New Perspectives on the Relativistically Rotating Disk and Non-time-orthogonal Reference Frames

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Abstract

The rotating disk problem is analyzed on the premise that proper interpretation of experimental evidence leads to the conclusion that the postulates upon which relativity theory is based, particularly the invariance of the speed of light, are not applicable to rotating frames. Different postulates based on the Sagnac experiment are proposed, and from these postulates a new relativistic theory of rotating frames is developed following steps similar to those initially followed by Einstein for rectilinear motion. The resulting theory agrees with all experiments, resolves problems with the traditional approach to the rotating disk, and exhibits both traditionally relativistic and non-relativistic characteristics. Of particular note, no Lorentz contraction exists on the rotating disk circumference, and the disk surface, contrary to the assertions of Einstein and others, is found to be Riemann flat. The variable speed of light found in the Sagnac experiment is then shown to be characteristic of non-time-orthogonal reference frames, of which the rotating frame is one. In addition, the widely accepted postulate for the equivalence of inertial and non-inertial standard rods with zero relative velocity, used liberally in prior rotating disk analyses, is shown to be invalid for such frames. Further, the new theory stands alone in correctly predicting what was heretofore considered a "spurious" non-null effect on the order of 10^{-13} found by Brillet and Hall in the most accurate Michelson-Morley type test to date. The presentation is simple and pedagogic in order to make it accessible to the non-specialist.

Key words: relativistic, rotating disk, Sagnac, rotating frame, non-time-orthogonal frame.

1 BACKGROUND

1.1 Perspectives of Einstein and Others

Albert Einstein never published a technical paper directly addressing the problem of the relativistically rotating rigid disk, although in private writings[1], in three books for the general audience[2],[3],[4], and as support for the use of generalized coordinates in his landmark 1916 paper[5], he purported that the space of such a rotating disk is curved, not flat. He further

attributed his early insights into general relativity theory to be a direct result of contemplating the curvature of such a rotating system. His perspective on the problem is revealed in a private 1919 letter found by Stachel[6] in the Einstein Archives at the Institute for Advanced Study in Princeton, New Jersey.

In that letter Einstein considers measuring rods laid out along the disk's radii and circumference and assumes Lorentz contraction exists along the circumference due to the tangential velocity $v = \omega r$ of the disk at a given radius r. In Einstein's words:

....imagine a "snapshot" taken from [the non-rotating frame] ... On this snapshot the radial measuring rods have the length l, the tangential ones, however, the length $l(1-v^2/c^2)^{1/2}$. The "circumference" [therefore is] $U = [2\pi r]/(1 - v^2/c^2)^{1/2}$.

He repeated this type of reasoning elsewhere [2, 3, 4], and used it to conclude that the rotating disk surface is not Euclidean since $U > 2\pi r$.

But not everyone agrees. In a 1951 letter[7] Einstein noted that Eddington and Lorentz considered the geometry on the disk to be flat, and he stated that he did not know what they meant. No references recounting Eddington's and Lorentz's thinking on the subject seem to be available, but others, such as Levy[8], appear to agree with them. Strauss[9] concludes that the space is curved, but argues that Einstein's logic was flawed, noting that

If the measuring rods laid along the circumference of the rotating disk are Lorentz contracted with respect to the inertial frame, so are the distances on the circumference they are supposed to measure; hence the two effects would cancel each other, and the ratio U/D *would turn out to equal π as in the Euclidean plane.*

Grøn [10], however, citing Møller [11] and Landau and Lifshitz [12], contends that this argument is wrong, Weber [13] supports Grøn's view, and Stachel [1] effectively concedes the point to Einstein.

1.2 Relevant Relativity Principles

Special relativity is restricted to inertial systems and is derived from two symmetry postulates:

1. The speed of light is the same for all inertial observers (it is invariant) and equals c.
2. There is no preferred inertial reference frame. (Velocity is relative, and the laws of nature are covariant, i.e., the same for all inertial observers.)

General relativity is applicable to non-inertial systems and is based on generalizations of the above two postulates embellished with other certain principles/assumptions, including:

1. The speed of light is invariant and equals c for non-inertial observers provided that it is measured locally by local standard clocks and measuring rods [14].
2. There is no preferred non-inertial frame. (The laws of nature are also covariant for non-inertial observers, although coordinate metrics different from those of special relativity are needed to represent those laws.)
3. Gravity and acceleration are *locally* indistinguishable, i.e., the equivalence principle. (Over finite distances, gravity can, however, be distinguished from acceleration due to the presence of gravitational tidal forces or geodesic deviation.)

4. Neither gravity nor acceleration changes the length of a standard rod or the rate of a standard clock relative to a nearby freely falling (inertial) standard rod or standard clock having the same velocity. (This is an assumption rarely emphasized in most texts, though Møller[11] makes the point clearly, and Einstein [15] emphasized it a number of times.) We will call this the "surrogate frames postulate" or when used with reference to standard rods, the "surrogate rods postulate."

The first general relativity point above is often a source of confusion, as it is sometimes said that the speed of light in general relativity can be different than c. This is true if, for example, one measures the speed of light near a massive star using a clock based on earth. (Time on such a clock is effectively the coordinate time in a Schwarzchild coordinate system.) As is well known, due to the intense gravitation field, the passage of time close to the star is dilated relative to earth time, and one would indeed calculate a light speed other than c. However, use of standard rods and clocks adjacent the light ray itself would result in a speed of precisely c.

Other confusion exists for scenarios where spacetime itself expands or contracts. For example, just after the big bang, space itself was expanding much like the surface of a balloon being blown up. A photon in space (analogous to an ant on the surface of the balloon) at a different location than an observer could then move away from the observer faster than c (analogous to faster than the ant can crawl on the surface) because the space (balloon surface) between the photon and the observer is itself expanding. Yet a photon spatially coincident with an observer could never be seen by that observer to have speed greater than c, and local standard rods and clocks adjacent any photon would find its speed equaling c regardless of the dynamical state of spacetime itself.

1.3 Nomenclature and Definitions

Upper-case letters herein shall refer to inertial systems; lower case to non-inertial systems. For example K shall designate the non-rotating (lab) frame; k, the rotating frame.

Flat spacetime will be referred to as "Minkowski space"; whereas the term "Minkowski metric" will be limited to refer only to a Minkowskian set of coordinates (Cartesian plus time) used within that flat space. Hence a Minkowski space need not be represented solely by a Minkowski metric, and we will in fact use cylindrical coordinates, i.e., (cT,R,Φ,Z), for the flat non-rotating inertial frame.

Though much of the paper may be understood with no working knowledge of differential geometry (the mathematics of general relativity), in Sec. 4 it is needed to derive certain results. These derivations are based on a Minkowski metric defined as

$$\eta_{\alpha,\beta} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \alpha,\beta = 0,1,2,3 \tag{1}$$

To eliminate confusion for the non-specialist, and to more readily compare results with those of Einstein and others, we employ cgs, not geometrized, units where $c =$2.998 X 10^{10} cm/sec.
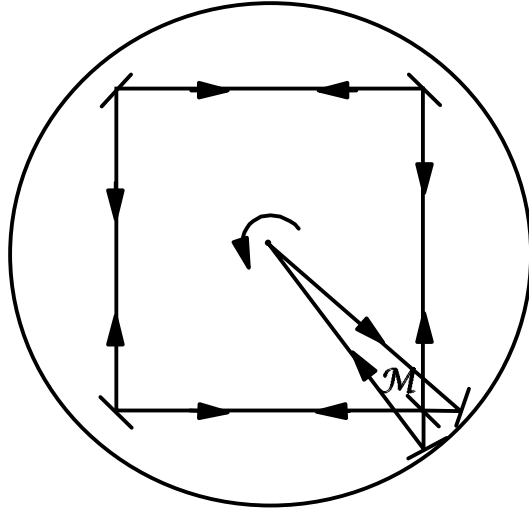
Fig. 1 The Sagnac Experiment

1.4 Rotating Disk Experimental Evidence

1.4.1 The speed of light.

In 1913 Sagnac [16] first demonstrated experimentally that rotating disks exhibit a remarkable property, the significance of which the present author believes has been completely overlooked ever since. That is, the local speed of a beam of light tangent to the disk circumference is not invariant, and not isotropic (as seen from the disk).

Fig. 1 depicts the Sagnac experiment schematically. A light beam is emitted radially from the center of a rotating disk and is split by a half silvered mirror M at radius r. From there one part of the beam is reflected by mirrors appropriately placed on the disk such that it travels in one direction around the circumference. The other half of the beam travels the same route over precisely the same distance, but in the opposite direction. The beams then meet up again and are reflected back to the center where interference of the two beams results in a fringing, i.e., a displacement of one light wave with respect to the other.

This is exactly the effect Michelson and Morley were first looking for, but could, due to now well known relativistic effects, never detect. Fringing results in either experiment would have implied different velocities of light in different directions. Hence, while the Michelson-Morley result indicated that for translational motion the speed of light is invariant and isotropic, the Sagnac experiment indicates that for rotational motion, no such conclusion may be drawn.

The results of Sagnac and others who have repeated his experiment have experimental accuracy only to first order in $v/c = \omega r/c$, and indicate that the speed of a light ray tangent to the circumference measured locally on the disk is equal to [17]

$$|v_{\text{light}}| \cong c \pm \omega r \tag{2}$$

where the approximately equal sign implies accuracy to first order, and the sign in front of the last term depends on the relative direction of the rim tangent and light ray velocities.

These results should, in fact, be expected. An inertial (non-rotating) observer of the two light rays would see each of them having the speed c, and during the time they are traveling around the circumference the disk would rotate some amount. Hence one ray would meet back up with the half-silvered mirror M before the other, and an observer fixed on the disk at M would conclude that the speeds in each direction were different. Further, the difference can be readily calculated, to first order, to be that shown in Eq. (2). (Selleri [18] makes the calculation rigorously to all orders.) Still further, the effect is *local* since angular velocity is constant and due to symmetry, any segment of a constant radius path (in the ideal experimental design) is equivalent to any other segment. Hence any global (average) speed effect measured by Sagnac over finite times and distances is equal to the local (infinitesimal) speed at any point on the circumference.

