Broken Symmetry and Spacetime

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Abstract

The phenomenon of broken spacetime symmetry in the quantum theory of infinite systems forces us to adopt an unorthodox ontology. We must abandon the standard conception of the physical meaning of these symmetries, or else deny the attractive "liberal" notion of which physical quantities are significant. A third option, more attractive but less well understood, is to abandon the existing (Halvorson-Clifton) notion of intertranslatability for quantum theories.

1 Introduction

Successful spacetime theories, including Newtonian as well as relativistic mechanics, are invariant under a wide range of symmetries. In certain physically contingent situations, these symmetries are broken by the matter content of spacetime. That is, a symmetry transformation that leaves spacetime unchanged fails to preserve the matter content.

This is little bother when the matter in question is classical. In that case it is easy to maintain a standard picture of (global) spacetime symmetries, according to which translating everything in the universe ten feet to my left, or rotating everything clockwise by 90 degrees, makes no physical difference at all. Instead such transformations amount to a change in notation – they change our *description* of physical reality (in terms of coordinates), but leave the reality itself unchanged.

When the matter content of spacetime is *quantum*, however, all of this comes into question – or at least it does if we make some plausible assumptions about which physical quantities to include in the content of quantum theory. So we must discard these plausible assumptions, or else alter the classical notion of spacetime symmetries and their nature.

In what follows I will motivate this dilemma and then explore some consequences of seizing either of its two horns.

2 Broken rotational symmetry

Physical theories, like physical and mathematical objects, possess *symmetries.* A symmetry of a thing is a group of transformations that map the thing to itself, or leave it unchanged. Thus a square drawn in a plane is symmetric under 90-degree rotations – performing such a rotation on the plane gives you an identical square. Similarly Newtonian mechanics is symmetric under *all* rotations. Whenever I let go of my pen above my table, Newtonian mechanics predicts that it will drop – no matter which direction the chair, the pen and I are oriented. Rotations leave the laws of physics unchanged.[1]

Imagine a Newtonian ball is placed on the "Mexican hat" surface depicted in Figure 1. What ground states (states of lowest energy) are possible for the ball? If we assume the only relevant force is gravity, the ball's total kinetic and potential energy will be lowest when it is stationary at a point of least possible elevation. This will include any point at the bottom of the "brim" of the Mexican hat. Since there are infinitely many such points, infinitely many ground states are available to the ball.

Now suppose the ball assumes one of these ground states. The system of the hat and the ball then fails to be invariant under rotations.[2] A 90-degree rotation, for example, will fail to map the system onto itself. Instead it is mapped to a system in which the ball has assumed a different ground state. Whenever a ground state fails to be invariant under a symmetry of the laws, we have a case of *spontaneous symmetry breaking.* Often it is said that multiple ground states are available to the system and symmetry transformations map between ground states.

Those familiar with the philosophy of space and time will be hesitant about this last claim, that multiple different ground states are possible for the ball. Of course if the ball

[1]It is a good – and philosophically loaded – question what this phrase means, but the assumption is normally made that all and only transformations which leave the Lagrangian unchanged are symmetries of "the physics." I will adopt this assumption in what follows.

[2]Throughout this example I refer only to rotations around the axis that points out the top of the hat.

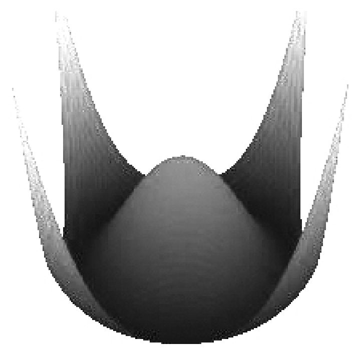

Figure 1: The Mexican Hat

and the Mexican hat are sitting on the table in front of me, it makes perfectly good sense to say that the ball could've been closer to me, or farther away. But suppose the ball-hat system is alone in an otherwise empty world (or in a world otherwise symmetric around the hat's axis). Then claiming that there are physically possible worlds in which the ball ends up at a different point along the brim of the hat commits us to some rather strange metaphysics.

