

Solving quantum field theories via curved spacetimes

Igor R. Klebanov and Juan M. Maldacena

Strongly interacting quantum field theories are notoriously difficult to work with, but new information about some of them is emerging from their surprising correspondence with gravitational theories.

Igor Klebanov is a professor of physics and associate director of the center for theoretical science at Princeton University in Princeton, New Jersey. **Juan Maldacena** is a professor at the Institute for Advanced Study in Princeton.

Eleven years ago several theorists proposed a remarkable correspondence between two seemingly different kinds of theories.¹ It is often called a duality because it is an equivalence between two different, “dual” descriptions of the same physics. On one side of the duality are certain quantum field theories (QFTs)—for example, Yang–Mills gauge theories similar to those in the standard model of particle physics. Such theories describe interacting particles moving in a flat d -dimensional spacetime. On the other side are theories that include gravity, like Albert Einstein’s general relativity or its string-theoretic generalizations. The gravitational theories are defined in a higher-dimensional spacetime containing at least the d dimensions of the particle theory plus one extra dimension of infinite extent. They often include a number of finite dimensions—in the form of a sphere, for example. Depending on the context, the correspondence is known as a gauge/gravity duality, gauge/string duality, or AdS/CFT (anti-de Sitter/conformal field theory) correspondence. Do not despair if the terminology is unclear. In this article we will attempt to describe in simple terms what the duality is and how and why it is useful for studying a variety of problems.

The gravitational theory involves a dynamic spacetime fluctuating around a special curved background: The gravitational potential energy rises sharply as the coordinate y of the extra dimension approaches infinity. To motivate the correspondence, we consider a class of curved spacetimes for which the gravitational potential energy has an absolute minimum at $y = 0$, as in figure 1. The correspondence asserts that the dynamics of the $d + 1$ -dimensional gravitational theory is encoded in a d -dimensional QFT. That should not be too surprising: In the presence of a potential energy rising sharply along a particular spatial direction, one expects the low-energy dynamics of a theory to become effectively confined to the remaining dimensions. The gauge/string duality, however, goes beyond that simple intuition because even motion along the extra dimension is reproduced by the corresponding d -dimensional QFT.

The duality is particularly useful for studying certain QFTs that are very strongly coupled because then the corresponding gravitational theory is defined in a weakly curved spacetime and readily analyzed using the methods of general relativity. On the other hand, the fact that weak curvature cor-

responds to strong coupling makes the duality hard to prove because it is difficult to do reliable computations in a strongly coupled field theory. Nonetheless, theorists have accumulated a great deal of evidence in its favor.

If one simply assumes that the duality is true, then it provides a wealth of new information about strongly coupled field theory. The collection of theories that can be solved using the duality is still rather limited; it does not contain any theory that describes a known physical system. Yet the theories that can be solved capture some essential features of theories realized in nature. One of the best-studied examples is a cousin of quantum chromodynamics, the field theory describing strong nuclear forces. The cousin, though, includes supersymmetry, a symmetry that pairs up bosons and fermions. Although superficially the cousin is rather different from

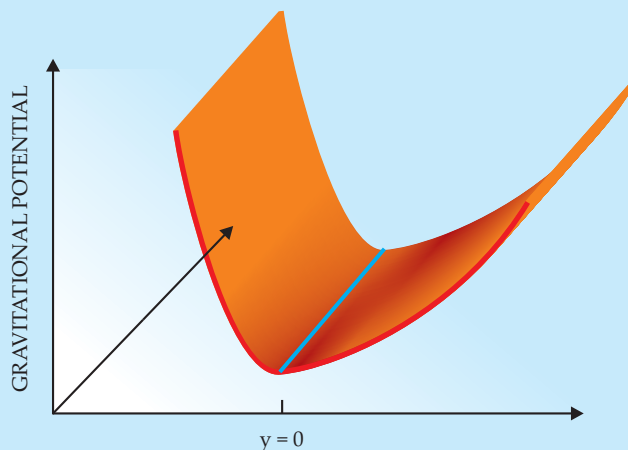


Figure 1. Particle in a potential energy well of a $d + 1$ -dimensional gravitational theory. If the potential energy has a deep minimum as a function of the “extra” dimension with coordinate y , then a low-energy quantum particle behaves as if it were moving in the remaining d dimensions. The gauge/gravity duality makes a more far-reaching statement: Even motion along the extra dimension can be described by a d -dimensional particle theory.