1.4.2 Absolute nature of rotational velocity.

The Michelson-Morley experiment also implied that translational velocities are relative, and that there is no preferred system of reference (no "ether").

Rotational velocities, on the contrary, are absolute. (The term "absolute" herein implies accordance with Mach's principle, i.e., absoluteness with respect to the distant galaxies, within Einstein's relativistic theory of flat spacetime). This is due, at least in part, to the absolute nature of the radially directed accelerations experienced by any rotating object. Hence, for rotational velocities there is a preferred frame, and it is the one in which no radial accelerations are experienced. Any observer, in any frame, can tell which system is the non-rotating one, i.e., the "preferred frame", and how much each of the other frames is rotating relative to it. This can be done by watching the motion of a Foucault pendulum, by noticing whether or not there is a Coriolis "force", or by a number of other means.

2 DIFFICULTIES WITH THE TRADITIONAL VIEW

2.1 The Postulates

The reader has no doubt noticed that the experimental results of Sec. 1.4 above appear to clash with the very postulates of Sec. 1.2 upon which the theory of relativity was founded. That is, all of the relativistic behavior with which we have become so familiar in the twentieth century, such as the Lorentz contraction and the lack of agreement on simultaneity, are a direct result of i) invariance of the speed of light, and ii) "reference frame democracy" (all frames are equal). Yet, the author contends, these postulates simply do not hold for rotating frames.

This point seems to have been overlooked for two reasons. Firstly, much of the literature [16] covering the Sagnac effect focuses on the fringe effect *per se*, and its concomitant mathematical description, without noting the significant implications such fringing has for the speed of light. Secondly, of those who were aware that this implied a variable speed of light, most probably glossed over the fact by assuming that in some manner it was merely another general relativistic (non-inertial systems) manifestation of the "light speed unequal to c effect". Yet, unlike the examples provided in Sec. 1.2, there is no expansion or contraction of spacetime associated with the rotating disk, and the fringing implies a true difference in the *local* measurement of light speed.

Therefore one should, indeed *must*, expect rotating frame behavior to differ from that of

translational motion. Relativistic effects such as the Lorentz contraction are not given *a priori*; they are derived. And they are derived from different empirically based principles than those governing rotational motion. Hence, we should be wary of conclusions drawn by simply applying derived tenets of relativity theory to rotating disks, as Einstein and others have done.

2.2 Geodesic Deviation

The Riemann curvature tensor (or simply "Riemann") R is a measure of the curvature of a given space. It is defined by virtue of the geodesic deviation equation of differential geometry [19],

$$\nabla_u \nabla_u n \ + \ R(..., u, n, u) \ = \ \mathbf{0}\,. \tag{3}$$

The first term above represents the *deviation* between *two* geodesics, i.e., the rate of change of the rate of change in proper distance of the perpendicular from the first geodesic to the second as one travels along the first. If R is zero, then Eq. (3) dictates that every pair of geodesics which are initially parallel will stay parallel along their entire length. The proper distance between them will never change, and they will never intersect. Since this is true only of flat spaces, a zero value for R means the space is flat. If $R \neq 0$, such as on the surface of a globe, two geodesics (e.g., lines of longitude) which start out parallel (at the equator), don't stay parallel (and cross at the poles). R is characteristic of the space itself, not the coordinate system used within that space. If, for example, $R = 0$ for a 2D flat space with Cartesian coordinate system, when we transform to a polar coordinate system, we still have $R = 0$.

It is commonly known [20, 21] that the four-dimensional (4D) spacetime of the rotating system (denoted k) is Riemann flat since $R = 0$ in the non-rotating frame (denoted K), and the rotating system coordinates are obtained by simply transforming the 4D coordinates of K into k. If Riemann is zero in K, it must also be zero in k. However, it does not necessarily follow that the subspace of the disk surface, embedded in the 4D space, is flat. By analogy, the 2D subspace surface of a sphere embedded in a flat 3D space is not itself flat.

Particles attached to the rotating disk undergo acceleration and hence do not follow geodesic paths. The path of a free particle or light ray, however, is a geodesic, and though it is straight as seen from K, it looks curved, even "corkscrew-like," as seen from k.

The question of flatness for the subspace of the disk can be addressed by considering two free particles traveling at the same velocity in K in the plane of the disk surface (i.e., the axial coordinate $Z = $ constant), and tracing out parallel lines in K. The observer in K sees them as straight and never intersecting. The rotating observer sees them as corkscrew-like and never intersecting. The point is that the geodesic equation, from which the Riemann tensor is defined, relates to the "never intersecting" part, not the "non-straight" part. In a curved space there is geodesic *deviation*. It says two geodesics *deviate* in their behavior. The two geodesics in question do not. Further, the two geodesics travel *in the plane of the disk surface*. Regardless of how one wishes to define the disk surface and all of the issues of simultaneity involved (see later sections), the basic fact remains that for $z = $ constant, the two geodesics do not deviate (i.e., they never cross). For a Riemann curved surface they must deviate. Therefore the disk surface is Riemann flat.

By analogy, two parallel geodesics which appear straight in an inertial system appear curved to an observer in a rectilinearly accelerating system. But they never appear to cross to the accelerating observer, and the proper distance between them never changes. As is well known, the space of a rectilinearly accelerating system is flat [22]. This is in full accord with Eq. (3)
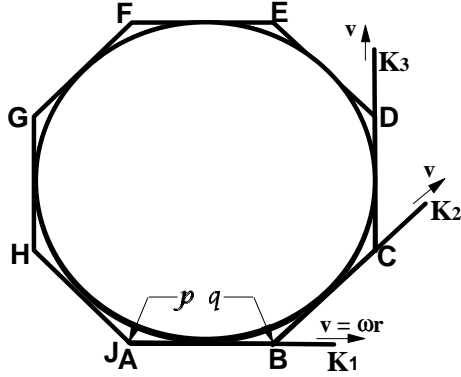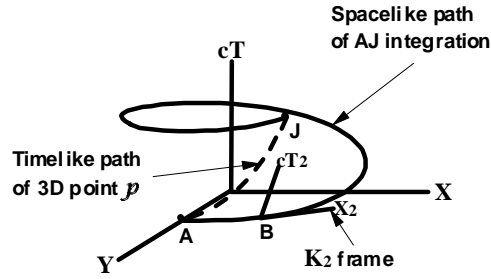
Fig. 2. Inertial tangent frames

Fig. 3. Tangent frames
integration path.

since geodesic deviation for such a system is zero, and so is Riemann, even though geodesics themselves do not look straight.

Geodesic deviation causes tidal forces, the stretching and compressing of a finite sized object in free fall (i.e., traveling a geodesic). Gravity tries to make one side of the object accelerate in a different direction, or at a different rate, than the other side. But a finite sized object traveling along a geodesic in the plane of the rotating disk would notexperience any tidal forces, and *all* observers, whether on the disk, the lab, or anywhere else, would agree there is no stress or strain within the object. Hence Riemann is zero along the path of the object, and the surface of the disk can not be curved.

2.3 Tangent Frames and the Discontinuity in Time

Applying traditional relativistic concepts directly to the rotating disk leads to another striking difficulty. It predicts a discontinuity in time on the surface of the disk, and in addition, the location of that discontinuity is arbitrary, being merely a function of the particular predilections of the observer. In other words, a continuous standard tape measure extending one circumference around the rim would not meet back up with itself at the same point in time. The logic leading to this conclusion follows.

In order to evaluate disk curvature, prior researchers have invoked the "surrogate frames postulate" (see general relativity principle 4 of Sec. 1.2) and used a series of inertial reference frames tangent to the disk rim with velocities equal to that of the rim edge (i.e., with $v= \omega r$). It is argued that since acceleration body forces do not affect standard rod length, rods in these inertial frames should be affected in precisely the same manner as rods aligned with, and attached to, the edge of the disk rim in k.

The problems with this approach can be illustrated with the aid of Fig. 2. Inertial measuring rods in inertial frames K_1 to K_8 with speeds ωr can be imagined as shown. For practical reasons we only show eight finite length rods, and we consider them as a symbolic representation of an infinite number of rods of infinitesimal length. A and B are *events* located in space at the endpoints of the K_1 rod which are simultaneous as seen from K_1; B and C are events located in space at the endpoints of the K_2 rod which are simultaneous in K_2; and so on for the other events C to J. A,B, ...J can be envisioned as flashes of light emitted by bulbs situated equidistantly around the disk rim.

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p is a spatial (three dimensional) point fixed to the disk at which both A and J occur. q is the spatial point on the disk at which B occurs. In principle, A, B, ... J, as well as p and q are located on the disk rim though they may not look so in Fig. 2 since the tangent rods shown are not infinitesimal in length.

Note that although events A and B are simultaneous as seen from K_1, they are not simultaneous as seen in K (via standard relativity theory for two inertial frames in relative motion). As seen from K, A occurs before B. Similarly, B occurs before C, and so on around the rim. If the events are light flashes, a ground based observer looking down on the disk would see the A flash, then B, then C, etc. Hence we conclude that as seen from K, A occurs before J even though A and J are both located at the same 3D point p fixed to the rim. As seen from K, during the time interval between events A and J the disk rotates, and hence the point p moves. (As an aside, Fig. 2 can now be seen to be merely symbolic since events A to J would not in actuality be seen from K to occur at the locations shown in Fig. 2. That is, by the time the K observer sees the B flash, the disk has rotated a little. It rotates a little more before he sees the C flash, etc.)

