Continuous quantum measurement of a double dot

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We consider the continuous measurement of a double quantum dot by a weakly coupled detector (tunnel point contact nearby). While the conventional approach describes the gradual system decoherence due to the measurement, we study the situation when the detector output is explicitly recorded that leads to the opposite effect: gradual purification of the double-dot density matrix. The nonlinear Langevin equation is derived for the random evolution of the density matrix which is reflected and caused by the stochastic detector output. Gradual collapse, gradual purification, and the quantum Zeno effect are naturally described by the equation. We also discuss the possible experiments to confirm the theory. [S0163-1829(99)01032-2]

The problem of quantum measurements has a long history; however, it still attracts considerable attention and even causes some controversy, mainly concerning the wave-function ''collapse'' (see, e.g., Refs. 1 and 2). Among various modern approaches to this problem let us mention the idea of replacing the collapse postulate by the gradual decoherence of the density matrix due to the interaction with the detector[3] and the approach of a stochastic evolution of the wave function (see, e.g., Refs. 4–11). The latter approach (which is used in the present paper) can describe the selective measurements for which the system evolution is conditioned on the particular measurement result (other keywords of the approach are quantum trajectories, quantum state diffusion, quantum jumps, etc.). The renewed interest in the measurement problem is justified by the development of experimental technique, which allows more and more experimental studies of quantum measurement in optics and mesoscopic structures.[12–17] The problem also has a close connection to the rapidly growing fields of quantum cryptography and quantum computing.[18]

In the recent experiment[15] with the ''which-path'' interferometer the suppression of Aharonov-Bohm interference due to the detection of which path an electron chooses, was observed. The weakly coupled quantum point contact was used as a detector. The interference suppression in this experiment can be quantitatively described by the decoherence due to the measurement process.[19–22]

We will consider a somewhat different setup: two quantum dots occupied by one electron and a weakly coupled detector (point contact nearby) measuring the position of the electron. The decoherence of the double-dot density matrix due to continuous measurement in this setup has been analyzed in Refs. 19 and 22. However, the decoherence approach cannot describe the detector output which is a separate problem analyzed in the present paper. We answer two interrelated questions: how the detector current behaves in time and what is the proper double-dot density matrix for a particular detector output. We show that the models of point contact considered in Refs. 19–21 describe an ideal detector. In this case the density matrix decoherence is just a consequence of averaging over all possible measurement results. For any particular detector output our equations allow the evolution of pure wave function to be followed. Moreover, a mixed density matrix can be gradually purified in the course of a continuous measurement.

Similar to Ref. 19 we describe the double-dot system and the measuring point contact by the Hamiltonian $\mathcal{H}=\mathcal{H}_{DD}+\mathcal{H}_{PC}+\mathcal{H}_{int}$, where $\mathcal{H}_{DD}=(\varepsilon/2)(c_1^\dagger c_1-c_2^\dagger c_2)+H(c_1^\dagger c_2+c_2^\dagger c_1)$ is the Hamiltonian of the double-dot, $\mathcal{H}_{PC}=\Sigma_l E_l a_l^\dagger a_l+\Sigma_r E_r a_r^\dagger a_r+\Sigma_{l,r}T(a_r^\dagger a_l+a_l^\dagger a_r)$ describes the tunneling through the point contact (T and H are real), and $\mathcal{H}_{int}=\Sigma_{l,r}\Delta T c_2^\dagger c_2(a_r^\dagger a_l+a_l^\dagger a_r)$, i.e., the tunneling matrix element for the point contact is T or $T+\Delta T$, depending on which dot is occupied. So, the average current $I_1=2\pi T^2\rho_l\rho_r e^2V/\hbar$ flows through the detector when the electron is in the first dot (V is the sufficiently large voltage across the tunnel contact, ρ_l and ρ_r are the densities of states), while the current is $I_2=I_1+\Delta I=2\pi(T+\Delta T)^2\rho_l\rho_r e^2V/\hbar$ when the second dot is occupied.

