

The Information Geometry of Space and Time

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Abstract. Is the geometry of space a macroscopic manifestation of an underlying microscopic statistical structure? Is geometrodynamics - the theory of gravity - derivable from general principles of inductive inference? Tentative answers are suggested by a model of geometrodynamics based on the statistical concepts of entropy, information geometry, and entropic dynamics. The model shows remarkable similarities with the 3+1 formulation of general relativity. For example, the dynamical degrees of freedom are those that specify the conformal geometry of space; there is a gauge symmetry under 3d diffeomorphisms; there is no reference to an external time; and the theory is time reversible. There is, in addition, a gauge symmetry under scale transformations. I conjecture that under a suitable choice of gauge one can recover the usual notion of a relativistic space-time.

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STATISTICAL GEOMETRODYNAMICS?

The point of view that has been prevalent among scientists is that the laws of physics mirror the laws of nature. The reflection might be imperfect, a mere approximation to the real thing, but it is a reflection nonetheless. The connection between physics and nature could, however, be less direct. The laws of physics could be mere rules for processing information about nature. If this second point of view turns out to be correct one would expect many aspects of physics to mirror the structure of theories of inference. Indeed, it should be possible to derive the “laws of physics” appropriate to a certain problem by applying standard rules of inference to the information that happens to be relevant to the problem at hand.¹

There is strong evidence that this second point of view is worth pursuing. For example, most of the formal structure of statistical mechanics can be explained as a consequence of the method of maximum entropy [3]. A second example is given by quantum mechanics. It is less well-known but nevertheless still true that many features of the quantum formalism which are usually introduced as postulates (the Hilbert spaces, linear and unitary time evolution, the Born probability rule, Hermitian observables, etc.) can be derived from principles of inference (consistency, entropy, and so on) once the subject matter has been correctly identified [7].

This paper explores the possibility that the general theory of relativity is a theory of this type; that it can be derived from an underlying “statistical geometrodynamics” in much the same way that thermodynamics can be explained in terms of an underlying

¹ Basic requirements of consistency, objectivity, universality, and honesty lead to the theory of probability [1][2] and to the method of maximum entropy [3]–[6] as the uniquely natural rules of inference.

statistical mechanics.

Our subject can be approached from a different direction. Modern developments in statistical inference [8][9] have shown that geometrical concepts turn out to be extremely natural tools to manipulate information. If physics is nothing but manipulating information about the world, then this suggests an explanation for the central role that geometry has always played in physics. It also suggests that it should be possible to explain basic geometrical notions such as spatial distance and temporal duration in terms of even more basic statistical notions.

In section 2 we take the first step towards specifying the subject matter. (The statistical geometrodynamics developed here is a model for empty vacuum; it does not include matter.) The difficulty is that space and time are invisible. What we see is not space but matter in space and it is not clear how to disentangle which properties should be attributed to the matter and which to space. The best one can do is sprinkle space with ideal test particles that are neutral to all interactions and are describable by a minimal number of attributes. Such purest form of matter is a dust of identical particles; they only interact gravitationally, and being identical the only attribute that distinguishes them is their position.

Then we introduce the main assumption: there is an intrinsic fuzziness to space which is revealed by an irreducible uncertainty in the location of the test particles. Thus, to each point in space we associate a probability distribution. The overall state of space – the macrostate – is defined by the product of the distributions associated to the individual points. The geometry of space is the geometry of all the distances between test particles and this geometry is of statistical origin [10]. Identical particles that are close together are easy to confuse, those that are far apart are easy to distinguish. The distance between two neighboring particles is the distinguishability distance between the corresponding probability distributions which is given by the Fisher-Rao metric [11]. A remarkable feature of this choice of distance is its uniqueness: the Fisher-Rao metric is the only metric that takes account of the fact that we deal with probability distributions and not with “structureless” points [12]. A second remarkable feature is that the information geometry we introduce does not define the full Riemannian geometry of space but only its conformal geometry. This appears at first to be a threat to the whole program but it turns to be just what we need in a theory of gravity [13].