According to the traditional treatment of the rotating disk, one then uses the K_i rods and integrates (adds the rod lengths) along the path AB ...J, moving sequentially from tangent inertial frame to tangent inertial frame. This path is represented by the solid line in Fig. 3, and one can visualize small Minkowski coordinate frames situated at every point along the curve AJ (see K_2 in Fig. 3) with integration taking place along a series of spatial axes (such as X_2 in Fig. 3). By doing this one arrives at a length for AJ, the presumed circumference of a disk of radius r, of precisely as predicted by Einstein and many others

$$\text{AJ} = \frac{2\pi r}{\sqrt{1 - \omega^2 r^2/c^2}}. \tag{4}$$

But consider that since point p moves along a timelike path as seen from K (see dotted line in Fig. 3), a time difference between events A and J must therefore exist as measured by a clock attached to point p. As a result, one end of a continuous tape measure riding with the rim of the disk would not meet back up with its other end at the same point in time. But any meaningful measurement of the circumference simply must have the same starting and ending event, and therefore must be a *closed path* in spacetime.

We have therefore shown that the tangent frames analysis approach leads to a discontinuity in time, a seemingly impossible physical situation. Even further, the spatial location of that discontinuity is completely arbitrary. It depends on where we choose our initial starting point p. This is a very serious dilemma for the traditional interpretation.

We conclude that simultaneity can not be defined in a consistent manner using local inertial clocks over any closed path where different parts of the path have different relative velocities. Hence, we are unable to measure the circumference of the rim with local inertial rods where the endpoints of all the rods are simultaneous as measured by local inertial clocks. Therefore, inertial frames tangent to the rim can not be used to measure the disk circumference, and conclusions made from so doing will not be valid.

This apparent violation of the heretofore seemingly sacrosanct "surrogate rods postulate" is addressed in Sec. 5.2.

2.4 Related Problems

Ehrenfest [23, 24] saw a paradox in the presumed circumferential Lorentz contraction effect which Einstein [25] and Grøn [26] attempted to resolve by claiming that the disk circumference tries to contract in Lorentz fashion, but can't, and so undergoes internal tensile stress. Other mechanically induced stresses aside, the disk presumably cannot be spun up to relativistic speeds without developing such stresses and, at high enough speeds, rupturing.

But one must then also argue that the time discontinuity of Fig. 3 is resisted in some way by a "tension" in the time component around the circumference. If the rods must be extended in order to meet up in space, then surely the endpoints of rods must be adjusted in time in order to meet up as well. Tensile stress may be a well known physical phenomenon in a material body in space, but there is certainly no such phenomenon associated with time.

A related problem is pointed out by several authors (see, for example Weber [13]). If light rays sent out around the circumference are used according to the standard Einstein synchronization procedure, one finds the clock at p at $360°$ to be out of synchronization with itself at $0°$. This leads to the restriction that one can only consider open paths on the disk surface. But then one must ask what prevents a physical disk based observer from traveling around one complete circumference? And what prevents her from laying down a continuous tape measure as she does so? And finally, how good a representation of the physical world is a model in which a clock can not be synchronized with itself?

3 NEW THEORY OF ROTATING FRAMES

In this section we re-derive key aspects of relativity theory for the rotating reference frame using two new postulates based on the Sagnac and other experiments. We follow logic similar to that employed by Einstein to derive special relativity for translational motion, but start from a different, but equally empirically justifiable, basis.

In Sec. 4 transformation techniques of differential geometry are utilized to rigorously derive all relevant characteristics of the rotating frame, including the exact form of Eq. (2), our first postulate below. The present Sec. 3, on the other hand, provides a physically meaningful, and simpler, derivation of certain of those characteristics.

3.1 New Postulates

We postulate the following:

1. The speed of light is not invariant between the ground and the rotating frame, and in the rotating frame is found to first order by the velocity addition law of Eq. (2)

$$|v_{\text{light}}| \cong c \pm \omega r$$

2. Observers can discern which frame is non-rotating (the "preferred frame").

3.2 Different Results

3.2.1 Simultaneity.

Fig. 4 depicts a means for defining simultaneity at any radius r on the disk. Light rays can be imagined as emitted simultaneously from the centerpoint of the disk, striking mirrors located at
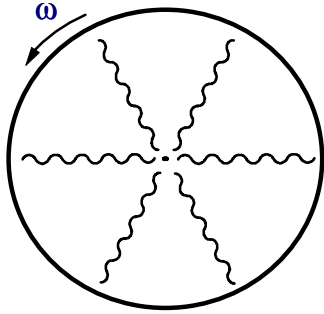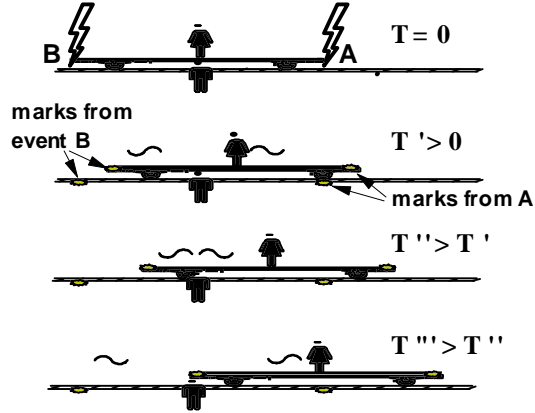
Fig. 4. Establishing simul-
taneity from the disk center

Fig. 5. Lorentzian vs
Galilean simultaneity

r, and being reflected back to the centerpoint. They all arrive back at the center at the same instant in time as measured by a clock located there, and one concludes that the events occurring when the light struck the mirrors are all simultaneous. These events are also simultaneous to observers in K.

The question then arises as to whether a non-inertial observer riding on the rim itself would agree that those same events are simultaneous. Standard relativity theory predicts she would not, since she and the K observer have relative velocity difference.

We answer this question by re-considering Einstein's famous *gedanken* experiment of the passing train shown in Fig. 5. Lightning strikes both ends of the car and leaves marks on both ends plus the ground. These events are A and B. Given the postulate that light has the same velocity as seen from the train or the ground, and given that both observers can later measure the distance to the brown marks left by lightning events and determine that each is 1/2 way between their respective marks, the train observer concludes that A occurred before B because she saw the A flash of light first. This she concludes because she knows the speed of light from both directions is the same for her. The ground observer sees each flash at the same instant and concludes the two events were simultaneous since the speed of light is also the same for him in both directions. This is case 1, the standard special relativity result.

For case 2, suppose instead that nature works in Galilean fashion and the light from A travels faster than the light from B as seen from the train frame. ($|V_A| = c + v$ and $|V_B| = c - v$ where v is the absolute value of the relative velocity between frames.) The train observer still sees the A flash first, and still later measures the distance to the marks and knows she is 1/2 way between them. But now she also knows that the light from A travels faster, so she would expect to see it first. The math is trivial. She concludes that A and B were indeed simultaneous, as does the ground observer.

As shown by the Sagnac experiment the speed of light on the circumference of the disk behaves as in the second case above. (Assume for the present that Eq. (2) is an exact equality. We will resolve the first order approximation issue in Sec. 4.) The observer on the disk knows she is rotating, knows she has tangential velocity relative to the inertial frame K, and knows from Eq. (2) the formula for calculating the velocity of light as seen by her (it is direct addition

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as for the train case 2 above.) She therefore concludes that two spatially proximate events on the circumference which are simultaneous in the ground frame are also simultaneous to her even though she sees one of them occur first.

Hence, whether measured from the center of the disk, or locally at any other point on the disk, simultaneity in the disk frame k is identical with that of K. (Selleri [18] agrees with this conclusion, although he takes a different route to get there.) So unlike systems with relative rectilinear velocities where there is no common agreement in simultaneity, systems with relative rotational velocities all do agree on simultaneity.

3.2.2 No Lorentz Contraction.

The Lorentz contraction is a direct result of non-agreement in simultaneity between frames. If there is agreement in simultaneity, there is no Lorentz contraction. To show this we need one additional, presumably inviolable, postulate. That is,

3. The proper spacetime length of any path is invariant under any transformation, i.e., it is the same for all observers.

Hence, for two frames in relative motion (notation should be obvious)

$$(\Delta s)^2 = -c^2(\Delta t)^2 + (\Delta l)^2 = -c^2(\Delta t')^2 + (\Delta l')^2 \tag{5}$$

For a rod at rest in the primed frame, an observer in the unprimed frame sees that rod such that its endpoints are *events* which for him occur simultaneously, i.e., $\Delta t = 0$. But in the primed system those events are not, according to standard relativity theory, simultaneous and $\Delta t' \neq 0$. This means $\Delta l \neq \Delta l'$, and results in Lorentz contraction [27].

If, however, the same two events could also appear simultaneous in the primed system, then $\Delta t' = 0$, and Δl must equal $\Delta l'$. This is, of course, not possible for two frames in relative *translational* motion, but, as we have shown, it is possible between two frames with different *rotational* motion.

Hence, if $\Delta l'$ is the length of a (short) standard measuring rod attached to the non-rotating K frame aligned tangentially to the disk rim, and Δl is the length of a similar rod attached tangentially to the rim, then neither rod looks shortened to observers in either frame. There is, therefore, no Lorentz contraction for rotating systems.

The reader should note carefully the distinction here between with the contention of Grøn [10] and others [28] that rods fixed to the disk will not contract since tension in the disk prevents them from so doing. In contradistinction, we show that there is simply no kinematic imperative for the rods to try to contract. No tension arises in the disk as it is spun up, and no relativistically induced rupturing occurs.

3.2.3 Time Dilation.

Although frames K and k agree on simultaneity, it can be shown that standard clocks in each run at different rates. (Note that two clocks running at different rates can nonetheless both agree on simultaneity, i.e., that no time elapsed off either one between two events.)

