BLACK HOLES AND GRAVITATIONAL WAVES: SPACETIME ENGINEERING

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1. Introduction

General relativity – Einstein's theory of relativistic gravitation – is the cornerstone of modern cosmology, the physics of neutron stars and black holes, the generation of gravitational radiation, and countless other cosmic phenomena in which strong-field gravitation is believed to play a dominant role. Yet the theory remains largely untested, except in the weak-field, slow-velocity regime. Moreover, solutions to Einstein's equations, except for a few idealized cases characterized by high degrees of symmetry, have not been obtained as yet for many of the important dynamical scenarios thought to occur in nature. Only now, with the advent of supercomputers, is it possible to tackle these highly nonlinear equations numerically and explore these scenarios in detail. That is the main goal of numerical relativity, the art and science of developing computer algorithms to solve Einstein's equations for physically realistic, high-velocity, strong-field systems. Numerical relativity also has a pressing goal – to calculate gravitational waveforms from promising astrophysical sources, in order to provide theoretical templates for the gravitational wave laser interferometers now under construction in the US, Europe and Japan.

The current focus of numerical relativity is handling two and three dimensional systems with strong dynamical fields. The underlying equations are multidimensional, highly nonlinear, coupled partial differential equations in space and time. They have in common with other areas of computational physics, like fluid dynamics and MHD, all of the usual problems associated with solving such nontrivial equations. However, solving Einstein's equations poses some additional complications that are unique to general relativity. The first complication concerns the choice of coordinates. In general relativity, coordinates are merely labels that distinguish points in spacetime; by themselves coordinate intervals have no physical significance. To use coordinate intervals to determine physically measureable (proper) distances and times requires the spacetime metric, but the metric is determined only after Einstein's equations have been solved. Moreover, as the integrations proceed, it often turns out that the original (arbitrary) choice of coordinates turns out to be bad, because, for example, singularities eventually are encountered in the equations. The gauge freedom inherent in general relativity – the ability to choose coordinates in an arbitrary way – is not always easy to exploit successfully in a numerical routine.

Another complication arising in numerical relativity involves the appearance of black holes. Black holes inevitably contain spacetime singularities – regions where the gravitational tidal field, the matter density and the spacetime curvature all become infinite. Encountering such singularities results in some of the terms in Einstein's equations becoming infinite, causing overflows in the computer output and premature termination of the numerical integration. Thus, when dealing with black holes, it is crucial to choose a technique which avoids the spacetime singularities inside.

Finally, one of the main goals of a numerical relativity simulation is to determine the gravitational radiation generated from a dynamical scenario. However, the gravitational wave components usually constitute small fractions of the background spacetime metric. Moreover, to extract the waves from the background requires that one probe the spacetime in the far-field or radiation zone, which is typically at large distance from the strong-field central source. Yet it is the strong-field region which usually consumes most the computational resources (e.g spatial grid) to guarantee accuracy. Furthermore, waiting for the wave to propagate to the far-field region usually takes nonnegligible integration time. Overcoming these difficulties to reliably measure the wave content thus requires that a code successfully cope with the problem of dynamic range inherent in such a simulation.

2. The Grand Challenge Efforts

The most outstanding unsolved problem in classical general relativity is the two-body problem. Consequently, significant computational effort is going into solving the coalescence of binary black holes and binary neutron stars. Obtaining the fully dynamical solutions to these scenarios is not only of theoretical interest, but it is also crucial to gravitational wave astronomy, since coalescing binaries are the most promising sources of gravitational waves detectable by the laser interferometers now being built. So important is this effort that in the US, both the NSF and NASA have funded Grand Challenge teams of computational physicists and computer scientists to work on these binary coalescence problems.

The 'holy grail' of these Grand Challenge efforts is the calculation of the complete gravitational waveform in each of the two polarization states, h_+ and $h_\times$, from binary inspiral and coalescence. Having the theoretical waveform as a template is crucial to the 'matched filter' technique of gravity wave detection by the laser interferometers under construction. It is also important for using the observed waveforms to probe the strong-field geometry around coalescing neutron stars and black holes.

Despite the importance of solving the coalescence problem and the intensive effort to date that has gone into obtaining the answer, *no computer code currently exists that can integrate two black holes or two neutron stars in binary orbit long enough to get a waveform out to an accuracy anywhere near 10%.* Nevertheless, considerable progress has been achieved. A number of computer modules crucial to solving the problem on parallel machines have been assembled and tested. Some new physical insights have emerged from simulations performed to date (see Section 3). Additionally, new formulations of Einstein's equations have emerged which recast them into a flux-conservative, first order, hyperbolic form where the only nonzero characteristic speed in that of light; (Choquet-Bruhat and York, 1995; Abrahams et. al., 1995). As a consequence, these equations may permit a more reliable "excision" of black holes and their nasty interior singularities from the numerical grid and the replacement of these regions by a well-behaved boundary condition ("horizon boundary conditions").

