A discussion on characteristics of the quantum vacuum

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Abstract: This paper will begin by considering the quantum vacuum at the cosmological scale to show that the gravitational coupling constant may be viewed as an emergent phenomenon, or rather a long wavelength consequence of the quantum vacuum. This cosmological viewpoint will be reconsidered on a microscopic scale in the presence of concentrations of "ordinary" matter to determine the impact on the energy state of the quantum vacuum. The derived relationship will be used to predict a radius of the hydrogen atom which will be compared with the Bohr radius for validation. The ramifications of this equation will be explored in the context of the predicted electron mass, the electrostatic force, and the energy density of the electric field around the hydrogen nucleus. It will finally be shown that this perturbed energy state of the quantum vacuum can be successfully modeled as a virtual electron-positron plasma, or the Dirac vacuum. © *2015 Physics Essays Publication*. [http://dx.doi.org/10.4006/0836-1398-28.4.496]

Résumé: Cet article commence en considérant le vide quantique à l'échelle cosmologique pour montrer que la constante de couplage gravitationnel peut être considérée comme un phénomène émergent, ou plutôt une conséquence de la longueur d'onde du vide quantique. Ce point de vue cosmologique sera réexaminé à une échelle microscopique en présence de concentrations de matière "ordinaire" pour déterminer l'impact sur l'état d'énergie du vide quantique. La relation dérivée servira à prévoir un rayon de l'atome d'hydrogène qui sera comparé avec le rayon de Bohr pour validation. Les ramifications de cette équation seront examinées dans le contexte de la masse de l'électron prévus, la force électrostatique et la densité d'énergie du champ électrique autour du noyau d'hydrogène. Il sera finalement montré que cet état d'énergie perturbée du vide quantique peut être modélisé avec succès comme un plasma virtuelle électron-positron, ou le vide de Dirac.

Key words: Gravity; Magnetohydrodynamics; Dark Energy Theory; Quantum Gravity Phenomenology.

I. BACKGROUND ON STANDARD MODEL OF COSMOLOGY

Prior to developing the central theme of the paper, it will be useful to present the reader with an executive summary of the characteristics and mathematical relationships central to what is now commonly referred to as the standard model of Big Bang cosmology, the Friedmann–Lemaître–Robertson–Walker (FLRW) metric. The Friedmann equations are analytic solutions of the Einstein field equations using the FLRW metric, and Eqs. (1) show some commonly used forms that include the cosmological constant,[1] Λ. In the equations, a is the scale factor, ρ is the density, p is the pressure, and k is the curvature

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}, \quad \text{and} \qquad (1)$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}.$$

The cosmological constant or vacuum energy can be subsumed into the density and pressure terms using the following lexicons: $\Lambda = 8\pi G\rho_{VAC}/c^2 = -8\pi G p_{VAC}/c^4$. Alternately, this can also be done with the following equivalent substitutions: $\rho \to \rho - \Lambda c^2/8\pi G$ and $p \to p + \Lambda c^4/8\pi G$. In this way, the density term ρ represents the total (energy) density of the universe consisting of matter, radiation, and vacuum energy (or dark energy). Based on observational data, the curvature k can be set to zero representing a flat universe. With these changes, the equations take on the simplified form (the left equation is sometimes referred to as the energy argument form, and the right equation is sometimes referred to the acceleration argument form)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}, \quad \text{and} \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right). \qquad (2)$$

The ratio $\dot{a}/a$ is defined as the Hubble parameter H which has a present measured value of $H_0 = 67.81$ km s^{-1} Mpc^{-1}.[2] The Hubble time is just the inverse of this relationship and has a value of $t_H = 14.42 \times 10^9$ yr and the actual age of the universe is $t_0 = 13.8 \times 10^9$ yr. The definition of the Hubble parameter H can be used with the energy form of Eq. (2) to yield $H^2 = 8\pi G\rho/3$ which can be rearranged to get an expression for the current critical density value as shown below

$$\rho_0 = \frac{3H_0^2}{8\pi G}. \qquad (3)$$

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The critical density value can be used to construct a normalized density parameter $\Omega = \rho/\rho_0$ that relates the density of a given component to the critical density. For example, the density parameter for matter (both baryonic matter and dark matter) today is $\Omega_M = \rho_M/\rho_0$ and the density parameter for the cosmological constant or vacuum energy today is $\Omega_\Lambda = \rho_\Lambda/\rho_0$. The published density parameter values for the spatially flat ($k = 0$) six-parameter Λ cold dark matter (CDM) cosmology model are $\Omega_M = 0.308$ and $\Omega_\Lambda = 0.692$ using the Planck temperature data combined with Planck lensing.[2,b)] The Hubble parameter value of $H_0 = 67.81$ km s^{-1} Mpc^{-1} indicates a critical density value of 8.643×10^{-27} kg m^{-3}. Using the stated density parameter values, the matter density contribution to the critical density is $\rho_M = 2.662 \times 10^{-27}$ kg m^{-3} and the vacuum energy contribution to the critical density is $\rho_\Lambda = 5.981 \times 10^{-27}$ kg m^{-3}. Although this narrative does not present any new findings or insights, it is provided to establish a framework of current understanding to support the subsequent discussion.