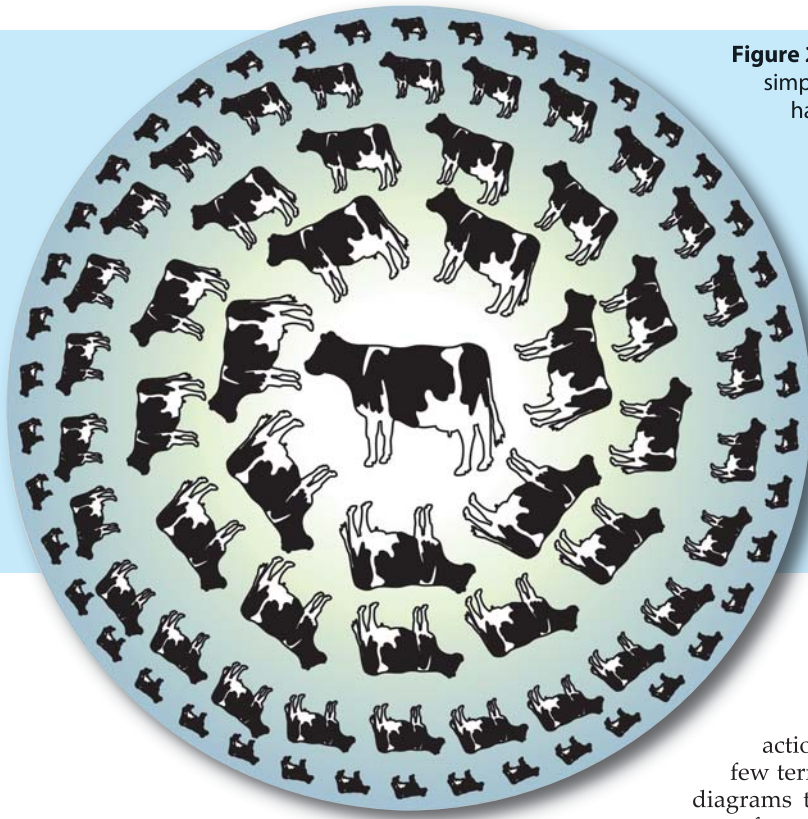


Figure 2. Hyperbolic space. Shown here is a map of the simplest negatively curved space. Each of the images has the same coordinate-invariant “proper” size; they appear to get smaller near the boundary because of their projection onto the flat surface of this page. A related phenomenon happens in standard Mercator-projection maps of Earth: Objects of a fixed proper size appear broader at high latitudes. The map of hyperbolic space is reminiscent of M. C. Escher’s *Circle Limit* woodcuts, but we have replaced Escher’s interlocking fish with cows to remind readers of the physics joke about the spherical cow as an idealization of a real one. In the anti-de Sitter/conformal field theory correspondence, theorists have really found a hyperbolic cow.

QCD, it has been useful for studying new phenomena in the physics of quarks and gluons at high temperatures, such as those that were observed not long ago at RHIC, the Relativistic Heavy Ion Collider, at Brookhaven National Laboratory.² Similar theories have provided model-building tools for physics beyond the standard model and have served as toy models for systems with quantum critical points that appear in condensed-matter physics.

Thus theories that are solvable via the duality provide approximations to physical systems that are useful for developing intuition about strongly coupled theories. The AdS/CFT correspondence has also been valuable for elucidating some theoretical paradoxes involving the quantum behavior of black holes. In fact, it was originally discovered in investigations of such questions. The duality also gives theorists new ways to study the quantization of gravity, which is one of the important issues at the frontier of theoretical physics.

QFT is ubiquitous and difficult

Quantum field theory, which considers quantized field fluctuations, was the result of merging special relativity with quantum mechanics. It has many physical applications. In particle physics, it underlies the famous standard model, which provides a precise description of the electromagnetic, weak, and strong interactions. In statistical mechanics, QFT successfully describes second-order phase transitions that occur, for example, in the vicinity of the critical point in a water–vapor phase diagram. In that application, the quantum field fluctuations are identified with the long-range statistical fluctuations in the 3D system under study; thus the field theory is defined in three spatial dimensions without time. Generally, many-body systems are well described by QFT when the typical length scales of fluctuations are much larger than the size of the system’s atomic constituents. Instead of varying temperature or pressure, one can achieve

long-wavelength fluctuations at zero temperature by varying properties of materials such as the doping concentration. The resulting behavior is called quantum criticality.

A QFT is often hard to analyze. If the interaction strength is weak, one can calculate the first few terms of a perturbative expansion using Feynman diagrams that provide a visualization of interactions in terms of particles locally splitting or joining with other particles. Yet in many interesting physical problems, the coupling is so strong that such an expansion is not feasible.