We make an important assumption of weak coupling between the double-dot and the detector (a better term would be the ''weakly responding'' detector),

$$|\Delta I|\ll I_0=(I_1+I_2)/2, \tag{1}$$

so that many electrons, $N\gtrsim(I_0/\Delta I)^2\gg 1$, should pass through the point contact before one can distinguish which dot is occupied. This assumption allows the classical description of the detector, namely, to neglect the coherence between the quantum states with different number of electrons passed through the detector.[23]

The decoherence rate $\Gamma_d=(\sqrt{I_1/e}-\sqrt{I_2/e})^2/2$ of the double-dot density matrix $\boldsymbol{\sigma}(t)$ due to the measurement by tunnel point contact has been calculated in Ref. 19. In the weakly responding limit (1) it can be replaced by $\Gamma_d=(\Delta I)^2/8eI_0$ or by the expression

$$\Gamma_d=(\Delta I)^2/4S_I, \tag{2}$$

where $S_I=2eI_0$ is the usual Schottky formula for the detector shot noise spectral density S_I. Equation (2) has also been obtained in Refs. 20–22 for the quantum point contact as a detector, the difference in that case is $S_I=2eI_0(1-\mathcal{T})$ where $\mathcal{T}$ is the transparency of the channel[24] (while above we implicitly assumed $\mathcal{T}\ll 1$).[25,26] Notice that the decoherence

rate (2) was derived in Refs. 19–22 without any account of the information provided by the detector, implicitly assuming that the measurement result is just ignored. Now let us study how this additional information affects the double-dot density matrix.

We start with the completely classical case in which there is no tunneling between dots ($H=0$) and the initial density matrix of the system does not have nondiagonal elements, $\sigma_{12}(0)=\sigma_{12}(t)=0$. We can assume that the electron is actually located in one of the dots, but it is not known in which one, and that is why we use probabilities $\sigma_{11}(0)$ and $\sigma_{22}(0)=1-\sigma_{11}(0)$. The detector output is the fluctuating current $I(t)$. The fluctuations grow when $I(t)$ is examined at smaller time scales, so some averaging in time (''low-pass filtering'') is necessary, at least in order to neglect the problem of individual electrons passing through the point contact. Let us always work at sufficiently low frequencies, $f\sim\tau^{-1}\ll S_I/e^2$, for which the possible frequency dependence of S_I can be neglected.

The probability P to have a particular value for the current averaged over time τ, $\langle I\rangle=\tau^{-1}\int_0^\tau I(t)dt$, is given by the distribution

$$P(\langle I\rangle,\tau)=\sigma_{11}(0)P_1(\langle I\rangle,\tau)+\sigma_{22}(0)P_2(\langle I\rangle,\tau), \quad (3)$$

$$P_i(\langle I\rangle,\tau)=(2\pi D)^{-1/2}\exp[-(\langle I\rangle-I_i)^2/2D], \quad (4)$$

where $D=S_I/2\tau$. Notice that these equations obviously do not change if we divide the time interval τ into pieces and integrate over all possible average currents for each piece (to consider only positive currents, the typical timescale τ should be sufficiently long, $S_I/\tau\ll I^2$, that is always satisfied within the assumed low frequency range).

After the measurement during time τ we acquire additional knowledge about the system and should change the probabilities σ_{ii} according to the standard Bayes formula[27] for *a posteriori* probability (taking into account particular detector result $\langle I\rangle$):

$$\sigma_{11}(\tau)=\sigma_{11}(0)\exp[-(\langle I\rangle-I_1)^2/2D]\{\sigma_{11}(0)\exp[-(\langle I\rangle-I_1)^2/2D]+\sigma_{22}(0)\exp[-(\langle I\rangle-I_2)^2/2D]\}^{-1},$$

$$\sigma_{22}(\tau)=1-\sigma_{11}(\tau). \quad (5)$$

Notice that we have considered so far the purely classical measurement and did not use any ''collapse'' postulate. Nevertheless, Eq. (5) can be interpreted as a gradual ''localization'' of the electron in one of the dots due to acquired information.

