Comments regarding recent articles on relativistically rotating frames

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In a recent paper on relativistically rotating disks, Weber[1] presents the prevailing view and appears to contend that one need simply apply traditional relativistic concepts directly and all problems and paradoxes disappear. After cordial and protracted communication with Professor Weber, the present writer remains convinced that the issue is, in fact, far from settled, and that the following inconsistencies remain unresolved by the standard ''solution.''

First, with regard to curvature, it is important to recognize that finite objects traveling geodesic paths (straight lines as seen from the lab) in the plane of the disk surface experience no tidal stresses, and this is true as seen by any observer, including those on the disk itself. Hence the disk surface must necessarily be Riemann flat, regardless of how one believes time should be defined on the disk. This is directly at odds with the traditional treatment.

Second, consider a continuous standard tape measure lying up against a ridge on the disk circumference. If we apply traditional relativity theory and instantaneous co-moving frames along the disk ridge, we find that the tape one circumference distance around the rim does not meet back up with itself at the same point in time. Although one may argue for local interpretation of standard relativity, at some point this interpretation must match up globally with physical reality. And a continuous tape measure that is temporally discontinuous cannot possibly be a physical reality.

Third, in Secs. V A and V D Weber reviews the traditional disk analysis tenet of the apparent impossibility of synchronizing a clock with itself via ''the usual way'' using light rays traveling around the disk circumference. But how can a coordinate system in which a clock is out of synchronization with itself be a reasonable representation of the real world?

In a recent article the present writer[2] has offered a theoretical solution to these conundrums that agrees with all experiments. In that paper the following fundamental point is emphasized.

Relativity theory is based on two postulates having their origin in the famous experiment of Michelson and Morley. These are (1) invariance of the speed of light, and (2) ''reference frame democracy,'' i.e., all inertial frames are equivalent; velocity is relative. The first of these carries over to general relativity provided light speed measurements are made locally with standard rods and clocks.

The Michelson–Morley results are applicable to frames in rectilinear (not rotational) motion, and all of the results of relativity such as Lorentz contraction, time dilation, and mass–energy dependence on speed are derived from the two postulates based on that experiment. They are not given *a priori*.

The Sagnac[3] experiment, on the other hand, is a Michelson–Morley-type experiment for rotational motion, and it showed that the local speed of light in a circumferential direction on rotating frames is not invariant.[4] Further, it has long been known that not all frames are equivalent for rotational motion, as any observer can determine which frame is the preferred or nonrotating one (e.g., it is the only one without a Coriolis ''force'').

The problem should be obvious, i.e., we cannot simply assume that effects such as Lorentz contraction exist *a priori* on the rotating disk. On the contrary, we have to start with new postulates based on Sagnac's results, not those of Michelson and Morley, and rederive the relativity theory for rotating frames following the same steps Einstein did for rectilinear motion.

In the paper[2] referenced above, the writer has done just that. The reference frame used is the (non-Minkowskian) rotating frame itself, not surrogate local Minkowskian comoving frames (which do not produce the same results). The analysis shows time dilation and mass–energy dependence on $v = \omega r$, just as in standard special relativity (and therefore agreeing with cyclotron experiments), but no Lorentz contraction along the disk rim. The disk surface turns out to be Riemann flat, in agreement with tidal force analysis, and not curved as argued by Einstein and others. Further, a continuous tape measure does indeed meet back up with itself at the same point in time.

The lack of synchronization of a clock with itself is also resolved, since the underlying and tacit assumption in the ''usual way'' of synchronizing is Einstein's first postulate that the speed of light is invariant, i.e., the same in both directions around the rim. But the Sagnac experiment shows that this is not true, and, in fact, to first order,

$$|v_{\text{light, circumference}}| = c \pm \omega r, \tag{1}$$

where the velocities in (1) are *physical* (not merely coordinate) values, i.e., they represent values that would be measured by standard physical instruments.

Further, the second relativity postulate does not apply either, as anyone can determine their angular velocity and their circumferential velocity (ωr) relative to the inertial frame in which their axis of rotation is fixed. When light rays are used to synchronize clocks around the circumference by observers knowing their circumferential velocity and the speed of light from (1) above, the synchronization turns out to be exactly what one finds by using light rays from a clock located at the disk center. Hence, a clock can be synchronized with itself using light rays traveling around the circumference, and there is no paradox at all.