To further complicate matters, the coupling typically depends on the characteristic energy of a process. In QCD, perturbative computations are possible at high energies—much higher than the proton rest energy of about 1 GeV—because the coupling weakens. They become useless, however, at energies comparable to the proton rest energy. Calculations in that strongly coupled regime are difficult. One solid path toward calculating masses of hadrons such as protons or heavy mesons is to approximate continuous spacetime by a discrete lattice of points. But that approach, called lattice QCD, requires a great deal of computing power (see the article by Carleton DeTar and Steven Gottlieb in *PHYSICS TODAY*, February 2004, page 45). Moreover, important quantities such as transport coefficients in high-temperature QCD cannot be accessed using standard numerical approaches. Many condensed-matter problems, too, involve strongly coupled QFT. Analytic techniques that deal with strongly coupled problems are few and valuable, even if they apply to only a narrow class of theories. The gauge/gravity duality offers such a technique.

The AdS/CFT correspondence

The QFTs relevant to many applications are approximately invariant under a scaling of all d spacetime coordinates: $x_\mu \rightarrow \lambda x_\mu$. (The subscript μ , which runs from 0 to $d - 1$, indexes the particular coordinate.) For example, QCD becomes nearly scale invariant at energies high enough that the coupling constant depends only logarithmically on energy. Condensed-matter systems close to second-order phase transitions or quantum critical points are also nearly scale invariant. Theories that are invariant under scaling typically also have other symmetries, including the spacetime inversion $x_\mu \rightarrow x_\mu/x^2$ that maps the origin to infinity. Combined with the Lorentz transformations and translations of special relativity, those transformations form the so-called conformal group. A

QFT invariant under that group is called a conformal field theory.

The conformal invariance is of great help in formulating an exact gauge/gravity duality because it translates into a geometrical symmetry of the dual space. If, in the QFT, all d dimensions are spatial, the dual space is the familiar $d + 1$ -dimensional hyperbolic, or Lobachevsky, space with constant negative curvature. Figure 2 is a map of that space. If the CFT is formulated in $d - 1$ space dimensions and one time dimension, the unique $d + 1$ -dimensional spacetime with the appropriate symmetry is anti-de Sitter space. For an introduction to that constant-negative-curvature spacetime, see box 1.

The intrinsic metric, or line element, of $d + 1$ -dimensional AdS space is

$$ds^2 = R^2 [e^{2y} (-dx_0^2 + dx_1^2 + dx_2^2 + \dots + dx_{d-1}^2) + dy^2], \quad (1)$$

where in the infinite extra dimension y ranges over all real numbers, and R is the curvature radius. The CFT corresponding to the AdS geometry is formulated in a flat d -dimensional spacetime parameterized by the coordinates x_0, \dots, x_{d-1} . That d -dimensional spacetime may be viewed as the boundary, at $y = \infty$, of the $d + 1$ -dimensional AdS space. The scale invariance of the QFT translates into a symmetry of the AdS metric under $x_\mu \rightarrow \lambda x_\mu, y \rightarrow y - \ln \lambda$.

What is the origin of the extra dimension in the gravitational theory? Imagine a localized state in the CFT with a given size and a position specified by the $d - 1$ spatial coordinates. By performing scale transformations, one can obtain states with all possible sizes, each with energy inversely proportional to the size. The extra coordinate y in the gravity theory labels the size of the state in the CFT, as shown in figure 3. More precisely, the size is proportional to e^{-y} .

In Einstein's gravity, the metric's time-time component g_{00} (that is, the piece that multiplies dx_0^2 in the line element) defines the gravitational potential energy V of a stationary object of mass m : $V = mc^2 \sqrt{-g_{00}}$, where c denotes the speed of light. For weak fields, $g_{00} \approx -(1 + 2\phi/c^2)$, with $m\phi$ the familiar Newtonian potential energy. In AdS space, V is proportional to e^y , so objects are pushed to smaller y . In the dual scale-invariant field theory, localized field configurations will thus expand to lower their energy.

Box 1. Anti-de Sitter space

Anti-de Sitter space is a spacetime with one time coordinate and, in general, d space coordinates. As a warm-up to looking at AdS space, however, we first present some curved two-dimensional surfaces.

The 2D unit sphere, which is positively curved, is defined as the set of points in 3D Euclidean space that are equidistant from the origin: $X^2 + Y^2 + Z^2 = R^2$, where R is the radius of the sphere. (For simplicity, in the remainder of this box we consider a unit-radius sphere and its analogues.) The metric, or line element, expresses an infinitesimal distance in terms of infinitesimal coordinate changes. It is simply obtained for the sphere from the metric of Euclidean space $ds^2 = (dX)^2 + (dY)^2 + (dZ)^2$ by restricting the coordinates to the sphere. Expressing the coordinates in terms of the standard polar and azimuthal angles, θ and φ —that is, setting $X = \sin \theta \cos \varphi$, $Y = \sin \theta \sin \varphi$, and $Z = \cos \theta$, one obtains the intrinsic metric of the sphere, $ds^2 = (d\theta)^2 + \sin^2 \theta (d\varphi)^2$.

