SPACE-TIME STRUCTURE IN HIGH ENERGY INTERACTIONS

D. Finkelstein*

Belfer Graduate School of Science
Yeshiva University, New York, New York

The preceding two talks were motivated by the courageous faith that our present ideas of the continuum and the gravitational field extend into the range of elementary particle sizes and far below. Equally interesting to high-energy physicists is the possibility that these ideas of space and time are already at the very edge of their domain and are wrong for shorter distances and times, and that high-energy experiments are a probe through which departure from the classical continuum can be discovered. The present talk is devoted to this alternative. The difficulty is that all our present theoretical work is based on a microscopic continuum and one is faced by the rather formidable problem of re-doing all physics in a continuum-free manner. Yet I think those who cope with the conceptual problems of quantum field theory for enough years eventually get sick of the ambiguities and divergencies that seem to derive from the continuum and are driven to seek some way out of this intellectual impasse. I would like to describe a program of this kind that I have been led into after vainly trying to extract some information about the small from extremely non-linear field theories.

The starting point here also is Riemann, who explicitly poses the question of whether the world is a continuous or a discrete manifold in his famous inaugural lecture. He points out certain philosophical advantages of the discrete manifold, in fact argues for it more strongly than for the continuous, and yet devotes his life to the continuous. Why?

* Young Men's Philanthropic League Professor of Physics, Supported in part by the National Science Foundation.

I think primarily on grounds of simplicity. Why is the continuous simpler than the discrete manifold? For one thing because it has more symmetries available to it. A continuum can have rotational symmetry, the world has rotational symmetry, a checkerboard cannot have rotational symmetry. Today, however, there are more options open to us than there were to Riemann. Faced with the apparent dilemma of the discrete and the continuum, our experience with quantum theory leads us to plunge boldly between these two with the synthesis called quantum. By a quantum I mean an object whose propositional calculus is isomorphic to the lattice of subspaces of a Hilbert space[1]. In present day quantum field theory, quantum concepts are injected at the top. We make up a complete classical picture of the world, a geometrical and dynamical structure which could in principle describe a real world, and then we take this beautiful theory and amend it by quantization. Is it not possible that in fact quantum concepts belong in the foundation? In particular, that instead of taking geometry and quantizing it, so to speak, we should take quanta and geometrize them? Should we not try to make a space-time theory in which from the start the elementary objects which make up the space-time are described by quantum laws and the space-time itself is assembled out of these by the quantum-logical procedures that we have mastered already in the quantum many-body problem?

I was forcefully introduced to this whole idea by Feynman some eight years ago. He didn't believe in continuum then. (He still doesn't believe in the continuum. See his "Character of Physical Laws")[2] . He suggested that a reasonable model for the world is a computer, a giant digital computer. The things we call events are processes of computation, and the fundamental fields represent stored information. The continuum theory of the world is totally absurd from this point of view. It imparts to each point of space-time an infinite memory capacity, in that an infinite number of bits are required to define a fundamental field like the electric vector potential. It takes an infinite channel capacity to communicate these numbers from one point of space to another. An infinite number of computations must be done by the computing element at each point to work out the field equations and pass on the output to the future.

The idea that the world is some kind of computer is not far removed, incidentally, from the idea that the world is some kind of brain; which is not far removed from the idea that the world is the mind of God; which stems back to the hermetic doctrines of the second century B. C. (I am indebted to Professor Jauch for this reference) so we are working in an ancient and honorable tradition.

The question is, however, can we make a computer which is compatible from the start with the principles of Quantum Theory and Relativity; and the first step to do this is to set up a little dictionary in which we translate the basic terms of physics into discrete or computer terms, instead of into continuum terms as we have had so much practice in doing. It will take a little more than half an hour to present a detailed dictionary of this kind. Let me simply indicate the spirit in which this can be done and the fact that to my surprise there seems to be no definite obstacle. I will indicate just a few of the possible models toward the end of my talk but I have the impression that I have wandered into open meadows where there are as yet no fences and unlimitable expanses of grass in which to graze.