In the paper it is also shown that the ''surrogate rods postulate'' (small coincident inertial and noninertial standard rods with zero relative velocity are equivalent), used liberally with co-moving frames in prior rotating disk analyses, is invalid for non-time-orthogonal frames, of which the rotating frame is one. In other words, Minkowski tangent frames can represent (curved or flat) time orthogonal frames locally, but not (curved or flat) non-time-orthogonal frames. This important fact appears never to have been realized before. As a corollary, this conclusion is true even in the large radius, small rotational velocity limit.

The derivation of all of these results is remarkably straightforward, provided one can put aside the unconscious predisposition toward a theory derived from different postulates than those shown by experiment to be applicable to rotating frames.

With regard to the traditional argument that ''...inertial frames used to interpret experiments are only approximate and invariably are part of a rotating system,'' for every supposed rotating system we are in (e.g., earth around sun, sun around galactic center, etc.) except one (earth surface around earth central axis), our frame is actually a freefall, or inertial, system and therefore Lorentzian. The only effective rotational velocity in that case is the earth surface velocity about its own (inertial) axis. Michelson and Gale[5] did, in fact, measure the Sagnac effect for the earth's surface velocity in the 1920s.

The most significant experiment, however, and the most accurate Michelson–Morley-type test to date, is that of Brillet and Hall.[6] They found a ''null'' effect at the $\Delta t/t = 3 \times 10^{-15}$ level, ostensibly verifying standard relativity theory to high order. However, in order to obtain this result they were forced to subtract out a ''spurious'' and persistent signal of approximate amplitude 2×10^{-13} at twice the rotation frequency of their apparatus. The theory developed by the present writer, in contradistinction to the standard theory, actually predicts just such an effect due to the earth surface velocity. For the Michelson–Morley test geometry this theory predicts a signal amplitude of 3.5×10^{-13}. For the Brillet and Hall test geometry, however, the light paths are not restricted to two perpendicular paths, and the resultant $\Delta t/t$ effect is diluted. Brillet and Hall do not specify pertinent light path dimensions, but from the sketch of their apparatus, one could expect a reduction in a signal of perhaps 30%–50%. This would result in a predicted amplitude range of $1.7-2.5\times10^{-13}$ and remarkably close agreement with the measured value.

With regard to electrodynamics, Ridgely[7] has recently used covariant constitutive equations in an elegant analysis to answer a troubling question cogently posed by Pellegrini and Swift.[8] Ridgely derives electrodynamic results for the rotating frame itself, not the co-moving frame(s), and finds that those results match what one would find by simply applying Maxwell's equations and traditional special relativity to the co-moving frame(s).

The conclusion is this. Only with use of the rotating frame itself (and associated transformations and metric) can one obtain internally consistent results that agree with all experiments. However, for the purposes of time dilation, mass–energy, and momentum calculations (as the writer has shown in his paper), and Maxwell's equations (as Ridgely has shown), one can get away with using traditional special relativity and local Minkowski co-moving frames. That is, in these cases Nature conspires to make both the rotating (nontraditional) and co-moving (traditional) frame solutions produce the same result for lab observers (i.e., mass–energy dependence on ωr, electric polarization, etc.). When it comes to matters of time (synchronization, simultaneity), space (curvature), and Michelson–Morley/Sagnac-type experiments, however, then analysis must be confined to the rotating frame itself, otherwise the above-delineated inconsistencies and inexplicable ''spurious'' experimental signals inevitably arise.

Thus, it appears that the rotating disk problem may have, at long last, been completely solved. According to Ridgely's and the present writer's analyses, no paradoxes remain, and all theory matches up with the physical world as we know it.

Finally, and perhaps ironically, the writer's analysis actually turns out to be completely consonant with special relativity. That is, unlike other attempts to reconcile the Sagnac results, it leaves Lorentz covariance and all other traditionally relativistic effects for *Minkowski* frames intact. Apparent differences, such as those described herein, manifest specifically for the non-Minkowskian rotating frame, and generally, are characteristic of non-time-orthogonal frames. That is, the underlying physics is the same, merely being seen from a different (time orthogonal versus non-time-orthogonal) point of view.