But the task of specifying the subject matter is not yet finished. A proper understanding of what we mean by a state requires that we be able to quantify the extent to which one state can be distinguished from another. In particular, the measure of time and dynamics itself derive from our capacity to measure change between one state that we call ‘earlier’ and another state that we call ‘later’ [14]. In section 3 we measure the change from one state to another using, once again, the Fisher-Rao metric. A peculiarity that arises when comparing the states of systems with a continuum of degrees of freedom turns out to be very significant. In such cases we have to make an explicit choice about which position in the later state corresponds, matches, or ultimately, is *the same as* a given position in the earlier state. The method of maximum entropy provides a natural

criterion to achieve the best match between two successive states². The resulting best-matching condition closely resembles the diffeomorphism constraint in the Hamiltonian formulation of general relativity [16].

It is interesting that the Fisher-Rao metric is used in two ways that are conceptually very different. One is to distinguish neighboring points, the other to distinguish successive states. The first is related to spatial distance, the second to temporal duration. This suggests an explanation of the old puzzle of how can space and time be so different physically and yet be represented mathematically in such a symmetrical way.

Having specified the states, in section 4 we tackle the dynamics. We ask: Given the initial and the final states, what trajectory is the system expected to follow? In the usual approach the dynamics is postulated. No further explanation is needed because “that’s the way nature is.” But this route is not open to us. We are just making inferences from relevant information and the expected trajectory is obtained, without additional postulates, from a principle of inference, the method of maximum entropy [10][17].

The resulting entropic dynamics is not identical with the general theory of relativity but there are remarkable similarities which strongly suggest that general relativity can be obtained in some appropriate limit.

THE INFORMATION GEOMETRY OF SPACE

Consider a cloud of identical test particles – specks of dust – suspended in an otherwise empty space. There are no rulers and no clocks, just dust. Being identical the particles are easy to confuse. The only distinction between two of them is that one happens to be here while the other is over there. To distinguish one speck of dust from another we assign labels or coordinates to each particle. We assume that three real numbers (y^1, y^2, y^3) are sufficient.

But particles can be mislabeled. Then the “true” coordinates y are unknown and one can only provide an estimate, x . Let $p(y|x)dy$ be the probability that the particle labeled x should have been labeled y . The labels x are introduced to distinguish one particle from another, but can we distinguish a particle at x from another at $x + dx$? If dx is small enough the corresponding probability distributions $p(y|x)$ and $p(y|x + dx)$ overlap considerably and it is easy to confuse them. We seek a quantitative measure of the extent to which these two distributions can be distinguished.

The following crude argument is intuitively appealing. Consider the relative difference,

$$\frac{p(y|x + dx) - p(y|x)}{p(y|x)} = \frac{\partial \log p(y|x)}{\partial x^i} dx^i. \quad (1)$$

The expected value of this relative difference does not provide us with the desired measure of distinguishability because it vanishes identically. However, the variance

² To borrow a term coined by Barbour, we might say that best-matching establishes a relation of “equilocality” [15].

which is positive definite does not vanish in general,

$$d\lambda^2 = \int d^3y p(y|x) \frac{\partial \log p(y|x)}{\partial x^i} \frac{\partial \log p(y|x)}{\partial x^j} dx^i dx^j \stackrel{\text{def}}{=} \gamma_{ij}(x) dx^i dx^j. \quad (2)$$

This is the measure of distinguishability we seek. Except for an overall multiplicative constant, the Fisher-Rao metric γ_{ij} is the only Riemannian metric that adequately reflects the underlying statistical nature of the manifold of distributions $p(y|x)$ [12].

We take the further step of interpreting $d\lambda$ as the *spatial* distance. Indeed, one would normally say that the reason it is easy to confuse two particles is that they happen to be too close together. We argue in the opposite direction and *explain* that the reason the particles at x and at $x + dx$ are close together is *because* they are difficult to distinguish.

The origin of the uncertainty is left unspecified. We assume, however, that any two particles at the same location in space are affected by the same irreducible uncertainty. Then the uncertainty is not linked to the particle, but to the place: the source of the uncertainty is a noise, a fluctuation or a fuzziness in space itself.