The time dilation effect can be demonstrated with the aid of the spacetime diagram of Fig. 6, which shows the helical path of a clock fixed on the disk as seen from frame K and that of

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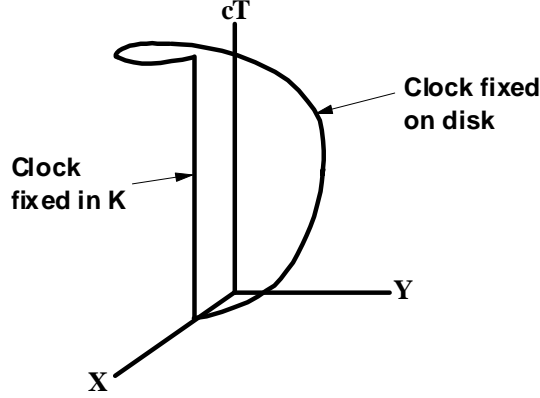
Fig. 6. Paths of clocks fixed in k and K

a clock fixed in K as seen from K. The moving clock travels the path of 3D point p of Fig. 3 extended for one full rotation. The path of that clock is a non-geodesic, while the path of its "twin" fixed in K is a geodesic, a straight line. The proper time passed for each clock is simply its path length (divided by ic), and this path length can be measured in any frame we choose since it is frame invariant. We choose frame K since it is the simplest.

In frame K, the proper *spatial* distance traversed by the disk fixed clock is $\Delta\sigma = 2\pi R = (\omega\Delta T)R$ where the time interval for one rotation is ΔT. Hence, the proper spacetime path length of the moving clock is

$$(\Delta s)^2 = -c^2(\Delta\tau)^2 = -c^2(\Delta T)^2 + (\Delta\sigma)^2 = -c^2(\Delta T)^2 + \omega^2 R^2(\Delta T)^2 \qquad (6)$$

Hence,

$$\Delta\tau = \Delta t = (1 - r^2\omega^2/c^2)^{1/2}\Delta T = (1 - v^2/c^2)^{1/2}\Delta T \qquad (7)$$

and the clock fixed in k on the disk rim runs slower than the K clock on the ground. Also, clocks run more slowly at greater radii, so it is not possible to synchronize standard clocks at different radii.

It is noteworthy that similar time dilation effects occur in translationally accelerating systems, yet it is readily shown [22] that such systems are nevertheless Riemann flat. (Acceleration does not cause spacetime curvature, gravity does.) Hence, time dilation in and of itself is not a sufficient condition for curvature.

Note also that, analogous with the rectilinearly accelerating system, it is not possible to synchronize standard clocks located at different radii in k since such clocks beat at different rates.

The analysis of a clock fixed on the disk at a certain radius r is similar to that of the traveling twin in the classic "twin paradox". Both twins live in Minkowski spaces, but the traveling twin follows a non-geodesic in spacetime (it must decelerate/accelerate to return to earth) and hence has a shorter elapsed time than the geodesic following sibling. This result, as in the rotating disk case, is independent of the reference frame of the observer since proper pathlength is invariant under transformation.

4 TRANSFORMATION THEORY

Einstein used his two postulates to derive the Lorentz transformation, from which all relevant relativistic characteristics may be found. If, conversely, he had known the Lorentz transformation first, he could have then derived his two postulates. In the present sec., we start with a reasonable guess at the correct transformation between rotating frames, analogous to the Lorentz transformation between translationally moving frames, and not only derive our original postulates, but predict other phenomena as well. As will be shown, these other phenomena are self consistent, do not lead to the difficulties delineated in Sec. 2, and agree with all known experiments.

4.1 Rotating Frame Metric and Transformations

Strauss [9], Franklin [29], Trocheries [30], and Takeno [31] have attempted to impose transformations between inertial and rotating frames which make an *a priori* assumption that the Lorentz contraction is operative and varies with the radius r of the disk (i.e., varies with the tangential velocity in the traditional special relativistic manner). These transformations appear to put the cart before the horse, i.e., they *start* with the Lorentz contraction built in.

An alternative, and more reasonable transformation (see Eqs. (8a-d) below) found in many sources [10,11,12,13,16,26,[32],[33]] (although with different interpretations and results than the present paper) makes no such assumption. It simply makes kinematic connections between the cylindrical rotating and non-rotating coordinate systems which are straightforward and seem most logical. If the transformation is correct, appropriate effects derivable from it should agree with experiment, and predicted results should be self consistent.

This coordinate transformation, where upper case coordinates represent the inertial frame K, lower case denote the rotating frame k, and the axis of rotation is coincident with both the Z and z axes, is

$$
\begin{aligned}
cT &= ct & &\text{(8a)} \\
R &= r & &\text{(8b)} \\
\Phi &= \phi + \omega t & &\text{(8c)} \\
Z &= z & &\text{(8d)}
\end{aligned}
\qquad (8)
$$

ω is the angular velocity of the disk, and t, the coordinate time for the rotating system, is the proper time of a standard clock located at the origin of the rotating coordinate frame, i.e., it is equivalent to any standard clock at rest in K. Note that t is only a coordinate. It is merely a label and cannot be expected to equal proper time at any given point on the disk (except, of course, at $r = 0$).

Assumptions upon which transformation (8) is based are: (i) The radial distance r measured in k can not be contracted as seen from K since velocity is always perpendicular to R, hence $R = r$. (ii) Radii in k (i.e., lines of constant ϕ and constant z) each are straight lines as seen from either k or K, move with rotational velocity ω, and are independent of r. (iii) The rotation has no effect on measurements in the direction of the axis of rotation, i.e., the Z direction, since, like the radial distance, $Z = z$ is perpendicular to velocity. Assumptions (i) and (iii) are apparently universally accepted by others. Assumption (ii) leads to Eq. (8c) and, as mentioned, has been considered by others.

The transformation (8) seems Galilean in nature, rather than relativistic, and if it is valid (as most researchers today feel that it is), we should not be surprised to find the disk exhibiting at least some Galilean characteristics.

To deduce the metric for the rotating system we begin with the line element for the standard cylindrical coordinate system of the Minkowski space K

$$ds^2 = -c^2 dT^2 + dR^2 + R^2 d\Phi^2 + dZ^2. \tag{9}$$

Finding dT, dR, $d\Phi$, and dZ from Eqs. (8), and inserting into Eq. (9), one obtains the metric of the coordinate grid in k. (Note this step incorporates postulate 3 of Sec. 3.2.2, i.e., ds is invariant.)

$$ds^2 = -c^2(1 - \tfrac{r^2\omega^2}{c^2})dt^2 + dr^2 + r^2 d\phi^2 + 2r^2\omega d\phi dt + dz^2$$

$$= g_{\alpha\beta} dx^\alpha dx^\beta \quad, \tag{10}$$

where the covariant form of the metric $g_{\alpha\beta}$ and its inverse, the contravariant matrix $g^{\alpha\beta}$, readily found via the standard cofactor method, are

$$g_{\alpha\beta} = \begin{bmatrix} -(1 - \frac{r^2\omega^2}{c^2}) & 0 & \frac{r^2\omega}{c} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{r^2\omega}{c} & 0 & r^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad g^{\alpha\beta} = \begin{bmatrix} -1 & 0 & \frac{\omega}{c} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\omega}{c} & 0 & (1 - \frac{r^2\omega^2}{c^2})/r^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{11}$$

For future reference, the comparable matrices in K are

$$G_{AB} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & R^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad G^{AB} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{R^2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

where sub and superscripts A and B as used here are upper case Greek letters for alpha and beta.

Note from Eqs. (10) and Eqs. ((11) that the rotating disk system is not orthogonal (the metric is not diagonal).

Taking the differentials in Eqs. (8), one can readily derive the matrix Λ_B^α which transforms contravariant components of vectors and tensors from K to k, and its inverse Λ_β^A which transforms contravariant vectors and tensors from k to K. These transformations between the two cylindrical coordinate systems are:

$$\Lambda_B^\alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{\omega}{c} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \Lambda_\beta^A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\omega}{c} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{13}$$

With the above metrics and transformations forming the basis of the new theory, we can proceed to derive the effects we would expect to see in the physical world.

4.2 Galilean Characteristics

4.2.1 Invariance of simultaneity.

From Eq. (8a) [or equivalently by comparing Eq. (9) and Eq. (10)], if $\Delta T = 0$ in K for two events, then $\Delta t = 0$ in k for the same two events. In other words the two events are simultaneous as seen from either system, in agreement with our earlier thought experiment based on physical reasoning. Note that the transformation between time coordinates of the present theory is much different than that of the Lorentz transformation. In the latter the coordinate time difference is dependent upon the locations of the two events; in the former, it is not.

4.2.2 No Lorentz contraction.

Note since Eq. (9) equals Eq. (10), the circumference of the disk for any radius $r = R$, at fixed time t and constant z (i.e., $dt = dT = dr = dR = dz = dZ = 0$), is $2\pi r(\Delta\Phi = \Delta\phi = 2\pi)$, implying that the rotating disk is indeed a flat space, and corroborating the physical reasoning of Sec. 3.2.2. (Some authors, most notably Grøn [10,26], contend that the non-time orthogonal nature of the rotating coordinate system negate this conclusion. We resolve this matter in Sec. 5 below and the Appendix.)

Note further that Lorentz contraction arises directly from the Lorentz transformation, yet Eqs. (8), the transformation now accepted as correct by virtually everyone in the field, is not the Lorentz transformation. There is, therefore, absolutely no reason (other than tradition) to tacitly assume that it must somehow give rise to Lorentz contraction.

4.2.3 Angular velocity addition.

Consider three co-axial reference frames, one of which is not rotating and designated by K, the second of which has rotational velocity ω_2 and designated by k_2, and the third of which has velocity $\omega_3 = 2\omega_2$ and is designated by k_3. ω_2 and ω_3 are measured relative to K. Note that an observer in k_2 sees the k_3 system rotate once relative to him, for each time interval that he rotates once relative to K. Hence $\omega_{3/2}$, the angular velocity of k_3 relative to k_2, has the same magnitude as ω_2, and therefore

$$\omega_3 = \omega_2 + \omega_{3/2} \tag{14}$$

Relationship Eq. (14) obviously holds in general, and demonstrates that rotational velocities for co-axial systems add directly, in Galilean fashion, frame to frame and not relativistically as do translational velocities. This not only lends further credence to the Galilean type transformation (8) employed herein, but also implies that there is no upper limit on angular velocity comparable to the luminal limitation on rectilinear velocities [34].