To begin the dictionary, it's helpful to organize the structure of physics into three groups of concepts:

At the top, dynamics - concepts clustering primarily about action, in which things like charge and mass figure.

In the level of dynamics we move freely with geometric concepts which make up the next lower level, in which the fundamental single quantity is probably the notion of time, and in which we also have such basic things as space-time events, a relation of causality, and the speed of light.

In dealing with both the dynamic and geometrical levels of physics we work with the tools of logic, which make up the deepest level. In a sense this program is an attempt to reduce all physics to logic. There is not that much difference between a logical recursion procedure and a finitary computer.

Now I indicate briefly the manner of translation at each of these levels, the logical, geometrical and the dynamical, in a way which I think makes the procedure fairly easy to follow for anyone else who cares to retrace my steps.

The first thing is to make our computer out of quantum elements. Computers are ordinarily made out of binary elements, bits, zeros and ones. Let us simply suppose that instead of dealing with these by the tools of ordinary Boolean algebra we use the algebra of subspaces of Hilbert space. In more familiar terms, if a bit can take on the values 0 and 1 it can also take on coherent superpositions of these states. In brief, the natural building block for a quantum computer is the spin $-\frac{1}{2}$ theory, as a natural building block for a classical computer is the set with two elements[3] .

Table I indicates in fair detail how the processes of propositional calculus which figure in the synthesis of computers in modern automata theory are to be translated from Boolean algebra into Hilbert space theory. Perhaps the only element of novelty involved is the need to go beyond the lowest-order propositional calculus. We make up computers out of things like words, sequences of bits, and we must look forward to making a path out of sequences of quantum elements for example, so we need the prescriptions for making such assemblages out of individuals.

Putting it differently, we need to go deeper into the propositional calculus and deal with systems having internal structure.

This is dealt with in a higher order or predicate calculus in ordinary logic, but in quantum mechanics we encounter the same problems every time we do many-body theory. We know how, given a theory of two objects, that is two Hilbert spaces, to make a theory of the <u>pair</u>, which is one of the basic steps in the synthesis of computers. Another of the important operations is going from an object to a new one called an arbitrary set of such objects. This is sometimes called the star process in automata synthesis. The corresponding procedure in set theory is going from a set S to 2^S . In a case where an object is described by a Hilbert space $\underset{\sim}{S}$ the immediate obstacle is: what do we mean by two to the power of a Hilbert space? In fact, there is a beautiful correspondence between the laws of set theory and the laws of the exterior algebra over a Hilbert space. This is the algebra that comes in whenever we do the theory of many fermions.

One way to express what I'm saying now is: when we learn that, say, the electron obeys Fermi-Dirac statistics, we can express this knowledge by saying the fundamental object of electron theory is a set of electrons; not for example a sequence of electrons, a basically different logical construct in which order is important, nor what I call a series of electrons, in which multiple membership is admitted. For a set, an element is either in it or isn't, and can't be in it twice, and this is reflected in the Fermi-Dirac statistics of electron theory.

Another important ingredient in the synthesis of automata is the theory of relations. A relation is a property of several objects. Having understood properties as subspaces of Hilbert space, and having understood several objects as meaning multiplied Hilbert spaces, there is no difficulty in formulating a quantum theory of relations. Some concepts of the classical theories do not admit direct quantum translations because of complimentarity. In particular the notion of a partial ordering, which is fundamental to the usual theory of automata, does not translate directly and I found it convenient to replace it by the notion of a precedence relation, an anti-symmetric transitive relation. In classical logic there is no particular advantage to using one rather than the other, but the definition I've just given of a precedence relation (the anti-symmetry and the transitivity) admit immediate formal extension into the theory of Hilbert spaces. By sticking to the things that possess such immediate translation, we guarantee a kind of correspondence. We know that when we go to a classical limit we will reconstruct the concepts of classical logic: it is only a question of neglecting commutators.