To assign an explicit $p(y|x)$ we consider what is perhaps the simplest possibility. We assume that $p(y|x)$ is sharply localized in a small neighborhood about x and within this very small region curvature effects can be neglected. We further assume that the information that is relevant to our problem is given by the expected values $\langle y^i \rangle = x^i$ and the covariance matrix $\langle (y^i - x^i)(y^j - x^j) \rangle = C^{ij}(x)$. This is physically reasonable: for each test particle we have estimates of its position and of a small margin of error. Since the underlying space is locally flat $p(y|x)$ can be determined maximizing entropy relative to a uniform measure. This leads to a Gaussian distribution,

$$p(y|x) = \frac{C^{1/2}}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} C_{ij} (y^j - x^j)(y^i - x^i) \right], \quad (3)$$

where C_{ij} is the inverse of the covariance coefficients C^{ij} , $C^{ik}C_{kj} = \delta^i_j$, and $C = \det C_{ij}$. The corresponding metric is obtained from eq.(2). For small uncertainties $C_{ij}(x)$ is constant within the region where $p(y|x)$ is appreciable and we get $\gamma_{ij}(x) = C_{ij}(x)$. The metric changes smoothly over space and, in general, space is curved. Connections, curvatures, and other aspects of the geometry can be computed in the standard way.

To summarize, to each point x in space we associate a probability distribution,

$$p(y|x, \gamma) = \frac{\gamma^{1/2}(x)}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} \gamma_{ij}(x) (y^j - x^j)(y^i - x^i) \right], \quad (4)$$

and the information geometry that derives from considerations of distinguishability among points (as revealed by appropriate test particles) leads us to introduce the metric field $\gamma_{ij}(x)$. The idea is general but was developed explicitly only for the special case of small uncertainties, that is, for test particles that are localized within regions much smaller than those where curvature effects become appreciable. Situations of extreme curvature found near singularities will not be addressed here.

But there is a feature of the distinguishability distance $d\lambda$ in (2) that is very significant: it is dimensionless. Indeed, in eq.(4) we can see that the metric $\gamma_{ij}(x)$ measures spatial lengths in units of the local uncertainty: if the local uncertainty is $\sigma(x)$, then the actual

Riemannian metric is $g_{ij}(x) = \sigma^2(x)\gamma_{ij}(x)$. This immediately raises the question of how to compare the uncertainties $\sigma(x)$ at two distant points. Information geometry only allows one to compare the lengths of small segments at the same place; it allows one to measure angles; it does not describe the full geometry of space; it only describes its conformal geometry. To assign a geometry to space we need to introduce an additional scalar field $\sigma(x)$.

One possibility, which we pursue in the rest of this paper, is that γ_{ij} only describes the conformal geometry of space and that *this is all we really need*. (Entropic dynamics is defined on a space of probability distributions, no additional structure is needed.) Perhaps the answer to the question of how to compare uncertainties at two different locations is: Why would we care? It is not that the irreducible uncertainty $\sigma(x)$ varies from point to point; perhaps such a comparison is objectively meaningless and therefore unnecessary. How can we define the length of an extended curve? Or, how can we compare distant lengths? We cannot. For most practical purposes this does not matter because usually we are only concerned with local distances and information geometry is quite adequate for this restricted purpose.

But if we strongly feel that we must compare distant lengths as a tool for reasoning, if we feel that we must define the length of curves for the sole purpose of constructing images, pictures, and models about the universe, then to satisfy this merely psychological urge, we can introduce a field $\sigma(x)$. In this case our predictions should not depend on the particular choice of $\sigma(x)$ which, being arbitrary, might as well be chosen to make our models convenient and simple. The selection of $\sigma(x)$ should be guided by purely esthetic considerations. Distance should be defined so that motion looks simple.