Now let us assume that the initial state is fully coherent, $|\sigma_{12}(0)|=\sqrt{\sigma_{11}(0)\sigma_{22}(0)}$, while still $H=0$. Since the detector is sensitive only to the position of an electron, the detector current will behave *exactly* the same way as in the case above. So, after the measurement during time τ we should assign the same values for $\sigma_{ii}(\tau)$ as in Eq. (5), but the question is not so trivial for the nondiagonal matrix element $\sigma_{12}(\tau)$. Nevertheless, we can write the upper bound

$$|\sigma_{12}(\tau)|\leqslant\sqrt{\sigma_{11}(\tau)\sigma_{22}(\tau)}. \quad (6)$$

If the actual measurement result is disregarded, then the upper bound for $|\sigma_{12}|$ can be calculated using the probability distribution of different outcomes given by Eq. (3) and the upper bound (6) for each realization,

$$|\langle\sigma_{12}(\tau)\rangle_r|\leqslant\int\sqrt{\sigma_{11}(\tau)\sigma_{22}(\tau)}P(\langle I\rangle,\tau)d\langle I\rangle=\sqrt{\sigma_{11}(0)\sigma_{22}(0)}\exp[-(\Delta I)^2\tau/4S_I] \quad (7)$$

(here $\langle\,\rangle_r$ means averaging over realizations). This upper bound exactly coincides[28] with the result given by the decoherence approach (2). This fact forces us to accept the somewhat surprising statement that Eq. (6) gives not only the upper bound, but the true value of the nondiagonal matrix element, i.e., the pure state *remains pure* (no decoherence occurs) during each particular measurement. (Actually, this is the usual statement for selective measurements,[5–11] i.e., when the detector output is taken into account.)

Simultaneously, we proved that the point contact detector considered theoretically in Refs. 19–21 (the model is confirmed experimentally[15]) causes the slowest possible decoherence of the measured system, and hence represents an ideal detector in this sense. In contrast, the result of Ref. 29 shows that a single-electron transistor[30] biased by relatively large voltage is not an ideal detector (the nonideal detector has also been considered in Ref. 22). Notice, however, that in the range of elastic cotunneling[31] the operation of the single-electron transistor is almost equivalent[29] to the case considered above, and, hence, it becomes an ideal detector.

If the initial state of the double dot is not purely coherent, $|\sigma_{12}(0)|<\sqrt{\sigma_{11}(0)\sigma_{22}(0)}$, it can be treated as the statistical combination of purely coherent and purely incoherent states with the same $\sigma_{11}(0)$ and $\sigma_{22}(0)$, then

$$\sigma_{12}(\tau)=\sigma_{12}(0)\exp\left(\frac{i\varepsilon\tau}{\hbar}\right)\left[\frac{\sigma_{11}(\tau)\sigma_{22}(\tau)}{\sigma_{11}(0)\sigma_{22}(0)}\right]^{1/2}. \quad (8)$$

Equations (5) and (8) are the central result of the present paper; they give the density matrix of the measured system (in the case $H=0$) with account of the measurement result.[32]

These equations can be also used to simulate the detector output $I(t)$ and the corresponding evolution of the density matrix. For example, in the Monte Carlo method we should first choose the timestep τ, satisfying inequalities $e^2/S_I\ll\tau\ll S_I/(\Delta I)^2$, and draw a random number for $\langle I\rangle$ according to the distribution (3). Then we update $\sigma_{11}(t)$ and $\sigma_{22}(t)$ using this value of $\langle I\rangle$ and repeat the procedure many times [the distribution for the current averaged over the interval $\Delta t=\tau$ is new every timestep because of changing $\sigma_{ii}(t)$ which are used in Eq. (3)]. The nondiagonal matrix element can be calculated at any time with Eq. (8).