To describe a negatively curved surface, or 2D hyperbolic

Correlation functions

In a CFT, the most important physical observables are correlation functions of local operators. A two-point correlation function, for example, is a suitable average over all field configurations of a product of operators at two separated points. In a theory of phase transitions, correlation functions encode the critical exponents and other useful information.

A universal operator, present in any field theory, is the symmetric stress-energy tensor $T_{\mu\nu}$. As in electrodynamics, its time-time component T_{00} gives the energy density of a field, the time-space components T_{0i} give the momentum density, and the space-space components T_{ij} encode the pressure and shear forces.

Remarkably, to calculate correlation functions of $T_{\mu\nu}$ in the dual description, one needs to consider propagation of gravitons in AdS space. The graviton carries the gravitational force, and so the $d + 1$ -dimensional dual theory must be gravitational. Inserting $T_{\mu\nu}(x)$ in a correlation function corresponds to a graviton emitted from point x on the boundary of the AdS space. Gravitons corresponding to insertions of stress-energy tensors meet in the interior and interact according to the rules of general relativity. Thus computation of the correlations of stress-energy components at different points in a field theory is translated into a problem of scattering gravitons in the interior of a curved spacetime. Other operators in the QFT correspond to other fields in the gravitational theory, and there is a one-to-one map between the two—a kind of dictionary relating the field-theory and gravity languages. The most precise correspondences found to date map CFT to string theory formulated in AdS space. String theories are described by general relativity only at low energies; at high energies, they contain a large additional set of string excitations.

In a CFT the conformal symmetry fixes the functional form of the two-point correlation function of the stress-energy tensor. But the function's normalization, N_{eff} , is an effective measure of the number of degrees of freedom in the CFT. In the gravity theory, the normalization is proportional to $R^{d-1}/\hbar G_N$, where G_N is Newton's constant in $d + 1$ dimensions and \hbar is Planck's constant. The inverse of the normalization controls the effective gravitational coupling in the dual gravitational theory; if that theory is to be weakly coupled, the QFT must contain many degrees of freedom.

Such QFTs arise naturally in generalizations of QCD

space, one needs to change only some signs: $X^2 + Y^2 - Z^2 = -1$, and $ds^2 = (dX)^2 + (dY)^2 - (dZ)^2$. One can define coordinates ρ and φ , through $X = \sinh \rho \cos \varphi$, $Y = \sinh \rho \sin \varphi$, and $Z = \cosh \rho$, and obtain the intrinsic metric $ds^2 = (d\rho)^2 + \sinh^2 \rho (d\varphi)^2$. Note that despite the negative signs in the above definition, the metric of the 2D hyperbolic space has only positive signs. The hyperbolic space is a 2D space, not a spacetime.

Analogously, one can define a 2D AdS space by $-X^2 - Y^2 + Z^2 = -1$, and $ds^2 = -(dX)^2 - (dY)^2 + (dZ)^2$. Introducing the coordinates x_0 and y defined through $X = e^y x_0$, $2Y = e^{-y} + e^y(1 - x_0^2)$, and $2Z = e^{-y} - e^y(1 + x_0^2)$, one finds the metric $ds^2 = -e^{2y}(dx_0)^2 + dy^2$. The negative sign of the first term signifies that x_0 is a time variable. Equation 1 in the main text displays the metric for a $d + 1$ -dimensional AdS space with constant curvature radius R .

In the gravity theories associated with the AdS/CFT correspondence, the important object is the AdS space itself; the flat space in which it is embedded plays no role.