4.2.4 Translational velocity addition.

Assume V^I are the components of the three velocity of an object as seen in the K cylindrical coordinate system, and U^A are the components of the four-velocity. For the same object, an observer in k measures v^i as components of the three velocity and u^α for the four-velocity. That is,

$$V^I = \frac{dX^I}{dT}, \qquad U^A = \frac{dX^A}{d\tau} = \frac{1}{\sqrt{1-v^2/c^2}} \begin{bmatrix} c \\ \frac{dR}{dT} = V^R \\ \frac{d\Phi}{dT} = V^\Phi \\ \frac{dZ}{dT} = V^Z \end{bmatrix}. \tag{15}$$

To find the three velocity addition law, we use the same procedure employed in special relativity to derive the relativistic velocity addition law. We begin by first transforming the four-vector dX^I to its counterpart in k, dx^α, using the appropriate transformation matrix from Eqs. (13).

$$\begin{bmatrix} cdt \\ dr \\ d\phi \\ dz \end{bmatrix} = dx^\alpha = \Lambda^\alpha_B dX^B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{\omega}{c} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cdT \\ dR \\ d\Phi \\ dZ \end{bmatrix} = \begin{bmatrix} cdT \\ dR \\ -\omega dT + d\Phi \\ dZ \end{bmatrix} \tag{16}$$

Three velocities in k are then found simply by dividing the spatial components of Eq. (16) by dt, and noting that $dt = dT$, i.e.,

$$v^i = \frac{dx^i}{dt} = \begin{bmatrix} \frac{dR}{dt} \\ -\omega \frac{dT}{dt} + \frac{d\Phi}{dt} \\ \frac{dZ}{dt} \end{bmatrix} = \begin{bmatrix} V^R \\ -\omega + V^\Phi \\ V^Z \end{bmatrix} \tag{17}$$

For an object with purely tangential velocity of magnitude V^{Tang} equal to RV^Φ one finds from Eq. (17) that

$$v^{\text{tang}} = rv^\phi = -\omega R + RV^\Phi = -\omega R + V^{\text{tang}} \tag{18}$$

a very Galilean-looking transformation.

It must be noted once again, however, that time derivatives above are with respect to coordinate time t, and for a disk fixed observer at any location other than $r = 0$, time dilation effects must be taken into account to reflect the actual velocities such an observer would measure with physical instruments. In practice this would mean dividing Eq. (18) by the factor $\sqrt{1 - \omega^2 r^2/c^2}$, i.e., by the factor local time differs from the coordinate time used in Eq. (17). Note that this does not change the directly additive quality of Eq. (18).

4.2.5 Lack of invariance of the speed of light.

Consider Eq. (18) where V^{Tang} represents the speed c of a light ray which could be propagating in the positive or negative Φ direction. Then

$$v^{\text{light,tang}} = -\omega R \pm c \tag{19}$$

This result is in remarkable agreement with the Sagnac experiment, and provides strong support for the validity of transformation (8).

Since velocities in Eq. (19) are coordinate velocities, we must divide both sides of the equation by the time dilation factor $\sqrt{1 - \omega^2 r^2/c^2}$ to represent the physical velocities a disk observer would actually measure using local standard clocks. By doing this we obtain the exact relationship for which the Sagnac result Eq. (2) was only a first order approximation. Hence, as

we assumed in Sec. 3.2.1, the exactly equal sign in Eq. (2) is correct if the velocities are taken as those which would actually be measured by an observer fixed to the disk; see Sec. 5.1 and Eq. (33).

By utilizing these velocities, rather than c, for light rays employed to synchronize clocks at a given radius, one then finds a clock at 360° is synchronized with itself at 0°. More generally, closed path integrations are fully allowable, and thereby consistent with what one would expect physically.

Note that we have derived our rotating frame postulates from transformation (8). Eq. (19) is the first postulate. By taking $\omega = 0$ in Eq. (19), we get our second. That is, the preferred frame is the one with isotropic light speed c, i.e., it is the inertial one.

In Sec. 5.1 we re-derive these results even more rigorously, and reconcile them with general relativity principle 1 in Sec. 1.2.

4.3 Lorentzian Characteristics

A plethora of cyclotron experiments demonstrates that rotating systems do indeed possess certain relativistic characteristics, such as time dilation (longer decay times) and mass-energy increase with speed. If transformation (8) is the correct one, then these effects must be predicted by it. The ensuing derivations do indeed confirm that transformation (8) is consistent with these experiments.

4.3.1 Time dilation.

From the metric of Eq. (10) with r, ϕ, and z constant and $ds^2 = -c^2 d\tau^2$, the proper time at any radius r is

$$d\tau = \sqrt{1 - \frac{r^2\omega^2}{c^2}}\, dt = \sqrt{1 - \frac{v^2}{c^2}}\, dT \qquad (20)$$

which corroborates the result Eq. (7) of Sec. 3.2.3. (Since speed is constant, finite "Δ" differences in Eq. (7) can be taken over to differentials "d".)

Note that the time dilation effect arises naturally from the simple and readily justifiable coordinate transformation (8), and was not "built in" from the start by assuming that it holds *a priori*. Further, time dilation does occur in the rotating frame in accordance with the standard relation of special relativity. However, unlike special relativity this effect is not symmetric between frames k and K. Observers in both systems agree that the rotating disk clocks run slower.

4.3.2 Path lengths of light and particles.

The pathlength of any object traveling in spacetime is invariant between frames in accordance with our "new" postulate 3, which is, of course, not really new but a fundamental principle of differential geometry. The path length of light, in particular, remains null as viewed from the rotating frame since $ds = 0$ in K, and hence ds must also $= 0$ in k.

4.3.3 Four-vectors.

The four-velocity and the four-momentum transform readily between the rotating and non-rotating systems also in accordance with basic principles of differential geometry/general rela-

tivity.

However, when making general transformations of four-vectors, one should keep two things in mind which are usually irrelevant for Minkowski metrics in Minkowski space, but are quite relevant for other metrics such as that of the rotating frame. Both of these relate to physical interpretation of the components of four-vectors (i.e., the quantities one would actually measure with instruments.)

The first of these concerns lies with the covariant or contravariant nature of the components. Since coordinate differences (e.g., dx^α) are expressed as contravariant quantities, and since four-velocity is simply the derivative of these coordinate differences with respect to the invariant scalar quantity τ (proper time), four-velocities only represent (proper) time derivatives of the coordinate values if they are expressed in contravariant form. In general, lowering the index of u^α via the metric $g_{\alpha\beta}$ gives components u_α which are *not* the time derivatives of the coordinate values. This is true because $g_{\alpha\beta}$ is not the identity matrix. Note that in inertial frames $g_{\alpha\beta}$ = $\eta_{\alpha\beta}$ (see Eq. (1)) which is, apart from the sign of the g_{00} component, an identity matrix. In a coordinate frame with such a Minkowski metric the covariant form of the four-velocity is identical to the contravariant form except for the sign of the timelike component. In other coordinate frames, however, the difference is much more significant, and care must be taken to work with the contravariant form of the four-velocity.

Four-momentum, on the other hand, must be treated in terms of its covariant components. This is because said four-momentum is the canonical conjugate of the four-velocity. In brief, if the Lagrangian of a given system is

$$L = L(x^\alpha, \dot{x}^\alpha, \tau) \qquad (21)$$

where dots over quantities represent derivatives with respect to τ, then the conjugate momentum is

$$p_\alpha = \frac{\partial L}{\partial \dot{x}^\alpha} \qquad (22)$$

Hence it is imperative that one use the covariant components of the four-momentum. Contravariant components, for all but a Minkowski metric, will not represent physical quantities such as energy, three momentum, etc.

Getting the correct contravariant or covariant components is not quite enough, however, in order to compare theoretical results with measured quantities. If a given basis vector does not have unit length, the magnitude of the corresponding component will not equal the physical quantity measured. For example, a vector with a single non-zero component value of 1 in a coordinate system where the corresponding basis vector for that component has length 3 does not have an absolute (physical) length equal to 1, but to three.

In general, therefore, (see Malvern [35], for example, for further explication), physical components are found from vector components via the relations

$$v^{\hat{\alpha}} = v^\alpha \sqrt{g_{\underline{\alpha\alpha}}} \qquad v_{\hat{\alpha}} = v_\alpha \sqrt{g^{\underline{\alpha\alpha}}} \qquad (23)$$

where carets over indices designate physical quantities, and underlining implies no summation.

Hence in order to compare theoretical component values with experiment, it is necessary to use contravariant components for coordinate differences and four-velocities, covariant components for four-momenta, and physical components of all component quantities whether covariant or contravariant.

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4.3.4 Mass-energy of a particle fixed on disk.

Consider a particle of mass m fixed on the disk at constant ϕ, r, and z. Since $d\phi = dr = dz = 0$, the four-momentum of the particle in k coordinate contravariant components (using metric Eq. (10)) is

$$p^\beta = mu^\beta = m\frac{dx^\beta}{d\tau} = m\begin{bmatrix} c\frac{dt}{d\tau} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{m}{\sqrt{1 - \frac{\omega^2 r^2}{c^2}}}\begin{bmatrix} c \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{24}$$

where $dt/d\tau$ is found from Eq. (20).