Using Eqs. (3)–(5), this Monte Carlo procedure can be easily reduced to the following nonlinear Langevin-type equation (equation for σ_{11} is sufficient):

$$\dot\sigma_{11}=\mathcal{R},\quad \mathcal{R}=-\sigma_{11}\sigma_{22}\frac{2\Delta I}{S_I}[I(t)-I_0] \quad (9)$$

$$=-\sigma_{11}\sigma_{22}\frac{2\Delta I}{S_I}\left[\frac{\sigma_{22}-\sigma_{11}}{2}\Delta I+\xi(t)\right], \quad (10)$$

where the random process $\xi(t)$ has zero average and ''white'' spectral density $S_\xi=S_I$. The second expression for

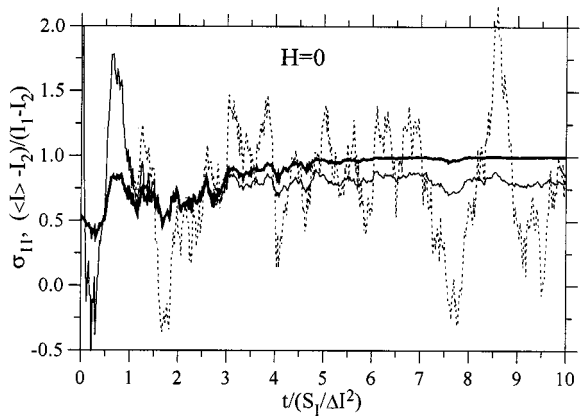

FIG. 1. Thick line: particular Monte Carlo realization of σ_{11} evolution in time during the measurement of uncoupled dots, $H=0$. The initial state is symmetric, $\sigma_{11}(0)=\sigma_{22}(0)=1/2$, while the measurement leads to gradual localization. Initially pure wave function remains pure at any time t. The thin line shows the corresponding detector current $\langle I\rangle$ averaged over the whole time interval starting from $t=0$ while the dashed line is the current averaged over the running window with duration $S_I/(\Delta I)^2$.

$\mathcal{R}$ allows the measurement to be simulated while the first one can be used to calculate the density matrix for given $I(t)$ [in case $H=0$ it can more easily be done using Eq. (5)].

Figure 1 shows a particular result of the Monte Carlo simulation for the symmetric initial state, $\sigma_{11}(0)=\sigma_{22}(0)=1/2$. The thick line shows the random evolution of $\sigma_{11}(t)$. Equation (10) describes the gradual localization in one of the dots (first dot in case of Fig. 1). Let us define the typical localization time as $\tau_{loc}=2S_I/(\Delta I)^2$ (we choose the exponential factor at $\sigma_{11}=\sigma_{22}=1/2$). Then it is exactly equal to the time $\tau_{dis}=2S_I/(\Delta I)^2$ necessary to distinguish between two states (defined as the shift of two Gaussians (4) from I_0 by one standard deviation), and $\tau_{loc}=\tau_d/2$ where $\tau_d=\Gamma_d^{-1}$. It is easy to prove that the probability of final localization in the first dot is equal to $\sigma_{11}(0)$, because $\sigma_{ii}(\tau)$ averaged over realizations is conserved [the deterministic flow of σ_{11} due to the first term in square brackets of Eq. (10) is exactly canceled on average by the dependence of the diffusion coefficient on σ_{11}].

The detector current $I(t)$ basically follows the evolution of $\sigma_{ii}(t)$ but also contains the noise which depends on the bandwidth. The dashed line in Fig. 1 shows the current $\langle I(t,t-\Delta t)\rangle=\Delta t^{-1}\int_{t-\Delta t}^{t}I(t)dt$ averaged over the ''running window'' with duration $\Delta t=S_I/(\Delta I)^2$, while the thin solid line is the current $\langle I(t,0)\rangle$ averaged starting from $t=0$.