What's so strange about this claim? For one thing there is no *qualitative* difference between a world in which the ball ends up at point P and one in which it's at P' (where these are two different points along the brim). One might respond that of course there is a difference – in one world the ball is at P, and in the other world it's at a different point P'. But this assumes some pre-existing means of identifying P and P' across worlds. In fact, since P' in the second world has all the same qualitative features that P has in the first world (the only qualitative properties that differentiate them are their locations relative to the ball), to suppose these two worlds are different is to suppose that spacetime points possess primitive identity. It must be that some facts other than the qualitative features of a point help determine whether that point is P.

Hoefer (1996) and Teller (1991), among others, argue persuasively against the notion that spacetime points possess primitive identity. There are two main arguments. First, primitive identity is metaphysically dispensable; everything about points that needs explaining can be explained in terms of their qualitative properties (Hoefer, 1996, 16-22). Second, we have good reason (intuitively, at least) to agree with Leibniz that spacetime symmetry transformations do not relate distinct possible worlds. For example, translating everything in a Newtonian spacetime in the same direction by three feet, or rotating everything by 90 degrees, does not really produce a different state of affairs. At best these amount to re-descriptions (in terms of different coordinates) of the same possibility. As Earman and Norton (1987) have shown, denying this principle of "Leibniz equivalence" entails that general relativity is indeterministic.

This leaves us with two acceptable ontologies for Newtonian and relativistic spacetime:

Relationism: There are no spacetime points; spatiotemporal properties are nothing more than (actual and possible) relations between physical objects.

Sophisticated Substantivalism: Spacetime points exist, but do not possess primitive identity or haecceity; physical states related by spacetime symmetry transformations describe the same possible world.

Both ontologies entail that in the classical symmetry-breaking case under discussion, there is a unique possible world with least energy, although the state space of our physical theory includes multiple ground states. So there must be a many-to-one correspondence between mathematical states and physical possibilities. Mathematically distinct ground states must be physically equivalent. More generally, the set of *excitations* from one ground state must be physically equivalent to the excitations from the other ground states; otherwise there would be modal differences between the ground states, implying that they are not after all physically equivalent.

Plausibly, two sets of states cannot be physically equivalent unless they are *intertranslatable* in the sense explained by Glymour (1971). There must at least be some way to (bijectively) map between the sets of states and the observable quantities defined on them that preserves (i) the set of physical quantities denoted by the observables and (ii) the values these quantities take on in each state. If this can't be done, it is impossible to claim that the two sets of states denote the same possibilities.

For the ball-hat system, a translation scheme is easy to produce. Suppose the system is described in cylindrical polar coordinates (r, θ, h). Then we simply perform the coordinate transformation $\theta \rightarrow \theta + \epsilon$ to translate between the ground state in which the ball rests at

$\theta = 0$ and the one in which it rests at $\theta = \epsilon$. This mapping also takes the set of states describing excitations from the former ground state to the set of excited states for the latter ground state. And it does so in such a way that the values of all classical observables (functions on the space of states) are preserved. The ground states of the ball-hat system thus meet our necessary condition for physical equivalence.

What's the upshot of all this? Mainly that the breaking of spacetime symmetries in classical physics poses no threat to the notion that global spacetime symmetry transformations never correspond to real physical changes. But this notion will come under threat when we consider broken symmetries in *quantum* physics.

3 Representations of quantum theory

The possibility of symmetry breaking in quantum theory depends on the existence of *inequivalent representations* of the theory. So before we explore some examples of quantum symmetry breaking, we would do well to review some basic facts about inequivalent representations. I will do my best to make these facts intelligible for neophytes, so many details will be omitted or relegated to footnotes.