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Response to ''Comments regarding recent articles on relativistically rotating frames'' [Am J. Phys. 67 (2), 158 (1999)]

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A full discussion of the many issues raised by Robert Klauber[1] is not possible in this short response. But I hope the following comments give some insight into the problem of the rotating disk and allow the reader to judge where and how we agree or disagree.

The spatial part of the invariant line interval in the laboratory frame (inertial frame) can be described by cylindrical coordinates. To go to the frame of the rotating disk, the azimuthal angle is replaced by $\phi+\omega t$ to get[2]

$$ds^2 = c^2(1-r^2\omega^2/c^2)dt^2 - dr^2 - r^2\, d\phi^2 - 2r^2\omega\, d\phi\, dt, \quad (1)$$

where ω is the angular velocity. The dependence on z, the coordinate along the axis of rotation, has been suppressed.

The angle ϕ has the range 0 to 2π. Coordinate clocks are taken to be fixed in position on the rotating frame, even though the time t that they read is the same as laboratory time.

These coordinates (r, ϕ, and t) are just one set of an infinite number that could be used to describe the rotating disk. For example, one can go about the disk changing the time on the clocks by setting $t'=t'(r,\phi,t)$ without leaving the frame of the rotating disk. Furthermore, a fixed r and ϕ give a point on the disk that has velocity $v=\omega r$ with respect to the laboratory. These coordinate markers (r and ϕ) can be changed to a new set of markers so that, for given values of these new markers, one again has a point fixed on the disk. All such coordinates describing the frame of the rotating disk are equally valid according to Einstein.[3]

Since the coordinate markers are arbitrary, one must be cautious in their interpretation of length and time intervals. A description of an experiment or a measurement can be made in any of these coordinates and, if done correctly, must yield the same result. As an example, the Schwarzschild line element can be written in terms of the traditional Schwarzschild coordinates or in isotropic coordinates. The transit times of radar signals reflected from an inner planet (a test of general relativity) look very different in terms of these two different coordinate systems. But the predicted numerical values of the transit times must be the same.[4]

The traditional way to describe a local event in a complicated geometry is to transform to the inertial frame that is instantaneously at rest with respect to the event. One has confidence in the interpretation of distances and time intervals in terms of the resulting Minkowski coordinates. Klauber objects to this procedure. Certainly this would be inappropriate for many studies of nonlocal or global properties of the metric.

Because of the cross term in the metric of Eq. (1), clocks at fixed r but different ϕ are not synchronized in the traditional way of sending light signals back and forth directly between the clocks.[5] Adjacent clocks on an *open* curve, however, can always be synchronized by adjusting the readings of the various clocks. In the case of the rotating disk this procedure cannot be extended globally; attempts to synchronize clocks on a closed curve lead to a discontinuity in time between two adjacent clocks. Klauber, however, uses a different synchronization in which the coordinate times t are synchronized as they stand. His method of synchronization is described following Eq. (1) of his comments.

A simple example may clarify how some of the conclusions of Klauber do not contradict the traditional view. Start with the invariant line interval of an inertial frame,

$$ds^2=c^2\ dt^2-dx^2, \tag{2}$$

and transform to a new frame by

$$x=x'+vt, \tag{3}$$

to get

$$ds^2=(c^2-v^2)dt^2-dx'^2-2v\ dt\ dx', \tag{4}$$

for the line interval described in the new coordinates. Equation (3) shows that every fixed point x' of the new frame travels with velocity v with respect to the original frame. For the propagation of light, set the invariant line interval equal to zero to find

$$\frac{dx'}{dt}=-v\pm c, \tag{5}$$

that is, the coordinate velocity is $(c-v)$ to the right and $-(c+v)$ to the left [compare with Eq. (1) of Klauber's comments]. These velocities do not contradict special relativity since, in the traditional view, the clocks reading time t are not synchronized, that is, clocks with larger x' have later times than if they were synchronized.