Now let us consider the general case of the double-dot with nonzero tunneling H. If the frequency Ω of ''internal'' oscillations is sufficiently low, $\Omega=(4H^2+\varepsilon^2)^{1/2}/\hbar\ll S_I/e^2$, we can use the same formalism just adding the evolution due to finite H (the product $\Omega\tau_{loc}$ is *arbitrary*). A particular realization can be either simulated by a Monte Carlo procedure similar to that outlined above [now update of $\sigma_{12}(t)$ using Eq. (8) should be necessarily done at each timestep, together with the evolution due to finite H] or equivalently described by the coupled Langevin equations

$$\dot{\sigma}_{11}=-\dot{\sigma}_{22}=(-2H/\hbar)\mathrm{Im}(\sigma_{12})+\mathcal{R}, \tag{11}$$

$$\dot{\sigma}_{12}=\frac{i\varepsilon}{\hbar}\sigma_{12}+\frac{iH}{\hbar}(\sigma_{11}-\sigma_{22})+\frac{\sigma_{22}-\sigma_{11}}{2\sigma_{11}\sigma_{22}}\mathcal{R}\sigma_{12}-\gamma_d\sigma_{12}, \tag{12}$$

where $\gamma_d=0$ for an ideal detector (see below). The alternative ''microscopic'' derivation of these equations can be done for the particular model of Ref. 19 and will be presented elsewhere.

Notice that in Eqs. (9)–(12) the derivative is defined as $\dot{\sigma}(t)=\lim_{\tau\to 0}[\sigma(t+\tau/2)-\sigma(t-\tau/2)]/\tau$ (Stratonovich formulation of the stochastic equations[33]). The equations would be different if the definition $[\sigma(t+\tau)-\sigma(t)]/\tau$ was used (Itô formulation). We use the former one because it gives the correct limit when the noise term $\xi(t)$ is replaced by a sequence of smooth functions[33] and also because the equations in Stratonovich formalism are physically more transparent since they do not contain extra terms arising due to $\mathcal{R}^2dt=\mathrm{const}$ [for example, the usual calculus rule $(fg)'=f'g+fg'$ is still valid]. To translate Eqs. (9)–(12) into Itô formalism, one would need to add the terms[33] $(S_I/2)F(dF/d\sigma)/2$, where F is the factor before $\xi(t)$. This would lead to extra terms $-(\sigma_{22}-\sigma_{11})\Delta I/2$ in square brackets of Eqs. (9)–(10) and extra term $-\sigma_{12}(\Delta I)^2/4S_I$ in Eq. (12). Notice that in Itô formalism the equations become linear [except for the terms proportional to $\xi(t)$].

The simplest way to avoid the possible confusion between two formulations of stochastic equations is to use the explicit calculation procedures (for finite τ) described above. However, the difference should be taken into account when results of other approaches to the stochastic wavefunction evolution[4–11] are compared. For example, this explains the apparent difference between Eqs. (9)–(12) and the results of Ref. 9 for a two-level system (with $\varepsilon=0$ and $\gamma_d=0$) derived in a different way. Among various approaches to selective quantum measurements, our approach is most closely related to the method of restricted path integral;[7] however, in some sense we consider the classical (not quantum) path integral. Let us also mention that the quantum nondemolition measurements[2] are outside the scope of our study, we consider only the measurements at the so-called ''standard quantum limit.''

Figure 2 shows the particular results of the Monte Carlo simulations for the double-dot with $\varepsilon=H$ and the different strength of the interaction with an ideal detector. The electron is initially located in the first dot, $\sigma_{11}(0)=1$. The dashed line shows the evolution of σ_{11} without detector. Notice that because $\varepsilon\neq 0$, the initial asymmetry of the electron location remains in this case for infinite time. When the interaction with detector, $\mathcal{C}=\hbar(\Delta I)^2/S_IH$, is relatively small (top solid line), the evolution of σ_{11} is close to that without the detector. However, the electron gradually ''forgets'' the initial asymmetry and the evolution can be described as the slow variation of the two-parametric phase of oscillations (recall that the wave function remains pure). In the decoherence approach (averaging over realizations) this corresponds to $\sigma_{11}\to 1/2$ at $t\to\infty$.[19]