CHANGE

We define the macrostate of space as a product over individual space points,³

$$P[\{y\}|\gamma] = \prod_x p(y|x, \gamma) . \tag{5}$$

To quantify the change from one state to another we use, once again, the Fisher-Rao metric, but a complication arises here. The comparison between two neighboring product states $P[\{y\}|\gamma]$ and $P[\{y\}|\gamma + \Delta\gamma]$ is carried out by comparing the individual factors and we need an explicit criterion to match factors in one state with factors in the other. For each position x in one state we must decide which is the matching x' in the other state. We must establish a relation of ‘equilocality’. Let us provisionally assume that a best-matching criterion has been found and that equilocal points have been assigned the same (or ‘commoving’) coordinates. Later we return to the question of specifying the “best-matching” criterion.

³ There is an assumption here that we do not need to keep track of information about correlations among degrees of freedom at different locations. Information about correlations may eventually turn out to be relevant (perhaps to account for non-gravitational interactions) and could be included in more elaborate statistical models of geometrodynamics.

Since the state (5) is a product, the change from $P[\{y\}|\gamma]$ to $P[\{y\}|\gamma + \Delta\gamma]$ is a sum where the contributions of the different degrees of freedom add in quadrature,

$$\Delta L^2 = \sum_x \Delta \ell^2(x), \quad (6)$$

where $\Delta \ell^2(x)$ measures the change from $p(y|x, \gamma)$ to its equilocal counterpart $p(y|x, \gamma + \Delta\gamma)$. For each position x , we have

$$\Delta \ell^2(x) = G^{ijkl} \Delta \gamma_{ij} \Delta \gamma_{kl}, \quad (7)$$

where, using eq.(4),

$$\begin{aligned} G^{ijkl} &= \int d^3y p(y|x, \gamma) \frac{\partial \log p(y|x, \gamma)}{\partial \gamma_{ij}} \frac{\partial \log p(y|x, \gamma)}{\partial \gamma_{kl}} \\ &= \frac{1}{4} (\gamma^{ik} \gamma^{jl} + \gamma^{il} \gamma^{jk}). \end{aligned} \quad (8)$$

We can write the sum in eq.(6) as an integral if we note that the density of distinguishable distributions is $\gamma^{1/2}$. In other words, the number of distinguishable distributions, or “distinguishable points”, within the coordinate interval dx is $dx \gamma^{1/2}$ (dx stands for d^3x).⁴ Then (6) is replaced by

$$\Delta L^2 = \int dx \gamma^{1/2} \Delta \ell^2 = \int dx \gamma^{1/2} G^{ijkl} \Delta \gamma_{ij} \Delta \gamma_{kl}. \quad (9)$$

Two points in space count as separate only to the extent that they can be distinguished. The effective number of spatial degrees of freedom, that is the number of “distinguishable points” in the coordinate interval dx is finite. This is neither due to an underlying discreteness in the structure of space nor to quantum effects, but due to the underlying intrinsic fuzziness of space.

To describe the change $\Delta \gamma_{ij}(x)$ at each location x it is convenient to introduce an arbitrary ‘time’ parameter t along the trajectory,

$$\Delta \gamma_{ij} = \gamma_{ij}(t + \Delta t, x) - \gamma_{ij}(t, x) = \partial_t \gamma_{ij} \Delta t, \quad (10)$$

$\partial_t \gamma_{ij}$ is the “velocity” of the metric in the special best-matched frame. Then eq.(9) becomes

$$\Delta L^2 = \int dx \gamma^{1/2} G^{ijkl} \partial_t \gamma_{ij} \partial_t \gamma_{kl} \Delta t^2. \quad (11)$$

Having computed the change in the special commoving frame where equilocal points have the same coordinates we now switch to an arbitrary coordinate frame where equilocal points at t and $t + \Delta t$ have coordinates x^i and $\tilde{x}^i = x^i - \beta^i(x) \Delta t$ respectively; equilocal points are “shifted” by $\beta^i \Delta t$. Next we consider the corresponding change in

⁴ Note that since we cannot compare distant lengths or distant volumes it makes no sense to say that $\int_{\mathcal{R}} dx \gamma^{1/2}$ measures the volume of an extended region \mathcal{R} ; it measures the number of distinguishable points in \mathcal{R} .