Inequivalent representations arise when we attempt to quantize the classical theory of an infinite system like a field or a set of infinitely many particles. At large length scales, classical physics provides a good approximation to the more fundamental quantum theory. Quantization is a means of beginning with a classical theory and constructing a corresponding quantum theory that agrees (approximately) with its predictions at large length scales.[3]

Physical quantities in quantum theory are represented by self-adjoint operators, also called *observables.* These operators act on a collection of state vectors, which must have the mathematical properties of a Hilbert space. For a physically possible state represented by a vector ψ in the Hilbert space, the quantity given by the operator $\hat{O}$ has its expected value given by

$$\psi(\hat{O}) = (\psi, \hat{O}\psi), \tag{1}$$

the inner product of ψ with the vector $\hat{O}\psi$.

[3] I will not discuss the measurement problem, which presents a significant challenge to this picture of quantum-classical correspondence.

In classical physics the canonical observables that characterize a state are position (q) and momentum (p). The first step in quantization is to represent these using self-adjoint operators $\hat{q}$ and $\hat{p}$. Position and momentum obey the Heisenberg uncertainty relation iff these operators obey the canonical commutation relations (CCRs)

$$[\hat{p}, \hat{q}] = \hat{p}\hat{q} - \hat{q}\hat{p} = i. \tag{2}$$

So for the vectors of a Hilbert space to represent quantum states, the position and momentum operators on that space must obey (2). We then say that the operators form a *representation* of the CCRs.

Strictly speaking, a representation of the CCRs cannot act on the entire Hilbert space; this is because the operators $\hat{p}, \hat{q}$ are unbounded and so are not well-defined on every vector ψ. But a representation of the CCRs in their Weyl form can do better. The exponentiated "Weyl operators" $\hat{U}(a) = e^{ia\hat{x}}, \hat{V}(b) = e^{ib\hat{p}}$ are well-defined on all ψ. So we can ensure that the CCRs hold true for all states by converting them to the Weyl relations,

$$\hat{U}(a)\hat{V}(b) = e^{iab}\hat{V}(b)\hat{U}(a). \tag{3}$$

A representation of the CCRs (in Weyl form) is then a set[4] of operators on a Hilbert space which obey (3). Quantization then amounts to the construction of such a representation.

Now we're ready to discuss inequivalent representations. In quantum mechanics with finitely many particles (and therefore finitely many degrees of freedom), the Weyl relations uniquely determine the set of all observables.[5] This makes it possible to suppose that the Weyl relations determine the set of all physical quantities for a finite quantum theory, along with the relations between them. So quantizing a finite classical theory gives a unique result. But this is not true for field theories and statistical mechanics, which require infinite systems. In such cases the Weyl relations do not uniquely determine the complete set of observables on a state space.

Instead, quantizing an infinite system results in a continuum of different Hilbert space

[4]More precisely, a pair of one-parameter unitary groups.

[5]Assuming only representations on separable Hilbert spaces are physical, this is established by the Stone-von Neumann theorem CITE.

representations.[6] Each representation possesses a different set of observables. Thus, if we suppose that all observables represent physically significant quantities, different representations disagree on which quantities there are. But there is some overlap: the Weyl operators $U(a), V(b)$ generate (via sums and products) a set of operators called the Weyl algebra.[7] And every representation of the CCRs includes this set of operators. In fact, the Weyl algebra is the largest set of operators shared by all representations.

At first glance, the choice of a representation seems arbitrary. For example, it appears to depend on whether one chooses an accelerating or inertial coordinate system in special relativity (see Halvorson and Clifton, 2001; Arageorgis *et al.*, 2003). A venerable tradition in physics, successful in relativity and gauge field theories, is to suppose that the fundamental physical quantities are invariant under arbitrary choices of perspective. This might motivate us to suppose that the fundamental quantities in quantum theory must be independent of the choice of representation. If so, the fundamental quantities must all correspond to operators in the Weyl algebra. Halvorson and Clifton (2001, PAGE) call this position *conservatism* about observables.[8]