So much for the bottom level[4]. The idea in each level is to reduce the concepts of the level to the smallest number of most operational concepts and translate them, in the hope that if we get their formal properties right, all else will follow. For example, instead of dealing with the whole continuum structure of wave functions, probability amplitudes, inner products, and the metric structure of Hilbert space, it was the yes-or-no structure, the implication relations $P \subset Q$, the relations defining the lattice of a Hilbert space that we singled out at the logical level. What are the corresponding things at the geometric level?

All the concepts of space and time can be expressed also in a theory of a partially ordered set. (It's odd that the same tool should work twice.) Space-time can be regarded as a <u>causal measure space</u>: that is to say, all the metric concepts can be expressed in terms of two things: the measure of a space-time volume, and the relation of causality. For example, the distance between two events can be defined in terms of the measure of the causal interval between the two events. The topology of space-time can be defined in terms of a system of neighborhoods consisting of causal intervals. In fact, I would say that relativity gains by this translation in that the theories of its entire structure, from the topological level up to the level of causality and geodesics, can all be expressed in terms of these two things in analogy with the way we develop the corresponding theories of the one-dimensional time axis; whereas in the ordinary development the topology and the metric structure are somehow divorced in that we do not use the things corresponding to spheres in general relativity to define this topology. It would give rather odd results if we said the distant stars are in topological contact with us just because we receive null rays from them.

So: Space-time is a causal measure space[5]. Where in the theory of quantum automata do we find the corresponding concepts? Right on the surface, waiting for us to grasp them.

First, the rule for translating measure is derived from the lower level. Measure is basically a logical concept in the theory of automata: you just count processes. So I will consider that our causal measure space is to have a measure derived by counting. In the quantum theory counting is done by the trace operation. Statements of location in space-time in a theory of this kind are represented in Hilbert space, just as statements of location in phase-space of classical mechanics are represented in Hilbert space in quantum mechanics. The only thing that remains is to specify a causal structure.

Where in automata do we find the causal relation but in the fact that some computations must take place before other computations, in the relation of <u>logical dependence.</u> In fact, in von Neumann's beautiful comparison of the computer and the brain, one comes very close to discussing the geometry of the brain or of the automaton in terms of just the one notion of what I will call logical precedence. The measurements that von Neumann makes on computers and brains he expresses in terms of arithmetic depth and arithmetic breadth. Arithmetic depth is the maximum number of logically dependent processes. Arithmetic breadth is the maximum number of logically independent, concurrent processes in the chain. For the computers of his day, these integer measurements of time and space were $\sim 10^8$ for time and ~ 1 for space. Today computers are still essentially one dimensional, having perhaps a logical breadth of ~ 10 . For the brain it's about 10^8 in both directions. You might say that man is a two-dimensional creature. If we wish to model space-time we will have to think even more in terms of such highly parallel or asynchronous computational models. Each event in space-time is somehow a calculation going on independently of those that occur in spacelike surfaces relative to it, and if you push the duration of the fundamental step down below 10^{-14}cm for safety, then the arithmetic depth of the universe is at least 10^{40} at present; and the arithmetic breadth of the universe at least 10^{120} at present, the cube of the former number of course, expressing the four-dimensional nature of the computational process that we must seek to model. The point is, given these two basic notions of cause and measure for automata a complete logical theory of the geometry of automata can be worked out.

Let me just mention two examples, one motivated entirely by the idea of a logical model without any consideration of relativistic invariance, and then a modification of this to make it exactly Lorentz invariant.

