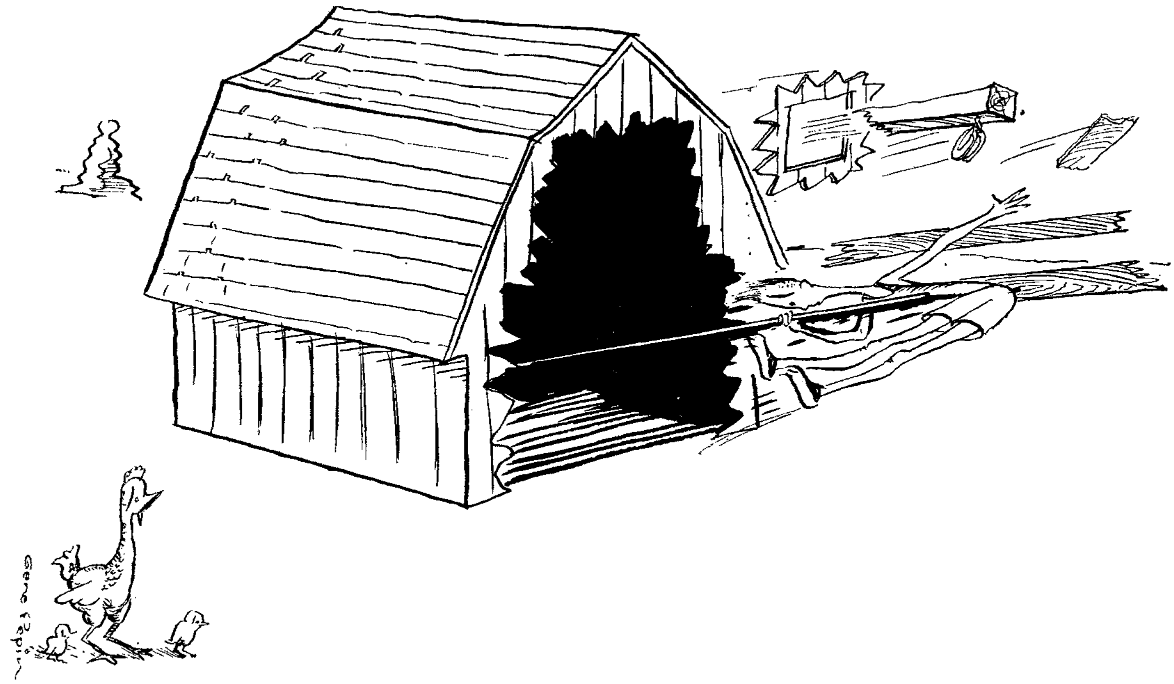

FIG. 2. Pole vaulter seen smashing backwards out of the barn after the door has been closed behind him. The pole being stopped by the rear wall regains its rest length and pushes back on the pole vaulter. The impulse reaches him after the door is closed because of the finite propagation time of the impulse shock wave in the pole. A large part of the front of the barn is knocked out because the door was closed and because both the rear wall and front door are assumed to be stronger than the front wall. Perhaps, this last illustration can be considered as emphasizing that the relativistic contraction effects are not to be considered as "mathematical fictions."

would be twice as long as any dimension of the barn. We also assume that a person at rest in the barn would see (i.e., measure) the pole as being very short and that after the pole vaulter enters the barn, the rest observer closes the door behind the pole vaulter! The pole vaulter, on the other hand, would see the barn as contracted and much smaller than the pole. The question is, how can the pole vaulter "explain" the fact that the door can be closed behind him?