But there is a problem with conservatism, namely that some observables of (seemingly) obvious physical importance are not elements of the Weyl algebra. Ruetsche (2003) focuses on temperature, which is only definable within a representation. Other examples include the stress-energy which is the source of Einstein's gravitational field, and the total number of particles (Ruetsche, 2002, 366-369). If we suppose there could be some matter of fact about the temperature of the universe, or the total number of particles it contains, we must reject conservatism. An obvious alternative is available. Each representation has a unique notion of "closeness" for operators, called its weak topology. We may expand the set of observables by including those which can be approximated arbitrary closely (according to the weak topology) by operators in the Weyl algebra. Halvorson and Clifton have named this approach *liberalism* about observables.[9]

This is perhaps the most basic dilemma in the interpretation of infinite quantum theory. If we wish to reify quantities like temperature and particle number, which play important roles in physical explanations, we must make a seemingly arbitrary choice of representation in

[6] The notion of "same representation" in play here is that of unitary equivalence. MORE MORE MORE

[7] This is a C*-algebra: MORE

[8] Conservatism about observables characterizes the view Ruetsche (2002) calls Algebraic Imperialism.

[9] In Ruetsche's parlance, liberalism is (ironically) one tenet of Hilbert Space Conservatism.

order to define the corresponding observables. Ruetsche (2002, 368) has suggested a possible solution: perhaps the choice of representation is not arbitrary after all. According to the Gelfand-Naimark-Segal (GNS) theorem, every quantum state is associated with a unique "home" representation of the Weyl relations.[10]. So we may suppose that a state instantiates those physical quantities defined as observables on its home (GNS) representation. On this view, which physical quantities are instantiated is a physically contingent fact, but it is determined by the state. This gives us one way of making liberalism about observables plausible.

The problem with this view – and with any view that entails liberalism about observables – is that it runs afoul of the standard picture of rotational symmetry discussed in Section 2. In the next section I will explain why.

4 Quantum symmetry breaking

To describe the possible outcomes of symmetry breaking in quantum theory, we must appeal to multiple inequivalent representations. To see why, recall from Section 2 that when a symmetry is broken there are multiple (mathematically distinct) ground states, related by symmetry transformations. In quantum theory, a given Hilbert space representation includes at most a single ground state. So for quantum symmetry breaking to occur, ground states in multiple inequivalent representations must be possible.

To see how this works, we must examine how symmetries are represented in quantum theory. In its most general form, a symmetry is given by a transformation (automorphism) of the Weyl algebra. A spacetime symmetry, for example, acts on the coordinates x, p. A transformation which takes x to Tx then takes $\hat{U}(a) = e^{ia\hat{x}}$ to $T\hat{U}(a) = e^{iaT\hat{x}}$; likewise for momentum and $V(b)$.

Symmetries must of course preserve expectation values, or else we could distinguish empirically between transformed and un-transformed systems. To meet this condition a symmetry must also act on states. In general for states ψ, observables $\hat{O}$ and symmetry transformations T, we require that $T\psi(T\hat{O}) = \psi(\hat{O})$.

Consider a ground state ψ_0 in its home (GNS) representation. If the ground state is invariant under a symmetry T – i.e., if $\psi_0(T\hat{O}) = T\psi_0(\hat{O}) = \psi_0(\hat{O})$ for all $\hat{O}$ – then T acts

[10] MORE ABOUT GNS

as a unitary operator on the GNS representation. Moreover, T *must* act unitarily if it is to preserve all of the inner products (ψ, ϕ) between state vectors.[11] Since these give the "transition probability" or likelihood of ψ collapsing into ϕ on measurement, they would appear to encode physically significant information that should be preserved by symmetries.

But if T is a broken symmetry, ground states are *not* invariant under T, so nothing guarantees that T will act like a unitary operator. In fact, whenever a symmetry of a quantum system is broken, T will fail to act like a unitary operator – the symmetry will not be *unitarily implementable.* This has been taken, for instance by Earman (2004), to be the defining feature of quantum symmetry breaking. Since T is not unitary, it maps between unitarily inequivalent representations. This is why inequivalent representations are required for the quantum description of symmetry breaking.