Let's consider the simplest kind of serial computation, which I call the binary code. Suppose we start from a single kind of binary digit representing two alternatives which you can think of as 0, a move forward in time and a step to the right or 1, a move forward in time and a step to the left in a kind of checkerboard diagram. We build a path as a sequence of such things, so let me call this basic thing out of which we will assemble space-time a link. Let us pass to the quantum theory by describing the link λ by a two-dimensional Hilbert space. The notion of a path π is then a well-defined Hilbert-space concept, a quantum sequence of such links: $\pi = \text{seq}\ \lambda$. For the end-point of the path, it suffices simply to ignore order, to say that two paths correspond to the same point in space-time, have the same end-point, if they differ only by a permutation of their links. The Hilbert space describing the point of space-time p is then gotten from the Hilbert space describing the path π in this primitive space-time by a symmetrization procedure that leads from the usual direct product to the familiar Bose-Einstein quantization. A point of space-time of this model is a Bose-Einstein ensemble or series of two-state objects, links described by a two-dimensional Hilbert space: $p = \text{ser}\ \lambda$.

I've given the lowest level of this model. The next thing is the causal structure: what does it mean to say one such ensemble p_1 comes before another p_2 in the assembly process? The simplest procedure is to say that $p_1 \subset p_2$ if the number of links of each kind in p_1 is smaller, so that you can get to p_2, intuitively speaking, by just adding more links and not subtracting. This can be expressed in terms of quantum symbolic logic quite trivially. Considering two points p_1, p_2 and a kind of link δ, for each point there is a number operator $n_\delta(1)$, $n_\delta(2)$. Then the primitive model for the causal relation is

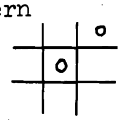
$$p_1 \subset p_2 \equiv \bigcap_\delta (n_\delta(1) < n_\delta(2))$$

If we go over to the classical limit of this quantum theory, which I do very childishly just by dropping the commutators, we obtain a classical causal measure space S, therefore a classical geometry which we can then look at as a geometrical object in itself. What is its structure? Its structure is the future null cone N^+ of special relativity with exactly the familiar Minkowsky measure and causal ordering together with what has to be counted as an internal coordinate, a single angle: $S = N^+ \times S^1$.

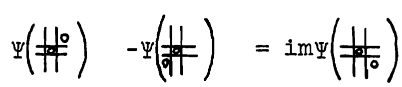
From the future null cone N^+ it is of course a triviality to assemble all of space-time by very simple formal procedures. In particular, two words in the binary code provide us with enough material to flesh out the convex closure of the null cone. The theory is not exactly covariant because the commutation relations of two harmonic oscillators, which is what we are dealing with here are not invariant under SL_2 even though the causal ordering C is invariant under SL_2. The non-covariance of the commutation relations disappears in the classical limit. That's why we end up with a Lorentz invariant space.

We then are faced with a decision between two models; one covariant only in the classical limit, which suggests that if we look at fine enough regions in space-time it is not inconsistent to imagine that departures from special relativistic covariance show up; the other, exactly covariant at all distances, obtained by replacing the commutation relations for two harmonic oscillators by the commutation relations of the Majorana representation. If you like you can say that what we have here is a new interpretation for the Majorana representation and algebra. The idempotents in it can be regarded as statements of location in a space which could be regarded as a quantized null cone in which the covariant relation $p_1 C p_2$ within the Majorana algebra plays the role of the causal order. So I call this exactly invariant theory the Majorana space-time. Two of these models suffice to make up a four-dimensional space-time, but when you double up on the number of external coordinates, you also multiply the number of internal coordinates. It turns out that the full structure of the algebra of the automaton generating what I would call two words in the binary code is that of the solid future cone C^+ multiplied not into S^1 but into $U(2,C)=S^1 \times SU_2$. These are internal coordinates in the simple sense that changing the values of these coordinates does not produce any causal separation.