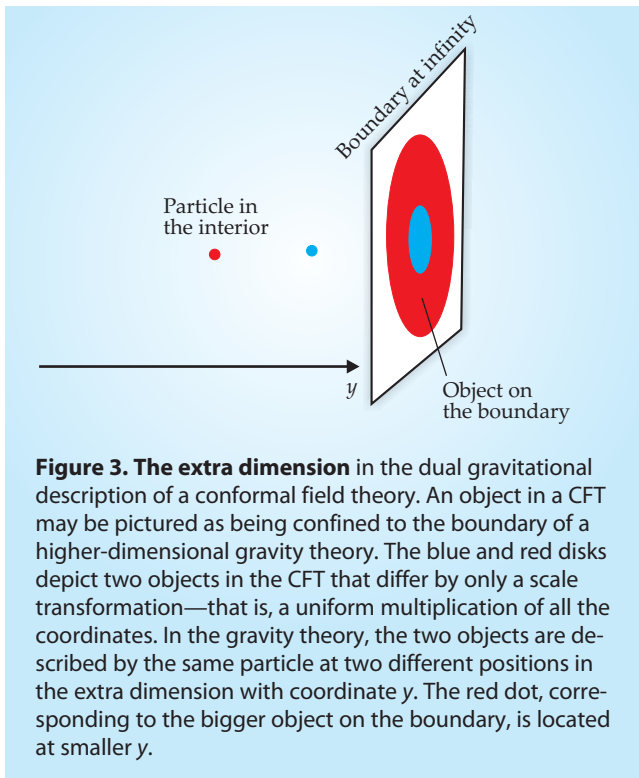


Figure 3. The extra dimension in the dual gravitational description of a conformal field theory. An object in a CFT may be pictured as being confined to the boundary of a higher-dimensional gravity theory. The blue and red disks depict two objects in the CFT that differ by only a scale transformation—that is, a uniform multiplication of all the coordinates. In the gravity theory, the two objects are described by the same particle at two different positions in the extra dimension with coordinate y . The red dot, corresponding to the bigger object on the boundary, is located at smaller y .

from three colors to a larger number of colors, N . The gluons that mediate color forces in those theories are analogues of photons, which mediate electromagnetic interactions. In contrast to uncharged photons, however, the gluons are themselves colored, and their number is $N^2 - 1$. Thus N_{eff} is comparable to N^2 .

But large N_{eff} alone is not enough to guarantee a simple gravity description. Gravity theories do not include light particles with large spin. On the other hand, in a weakly coupled QFT, one often has operators with large spin. The mismatch may be removed by turning up the interaction strength g , the QFT analogue of the electric charge e in electrodynamics. To see that a stronger interaction simplifies the dual gravitational theory, note that the curvature radius of AdS space is proportional to $(g^2 N)^{1/4}$ multiplied by the fundamental string-theory length scale. Thus the spacetime in a gravitational theory that is dual to a field theory with $g^2 N \gg 1$ is weakly curved and one can reliably study the gravitational theory with the methods of general relativity. Conversely, if the field theory coupling is weak—that is, $g^2 N \ll 1$ —then the radius of curvature is smaller than the string length. In that regime, general relativity is not a good approximation.

The most tractable correspondences therefore involve strongly coupled CFTs with many color states. Luckily, theorists have constructed many such theories. The simplest and most studied of them is the gauge theory in four spacetime dimensions with the greatest possible number of supersymmetries that pair up bosonic and fermionic fields. Although supersymmetry is not fundamental to gauge/gravity duality, it is a technical tool that simplifies its analysis.

Black holes and thermal field theories

A fascinating aspect of general relativity is the existence of black holes, which are surrounded by event horizons. Any classical object that falls through such a horizon cannot return, but Stephen Hawking demonstrated that the horizon itself radiates with a characteristic temperature T . Now sup-

pose that far away from the black hole—in higher dimensions it would be a black membrane—the $d + 1$ -dimensional spacetime looks like AdS space. One can then consider a black membrane with a horizon located at a fixed position y_0 and extended in the $d - 1$ spatial dimensions, as illustrated in figure 4. The gravitational theory in the black membrane spacetime is again described by a d -dimensional QFT, but now the QFT has been heated up—to the same temperature T as the horizon!

The duality connecting black holes with strongly interacting QFTs has many interesting implications. As shown by Jacob Bekenstein and Hawking, a black hole has an entropy proportional to the area of its event horizon. Computing the finite-temperature entropy for a strongly interacting QFT is generally difficult, since particle interactions make important contributions to the free energy and therefore to the entropy. Yet when the QFT has a dual gravity theory, its entropy can be found simply through calculating the area of the horizon.

What does the “blackness” of the horizon correspond to in the QFT? Black holes are excellent particle absorbers. Any fluctuation of the black hole geometry decays exponentially, since waves near the horizon are swallowed by the black hole. In the finite-temperature QFT, that phenomenon corresponds to the rapid thermalization of fluctuations.