the metric. Under the infinitesimal shift $\tilde{x}^i = x^i - \beta^i(x)\Delta t$ the metric at $t + \Delta t$ transforms into $\tilde{\gamma}_{ij}$,

$$\gamma_{ij}(t + \Delta t, x) = \tilde{\gamma}_{ij}(t + \Delta t, x) - (\nabla_i \beta_j + \nabla_j \beta_i) \Delta t, \quad (12)$$

where $\nabla_i \beta_j = \partial_i \beta_j - \Gamma_{ij}^k \beta_k$ is the covariant derivative associated to the metric γ_{ij} . In the new frame, setting $\tilde{\gamma}_{ij}(t + \Delta t, x) - \gamma_{ij}(t, x) = \Delta \gamma_{ij}$ the change in γ_{ij} between equilocal points is expressed as

$$\Delta \beta \gamma_{ij} = \Delta \gamma_{ij} - (\nabla_i \beta_j + \nabla_j \beta_i) \Delta t, \quad (13)$$

or, $\Delta \beta \gamma_{ij} = \dot{\gamma}_{ij} \Delta t$, where

$$\dot{\gamma}_{ij} \stackrel{\text{def}}{=} \partial_t \gamma_{ij} - \nabla_i \beta_j - \nabla_j \beta_i. \quad (14)$$

In terms of the transformed coordinates the change ΔL^2 retains the same form as before, eq.(11), except that the new best-matched velocities $\dot{\gamma}_{ij}$ are the coordinate velocities $\partial_t \gamma_{ij}$ suitably “corrected” by the shift β^i ,

$$\Delta \beta L^2 = \int dx \gamma^{1/2} G^{ijkl} \dot{\gamma}_{ij} \dot{\gamma}_{kl} \Delta t^2. \quad (15)$$

Note that ΔL^2 depends only on the initial and final states and is invariant under time reparametrization $t \rightarrow t' = f(t, x)$.

Now we address the problem of specifying the best-matching criterion. For given velocities $\partial_t \gamma_{ij}$ our estimate of the actual change ΔL^2 can be artificially altered by different choices of the shift β^i . We have to decide which values of β^i provide the best equilocality match.

The problem of selecting the optimal shift can be tackled as a problem of inference: the “prior” state of information is described by the earlier distribution $P_t = P[\{y\}|\gamma]$, and we are given the new information that the “posterior” state belongs to the later “trial” family of distributions $P_{t+\Delta t} = P[\{y\}|\gamma + \Delta \gamma]$. The trial distributions are essentially identical except for diffeomorphisms – the spatial shifts $\beta^i \Delta t$. Which one do we choose? *We choose the distribution that does the least violence to our prior beliefs while fully accommodating the new information.* Phrased in this way it is clear that this is the kind of question the method of maximum entropy was designed to answer: *Best matching reflects the least change.*

The actual change ΔL^2 between the two successive states is obtained by maximizing the appropriate relative entropy or minimizing the corresponding $\Delta \beta L^2$ over all choices of β^i ,

$$\Delta L^2 = \min_{\beta} \Delta \beta L^2. \quad (16)$$

Vary with respect to β ,

$$\delta(\Delta \beta L^2) = 2 \int dx \gamma^{1/2} G^{ijkl} \dot{\gamma}_{ij} \delta \dot{\gamma}_{kl} \Delta t^2 = 0. \quad (17)$$

Next use $\delta\dot{\gamma}_{kl} = -\nabla_k\delta\beta_l - \nabla_l\delta\beta_k$ and integrate by parts to get

$$\nabla_l(2G^{ijkl}\dot{\gamma}_{ij}) = 0 \quad \text{or} \quad \nabla_l\dot{\gamma}^{kl} = 0, \quad (18)$$

where we used eq.(8) and

$$\dot{\gamma}^{kl} = \partial_t\gamma^{kl} + \nabla^k\beta^l + \nabla^l\beta^k. \quad (19)$$

Eqs.(18) are the differential equations that determine the shift β^i that establishes the best matching and equilocality between the given initial and final geometries γ_{ij} and $\gamma_{ij} + \Delta\gamma_{ij}$. Alternatively, we can consider these equations as constraints on the allowed change $\Delta\gamma_{ij} = \partial_t\gamma_{ij}\Delta t$ for a given shift β^i .