Let's ascend to the dynamical level, where the action is. What is the form in which the laws of nature should be expressed in such a space-time? Guided by our success in dealing with partial orderings of the two lower levels, it is suggested that we try and express the laws of mechanics also by the theory of a partially ordered set. (It is encouraging that in the theory of thermodynamics, which really belongs to the same level as dynamics, such a formulation in terms of partial ordering is actually more unified and beautiful than that, say, in terms of an entropy principle, or the more classical ones. All of the thermodynamics has been expressed as a theory of a partially ordered set in which the objects are the states and the partial ordering is the relation of the existence of a natural process going from one state to another. The entropy appears as a valuation of this partial ordering in the same way that time appears as a valuation of the causal ordering and measure appears as a valuation of the implicative ordering of the previous two levels)[6] Feynman has exhibited a model of quantum theory in a discrete space, which is a big step towards a quantum space. It is in fact a two-dimensional model of the Dirac equation. If you think of a two-dimensional space-time in the form of a checkerboard, if you suppose that only the black squares can be occupied as in the game of checkers and that a man can step forward to the right or step forward to the left, then again a path is obviously a binary sequence, and the elementary link is a binary digit. Feynman pointed out that the Dirac equation on this simple model could be derived from the law that the transition amplitude for a path is $(im)^R$ where R is the number of reflections of motion along the path[7]. The difference form of this law is the statement that the amplitude Ψ for the pattern



is a superposition of the amplitudes for two preceding patterns with a certain coherent phase:



This is in fact the Dirac equation in this space, although it is not entirely familiar looking. In the case where we go over to wave-functions which change slightly over a single square, so that their changes can be accurately represented in terms of derivatives, the first two amplitudes give $\alpha^{\mu}\partial_{\mu}$ and the third is just the mass term in the Dirac equation. The mass is here given an immediate numerical interpretation as the probability amplitude of jitter (zitter) per chronon, the probability amplitude for reflection of motion in one unit of time on this lattice. The argument in the amplitude Ψ is not merely a point in space, but a point and a direction, which is sufficient to define a path in the classical motion.

The space of paths of this model is the configuration space of the linear Ising model and the quantity R in Feynman's transition amplitude is the pair Hamiltonian for the linear Ising model. The generalization from two discrete dimensions to a covariant model is simply to replace the Ising model by the linear Heisenberg model, the Ising Hamiltonian operator by the Heisenberg pair Hamiltonian.

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TABLE I. CONCEPTS OF QUANTUM LOGIC

Concept	Notation	Representation
I. Propositional System		
System, Object	a,b,..., A,B...	
Proposition	P,Q, ...	subspace
Implies	$\subset$	inclusion
Identity proposition	I, I_a	the Hilbert space
Null proposition	ϕ, ϕ_a,	the zero vector
Or, adjunction	$\cup$	span
And, conjunction	$\cap$	intersection
Not	$\sim$	orthocomplement
Disjoint	$\perp$	orthogonal
Measure	$\lvert \underline{P} \rvert$	dimension
Quantity, coordinate, variable	p,q,...	operator
Function of a quantity	$f(p)$, p^2,...	cf. functional calculus
Propositional function of a quantity	$P(p), p>0, \ldots$	"predicate"
Point	σ	singlet, pure state
II. Calculus of Propositional Systems		
Sum, disjunction	$a + b$, Σa_n	direct sum
Product	$a\ b$, Πa_m	direct product
Theory of Binary Relations		
Relation	R, aRb	subspace of $I_a x I_b$
Transpose	$R^T, aR^Tb \equiv bRa$	exchange subspace
Antisymmetry	$R \perp R^T$	
Transitive	$aRb \cap bRc \subset aRc$	
Function, Mapping	$F: (P \subset Q) \subset (F(P) \subset F(Q)), \lvert F(P) \rvert \leq \lvert P \rvert$	linear transformation
Assemblies		
Set of a's	set a, 2^a	(F.D) exterior algebra on I_a
Series of a's	ser a	(B.E.) symmetric tensors on I_a
Sequence of a's	seq a	(M.B.) tensors on I_a