Thermodynamic and transport properties of a strongly coupled QFT are related to properties of the black hole geometry. Thus the computation of a transport coefficient, such as the shear viscosity, becomes tractable in the gravitational formulation—one has to solve a certain wave equation in the black hole geometry. Calculations along those lines have been used to gain intuition about complicated finite-temperature QCD problems that can be realized experimentally. RHIC is believed to have produced a new state of matter, which was

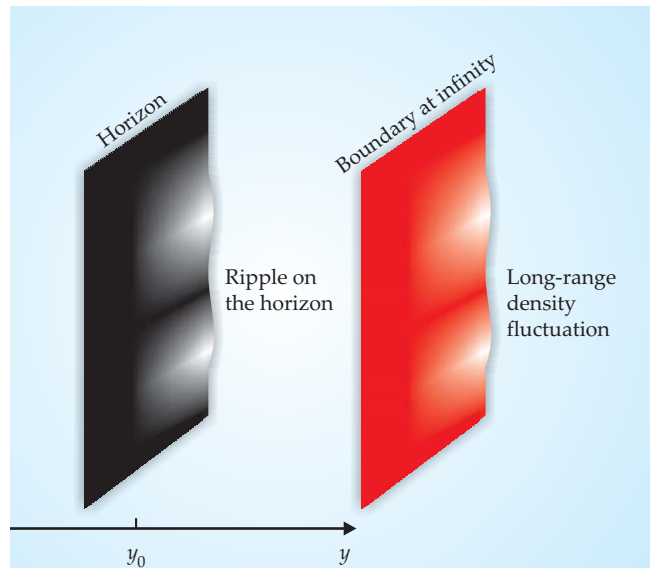


Figure 4. The black hole duality. A strongly interacting quantum field theory at a finite temperature is related to a black membrane—the higher-dimensional analogue of a black hole—in the dual gravity system. Thermodynamic properties of the strongly coupled QFT are related to geometric properties of the horizon. For example, a long-range fluctuation in the density of the thermal system corresponds to a ripple on the event horizon surrounding the black membrane. This ripple is absorbed by the black membrane; the corresponding phenomenon in the thermal QFT is thermalization.

originally dubbed the quark–gluon plasma. Experiments at RHIC established that the matter behaves more like a liquid than a plasma because its viscosity is much smaller than had been expected.² Interestingly, when the gauge/gravity duality is applicable, it also predicts a low value for the viscosity (see reference 3 and PHYSICS TODAY, May 2005, page 23). One can compute other quantities, such as the energy loss that a very energetic quark will suffer as it propagates through the medium. In some cases the finite-temperature gauge/gravity duality has motivated the study of new strong-coupling phenomena that may be experimentally observable.

A confining potential-energy well

Gauge/string duality also gives new insights into color confinement, a deep and mysterious aspect of QCD. Even though QCD is formulated in terms of the fundamental colored particles—quarks and gluons—nobody has observed a free one; they are always confined inside color-neutral particles such as protons, neutrons, and pions. The way in which hadrons are made of quarks and gluons is quite different from the way in which nuclei are made of neutrons and protons. Those nuclear constituents are often bound together tightly, but hitting the nucleus hard enough will liberate them. No matter

how hard one hits a hadron, though, it will decay only into other hadrons, not into free quarks and gluons. That phenomenon has numerical support from lattice simulations of gauge theory, but its theoretical proof is still missing.

Once again, gauge/string duality provides new insights. Admittedly, no simple string theory dual to QCD has been found, but some confining gauge theories do have tractable dual formulations.⁴ Those theories are not scale invariant; they have a preferred length scale that is comparable to the size of the lightest bound states. For example, in QCD the scale is about that of the proton radius, 10^{-15} m. When the QFT is not scale invariant, the dual spacetime has nonconstant curvature and a “warped” metric of the form

$$ds^2 = e^{2A(y)} (-dx_0^2 + dx_1^2 + \dots + dx_{d-1}^2) + dy^2. \quad (2)$$

In such a spacetime, the gravitational energy of a massive particle is proportional to $e^{A(y)}$. The function $A(y)$ is such that for large y , the energy approaches the exponential e^y found in the AdS metric, but its form becomes more complicated as y decreases. It attains its absolute minimum, $e^{A(0)}$, at $y = 0$. Thus massive objects fall to the bottom of the gravita-

Box 2. String theory and QCD

Relations between string theory and strong interactions are older than quantum chromodynamics (QCD) itself. In fact, string theory was born in the late 1960s to describe hadronic spectra and interactions. The motivation was partly experimental. Some observed hadronic states exhibited a linear relation between mass squared and spin; relativistic strings could explain that relation. But it did not take long for the string models to run into problems: The simplest string theories could be defined only in 10 spacetime dimensions (for theories containing bosons and fermions) or 26 dimensions (for theories with bosons only), and they had to include quantum gravity.⁸

Furthermore, deep inelastic scattering experiments were showing signs of more fundamental constituents inside the hadrons, a result that seemed incompatible with a string description. Those experiments provided an important piece of information: Strong interactions become weaker at high energies. The only known theories in four spacetime dimensions with that property, called asymptotic freedom, are the Yang–Mills gauge theories (see PHYSICS TODAY, December 2004, page 21). In the early 1970s, the $SU(3)$ Yang–Mills gauge theory (that is, QCD) was proposed as the exact description of strong interactions, and it is now an integral part of the standard model. Yet QCD is hard to solve because the coupling g becomes strong at low energies.