ENTROPIC DYNAMICS

The dynamical question is ‘‘Given initial and final states, what trajectory is the system expected to follow?’’ The answer [10][17] follows from the implicit assumption that there exists a continuous trajectory which reduces the problem of studying large changes to the simpler problem of studying small changes. Consider then the short segment of the trajectory between the states P_t and $P_{t+\Delta t}$. The idea is that in going from one to the other the system must pass through a halfway point, and also through a state that lies a third of the way, and so on. More generally, the trajectory is composed of states such that having travelled a distance dL from the initial P_t , there remains a distance ωdL to the final $P_{t+\Delta t}$, with $0 < \omega < \infty$. The trajectory is the set of states obtained as ω sweeps from 0 to ∞ . However, in the case of geometrodynamics we know much more than just that the product state eq.(5) must evolve through a continuous sequence of intermediate states. *We also know that each and every one of the individual factors must evolve continuously through a sequence of intermediate states to reach the corresponding final state.* This means that instead of one parameter ω there are many such parameters, one for each position x . In other words, the intermediate states P_ω interpolating between the initial P_t and the final $P_{t+\Delta t}$ should be labeled by a function $\omega(x) = w\zeta(x)$ where $\zeta(x)$ is a fixed positive function and the parameter w varies from 0 to ∞ .

There is no single trajectory; each choice of the function $\zeta(x)$ defines one possible trajectory. In a sense, the system follows many alternative paths simultaneously – this is Wheeler’s many-fingered time – and physical predictions are independent of the choice of the arbitrary function $\zeta(x)$. The path-independence is very significant because the product state P_t provides us with the only definition of what an ‘‘instant’’ is, of what state $p(y|x')$ of a distant test particle at x' we can agree to call simultaneous with a certain state $p(y|x)$ of the test particle at x . Therefore, if there is no unique sequence of intermediate states, then there is no unique, absolute definition of simultaneity. We see here a ‘‘foliation’’ invariance, a rudimentary form of local Lorentz invariance.

Let t be the ‘‘time’’ parameter labeling successive intermediate states. The initial state is $\gamma_{ij}(t, x) = \gamma_{ij}(x)$, the final state is $\gamma_{ij}(t + \Delta t, x) = \gamma_{ij}(x) + \Delta\gamma_{ij}(x)$, and the intermediate states are of the form $\gamma_{ij}(t + dt, x) = \gamma_{ij}(x) + d\gamma_{ij}(x)$. For appropriate choices of the shift the best-matched changes corresponding to $\Delta\gamma_{ij}$ and $d\gamma_{ij}$ are given

by eq.(13) and

$$d_{\beta}\gamma_{ij} = d\gamma_{ij} - (\nabla_i\beta_j + \nabla_j\beta_i) dt . \quad (20)$$

To determine the intermediate state P_{t+dt} one varies $d\gamma_{ij}$ to maximize the relative entropy

$$S[P_{t+dt}|P_t] = -\frac{1}{2}dL^2 = -\frac{1}{2} \int dx \gamma^{1/2} d\ell^2(x) , \quad (21)$$

subject to independent constraints at each point x . For each of the factors in the product state P_{t+dt} we require that if the distance to the initial state is $d\ell(x)$ then the distance that remains to be covered to reach the final state is $d\ell_f(x) = \omega(x)d\ell(x)$ where

$$d\ell^2(x) = G^{ijkl} d_{\beta}\gamma_{ij} d_{\beta}\gamma_{kl} , \quad (22)$$

and

$$d\ell_f^2(x) = G^{ijkl} [\Delta_{\beta}\gamma_{ij} - d_{\beta}\gamma_{ij}] [\Delta_{\beta}\gamma_{kl} - d_{\beta}\gamma_{kl}] . \quad (23)$$

Introducing Lagrange multipliers $\lambda(x)$, the basic variational principle of entropic dynamics is

$$0 = \delta \int dx \gamma^{1/2} \left[d\ell^2 + \lambda \left(d\ell_f^2 - \omega^2 d\ell^2 \right) \right] . \quad (24)$$