When the coupling with the detector increases, the evolution significantly changes (middle and bottom curves in Fig. 2). First, the transition between dots slows down (quantum Zeno effect[34,5,6,8,9,12,19,22,29]). Second, while the frequency of transitions decreases with increasing interaction with the de-

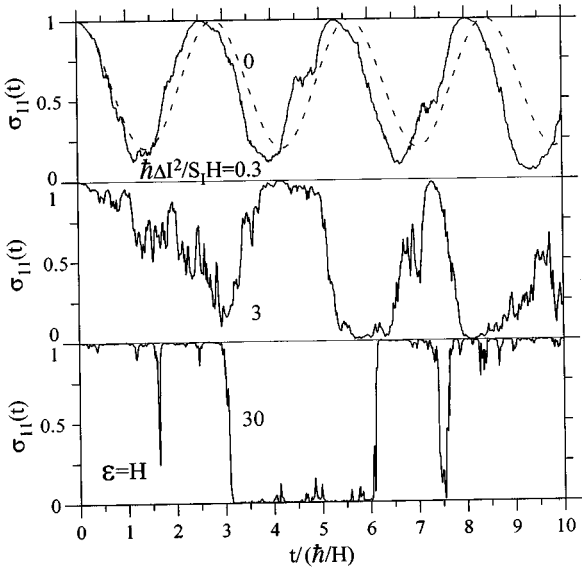

FIG. 2. Random evolution of σ_{11} (particular Monte Carlo realizations) for asymmetric double dot, $\varepsilon = H$, with the electron initially in the first dot, $\sigma_{11}(0) = 1$, for different strength of coupling with detector: $\mathcal{C} = \hbar(\Delta I)^2/S_I H = 0.3$, 3, and 30 from top to bottom. The dashed line represents $\mathcal{C} = 0$ (unmeasured double dot). Increasing coupling with detector destroys the quantum oscillations (while the wave function remains pure at any t), slows down the transitions between states (quantum Zeno effect), and for $\mathcal{C} \gg 1$ leads to uncorrelated jumps between well localized states.

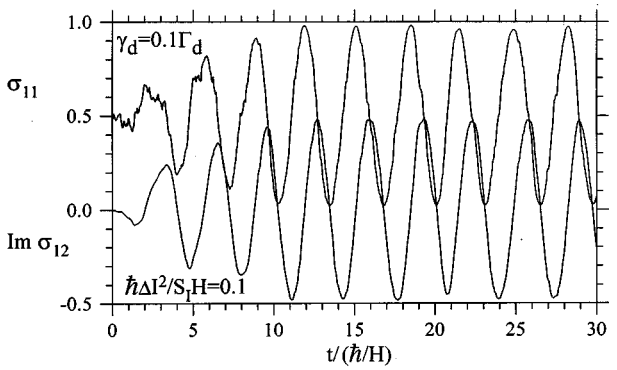

FIG. 3. Gradual purification of the density matrix [$\sigma_{11}(t)$ and $\mathrm{Im}\sigma_{12}(t)$ are shown] of the symmetric double dot ($\varepsilon = 0$) measured by a slightly nonideal ($\gamma_d/\Gamma_d = 0.1$) weakly coupled ($\mathcal{C} = 0.1$) detector.

tector, the time of a transition also decreases, so eventually we can talk about uncorrelated ''quantum jumps'' between states.

In a regime of small coupling with a detector, $\mathcal{C} \ll 1$, the detector output is too noisy to follow the evolution of σ_{ii} and, correspondingly, only slightly affects the oscillations (the presence of quantum oscillations in the double-dot can be noticed only as a relatively small peak in the spectral density of the detector current). In contrast, when $\mathcal{C} \gg 1$ the detector accurately indicates the position of electron and simultaneously destroys the oscillations.

