Translation:Concerning the Definition of the Rigid Body in the Kinematics of the Principle of Relativity

Concerning the Definition of Rigid Bodies in the Kinematics of the Principle of Relativity.

By M. Born.

In my paper "The theory of the rigid electron in the kinematics of the principle of relativity[1], I have stated a definition of the "rigid" body satisfying the relativity principle, and I have shown that it suffices to simplify the theory of electrodynamic mass of electrons to a certain extent. In the course of further developing the consequences of this definition, it followed that this "rigid" body in no way is a real analogous to the rigid body of ordinary mechanics in all respects;[2] since this circumstance has been discussed from several sides, I would like to present my views concerning the physical usefulness of this concept. My definition amounted to saying, that the wordlines of the points of the body form a bundle of equidistant curves in four-dimensional space $xyzt$, when a measure determination with the line element $dx^2 + dy^2 + dz^2 - c^2 dt^2$ is introduced into it. Though any of such worldline bundles can in general be generated by guiding a three-dimensional linear space along an arbitrarily chosen worldline, so that it is constantly normal to it, exactly in the same way (in § 4 of my previously cited paper) as the most general translatory motion is generated by slipping a normal line along a plane worldline. From that it follows that the motion of a "rigid" body is in general completely determined by *one* worldline, i.e. by the motion of *one* point. This result was derived in a very complete and mathematically elegant way by G. Herglotz in his paper "On Bodies that are to be Designated as 'Rigid' from the Standpoint of the Relativity Principle"[3], and he discussed the exceptional cases at which a larger freedom of motion is present.

Since the new concept of rigidity consequently doesn't contain the necessary 6 degrees of freedom, it certainly cannot be applied to ordinary material rigid bodies.

However, in my point of view, there is principally nothing in the way to employ it as the foundation of the dynamics of the electron.

Above all it is to be considered, that every assumption concerning the kinematic constitution of the electron is purely hypothetical. One can deny such an intrusion into the interior of the electron from the outset;[4] but on one hand it were experimental facts, namely the dependence of mass on velocity observed at cathode and Becquerel rays, which suggested an explanation in this way, and in addition this denial would hit Abraham's theory in the same way. The latter contradicts the relativity principle; when this principle is proven as the foundation of the description of nature (for which all indications

speak), then a modification of Abraham's theory will be necessary. Therefore one will try to state a mechanics of the electron which is as closely connected to this theory as possible, and I think that this directly leads to my approach.

An electrodynamic explanation of the inertia-phenomenons of convectively moving electricity from the standpoint of the relativity principle thus makes the new definition of rigidity extraordinarily near at hand. On the other hand, no contradiction can be derived from observational facts against the theory developed from that. I only would like to allude to some points. First, my theory namely provides the constancy of electrodynamic rest mass and thus the validity of LORENTZ's formula for the apparent mass with respect to all observable motions of free electrons; according to the newest measurements[5], the observations are represented by this formula at least as good as by Abraham's formula. *However, at the other places where the electrons are used for the explanation of phenomena, it certainly suffices everywhere to interpret them as point charges.* If one now ascribes the rigidity as defined by me to the electrons, then they are moving "as one point" according to the things previously said. There is no phenomenon at all, for which rotations of electrons have been used to explain them. If one considers furthermore that the rotations of electrons in Abraham's theory give rise to those very strange force-free self-oscillations, as discovered by Herglotz[6] and Sommerfeld[7], then one won't see the circumstance as disadvantage, that the new theory excludes these forms of inconceivable and theoretically strange forms of motion. We of course have to wait, as to whether new observational facts will be in contradiction with my hypothesis.

As regards the ordinary material bodies, rigidity plays an essentially different role as in the kinematics of the electron; because here, the rigid body is not a true continuum, but a limiting case (nowhere strictly realized in nature) of the deformable medium formed by an immense number of atoms and electrons. It will require an essentially new way of treatment in order to include this ideal case into the system of the relativity principle. This question is surely to be seen as still open. If one imagines the material bodies as being composed of atoms and electrons, which e.g. are "rigid" in my sense, then they will follow curvilinear paths (which are surely consistent with their kinematics) when the whole body is rotating, and also the rotational axis as an abstraction provides no difficulty at that occasion.

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1. M. Born, Ann. d. Phys. 30, 1, 1909
2. For instance, P. Ehrenfest (this journal, 918, 1909) shows in a very simple manner, that a resting body can never be brought into uniform rotation; I already discussed this fact with A. Einstein at the meeting of natural scientists in Salzburg.
3. G. Herglotz, Ann. d. Phys. 31, 393, 1910.
4. Compare the explanations of M. Planck at the discussion concerning the lecture by A. Einstein at the meeting of natural scientists in Salzburg. This journal 10, 825, 1909.
5. A. H. Bucherer, Ann. d. Phys. (4), 28, 513-536, 1909; E. Hupka, Verh. d. D. Phys. Ges. 11, 249-258, 1909.
6. G. Herglotz, Nachr. v. d. Kgl. Ges. d. Wissensch. zu Göttingen, Math.-phys. Kl. 1903, S. 357.
7. A. Sommerfeld, Nachr. v. d. Kgl. Ges. d. Wissensch. zu Göttingen, Math.-phys. Kl. 1904, S. 353.