The mass-energy (non-physical), except for a factor $-c$, and three momenta (non-physical) are the four-dimensional conjugate momenta of the dx^β and are the components of the covariant four-momentum vector

$$p_\alpha = g_{\alpha\beta}p^\beta = \begin{bmatrix} -(1 - \frac{r^2\omega^2}{c^2}) & 0 & \frac{r^2\omega}{c} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{r^2\omega}{c} & 0 & r^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \frac{mc}{\sqrt{1 - \frac{\omega^2 r^2}{c^2}}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{mc}{\sqrt{1 - \frac{\omega^2 r^2}{c^2}}}\begin{bmatrix} -(1 - \frac{r^2\omega^2}{c^2}) \\ 0 \\ \frac{r^2\omega}{c} \\ 0 \end{bmatrix} \tag{25}$$

The physical energy e of the particle as measured in k is therefore

$$e = -p_{\hat{0}}c = -p_0\sqrt{-g^{00}}c = -p_0 c = mc^2\sqrt{1 - \frac{\omega^2 r^2}{c^2}}$$

$$= mc^2 - \tfrac{1}{2}m\omega^2 r^2 - \text{(higher order terms)} \tag{26}$$

$$= mc^2 + V_{class} + \ldots\ldots .$$

where V_{class} is the classical potential for the particle as seen from the rotating frame, and V_{class} plus the higher-order terms is the relativistic potential energy.

The energy of the particle as seen from K can be found by using the second of Eqs. (13) to transform p^β of Eq. (24) into the four-momentum of the inertial frame P^B. By then using G_{AB}, the metric of K, P_A is found to be

$$P_A = G_{AB}P^B = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & R^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \frac{mc}{\sqrt{1 - \frac{\omega^2 R^2}{c^2}}}\begin{bmatrix} 1 \\ 0 \\ \frac{\omega}{c} \\ 0 \end{bmatrix} \tag{27}$$

The mass-energy E of the particle as measured from K is therefore

$$E \;=\; -P_{\hat{0}}\,c \;=\; -P_0\sqrt{G^{00}}\,c \;=\; -P_0 c \;=\; \frac{mc^2}{\sqrt{1-\frac{\omega^2 R^2}{c^2}}}$$

$$\;=\; \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \;=\; mc^2 \;+\; \tfrac{1}{2}m\omega^2 R^2 \;+\; \text{.......} \quad . \tag{28}$$

in full accord with the relativistic mass-energy effect. Note that the total energy in both k and K becomes imaginary at $r = c/\omega$ where the tangential disk speed reaches that of light. That these results were obtained from the transformations (8) (i.e., Eqs. (13) derived without *ad hoc* Lorentz factors thrown in, supports the contention that those transformations are indeed the correct ones.

Note also that P_Φ turns out to be the relativistic angular momentum, the conjugate momentum of Φ, as it must be if the transformation employed is correct.

$$P_\Phi \;=\; \frac{mR^2\omega}{\sqrt{1-\frac{\omega^2 R^2}{c^2}}} \;=\; \frac{mvR}{\sqrt{1-\frac{v^2}{c^2}}} \tag{29}$$

Further, p_ϕ, the relativistic angular momentum as seen from k (see Eq. (25)), has the same value as P_Φ, and is non-zero even though the four-velocity component u^ϕ in k is zero.

5 RAMIFICATIONS OF NON-TIME-ORTHOGONALITY

5.1 The Speed of Light

Consider the line element of Eq. (10) for a ray of light directed tangentially at radius r (with velocity c in K). $dz = dr = 0$, and

$$ds^2 = 0 = \;-\; c^2(1-\tfrac{\omega^2 r^2}{c^2})dt^2 + r^2 d\phi^2 + 2r^2\omega\, d\phi\, dt \quad . \tag{30}$$

Solving Eq. (30) for $d\phi$ via the standard quadratic equation formula and dividing the result by dt, one obtains

$$\frac{d\phi}{dt} \;=\; -\omega \;\pm\; \frac{c}{r} \quad , \qquad v^{\text{tang}} \;=\; \frac{r d\phi}{dt} \;=\; -r\omega \;+\; c \,. \tag{31}$$

The same result as Eq. (19).

For velocities in terms of local times on the rim, substitute

$$dt = \frac{dt_l}{\sqrt{1-\frac{\omega^2 r^2}{c^2}}} \quad , \tag{32}$$

where dt_l is time as measured by local standard clocks at r, and hence

$$v^{\text{tang,phys}} \;=\; \frac{-r\omega \;\pm\; c}{\sqrt{1-\frac{\omega^2 r^2}{c^2}}} \tag{33}$$

is the exact expression for the first-order approximation Eq. (2) of the Sagnac experiment.

Note that without the off diagonal (non-orthogonal) terms of the metric in Eq. (30) the physical velocity above measured by local standard clocks would be c. (Delete the last term in Eq. (30) and substitute the relation Eq. (32) for dt in terms of dt_l.)
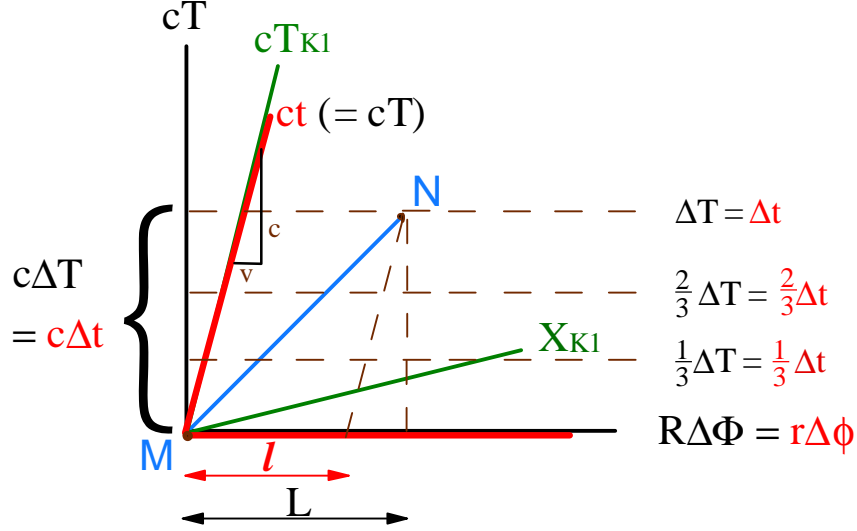
Figure 7. Speed of Light in Three Reference Frames at r = R

Fig. 7 helps to explain this effect of non-orthogonality graphically. The coordinate axes shown as perpendiculars represent K; the slanted coordinate lines represent the local inertial frame at r, K_1; the line MN represents the null path of a light ray; and the bold lines represent the non-orthogonal coordinate axes for k at the location r. Note that the coordinate time axes of k and K_1 are coincident, as are the spatial axes of k and K. Coordinate times T (in K) and t (in k) are equal (see dashed horizontal lines representing different values of $cT = ct$). Physical spatial distance in K ($= R\Delta\Phi$) and in k ($=r\Delta\phi$) are equal, and we consider Δ values as small. Coordinate systems for K and K_1 are orthogonal in Minkowski space, the coordinate system for k at r is not. In all three systems the path length of MN is zero, since pathlength is invariant.

Observe that for a given amount of coordinate time (which is the same in both k and K, i.e., $c\Delta T = c\Delta t$), the light ray travels a certain spatial distance l in k, but a greater spatial distance L in K. Hence the speed of light measured in k is less than that in K, and this corresponds with the plus sign before the c in Eq. (31). For a light ray in the opposite direction (minus sign in Eq. (31)) one can show graphically (with a light ray MN$^/$ in the second quadrant of Fig. 7 at right angles to MN) that the corresponding l distance is greater than L and hence the velocity for that ray would be greater in k than in K.

Given that the slope of MN is unity, $L = c\Delta T$. Dividing this by ΔT, one gets the speed of light in K as c. The k time axis has slope $c/v = c/\omega r$, so $l = L - c\Delta T(\omega r/c)$. Dividing this by ΔT, one arrives at Eq. (31) for the coordinate speed of light in k (with the plus sign for c since light ray MN is traveling in the direction of disk rotation).

Note that in both inertial frames K and K_1, the speed of light ray MN equals c. We can therefore conclude that general relativity principle 1 remains valid, provided we constrain it to refer to time orthogonal frames (such as the Minkowski, Schwarzchild, and Friedman geometries). It does not hold for non-time-orthogonal frames.

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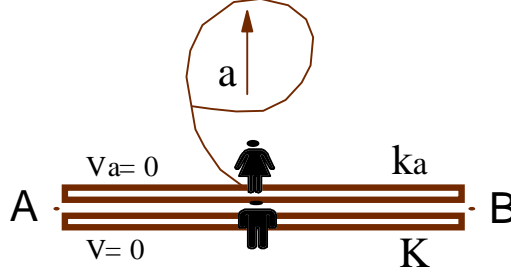
Fig. 8. Equivalent Inertial and Non-Inertial Rods

5.2 The "Surrogate Rods Postulate"

Non-time-orthogonality also reconciles the results of Sec. 2.3 (tangent frames can not be used to measure the circumference) with the heretofore seemingly universal applicability of the surrogate rods postulate of Sec. 1.2. We note first, however, that the tangent frames do not, strictly speaking, have the same velocity as the disk rim. The rim segment, in addition to its linear velocity component $v = \omega r$, has an angular velocity ω which the tangent frame does not. Hence, unlike other successful applications of the surrogate rods postulate, the tangent frames here do *not* mimic the rim frame velocity in all regards. Therefore, they can not, in the truest sense be considered "co-moving" as many prior researchers have assumed. To see the effect of this in terms of non-time-orthogonality, we first consider the underlying principles on which the surrogate rods postulate is based.

Fig. 8 shows two spatially coincident standard rods with zero relative velocity, the first of which is fixed in a rectilinearly accelerating frame k_a, and the second of which is fixed in an inertial frame K. Consider two light flash events A and B located at the endpoints of both rods. An observer in K at the centerpoint of the inertial rod sees both flashes at the same time and concludes they were simultaneous as seen from the K frame. Likewise, an observer on the k_a rod halfway between A and B would see them at the same time as well and know that the events were also simultaneous as seen from the k_a frame. That is, $\Delta t_a = \Delta T = 0$ between A and B. Since the proper spacetime length between the two events is the same as seen from both frames (i.e., $\Delta s_a = \Delta S$), then the spatial length between them must also be equal, and the length measured by rods in k_a and K between A and B are equal. Similar arguments can be made for acceleration in the rod lengthwise direction, as well as for gravitational body forces induced by a massive body.