We already discussed broken rotational symmetry in classical physics. This sometimes occurs in infinite quantum theory as well. One example occurs in the case of *coherent states* which arise in the so-called infrared problem in quantum field theory (see Roepstorff, 1970). Though I expect this example will be unfamiliar to most readers, it is formally simple. To construct a coherent state with broken rotational symmetry, we begin with a rotationally invariant state ψ_0. In field theory, there are infinitely many configuration and momentum observables $\hat{q}_i, \hat{p}_j$ which obey the CCRs (2) for $i = j$. From these we can construct Weyl operators $\hat{U}_i(a), \hat{V}_j(b)$ obeying (3). ψ_0 then assigns expectation value $\psi_0(W)$ to each Weyl operator W.

A transformation which takes ψ_0 to another state that also obeys the CCRs is called a *canonical transformation.* Among these are the *coherent transformations*, which take the momentum expectation values $\psi(p_j)$ to $\psi(p_j) + \lambda_j$, for given scalar λ_j's. In terms of the Weyl operators, a coherent transformation takes $V_j(b)$ to $e^{i\lambda_j} V_j(b)$. Call the transformed state ψ_λ. Like all states, ψ_λ has a "home" GNS representation. ψ_λ's home representation is unitarily equivalent to ψ_0's iff

$$\sum_j |\lambda_j|^2 < \infty. \tag{4}$$

For some choices of λ_j, applying a coherent transformation to ψ_0 will give us a state with broken rotational symmetry. To see how this happens, note first that symmetries are most

[11] This is established for Hilbert spaces of dimension greater than two by Wigner's theorem (Araki, 1999, 49-53).

generally given by transformations (automorphisms) on the Weyl algebra. Thus a rotation R transforms the Weyl operators $V_j(b)$ to $R(V_j(b))$. For a coherent state ψ_λ, this means that R transforms $V_j(b) + \lambda_j$ to $R(V_j(b)) + R(\lambda_j)$. Now suppose that the λ_j are chosen so that

$$\sum_j |\lambda_j - R(\lambda_j)|^2 = \infty. \tag{5}$$

Then ψ_λ and the rotated state $R(\psi_\lambda)$ have inequivalent GNS representations. Rotational symmetry transformations take ψ_λ outside its home representation; thus, the rotational symmetry is broken for such a state.

A mathematically more difficult but physically more familiar case is that of ferromagnetism, detailed in Ruetsche (forthcoming). The main payoff is just this: suppose ψ_M is the ground state of an infinitely long bar magnet. In such a case the magnet will be polarized in the direction of its North pole. Then the rotated state $R(\psi_M)$ is polarized along a different axis and lives in a different, unitarily inequivalent representation from ψ_M. No unitary operator can map between the set of states reachable by exciting the ground state ψ_M and those reachable from $R(\psi_M)$. Again rotational symmetry is broken.

5 Intertranslatability of representations

We saw at the end of Section 2 that two states, or more generally two sets of possible states, can only be physically equivalent if they are intertranslatable. Take a state with rotationally broken symmetry like our infinitely long magnet ψ_M. Could the state ψ_M be physically equivalent to the rotated state $R(\psi_M)$? Moreover, could the set of excited states reachable from ψ_M be physically equivalent to the states reachable from $R(\psi_M)$?

Before we answer these questions, let's try to see what's at stake. Recall the two most plausible ontologies for spacetime, relationism and (sophisticated) substantivalism. Both entail that spacetime symmetry transformations do not represent real physical changes in a system. This means a physically possible world is, on these views, given by an *equivalence class* of solutions, according to which any two solutions related by a symmetry are (physically) equivalent. If we generalize this principle to sets of solutions, it likewise follows that if a particular symmetry transformation gives a bijection between two such sets, these sets should be physically equivalent. All of this appears straightforwardly inconsistent with the possibility that a spacetime symmetry transformation – for instance, a global rotation – could relate two physically inequivalent states or sets of states.