Concept	Notation	Representation
Quantifiers		
Numerical	NP, $Na\ Pa$	$\Sigma(P\ \Psi)\ \Psi^*$
Universal	$\bigcap P$, $\bigcap a\ Pa$, $\forall a\ Pa$	$N \sim P=0$
Existential	$\bigcup P$, $\bigcup a\ Pa$, $\exists a\ Pa$	$NP \neq 0$

TABLE II. CONCEPTS OF GEOMETRY OF LOGICAL NETS

Concept	Notation	Representation
Point (Event)	p	Computational step
Measure (of point set)	$\|P\|$	Cardinality
Causal precedence	pCp'	Logical precedence
Time-like path	π	Maximal well-ordered set
Space-like surface	Σ	Maximal non-ordered set
Metric	$\rho(p,p')$	$\|\{p'' \mid pCp'Cp''\}\|$

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DISCUSSION

JAUCH The exposition of Professor Finkelstein offers such dazzling possibilities that if I make a simple question or two it will not do justice to all the richness of what he has presented us; but I would like to make one statement and one question.

First, the motivation of all this was given in terms of Riemann's question about the replacement of the continuum by discreteness. However, when you start out you throw continuity out the front door and it comes in the back door again, namely through Hilbert space. Hilbert space, of course, is constructed with coefficients from the continuum, and so continuity comes in; at the lower level to be sure, but still it is there, somehow you will not be able to get rid of it, and you have to live with it.

The second is a question, simply a technical question. I did not quite understand: did you get the three-plus-one dimensionality of space, or is that an input that you have to put in?

FINKELSTEIN It was put in by hand when I said the binary code. If you take the singulary code, you get a one-dimensional space-time consisting of nothing but a time axis, plus a single internal degree of freedom having nothing to do with causality, a kind of Newtonian world. If you wanted, say, a nine-dimensional world you'd only have to use a ternary code with three basic characters.

As for the continuity, remember it is not discreteness which is the goal, but finiteness. I don't ever want to have to do an integral again as long as I live. I want to do nothing but finite sums, and if we work with finite dimensional Hilbert spaces, we find nothing but finite sums to be computed, even though they possess the full continuous symmetry group of, in this case, the Lorentz group. Incidentally, I should mention that in this kind of a model, and in fact all the ones I've exhibited, time is the number of chronons. This is also the dimensionality of the Hilbert space in which one need work up to a given time. At the present epoch there is no evidence then that one needs a Hilbert space of dimension more than 10^{42} to describe all statements about location in space-time; and if you want, say, to discuss sets of points in space-time as in electron theory, you're still down to a dimension of only $2^{(10^{42})}$, which is considerably less than infinity.

COLEMAN It is not clear in the scheme precisely what one means when talking about the present epoch. Clearly to speak of this being a certain time rather than some other time is rather like a statement saying that the value of the electric field at this point is a certain value rather than another value, since you have, so to speak, not only quantized the field but quantized the argument of the field. Now we know when discussing the electric field in conventional quantum mechanical theories even though the current classical value of the electric field is some certain value, we have to include the possibility of the value of the electric field being arbitrarily large. So, I wonder if this happens in your scheme, and if so, if you might not be led back to an infinite dimensional Hilbert space after all.

FINKELSTEIN And the answer is, I don't fully know. I've been worried about the fact that the most primitive sorts of models that I've made up all possess a cosmological origin. That's why I could speak of the time being finite, and so forth. It's possible with some sweat to make up models which lack this and then, not surprisingly, they operate in infinite-dimensional Hilbert spaces, so again one is confronted with the danger of having to do at least an infinite sum. Right now I am more interested in finiteness than even in preserving time-translation invariance. There's much more evidence for one than the other.

COLEMAN So your models have possessed Lorentz but not Poincaré covariance. Is that correct?