Variations of $d\gamma_{kl}$ give

$$d_{\beta}\gamma_{ij}(x) = \chi(x) \Delta_{\beta}\gamma_{ij}(x) \quad \text{where} \quad \chi(x) = \frac{\lambda(x)}{1 + \lambda(x) (1 - \omega^2(x))} . \quad (25)$$

The Lagrange multipliers $\lambda(x)$ are determined so that the constraints $d\ell_f = \omega d\ell$ hold. We get

$$d\ell(x) = \chi \Delta\ell(x) \quad \text{and} \quad \chi(x) = \frac{1}{1 + \omega(x)} , \quad (26)$$

and conclude that the selected intermediate state $d_{\beta}\gamma_{ij}$ is such that

$$d\ell(x) + d\ell_f(x) = \Delta\ell(x) , \quad (27)$$

which means that the metric at the point x (the metric γ) evolves along geodesics in its individual configuration space. Degrees of freedom at different locations do not, however, evolve independently of each other; they are coupled through the diffeomorphism constraint eq.(18) which decides, at each moment in time, which spatial points are equi-local. Note that the trajectory described by (27) is explicitly independent of $\omega(x)$; this is foliation invariance.

Having derived a model of statistical geometrodynamics by applying standard rules of inference to the information codified in the states of the system, we can now summarize the dynamics by introducing an action that leads to the same equations of motion. The proposed action is

$$J = \int_{t_i}^{t_f} dt \int dx \gamma^{1/2} (G^{ijkl} \dot{\gamma}_{ij} \dot{\gamma}_{kl})^{1/2} . \quad (28)$$

Our next step should be to explore the consequences of this statistical geometrodynamics and establish the relation if any between this theory and Einstein's General Relativity, but this is a subject for future work.

CONCLUSIONS AND SOME COMMENTS

The model of statistical geometrodynamics (SGD) developed here combines two basic ideas. First, the geometry of space is of statistical origin and is explained in terms of the distinguishability metric of Fisher and Rao. Second, the dynamics of this geometry is derived purely on the basis of principles of inference; there is no need to postulate additional “laws of nature.”

The similarities with the general theory of relativity (GR) suggest that GR can be obtained in some appropriate limit. For example, just as in GR the dynamical degrees of freedom are those that specify the conformal geometry of space [13]. The best-matching condition corresponds to the diffeomorphism constraint in the Hamiltonian formulation of GR [16]. There is no reference to an external time; there is a natural intrinsic time defined by the change of the system itself which, just as in GR, can only be obtained after the equations of motion are solved [18]. Despite being derived by maximizing entropies the theory is time reversible.

Perhaps the feature of SGD that does most violence to our intuition is its scale invariance. The scale factor $\sigma(x)$ needed to assign a Riemannian geometry to space is arbitrary and its choice should be dictated by convenience. This gauge invariance can be used to great advantage. The essence of the dynamics of GR lies in the embeddability of space in spacetime: any model that uses only the metric tensor to describe the changing geometry of space as it evolves in spacetime is equivalent to GR [19]. I conjecture that the $\sigma(x)$ can be chosen so that the evolving geometry of space sweeps out a four-dimensional spacetime – which amounts to choosing the gauge so that the appropriate Gauss-Codazzi equations are satisfied. In this particular gauge SGD should coincide with GR, in other words, we will have accomplished our goal of deriving macroscopic GR from a more basic microscopic statistical theory.

If true statistical geometrodynamics would have a number of implications for physics. Perhaps the most interesting are the revision it requires of the notion of distance and the recognition that spacetime is not a fundamental notion. The statistical nature of geometry would provide a decoherence mechanism that could act as a regulator for the infinities pervading quantum field theories; it might bear on the subject of CP violation and matter-antimatter asymmetry; Lorentz and CPT symmetries would only have statistical validity. On the other hand, the scale invariance might bear on cosmological issues such as the early inflation and the late accelerated expansion of the universe.

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