This means that if the set of possibilities in which our magnet is polarized along the $+z$ direction is physically inequivalent to the set in which it's polarized along $+x$, both sophisticated substantivalism and relationism are in trouble. So we might hope that sensible ontologies for quantum theory, including liberalism about observables, would at least permit us to translate between the $+z$-polarized states and the $+x$ states. But when rotational symmetry is broken, liberalism about observables does not permit this.

This is a consequence of the unitary inequivalence between the $+z$ ground state's GNS representation and the $+x$ representation. A theorem of Halvorson and Clifton (2001, Prop. 4) entails that

- Assuming liberalism about observables, there is a translation scheme between two (irreducible) representations only if the representations are unitarily equivalent.

Unless some metaphysical fix can be found, our best ontologies of spacetime must be incompatible with liberalism.

6 Metaphysical fixes

A similar problem for relationism and sophisticated substantivalism (call the disjunction of these views the *standard picture*) has arisen before. It was prompted by the case of parity asymmetry in the standard model. In this case a fix was found; perhaps it can be extended to cover the present case.

The earlier problem arose from the fact that the gauge theory describing the weak nuclear interaction is not invariant under parity transformations. *Parity,* or reflection across one spatial axis, is a transformation which changes the handedness (or chirality) of structures like hands which possess incongruent counterparts. Some quantum fields, including the Dirac electron, possess this sort of handedness. And the gauge bosons ("force-carrying" fields) of the weak interaction couple only with left-handed, as opposed to right-handed, Dirac fields.

This is curious when we reflect for a moment on what a parity transformation amounts to. To invert parity, we pick a spatial axis (x, say) and an origin; then we reverse the coordinates of each point along that axis (taking x to $-x$). This is nothing but a change of coordinates, so the standard picture ought to entail that no two worlds related by a parity transformation are distinct possibilities. But here are two such worlds: one in which the weak bosons couple with right-handed Dirac fields, and one in which the bosons couple with left-handed Dirac fields. Weak theory tells us that while the first world is physically impossible, the second is possible. Since the same world can't be both possible and impossible, parity transformations must sometimes relate *distinct* worlds.

For a fix to work, some mistake must be found in the above reasoning. The project of rectifying this mistake was completed in stages by Hoefer (2000), Huggett (2000) and Pooley (2003). Briefly, the solution is as follows. For the above reasoning to be correct, there must be some intrinsic difference between right- and left-handed Dirac fields. But the handedness of fields is defined only in *relation* to a conventional, right-handed coordinate system (one in which we use the "right-hand rule" to compute cross products). In particular, the theory predicts no difference between right-handed fields written in right-handed coordinates and left-handed fields written in left-handed coordinates.

This means that, while it is physically necessary in weak theory that there *be* a preferred handedness, it is a matter of convention whether that handedness is right or left. The theory asserts merely that there exists a preferred handedness, not which handedness it is. We can thus maintain, in accordance with the standard picture, that there is no matter of fact about "which" handedness is preferred. So, while the theory entails a physical difference between electrons of the preferred handedness and electrons with the opposite handedness, it does *not* entail a difference between right- and left-handed electrons, because there is no matter of fact about whether the preferred handedness is right or left. In a world with a single electron, there will be no fact of the matter about whether the electron is right or left handed, but it *will* be determinate whether the electron has a handedness which allows it to couple with the weak bosons.

Let's see if a similar fix can apply to our example of a quantum magnet with broken rotational symmetry. Our dilemma, remember, is that the theory seems to predict a physical difference between the possible magnets polarized along the $+z$ direction and those polarized along $+x$. But, analogous to the parity case, these claims are being made within a coordinate system in which we've conventionally "painted on" x and z axes. These names for the axes are pure convention, and could have been reversed without changing the physics. So we may conclude, as in the parity case, that the physics is not telling us about any intrinsic difference between $+x$ magnets and $+z$ magnets. Rather, it is telling us that *some* direction is in fact preferred, and there is a physical difference between systems oriented along *that* direction and systems oriented along a *different* direction. But none of this entails that coordinate changes represent real physical changes.