3. Applications: Topology of a Black Hole Event Horizon

Two definitive results that emerged from the early phase of the Binary Black Hole Grand Challenge work have to do with the topology of the black hole event horizon. The event horizon is the surface of a black hole; events which occur inside cannot be seen from the outside because nothing can escape the interior, including light signals. The first result of the simulations dealt with the event horizon formed from the merger of two identical, nonspinning black holes which collide head-on. The second dealt with the formation of the event horizon from the collapse of a rotating toroid. Both results were obtained by performing simulations with a mean-field, particle simulation code in axisymmetry (Shapiro and Teukolsky, 1992; Abrahams et. al., 1994). The code solved the collisionless Boltzmann equation for the matter by integrating particle geodesic equations in the mean gravitational field of the distribution. The matter provided the source terms for the gravitational field (ADM) equations, which were solved by finite differencing.

3.1. HEAD-ON COLLISION AND COALESCENCE OF TWO BLACK HOLES

Consider the the head-on collision of two identical clusters of collisionless particles (Shapiro and Teukolsky, 1992). These particles may represent stars in a cluster, or any other form of collisionless matter which interacts exclusively by gravity. At $t = 0$ the velocity dispersion in each cluster is zero, so that each cluster collapses on itself to form a black hole prior to merger. In this way we set up a simulation of a head-on collision of black holes.

In Figure 1 we show spatial snapshots of the collision at three different instants of time. The upper figure shows the initial configurations, which are hurling towards each other at velocity $v = 0.15c$. The middle frame shows the formation of two black holes from the collapse of each of the clusters. Note how the tidal field of the companion causes a distortion in the shape of the black hole, giving rise to the "hour glass" appearance of the merging holes when they first come into contact. The bottom frame shows the configuration at late times, when the two black holes have coalesced into a single, spherical Schwarzschild hole which encompasses all of the matter.

Figure 2 is a spacetime diagram of the collision and merger. The time axis is along the vertical direction, while spacelike hypersurfaces (with one of the spatial directions suppressed) are horizontal planes at any instant of time. The collision axis goes from left to right and the black hole horizon is shown as the dark shaded surface. Some of the light rays which generate the horizon ("null generators of the horizon") are shown. Their trajectories were traced by numerically propagating light rays in the background curved spacetime. The inset shows a closeup view of the formation and merger of the two horizons and how the rays enter the horizons at those early events.

Figure 2 is the famous "pair of pants" picture of the event horizon for coalescing black holes that was sketched in general relativity textbooks more than 25 years ago (Hawking and Ellis, 1973; Misner et. al., 1973). The figure shown here is the first real calculation of such a diagram. Many things were known about the the topology of the merging horizons and the null generators prior to these numerical simulations, but a few important details were not. It was well known that the black hole is spanned by null generators, which can intersect or cross each other only at those points at which they enter the horizon. Once on the horizon, a null generator can never propagate off, nor can it ever cross another null generator. These properties were all understood and nicely corroborated by the simulation. But in addition, the simulation revealed for the first time a line of crossover points for the null generators extending from the "crotch" on the "pair of pants" down along each inside trouser seam, around each bottom, and continuing a small distance up each outside "seam" (see Figure 2). The points on the outside seam at which the line of crossover terminates are caustics,

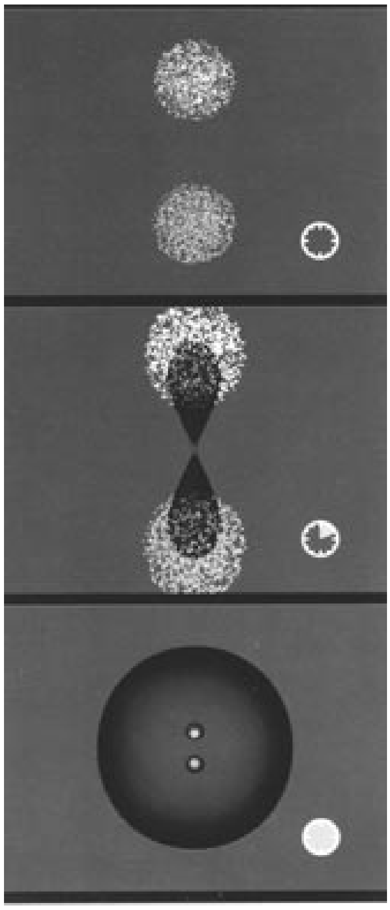

Figure 1. The collision and coalescence of two black holes formed by the collapse of two spheres of particles. The black hole event horizon is shown by the dark shaded surface. The clock in each snapshot indicates the fraction of time elapsed during the simulation.

where the intensity of the intersecting light rays becomes infinite. For a single, isolated cluster undergoing spherical collapse to a black hole, the caustic would arise at the base of the spacetime diagram at the point at which the horizon first forms, and there would be no crossovers. Here, the gravitational tidal field of the colliding black holes shifts the location of the caustic and produces the line of crossovers. An analysis of the simulation (Matzner et. al., 1995) shows that the line of crossovers is spacelike, which means that they cannot be traced by light rays or particles moving slower than c. As this line approaches the caustics on the sides, it becomes asymptotically light-like.