Equations (11) and (12) with the term $\mathcal{R}$ given by Eq. (9) can be used to obtain the evolution of the density matrix in an experiment provided the detector output $I(t)$ and initial condition $\sigma_{ij}(0)$ are known. Notice that even if the initial state is completely random, $\sigma_{11} = \sigma_{22} = 1/2$, $\sigma_{12} = 0$, the nondiagonal matrix element gradually appears during the measurement, so that sufficiently long observation with an ideal detector leads to almost pure wave function for the double dot. Such a purification of the density matrix described by Eqs. (11) and (12) is analogous to the localization at $H = 0$.

Equations (11) and (12) can be generalized for a nonideal detector, $\Gamma_d > (\Delta I)^2/4S_I$ (as in Refs. 22 and 29), which gives less information than possible in principle. Let us model it as two ideal detectors ''in parallel'' with unaccessible output of the second detector. Then the information loss can be represented by the extra decoherence term $-\gamma_d \sigma_{12}$ in Eq. (12) where $\gamma_d = \Gamma_d - (\Delta I)^2/4S_I$.[35] The limiting case of a nonideal detector is the detector with no output (just an environment, $\Delta I = 0$) or with disregarded output. Then Eqs. (11) and (12) reduce to the standard decoherence approach.

For a nonideal detector it is meaningful to keep our old definition of the localization time, $\tau_{loc} = \tau_{dis} = 2S_I/(\Delta I)^2$, while $\tau_d < 2\tau_{loc}$. So, we consider localization time not as a real physical quantity but as a quantity related to the observer's information. Similarly, the effective decoherence time is defined as $\tau_d' = \gamma_d^{-1}$.

Figure 3 shows a particular realization of random evolution of σ_{11} and $\mathrm{Im}\sigma_{12}$ for a symmetric double dot measured by weakly coupled ($\mathcal{C} = 0.1$) nonideal detector with $\gamma_d/\Gamma_d = 0.1$. We start from a maximally mixed state, $\sigma_{11}(0) = 0.5$, $\sigma_{12}(0) = 0$, and the figure shows the gradual purification of the density matrix in a course of measurement (notice that $\mathrm{Re}\sigma_{12}(t) = 0$ because $\varepsilon = 0$). The nonideality of the detector does not allow the complete purification: oscillations of $\mathrm{Im}\sigma_{12}(t)$ do not reach ± 0.5 limit, as it would be in the case of ideal detector.

Let us mention that following the ''orthodox'' (Copenhagen) point of view, we do not attempt to distinguish between the ''real'' density matrix and the density matrix which can be known by the observer. For example, the evolution of σ_{11} due to the measurement in case of no tunneling between dots ($H = 0$) can be interpreted both as a real process or just as a gradual acquiring of information about the electron position. Another example is the case of the nonideal detector. We can interpret the term $-\gamma_d \sigma_{12}$ in Eq. (12) as real decoherence; however, it is also possible to argue that it just represents the partial loss of information inside the imperfect detector, so that perhaps the pure density matrix could be restored if some hidden traces left in the detector had been analyzed. Developing this example further, let us imagine that two observers have different levels of access to the detector information, then the density matrix for them will be different. Actually, this just means that the observer with less information will not be able to make as many (or as accurate) predictions as the other one. Nevertheless, he still can treat his density matrix as a real one for all purposes. The limiting case when the observer does not have any information about the detector output (or this information is ignored in the experiment) is equivalent to averaging over all possible realizations, i.e., to the standard decoherence approach.

So, if different realizations of the detector output are effectively averaged in an experiment (as in Ref. 15), the decoherence approach is suitable. In contrast, if the single re-

alization of the detector current is recorded (and somehow used) in an experiment, then the proper description is given by Eqs. (11) and (12). The simplest experimental idea is just to measure $I(t)$ when $\mathcal{C}$ is not too small and check if it is consistent with these equations. However, it would be much more interesting to devise an experiment in which the subsequent system evolution depends on the preceding measurement result.