Strings, though, may come to the rescue. In QCD they form as color electric flux tubes that are well established through numerical lattice simulations. Also, QCD itself simplifies if the number of colors is changed from three to a large number N , provided that g^2N is held fixed.⁹ A simple argument shows that in the large- N limit, the gauge theory should become a kind of string theory, with splitting and joining interactions suppressed by $1/N$. The argument, however, does not specify the string theory. Theorists initially assumed that the string theory should be defined in the four spacetime dimensions of the gauge theory. But Alexander Polyakov later conjectured that one needs at least five dimensions and a curved spacetime.¹⁰

Meanwhile, theorists continued to study string theory as a theory of quantum gravity. They intently studied 10D string theory and came to understand many of its properties in great

detail. In the mid-1990s Joseph Polchinski found that the theory contains various spatially extended objects called D-branes.¹¹ One type of D-brane has three infinite spatial dimensions, just like the observed world. At low energies, N coincident D-branes of that type are described in terms of a 4D $SU(N)$ gauge field theory similar to QCD, except with extra symmetries—supersymmetry and conformal invariance. But the stack of D-branes curves the 10D spacetime in which it is embedded. In the region near the stack, half of the dimensions form a 5D anti-de Sitter space, and the rest describe a 5D sphere. So, one point of view on the dynamics of the D-branes uses the 4D gauge theory; the other, the 10D string theory in the curved space (see PHYSICS TODAY, August 1998, page 20). The conjecture that the two theories are exactly equivalent provides the most solid example of the gauge/string duality. Recent tests of the correspondence¹² involve calculations of certain quantities in the supersymmetric gauge theory as functions of g^2N . Remarkably, they rely on Bethe ansatz methods that are frequently used to study spin chains arising in condensed-matter physics (see the article by Murray Batchelor, PHYSICS TODAY, January 2007, page 36). For large g^2N , the results agree with the predictions of string theory.

Similar gauge/string dualities have been found for theories that are not scale invariant, but QCD itself has so far evaded a solution. Due to asymptotic freedom, QCD is weakly coupled at high energy. The dual description of weakly coupled theories requires highly curved spacetimes for which higher powers of curvature are more important than the leading power. At high energy, though, one doesn’t need to resort to the dual calculation; the theory is weakly coupled. On the other hand, at low energies the coupling is strong enough to invalidate naive perturbative computations, but not so strong that QCD has a weakly curved dual description. General relativity is thus not enough to analyze the dual theory; one needs the full apparatus of string theory, which has so far been too difficult to implement. Nonetheless, the tools developed during the past 10 years or so have generally improved physicists’ understanding of strongly coupled gauge theory.

tional well, where they retain a nonzero energy proportional to $e^{A(0)}$. Note that in QCD most of the proton mass comes from the effects of confinement; the masses of the quarks make a relatively small contribution.

Theorists have posited that as a heavy quark–antiquark pair separates in a confining theory, a color electric flux tube forms with constant energy per unit length and a characteristic thickness. That object, often called a confining string, has been observed in numerical simulations. When such a string is spinning, it nicely explains the masses of high-spin mesons. (For more on how string theory is connected to QCD, see box 2.) How is the confining flux tube manifested in the dual, string-theoretic description of the gauge theory? The answer is unexpectedly simple: It is the fundamental string located at the bottom of the potential-energy well shown in figure 5. Indeed, the energy per unit length of a string at fixed y and stretched along one of the spatial directions in the metric of equation 2 is proportional to $T_s e^{2A(y)}$, where T_s is the fundamental-string tension. Such a string naturally falls to the bottom of the gravitational potential-energy well, where it retains a nonvanishing tension $T_s e^{2A(0)}$. Since all confining gauge theories are expected to generate flux tubes, their dual formulations cannot be given merely in terms of a gravitational field theory—they must involve string theory.

A variety of applications

The gauge/gravity duality has found theoretical applications to physics ranging from the highest to the lowest conceivable energies. At the high end, theorists are exploring physics at the Planck scale of about 10^{19} GeV, which necessarily involves quantized fluctuations of spacetime geometry. Gauge theories with N colors are dual to quantum gravity in curved spacetime, with the effective gravitational coupling being of order $1/N$. Those gauge theories have conventional quantum mechanics, with unitary time evolution of wavefunctions. Therefore, the duality provides the most solid argument to date for why quantum gravity in general, and a black hole in particular, does not destroy information.