This fix would work perfectly well, *if* the magnet case were properly analogous to the weak theory. But it isn't. The existence of a preferred direction in weak theory is ensured by the parity-asymmetry of the laws. But in the case of our magnet, the rotational *symmetry* of the laws should similarly ensure the *nonexistence* of a preferred direction.

To deny this is to deny any substantive difference between spontaneously broken symmetry and asymmetry. In cases of broken rotational symmetry, the laws still entail the nonexistence of a preferred direction, so any metaphysical fix appealing to such a direction is in violation of the laws. The problem for liberalism is not just that it requires solutions related by rotations to be physically distinct. It requires that they be distinct solutions *to a theory that is symmetric under rotations.* We cannot solve the problem by denying that rotation is a symmetry of the theory, which we must do in order to apply the parity-fix.

7 Deeper problems

It's beginning to appear as if there's no way out except to deny either liberalism or the standard picture of spacetime symmetries. But I'm not about to ask you to choose. This appearance of inevitability is an artifact of the way I've presented the problem so far.

It's time to broaden the scope of our inquiry beyond the carefully chosen question of whether the $+z$-polarized states of a magnet are, for the liberal about observables, physically equivalent to the $+x$-polarized states. To make sure that denying liberalism will get us out of my dilemma, we must pose the same question to the *conservative* about observables. And this is where things become truly strange. The conservative's answer is no different from the liberal's.

This fact was established by Halvorson and Clifton (2001), but the reason why isn't obvious from their presentation. The important result for the conservative about observables is the following:

- On the "conservative approach to states," there is a translation scheme between two (irreducible) representations only if they are unitarily equivalent. (Halvorson and Clifton, 2001, Prop. 3)

A *conservative-about-states* is supposed to hold that the density operators of *one* of the two representations (the representation's folium) gives an exhaustive list of the physically possible states. Nothing about conservatism about observables entails conservatism-about-states, so why does this result matter?

The answer has to do with how Clifton and Halvorson are thinking of translation. They view it as a relationship between complete (exhaustive) sets of physically possible states. This means that, in establishing that two representations are intertranslatable for the conservative-about-states only if unitarily equivalent, they've also established something else. Their result could not hold unless the folia of two representations are intertranslatable only if the representations are unitarily equivalent. And this must be true, *independent* of whether one actually is a conservative-about-states. Even someone "liberal" about states, who counts many more states as physically possible, must still agree that the sets of states counted as possible by two conservatives-about-states are intertranslatable only if they're unitarily equivalent. It's just that he disagrees with the conservatives about whether these states are the only possible ones.

Nothing about the above reasoning presumes liberalism about observables. On the assumptions of Halvorson and Clifton, it is therefore true, even for the conservative about observables, that the states of two disjoint folia are physically inequivalent. Since the $+z$-polarized states of a magnet form a folium disjoint from its $+x$-polarized states, even the conservative must agree that these two sets of possibilities are not physically equivalent.

I imagine you'll agree that it's time to examine the assumptions of Halvorson and Clifton.

Glymour's notion of translation between theories is their starting point. This requires that any translation map the primitive terms (fundamental quantities, in physics) of one theory to those of the other, so that the theorems (predictions) of the two theories are logically equivalent. So to translate between the toy 'theories' given by two representations, Halvorson and Clifton must identify the primitives and theorems of a given quantum theory. Finding the theorems is easy; these are just the predictions each state makes about the values of the observables. So Clifton and Halvorson require that any translation scheme include bijective mappings β between the states of the two toy 'theories' and α between their observables, such that, for all observables $\hat{A}$ and states ω,

$$\beta(\omega)(\alpha(\hat{A})) = \omega(\hat{A}). \tag{6}$$

(Note that which operators A count as observables will depend on whether you're a conservative or liberal about observables.) This strikes me as a straightforward and reasonable requirement for any translation scheme.