For example, let us first prepare the double dot in the symmetric coherent state, $\sigma_{11}=\sigma_{22}=|\sigma_{12}|=1/2$, make $H=0$ (raise the barrier), and begin measurement with an almost ideal detector. According to our formalism, after some time τ (the most interesting case is $\tau\sim\tau_{loc}$) the wave function remains pure but becomes asymmetric and can be calculated with Eqs. (5) and (8). To prove this, an experimentalist can use the knowledge of the wave function to move the electron into the first dot with probability equal to unity. Namely, he switches off the detector at $t=\tau$, reduces the barrier (to create finite H), and creates the energy difference $\varepsilon=[(1-4|\sigma_{12}|^2)^{1/2}-1]H\mathrm{Re}\,\sigma_{12}/|\sigma_{12}|^2$; then after the time period $\Delta t=[\pi-\arcsin(\mathrm{Im}\sigma_{12}\hbar\Omega/H)]/\Omega$ the ''whole'' electron will be moved to the first dot, which can be checked by the detector switched on again. [Alternatively, using the knowledge of $\sigma_{ij}(\tau)$ an experimentalist can produce exactly the ground state of the double-dot system and check it, for example, by photon absorption.]

Another experimental idea is to demonstrate the gradual purification of the double-dot density matrix. Let us start with a completely random state ($\sigma_{11}=\sigma_{22}=1/2$, $\sigma_{12}=0$) of the double dot with finite H. Then using the detector output $I(t)$ and Eqs. (11) and (12) it is possible to calculate the evolution of the density matrix. These calculations will show the gradual purification (the most interesting case is $\Omega\tau_{loc}\lesssim 1$), eventually ending up with almost pure wave function with precisely known *phase* of quantum oscillations. The final check of the wave function can be similar to that considered above. However, it can be even simpler, because with the knowledge of the phase of oscillations it is easy to stop the evolution by raising the barrier when the electron is with certainty in the first dot. If rapid calculations (by some analog on-chip circuit) are not available, the barrier control can be random, while appropriate cases can be selected later.

An experiment of this kind could verify the formalism developed in the present paper. While such an experiment is still a challenge for present-day technology, we hope that it can be realized in the near future.

In conclusion, we have developed a simple formalism for the evolution of the double-dot density matrix with an account of the result of the continuous measurement by weakly coupled (weakly responding) point contact. In contrast to most previous studies on the selective quantum measurements, our equations treat mixed states and allow the consideration of a nonideal detector. The equations show the gradual purification of initially mixed state of the double-dot due to continuous quantum measurement. This effect can be studied experimentally in various mesoscopic setups.

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[26] Modification of Eq. (2) in the case when the phase of transmitted and reflected electrons in the detector is sensitive to the double-dot state has been discussed in Ref. 22. We assume that there is no such a dependence in our case.
[27] *A posteriori* probability $\tilde{p}(A)$ of an event A given that the event F has happened, is equal to $\tilde{p}(A)=p(A)p(F|A)/\Sigma_B[p(B)p(F|B)]$, where $p(A)$ is *a priori* probability and $p(F|A)$ is the conditional probability of event F given event A.

[28] If a similar approach is formally applied to the case $\Delta I \sim I_0$, we obtain $|\langle \sigma_{12}(\tau)\rangle_r|/[\sigma_{11}(0)\sigma_{22}(0)]^{1/2} \leqslant [2\sqrt{S_{I1}S_{I2}}/(S_{I1}+S_{I2})]^{1/2}\exp[-\tau(\Delta I)^2/2(S_{I1}+S_{I2})]$. Then the corresponding result of Ref. 19 is smaller than this upper bound for large τ, while it is above the limit for $\tau \lesssim \Gamma_d^{-1} \sim e/I_0$. The latter unphysical situation is because the detector cannot be described classically at small τ.

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