At the low-energy end, QFTs often arise in the low-temperature domain of condensed-matter physics. One much-explored class of examples concerns the behavior of systems at quantum critical points.⁵ The relevant zero-temperature, scale-invariant theories are analogous to field theories describing second-order phase transitions, but they are formulated in d spacetime dimensions rather than in just spatial dimensions. In quantum critical systems, one is interested in computing transport properties at zero or finite temperatures. The theories with tractable dual gravity descriptions can be viewed as toy models for which such computations are feasible at strong coupling. So far, those manageable theories don't correspond to real-world condensed-matter systems, but they may capture some of those systems' important features. For a study of transport properties near quantum criticality in that vein, see reference 6.

Now is an exciting time in particle physics—the Large Hadron Collider at CERN will soon reach its peak collision energy of 14 TeV. We are confident that the current standard model of particle physics is not the final word about nature at high energies, and the LHC's experiments may well shed light on the new layer of physics. Those who build particle models explore many different scenarios, but they are often limited by difficulties associated with strongly coupled QFT. The gauge/gravity duality enables them to investigate alternative, strongly coupled theories. And in fact, the warped AdS-like geometries of equation 2 were also introduced for a number of phenomenological reasons, in particular to ex-

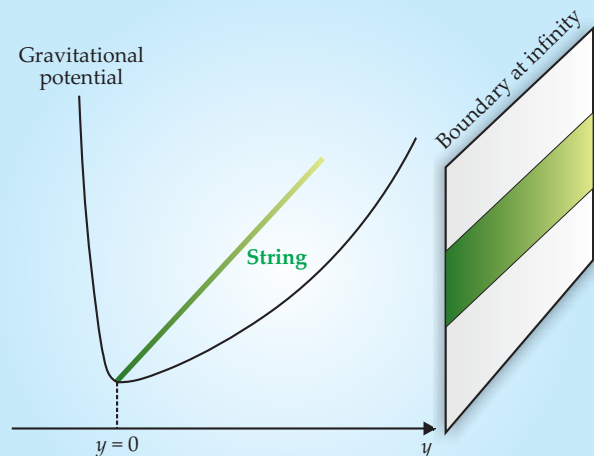


Figure 5. A string theory of quark confinement. A long fundamental string that has fallen to the bottom of the gravitational potential energy well describes the so-called color flux tube in a confining gauge theory at the boundary of the extra dimension. Such strings explain why the potential energy between a quark and an antiquark rises linearly with their separation and prevents the two particles from escaping to freedom. The thickness of the string in the gauge theory is related to the position of the fundamental string in the y direction.

plain why the scale of weak interactions, 250 GeV, is so much smaller than the Planck scale.⁷

The gauge/gravity duality has enabled field theorists to explore new possibilities away from weak coupling. Some strongly coupled field theories can now be solved via their dual curved spacetimes and provide a “hyperbolic cow” approximation to interesting physical systems. We are optimistic that the future will reveal even closer connections between gauge/gravity duality and nature.

References

1. J. M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998); J. Maldacena, *Int. J. Theor. Phys.* **38**, 1113 (1999); S. S. Gubser, I. R. Klebanov, A. M. Polyakov, *Phys. Lett. B* **428**, 105 (1998); E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998).
2. I. Arsene et al., BRAHMS collaboration, *Nucl. Phys. A* **757**, 1 (2005); B. B. Back et al., PHOBOS collaboration, *Nucl. Phys. A* **757**, 28 (2005); J. Adams et al., STAR collaboration, *Nucl. Phys. A* **757**, 102 (2005); K. Adcox et al., PHENIX collaboration, *Nucl. Phys. A* **757**, 184 (2005).
3. G. Policastro, D. T. Son, A. O. Starinets, *Phys. Rev. Lett.* **87**, 081601 (2001).
4. I. R. Klebanov, M. J. Strassler, *J. High Energy Phys.* **2000**(08), 052 (2000).
5. S. Sachdev, *Quantum Phase Transitions*, Cambridge U. Press, New York (1999).
6. C. P. Herzog, P. Kovtun, S. Sachdev, D. T. Son, *Phys. Rev. D* **75**, 085020 (2007).
7. L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999).
8. T. Yoneya, *Lett. Nuovo Cimento.* **8**, 951 (1973); J. Scherk, J. H. Schwarz, *Nucl. Phys. B* **81**, 118 (1974).
9. G. 't Hooft, *Nucl. Phys. B* **72**, 461 (1974).
10. A. M. Polyakov, *Nucl. Phys. B Proc. Suppl.* **68**, 1 (1998).
11. J. Polchinski, *Phys. Rev. Lett.* **75**, 4724 (1995).
12. N. Beisert, B. Eden, M. Staudacher, *J. Stat. Mech.* **2007**, P01021 (2007). ■