There is no contraction between the coordinate markers of the two frames with an observation made at the same coordinate time t. This follows directly from Eq. (3),

$$\Delta x=\Delta x', \quad \text{for } \Delta t=0. \tag{6}$$

But the question arises as to what is the actual distance in the spatial coordinate interval $\Delta x'$ of the moving frame. How would an observer on this frame set about making measurements and doing experiments so that the results are intelligible when communicated to other observers on different frames? An atomic clock can be used to measure time and the SI meter, defined as the distance traveled by light in vacuum during a time of 1/299,792,458 s, can be used for distance. With this definition of distance, an observer on the moving frame can measure the length of the interval $\Delta x'$ by recording the time for a light signal to go back and forth over the interval. The distance[5] is simply one-half of the proper time interval elapsed multiplied by c. This gives

$$\text{Distance}=\frac{\Delta x'}{\sqrt{1-v^2/c^2}}, \tag{7}$$

that is, the coordinate system appears to be *stretched*. Then if a new coordinate system for the moving frame were laid out with the same standard meter as used in the original inertial frame, the new coordinate intervals on the moving frame would appear *contracted*, as observed from the lab.

Unlike the clocks on the rotating disk, all the clocks on the moving frame with a metric described by Eq. (4) can be reset to eliminate the cross term. Then, rescaling the spatial and time coordinates one arrives at the Minkowski metric. The overall transformation is the Lorentz transformation, as expected.

The same measurement of distance can be used on the rotating disk; one finds that

$$\text{Distance}=\frac{r\Delta\phi}{\sqrt{(1-v^2/c^2)}}, \tag{8}$$

for the coordinate interval $r\Delta\phi$, while the distance in the radial direction is Δr. Measured in this way, the distance around the rim of the rotating disk divided by the radius is greater than 2π, that is, the geometry is non-Euclidean.

The experiment of Brillet and Hall is a test of the isotropy of space.[6] They measure the apparent length of the cavity of Fabry–Perot interferometer mounted horizontally on a table that is rotated about the vertical at a rate f (about once every 10 s). The condition of standing waves within the cavity will change if the propagation of light varies due to a preferred direction of space. Such an anisotropy would show up as a signal at rotation frequency $2f$. Brillet and Hall obtained a null result after subtracting a spurious signal at frequency $2f$ from their data. The cause of this signal is not explicitly stated in their paper. Klauber attributes the spurious signal to the effects of the rotating frame of the earth. However, with the length of the cavity in terms of the metric as given in Eq. (8), one obtains the null result expected for spatial isotropy; that is, there is no apparent change in length of the cavity with orientation. Klauber does not agree with this result since he does not accept the distance formula of Eq. (8).

Since Eq. (8) is good for small distances and is appropriate for a *local* experiment, there is the possibility that nonlocal effects of the metric could contribute to the spurious signal. The sensitivity of the instruments may be such that, even though the experiment is of short duration and spatial extent, nonlocal effects of the metric are observed. Using the metric of Eq. (1) for the propagation of light, one finds that any nonlocal effects due to rotation are negligible.

The most reasonable explanation of the spurious signal is the actual change in length of the cavity due to the varying gravitational stretching of the interferometer. This variation comes about because the axis of rotation of the interferometer is not perfectly vertical. Brillet and Hall state that this is one of two major factors that limit the sensitivity of the experiment. This stretching produces a strong signal at the table rotation frequency f. This strong signal can be largely eliminated since the signal of interest is at twice the rotation frequency. But Brillet and Hall refer to the strong signal as ''nearly'' sinusoidal so one expects higher harmonics. A second harmonic down by a factor of 12 would be approximately the strength of the spurious signal at frequency $2f$. No further explanation of this signal is warranted without further analysis of the data.

[1] R. D. Klauber, ''Comments regarding recent articles on relativistically rotating frames,'' Am. J. Phys. **67** (1999).

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EXAMS

I believe that perhaps one of the most potent influences tending to the development of mediocrity in thought is to be found in the necessity of testing the progress of the student as he learns, in the examination system, for example. If it is necessary every few weeks so set a group of half a dozen questions to test what the student has acquired, it is much easier to have questions which permit an answer in terms of facts, or in a standardized system of words invented to describe principles, than it is to set questions which necessitate answers which come from the brain rather than from the memory. It is convenient for the examiner if the answers are all more or less alike in method and wording.

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