Their other requirement is more suspect. They suggest:

> Think of the Weyl operators $[\hat{U}(a), \hat{V}(b)]$ as the primitives of our two 'theories,' in analogy with the way the natural numbers can be regarded as the primitives of a 'theory' of real numbers. (Halvorson and Clifton, 2001, PAGE)

The analogy is that all observables can be constructed, either by algebraic operations or taking limits, from Weyl operators and series of Weyl operators. They then require that, for all such "primitive" Weyl operators W,

$$\alpha(\pi_1(W) = \pi_2(W). \tag{7}$$

This is the condition responsible for their Proposition 3, that for the conservative-about-states two representations are intertranslatable only if unitarily equivalent. It requires that any translation take each Weyl operator in representation π_1 to *the very same* Weyl operator in π_2. In particular, it rules out the possibility that some Weyl operator in π_1 should be mapped in translation to a *different* Weyl operator in π_2. This is violated by symmetry transformations, which often take one Weyl operator to a different one which is (for instance) localized in a different region of spacetime. Sometimes a symmetry can guarantee the existence of a translation scheme in Halvorson and Clifton's sense – but only if the symmetry is unitarily implementable.

Immediately, one worries that Halvorson and Clifton may be equivocating in their use of 'primitive,' when justifying the Weyl operators as the primitive terms in quantum theory. Is the sense in which a *physical theory's* basic terms are primitive really the same as the sense in which the natural numbers are primitive in number theory?

This general worry is supported by some concrete concerns about their specific choice of primitives. First: why shouldn't all observables count as primitives? Second: why must our two theories agree on the numerical values of the primitives? Together, the criteria (6, 7) require that a translation map every state to one that agrees with it on the expectation values of all Weyl operators. But why can't different numerical expectation values represent the same physical value of a primitive quantity, in different theoretical "languages" like the ones we're trying to translate?

Let's step back and look at the dialectic. We've seen that the Halvorson-Clifton translation criteria (6, 7) by themselves are enough to generate the problem raised in section 5 for both the liberal and the conservative about observables. Together with the physical possibility of broken spacetime symmetry, these criteria entail that states related by global rotations of everything in the universe can fail to be physically equivalent. This result is in serious tension with the best available metaphysics for spacetime. We've also seen that independent concerns arise, to the effect that the Halvorson-Clifton criteria may be too stringent. Some response may be possible, on behalf of their criteria, but in light of the serious dilemma for the standard picture of spacetime that results from accepting the criteria, I think the best option is clearly to abandon them.

I should note that Halvorson now agrees with me, writing in a recent erratum that

> The original definition of physical equivalence assumes that the two observers already have established a translation scheme between their primitives i.e. $[\pi_1(A)]$ corresponds to $[\pi_2(A)]$. But this requirement is overly restrictive, because once the $[\pi_1(A) \to \pi_2(A)$ correspondence is fixed, the translation scheme between states is also fixed (i.e. the two theories are intertranslatable only if they have the same states relative to the fixed correspondence... $CITE$

In other words, their criteria placed too strong a restriction on the admissible translation schemes, effectively limiting them to a single method. That said, their use in Halvorson and Clifton (2001) was restricted to the section on physical inequivalence of representations, and does not affect that paper's other arguments about the observer-dependence (and thus the nonexistence) of particles in QFT.[12]

[12] Elsewhere, the Halvorson-Clifton translation criteria *do* feature importantly in one of my three arguments against field interpretations of QFT (Baker, forthcoming). In that paper, I argue that a field interpretation must presume the Halvorson-Clifton criteria, since all bounded functions of the fields (i.e. Weyl operators) must be physically significant on such an interpretation. I am now less certain of this. It is certainly true that any translation scheme on the field approach must, as I argue, preserve all bounded functions of the

8 Rethinking intertranslatability

In an important sense, our philosophical picture of QFT is left adrift by this new result.

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fields. But in light of my arguments in this section, it is unclear whether a translation could in some sense "preserve" these bounded functions without holding fixed all the expectation values of the Weyl operators.

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