Hence the surrogate rods postulate is merely a restatement of the proper spacetime path length invariance postulate for the special case where Δ(time) = 0 for both observers (which it is for zero relative velocity and time orthogonal frames). Figs. 9 and 10 reveal what this means in the context of non-time-orthogonal reference frames.

Fig. 9 depicts two inertial reference frames, K and K_1, in relative motion, and serves as a review of the cause of the Lorentz contraction effect. A rod fixed in K_1 has length $L_1 = L$ as seen from K_1. The endpoints (and all points between) of L_1 move with velocity v relative to K along world lines parallel to the K_1 time axis. Lorentz contraction arises because the observer in K sees rod endpoint events as A and C, whereas the observer in K_1 sees them as A and B. L_1', the distance between A and C is less than L, the distance between A and B, by the Lorentz
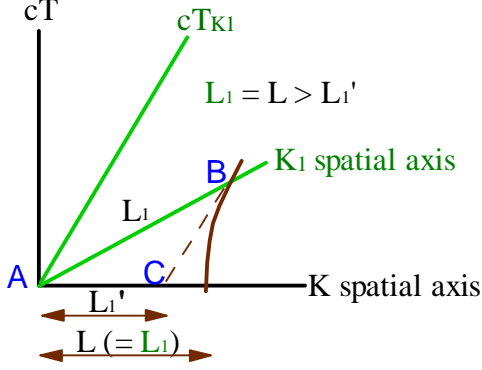
**Fig. 9. Standard Rods in
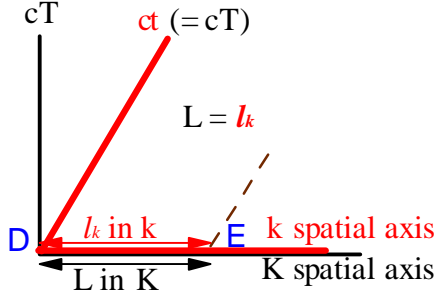Two Inertial Frames**

**Fig. 10. Standard Rod in
Non-time-orthogonal Frame**

contraction factor. Though it is beside the point we are in the process of making, the Lorentz contraction effect is thus seen to be little more than an optical illusion fostered on us by lack of agreement in simultaneity. No Lorentz contracted object ever feels compressed.

In contrast with Fig. 9, Fig. 10 shows the rotating coordinate frame k at radius r superimposed with the non-rotating inertial frame K. In Fig. 10 we show two circumferentially aligned standard rods, one designated by l_k which rides with the disk and the other, L, fixed in K. When both rods are at rest in the same inertial system, they have equal length, i.e., $l_k = L$. When the disk is spinning they also have equal length, as seen by both k and K observers, since, as discussed earlier, the same endpoint events (D and E) of both rods are seen as simultaneous in both frames. Yet, due to the special nature of the non-time-orthogonal frame k at r, the l_k rod has a non-zero velocity relative to the L rod. Every point on the l_k rod in Fig. 10 moves with the same velocity as every point on the L_1 rod in Fig. 9. Yet L_1 looks contracted from K, whereas l_k does not, i.e.,

$$L_1 = L = l_k > L'_1 \tag{34}$$

That is, even though L_1 and l_k have the same velocity as seen from K, they do not have the same spatial length as seen from K. Prior researchers have almost universally assumed they do.

That L_1 looks contracted as seen from k can be corroborated by superimposing the k frame of Fig. 10 with the K_1 frame of Fig. 9. Hence, two rods with the same velocity, one in a time orthogonal frame and one in a non-time-orthogonal frame do not have equal lengths. We conclude that the surrogate rods postulate is only valid for time orthogonal frames.

5.3 The Large Radius, Small ω Limit

Several researchers [13,18] have considered the limiting case of very large radius, small angular velocity, with large circumferential velocity $v = \omega r$. In this case acceleration v^2/r approaches zero, and it is argued there is no way to discern between such a frame and an inertial frame. Hence, a circumferential segment of the rotating frame in such case must approximate an inertial (Lorentzian) frame.

The answer to this conundrum lies in non-time-orthogonality. From Fig. 7 it can be seen that the slope of the time axis in the k frame is c/v, and as we have shown, the Sagnac effect, the

lack of Lorentz contraction, and all other peculiarities of the rotating frame are derivable from non-time-orthogonality, i.e., the slope of that axis. But in taking the limit described, v remains constant, and hence so does the slope of the time axis. Therefore, all of the non-Lorentzian phenomena heretofore described for rotating frames are unmitigated in passing to the limit. (See also Sec. 6.)

Contrary to what many claim, an observer on this limiting case frame can determine she is rotating. In fact, three experiments can reveal this. The first is the Sagnac experiment. The second is described in Section 6.2 below. The third involves measuring the mass of a known entity such as an electron which varies relativistically with the potential energy, i.e., as a function of v^2 alone as in Eq. (26), and hence one can readily determine v.

6 THE NEW THEORY AND EXPERIMENT

6.1 Michelson-Morley Revisited

Given the speed of our planet around its sun, and the speed of our solar system around its galactic center, one might ask why measurements on our planet (which could be considered as part of a frame rotating about the center of each of these systems) do not seem to exhibit the aforementioned non-Lorentzian properties. In particular, why did Michelson and Morley not find the speed of light in the direction of galactic rotation different from that in other directions? Note that given the Sagnac results, this question has an empirical imperative which is independent of any theory, i.e., any particular rotating frame analysis.

The answer, the author submits, is that bodies in gravitational orbits follow geodesics, i.e., they are in "free fall". That is, they are in locally inertial frames and therefore obey Lorentzian mechanics. Objects fixed in "true" rotational frames, on the other hand, are held in place by non-gravitational forces, do not travel geodesic paths, and exhibit Sagnac type characteristics. Hence, the only effective rotational velocity for the earth is the earth surface velocity about its own (inertial) axis. Michelson and Gale [36] did in fact measure the Sagnac effect for the earth's surface velocity in the 1920's. And in order to be maintain accuracy, the Global Positioning System must apply a Sagnac velocity correction to its electromagnetic signals [37].

6.2 Modern Michelson-Morley Experiments

The most significant experiment, however, and the most accurate Michelson-Morley type test to date is that of Brillet and Hall [38]. They found a "null" effect at the $\Delta t/t = 3 \text{X} 10^{-15}$ level, ostensibly verifying standard relativity theory to high order. However, to obtain this result they subtracted out a persistent "spurious" signal of amplitude $2 \text{X} 10^{-13}$ at twice the apparatus rotation frequency.

Compare this anomalous signal to that predicted by the presently proposed theory. The velocity of the earth surface at 40° latitude, where the Brillet and Hall experiment was performed, is .355 km/sec. If the speed of light is truly increased or decreased by this amount in the direction of rotation, then a Michelson-Morley experiment (see Fig. 11) with one leg in the direction of the velocity would yield [39]

$$\frac{\Delta t}{t} \cong \tfrac{1}{2} \frac{v^2}{c^2} \qquad (35)$$
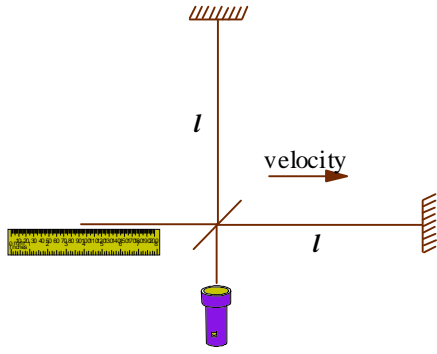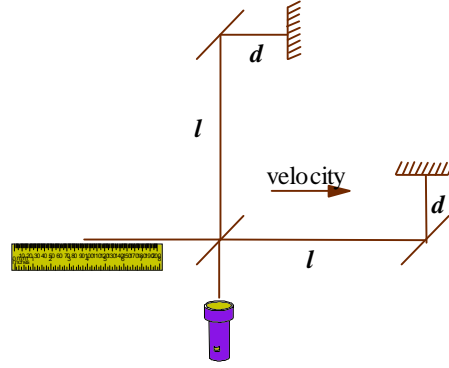
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Figure 11. Michelson-Morley Experiment

Figure 12. Modified Michelson-Morley Experiment with Decreased Sensitivity

where t is the round-trip time for one leg and Δt is the difference in time taken between the leg aligned with the velocity vector and the leg perpendicular to that vector.

Brillet and Hall rotated their equipment about a vertical axis (always perpendicular to the earth surface velocity). In such a case Eq. (35) would yield the peak-to-peak amplitude of the signal. To compare with Brillet and Hall's reported single peak amplitude signal, one must then divide Eq. (35) by two. Using .355 km/sec in Eq. (35) and dividing by two, one gets $\Delta t/t = 3.5 \times 10^{-13}$. Fig. 12 can help to explain the reason for the discrepancy in this number and the reported value.

In Fig. 12 the two light rays no longer travel solely on two perpendicular paths. In the Brillet and Hall experimental apparatus a similar configuration to that of Fig. 12 was used, apparently to accommodate the laser equipment which provided such extraordinary accuracy. (See Fig. 1 in Brillet and Hall.) Note that if the two shorter legs d were each 25% the length of the primary legs l, then the time t in Eq. (35) would be 1.25 times that of Fig. 11. Note also that the lower of the d legs is aligned perpendicularly to the velocity, whereas the lower path is intended to monitor light speed in the direction of the velocity vector. If both it and the upper d leg were 25% of the primary legs in length, one could therefore expect a 25% reduction in Δt as well. Hence the total signal $\Delta t/t$ would be a reduced by a factor of $.75/1.25 = 60\%$.