FINKELSTEIN Right. But in the continuum limit, in one model, you do have the solid future light cone, and as long as you translate within it, you have exact Poincaré invariance - translation as well as Lorentz - but the continuum limit breaks down.

COLEMAN No, but truly the Poincaré things are not unitarily implementable.

FINKELSTEIN Exactly. Back in the quantum theory there must be some remnants of this invariance and I have not fully explored it yet. There is always available something like the future time-translation. This is the semi-group, rather than group, of future-time-like translations by discrete amounts, multiples of course of the fundamental constant τ. The spectra that one gets for the coordinates in the quantum theory before going to the classical limits varies slightly from model to model, but typically, in spite of the exact Lorentz covariance, one might have that t is an integer multiple of the fundamental constant, whereas x,y and z possess purely continuous spectra.

WIGNER I am a good deal confused by a number of things that you said, particularly about the invariance of the theory. You don't postulate Poincaré invariance, but you do postulate Lorentz invariance and time-displacement invariance, or did I mishear that?

FINKELSTEIN In fact, I've described several theories so that it's understandable that the hypotheses could get garbled. In my initial work I postulated no invariance at all. I simply looked for quantum models of binary computation procedures, and was rather shocked to discover that the simplest non-trivial model possessed Lorentz invariance in the classical limit. Then I noticed that one could restore full Lorentz invariance in the quantum theory by a slight change in the commutation relations and exhibit a whole class of other models. These models still lack time-transitional invariance as unitary transformations, as one might expect for a theory which contains only the future light cone. A time-translation does exist, which is not unitary but an isometric linear transformation in the quantum theory.

WIGNER I see, but you don't have time-translation invariance.

FINKELSTEIN Time-translation is not represented by a unitary transformation. Time-translation by discrete quantities is represented by an operation something like the excitation operator for a harmonic oscillator; it doesn't have an inverse.

WIGNER The question which is not terribly clear is simply this: if you have time-translation invariance and Lorentz invariance, by the combination of the two you have also invariance with respect to every other translation. Now, if one assumes invariance in this way, unless one restricts the group terribly strongly, one obtains a dense manifold of Poincaré transformations. And this is almost the same thing as true Poincaré invariance. You see, if I may talk about distant past, the Poincaré group's representations were investigated. The assumptions were not that there is space-time, only that there is invariance, and that led somewhat disappointingly to space-time. Now naturally, one wants to restrict somewhat the group, but there is no restriction of the Poincaré group which is not everywhere dense and which contains ...

<u>FINKELSTEIN</u> Right. And of course this question of symmetry is crucial to any model of this kind. It is the first thing one has to rub one's nose in. All that I can say is that, no, we do not have the group of time translational invariance. We have only a semi-group, in the models I've exhibited here. We don't have inverses. We can go into the future, but not into the past. The representation of the step into the future is also not by a unitary operator. So these are not even unitary representations of semi-groups that we have here, but isometries, which preserve the length but have no inverse. You can't go home again.

<u>COLEMAN</u> I can give an example that may clarify Professor Wigner's problem although it does not have the full complexity of structure of Dr. Finkelstein's model. If you consider the Hilbert space of all square-integrable functions whose support is the interior of the forward light cone, that is a legitimate Hilbert space. On that Hilbert space, in the natural way, Lorentz transormations act as unitary transformations. They transform the points and induce a change in the functions that doesn't change the measure. However if you consider translations in the forward direction, these don't change the norm of functions, but they map the full space into a subspace, and therefore do not have an everywhere defined inverse. So therefore you have the Lorentz Group represented in the usual way, but a subset of the Poincaré group, to wit translations with vectors that lie in the forward light cone, is represented not by unitary transformations but by isometries from the whole space into a smaller space. And that's sort

of group-theoretical structure that comes up here.

<u>FINKLESTEIN</u> Incidentally, it's typical of automata that for them the passage of time is a semi-group rather than a group of transformations.

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