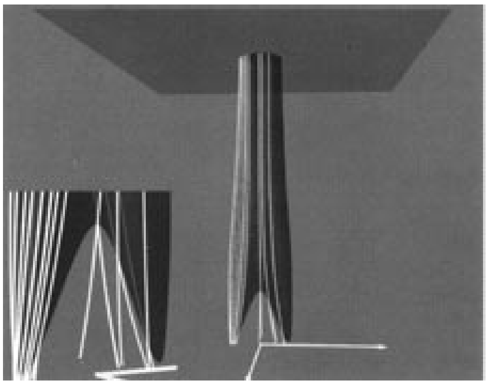

Figure 2. Spacetime diagram for the collision shown in Figure 1. Some of the light rays generating the horizon are drawn. The collision axis goes from left to right and the time axis is vertical. The inset is a blow up of the caustic and crossover structure at the birth of the horizon.

3.2. TOROIDAL BLACK HOLES AND TOPOLOGICAL CENSORSHIP

Consider the collapse of rotating toroidal clusters of collisionless particles to Kerr black holes. One of the interesting findings of our numerical simulations was a case in which the black hole event horizon emerges as a toroid (Abrahams et. al., 1994; Hughes et. al., 1994). The initial matter distribution is based on a solution for a relativistic toroidal cluster in dynamical equilibrium. Collapse was induced by reducing the angular velocity of each particle by a factor of two, resulting in a nonequilibrium cluster with a total angular momentum of $J/M^2 = 0.7$.

The spactime diagram for the collapse looks quite similar to the diagram plotted in Figure 2. The line of crossover points at which light rays enter the horizon, and the cusp formed by rays at the point at which the line of crossovers terminates, are all present as in Figure 2. The toroidal horizon first forms entirely within the vacuum, between the origin and the inner edge of the collapsing, toroidal cluster. The horizon then expands to fill up the "doughnut hole," becoming topologically spherical at about the instant when the outer edge of the horizon reaches the inner edge of the matter toroid.

When the results of the numerical simulation were first reported, it appeared that they might be in conflict with recent theorems regarding "topological censorship." These theorems permit a nonstationary black hole to have the topology of a two-sphere or a torus (Gannon, 1976), but a torus can persist only for a short time. According to topological censorship (Friedmann et. al., 1993; Jacobson and Venkataramani, 1995) the hole in a toroidal horizon must close up quickly, before a light ray can pass through. Moreover, the black hole must be two-sphere if no null generators enter the horizon at later times (Browdy and Galloway, 1995).

Once the numerical simulations were studied in detail and the geometry of the transient toroidal event horizon was analyzed by means of a spacetime diagram like Figure 2, the results were shown to be completely consistent with the theorems concerning the topology of black hole horizons (Shapiro et. al., 1995). At late times, when equilibrium has been reached, the topology is spherical, in agreement with the theorem of Hawking (1972). At early times, the topology is temporarily toroidal. However, the line of crossovers traced out by a point on the inner rim of the torus is *spacelike*. That implies that the "hole" in the torus indeed closes up faster than the speed of light, in compliance with topological censorship. Finally, at intermedidate times, when the horizon is spanned by its full complement of null generators, the rotating black hole has a spherical topology, in accord with the theorems of Browdy and Galloway (1995).

4. Conclusions

Numerical relativity is only in its infancy, or at most very early adolescence. It is still under development as a valuable scientific tool to solve Einstein's equations of general relativity for realistic, dynamical spacetimes involving strong-field sources like black holes. Already it has been used successfully to determine gravitational waveforms for simple cases characterized by high degrees of spatial symmetry, like head-on collisions and axisymmetric collapse. Its refinement is absolutely crucial for interpretating the waveforms likely to be observed by the new gravitational wave laser interferometers now under construction, like LIGO, VIRGO, GEO and TAMA. These waveforms will arise from the inspiral and coalescence of binary black holes and neutron stars.

Numerical relativity has also demonstrated that beyond the calculation of accurate numerical solutions, it can provide *qualitative* insight into Einstein's equations in those cases where uncertainty still prevails. It can even be helpful as a guide to proving (or disproving) theorems about strong-field spacetimes in those instances where analytic means alone have not proven adequate. Numerical relativity is an important component of modern computational astrophysics whose role is likely to mushroom in the not too distant future.

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