Although Brillet and Hall did not provide the pertinent dimensions in their paper, from the sketch of their equipment one can conclude that the 25% figure used above may be fairly accurate. To account for this approximation we can assume a signal strength modification factor of 50% to 70% . For these percentages, the expected signal range is $1.7\text{-}2.5 \times 10^{-13}$, in remarkable agreement with the measured value of 2×10^{-13}.

Note also that in the Brillet and Hall experiment signals were exchanged electronically between the two legs, and that actual fringing at a single location was not measured. Hence, we may expect the variation in light speed to affect these transmission signals as well, introducing additional error.

The author also investigated possible mitigating effects from "frame dragging" (a $t\phi$ off diagonal term appearing in the metric due to the earth's angular momentum), and the chord path effect (the light ray parallel to the velocity actually travels a chord of the arc length, not the arc length itself). These were found to be negligible to orders of magnitude well beyond 10^{-13}.

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For completeness, we also mention the results of Hils and Hall [40] which found no variations to an order of $2\text{X}10^{-13}$ (only coincidentally the same number as above.) However, the Hils and Hall apparatus was fixed relative to the earth's surface and hence was immune to variations of the type predicted by the present theory (which would manifest only if the apparatus were rotated relative to the earth surface.)

6.3 A Proposed Experiment

We propose another experiment using a combined Sagnac and Michelson-Morley type apparatus in order to further test the present theory. For this experiment light is emitted from a rotating disk center in the manner of the Sagnac experiment, but a Michelson-Morley type apparatus is mounted on the disk rim. When the light from the center reaches the rim it is split into two components. One of these travels along the circumference a short distance and then is reflected directly backwards (rather than further out around the rim.) The other component of the light is reflected in the z direction (perpendicular to the disk) an equal distance and reflected directly backwards as well. The two returning components are then deflected toward the disk center where fringe effects are measured. Accelerating the disk, and accounting for elastic deformation, one should find the degree of fringing varies. The standard theory predicts no such variation.

7 ELECTRODYNAMICS, MECHANICS, AND SPACETIME

With regard to electrodynamics, Ridgely [33] has recently used covariant constitutive equations in an elegant analysis to answer a troubling question cogently posed by Pellegrini and Swift [32]. He uses transformation (8) to derive electrodynamic results for the rotating frame k, not the tangent frames K_i, and finds that those results match what one would find by simply applying Maxwell's equations and traditional special relativity to the tangent frames.

We can conclude the following. Only with the theoretical approach shown herein can one obtain internally consistent results which agree with all experiments. However, for the purposes of mass-energy, momentum, and time dilation calculations (shown herein) and Maxwell's equations (shown by Ridgely), one can get away with assuming that the tangent frames represent the rotating frame and using traditional special relativity. That is, in these cases the laws of nature conspire to make both the present and the traditional analysis produce the same result for observers in K (i.e., mass-energy dependence on ωr, electric polarization, etc.). When it comes to matters of time (synchronization, simultaneity), space (curvature), and

Michelson-Morley/Sagnac type experiments, however, then analysis must be confined to the rotating frame itself, else the inconsistencies of Sec. 2 and inexplicable "spurious" experimental signals inevitably arise.

It therefore appears that the rotating disk problem may have, at long last, been completely solved. According to Ridgely's results and the theory proposed herein, no paradoxes remain and all theory matches up with the physical world as we know it.

8 SUMMARY AND CONCLUSIONS

8.1 New Theory Predictions

The lack of Lorentz contraction, agreement in simultaneity, flatness of the disk surface, non-invariant/non-isotropic speed of light, and time dilation can all be derived in two different ways: (i) directly from transformation (8); and (ii) from the Sagnac experiment. All but the asterisked phenomena summarized below can be determined in at least two ways (i.e., transformation (8), experiment, or thought experiment based on Sagnac). An asterisk (*) indicates the conclusion depends only on the validity of transformation (8).

1. The subspace surface of a rotating disk is flat.

2. No circumferential Lorentz contraction exists. Further, no relativistically induced tensile stress develops as there is no kinematic imperative for the disk circumference to try to Lorentz contract.

3. Observers anywhere in the rotating frame and observers in the non-rotating frame all agree on simultaneity.

4. Velocities (angular and translational*) add directly frame-to-frame and not relativistically.

5. Angular velocities are absolute and have no upper speed limitation.

6. Rods in inertial frames with velocities equal to the tangent velocities at a given disk radius can not be used to measure the circumference, since
a. the "surrogate frames postulate" for equivalence of inertial and non-inertial standard rods is not valid for the rotating frame, and is generally invalid for any non-time-orthogonal frame, and
b. doing so leads to a discontinuity in time.

7. Light has a null path length, yet the local speed of light in the rotating frame (and all non-time-orthogonal frames) is not isotropic and generally not equal to c.

8. Time dilation does occur, but it is not symmetric, i.e., rotating and non-rotating observers agree that time dilation occurs on the disk relative to the stationary frame.

9. A particle fixed on the disk exhibits relativistic mass-energy dependence on tangential velocity. (No asterisk since cyclotron experiments validate this effect.)

10. Only the theory proposed herein yields self consistent results which completely conform with physical reality. However, use of tangent frames and traditional special relativity produce the same results for a certain subset of phenomena.

8.2 Comparison of Various Perspectives

The proposed theory resolves all difficulties with the traditional disk analysis delineated in Sec. 2.

Both the proposed theory and the traditional analysis agree with cyclotron experiments, i.e., they both predict time dilation (longer particle decay times), as well as relativistic mass-energy dependence on speed. Both theories are also consonant with the Phipps [41] experiment which has been used to discount certain other prior approaches to the problem not based on transformation (8). Importantly, however, the new theory predicts the results of the Brillet and Hall [38] experiment which the standard theory does not.

The new theory also agrees with part of Grøn's first work [42] on the standard approach, where he uses transformation (8) and concludes from it that simultaneity on the disk and in the lab are the same. However his latter paper [10] employs tangent frames analysis and recounts

purported difficulties in accelerating the disk which are the direct result of disagreement in simultaneity. In the presently proposed theory no such kinematic restriction on disk acceleration exists, and there is no mechanism by which any such disk would rupture from relativistically induced tensile stress, as has been contended by Einstein, Grøn, and others.

Table I summarizes the similarities and differences between the approaches of Einstein, Grøn, and the proposed theory.

TABLE I. COMPARISON OF VARIOUS DISK ANALYSES

	Einstein	Grøn	ThisPaper
Postulates agree with experiment?	No	No	Yes
Discontinuity in time?	Yes	Yes & No	No
Clock synchronized with itself?	No	No	Yes
Closed paths allowable?	No	No	Yes
"Tension" in time required?	Yes	Yes	No
Predicts Brillet & Hall anomaly?	No	No	Yes
Agrees with cyclotron experiments?	Yes	Yes	Yes
Relativistic mass-energy?	Yes	Yes	Yes
Time dilation on disk?	Yes	Yes	Yes
Lorentz contraction effect?	Yes	Yes	No
Disk surface is curved?	Yes	Yes	No
Relativistically induced disk stress?	Yes	Yes	No
Same simultaneity: disk and lab?	No	Yes & No	Yes
Time as defined is observable?*	No	No	Yes
Transformation is Galilean type?	No	Yes	Yes
Restricts surrogate rods principle?	No	No	Yes
Speed of light $= c$ on disk?	Yes	No†	No
Agrees with Phipps experiment?	Not treated	Yes	Yes

*See Appendix

† Result derived from transformation (8), but effect on relativity postulates not considered.

1   APPENDIX: THE METRIC OF PRIOR TREATMENTS

Landau and Lifshitz [12], Møller [11], Strauss [9], and Grøn [10] have all discussed a three dimensional submetric of the four-dimensional rotating frame metric defined by

$$\gamma_{ij} = g_{ij} - \frac{g_{0i}g_{0j}}{g_{00}} \quad . \tag{A1}$$

So defined, γ_{ij} represents the spatial metric which is locally orthogonal to the local proper time axis. Some of these authors have then used this metric to determine whether the space of the rotating disk is flat or not, and have concluded that it is curved. In fact, using the metric of Eq. (11) for $g_{\alpha\beta}$ in Eq. (A1) one finds

$$\gamma_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{r^2}{1-\frac{r^2\omega^2}{c^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A2}$$

A line element around the circumference then becomes

$$ds = \frac{r}{\sqrt{1-\frac{r^2\omega^2}{c^2}}}d\phi = \frac{r}{\sqrt{1-\frac{v^2}{c^2}}}d\phi \tag{A3}$$

From Eq. (A3) it is obvious that the circumference C is not equal to $2\pi r$, and in fact is equal to what Einstein and some of the above authors have claimed.

Further, Riemann for the metric of Eq. (A2) is non-zero.

However, the metric Eq. (A2) is derived for a differential line element having simultaneous starting and ending points as measured by local inertial clocks. In other words it assumes that a local inertial frame is aligned with ds and that measurement is carried out such that the endpoints of ds are simultaneous in that inertial frame. But this is nothing other than the type of integration path we investigated herein with the aid of Fig. 3 (solid line). As demonstrated, such an integration cannot be carried out around a closed path on the surface of the disk wherein the starting and ending points are simultaneous, and hence, it is meaningless as a physical measure of the disk circumference.

Adler, Bazin, and Schiffer [43] use the time transformation

$$dt* = dt - \frac{\omega r^2}{c^2-\omega^2 r^2}d\phi \tag{A4}$$

in Eq. (10) and obtain the same metric Eq. (A2), where the new coordinate time is then t^*.

However, the transformation Eq. (A4) is like any transformation in that it effectively shifts one to a different reference frame. Hence the time t^* as defined no longer represents time on the rotating frame itself, but some other time. This other time definition, we contend, has no meaning in the sense of being actually observable in the physical world by any possible observer. In essence, it represents an observer who miraculously skips from tangent inertial frame to tangent inertial frame without the concomitant acceleration and rotation associated with the

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disk itself. Not only is this not possible, but such a definition of time leads to a temporal discontinuity, as we have shown.

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