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E. Dewan, and M. Beran

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Note on Stress Effects due to Relativistic Contraction

E. Dewan and M. Beran
Air Force Cambridge Research Center, Bedford, Massachusetts
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In this note a thought experiment will be used to show that relativistic contraction can introduce stress effects in a moving body.

WRITERS in the field of special relativity have often given the impression that the Lorentz-Fitzgerald contraction is merely the result of certain abstract mathematical transformations and, as such, is merely an apparent effect without having, in a sense, "real existence." We shall show that such a point of view could be misleading since under certain circumstances relativistic contraction can cause measurable stresses. In the following thought experiment we shall assume for simplicity that (a) with respect to any inertial frame all accelerations are of such magnitude that, to the first order, special relativistic velocity effects are essentially independent of acceleration, and (b) there is no gravitation field present. Although the thought experiment will involve acceleration, general relativity need not be used since all observations will be assumed to be made with respect to inertial frames.

Consider two *identically constructed* rockets at rest in an inertial frame S. Let them face the same direction and be situated one behind the other. If we suppose that at a prearranged time both rockets are simultaneously (with respect to S) fired up, then their velocities with respect to S are always equal throughout the remainder of the experiment (even though they are functions of time). This means, by definition, *that with respect to S* the distance between the two rockets does not change even when they speed up to relativistic velocities. Since this fact confuses some students, we discuss it at some length in the Appendix.

Now suppose that one end of a silk thread is attached to the back of the first rocket and the other end to the front of the second rocket and that the experiment is repeated, assuming that the thread does not affect the motion of the rockets. According to the special theory the thread must contract with respect to S because it has a velocity with respect to S. However, since the rockets maintain a constant distance apart with respect to S, the thread (which we have assumed to be taut at the start) cannot contract; therefore a stress must form until for high enough velocities the thread finally reaches its elastic limit and breaks.

This would appear differently to observers in various inertial frames along the track, the origins of which frames have velocities instantaneously equal to that of the front rocket as it accelerates. Since $t'=(t-Vx/c^2)/(1-V^2/c^2)^{\frac{1}{2}}$ (where the symbols have the usual meaning) each frame used here has a different synchronization scheme because of the Vx/c^2 factor. It can be shown that as V increases, the front rocket will not only appear to be a larger distance from the back rocket with respect to an instantaneous inertial frame, but also to have started at an earlier time.

This thought experiment, which demonstrates direct relativistic stress effects, may be generalized. One may conclude that whenever a body is constrained to move in such a way that all parts of it have the same acceleration with respect to an inertial frame (or, alternatively, in such a way that with respect to an inertial frame its dimensions are fixed, and there is no rotation), then such a body must in general experience relativistic stresses.

The details of rotational relativistic stress effects are certainly more complicated than the foregoing; however, if a rotating object such as a disk were constrained in such a way that its dimensions with respect to an inertial frame could not change, then here too stresses would be set up.[1]

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[1] Landau and Lifshitz, *Classical Theory of Fields* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1951), p. 24.

We conclude then that if a body moves in an arbitrary manner with respect to an inertial frame and if, with respect to this frame, its shape and size are in some way held constant, then such a body will experience, in addition to the usual stresses, a stress which can be ascribed to purely relativistic contraction effects. In the light of some of the past literature on similar subjects it is not unlikely that objections to the validity of the foregoing conclusions will be made by stating that since accelerations are introduced the special theory is not adequate to deal with the problem. However, accelerations may be consistently introduced into the theory provided they are viewed from an inertial frame. The special theory is as adequate to deal with this problem as it is with the resolution of the clock paradox.

APPENDIX

At first sight the fact that the distance between the two rockets will not change might seem to be a violation of the Lorentz transformation because the latter implies that a fast-moving object contracts in the direction of its velocity. Thus one may be tempted to regard the following distances as being of the same sort: (a) the distance between two ends of a connected rod, and (b) the distance between two objects which are *not connected* but each of which *independently* and simultaneously moves with the same velocity with respect to an inertial frame. That is to say, it is easy to think that *both* of these distances must contract at high velocities. It can be shown, however, that this is not correct. One way to show it is by the following argument.

Suppose that the distance with respect to S between two rockets *were* to contract as they reached higher velocities. This would imply that with respect to S there would be a small relative velocity between the rockets; that is, if the distance between the two rockets were to change with respect to S, then they could not have the same velocity with respect to S. This contradicts the initial assumption that both rockets are identically constructed, for the latter implies that they must have velocities which, with respect to S, must be equal at all times (i.e., identical construction implies that since the velocities change in time, they must change in exactly the same manner for both rockets).

If this argument is not convincing one can reason as follows: The Lorentz contraction is given by $l_s = l_0(1 - V^2/c^2)^{\frac{1}{2}}$ where l_0 is the proper length, l_s is the length measured with respect to S, and V is the velocity of the instantaneous rest frame of the system in question. If the distance between the two rockets were to contract, the amount of contraction would depend upon their initial distance apart (l_0). As l_0 is increased, the difference between the velocities of the two rockets must also increase to take into account the increase in the amount of contraction. This brings up the following questions which clearly bring out the untenability of this assumption: "How do the two rockets obtain information, i.e., "know," about the distance between themselves? By what mechanism do they adjust their velocities in a way that depends upon l_0? At very high accelerations, does the front rocket go backwards at first or does the back rocket catch up to it in order to reduce the distance between them? If we have *three* rockets lined up, then which ones have the larger and smaller velocities to give the contractions?," etc.

These difficulties do not arise in the case of a connected rod. The reason the rod contracts is because the distance between its ends is defined with respect to its "proper Lorentz frame." If the rod is assumed to be "rigid" in the relativistic sense, then it is *this distance* which must remain constant. This implies a contraction when measurements are made from a frame moving with respect to the rod. The fact that the rod is *connected* allows the possibility for it to define a rest length. *Two unconnected objects which move in a prescribed manner with respect to an inertial frame need not satisfy constraints which are defined with respect to an instantaneous rest frame.*