II. THOUGHT EXPERIMENT: GRAVITATIONAL COUPLING CONSTANT

Consider the following thought experiment: what would an inertial observer in deep space far away from any concentrations of ordinary matter find if the vacuum energy were to be integrated over the Hubble sphere which is the spherical surface area defined by the Hubble radius or Hubble length, c/H_0 or ct_H? The Hubble time is the linear expansion time of the universe, and the surface defined by the Hubble sphere for the observer is the point at which objects are receding away from that observer at the speed of light. Beyond the Hubble sphere, objects recede away from the observer at greater than the speed of light due to the expansion of the universe. The integral for this thought experiment is shown below

$$\iint_S \Omega_\Lambda \rho_0 c^2 dS \to \int_0^{2\pi}\int_0^{\pi} \Omega_\Lambda \rho_0 c^2 (ct_H)^2 \sin\theta d\theta d\varphi = 4\pi c^2 t_H^2 \Omega_\Lambda \rho_0 c^2. \tag{4}$$

The cosmological parameter values provided in Sec. I can be used to evaluate the expression, and the resultant value turns out to be very close to the Planck force

$$4\pi c^2 t_H^2 \Omega_\Lambda \rho_0 c^2 \approx \frac{c^4}{G}. \tag{5}$$

If the density parameter for vacuum energy is replaced with the value 2/3 (which is close to the measured value of $\Omega_\Lambda = 0.692$), then the equation is exact match for the Planck force

$$4\pi c^2 t_H^2 \frac{2}{3} \rho_0 c^2 = \frac{c^4}{G}. \tag{6}$$

The equation can be rearranged to solve for the gravitational coupling constant, G

$$G = \frac{1}{4\pi t_H^2 \frac{2}{3} \rho_0}. \tag{7}$$

The above equation may have a familiar look to the reader. Although this equation is the result of a little thought experiment for an inertial observer in deep space considering the local impact of the macroscopic surface integral of the vacuum energy out to the Hubble sphere, this equation can also be rearranged into the form for the critical density shown in Eq. (3) just discussed in Sec. I: $\rho_0 = (3H_0^2)/(8\pi G)$.

What is to be made of Eq. (7)? It suggests that the value for the gravitational coupling constant could be viewed as a long wavelength consequence (ct_H) of vacuum energy. Said another way, the value of the physical coupling constant that determines the interaction strength of the gravitational force is an emergent phenomenon resulting from long wavelength dynamics of the quantum vacuum expressed out to the Hubble sphere for an inertial observer. Rather than being a fundamental force that is a result of an intrinsic characteristic of matter, gravity ends up being a secondary effect that results from gradients in the vacuum energy that are a result of the presence of baryonic matter concentrated at a particular location. Two concentrations of baryonic matter in proximity to one another will each perceive the gradient in the vacuum energy resulting from its neighbor, and these gradients result in the two concentrations of baryonic matter accelerating toward one another. This resultant acceleration is what is termed the gravitational attraction between the two bodies. Gravity being an emergent force is not a new idea, and the literature has a number of papers discussing different approaches to the topic.[4–6]

Another item that bears further discussion is what to make of the *ad hoc* 2/3 substitution between Eqs. (5) and (6)? Although it resulted in Eq. (7) yielding an exact match for the gravitational coupling constant, it could be viewed as simply splitting up the $8\pi/3$ term from the definition of the critical density shown in Eq. (3) into a 4π and 2/3 term, so there should be no surprise that Eq. (7) is exact. However, the thought experiment was developed independently and with a different motivation from Eq. (3), and has identified an alternative physical explanation for the origin of the critical density—or expressed more intuitively by Eq. (7), an origin for the gravitational coupling constant. It may also be suggesting from first principles that the vacuum energy density parameter is exactly 2/3.

III. LOCAL PERTURBATION OF QUANTUM VACUUM

In the preceding thought experiment, the gravitational coupling constant was shown to have a possible connection to the Hubble length, and an alternative physical

[b)]For comparison with 2015 PLANCK data presented in the narrative, the Wilkinson Microwave Anisotropy Probe (WMAP) mission final 2013 report provides the following ΛCDM cosmological parameter values: $\Omega_M = 0.287$, $\Omega_\Lambda = 0.713$, $H_0 = 69.32$ km s^{-1} Mpc^{-1}, and universe age of 13.77×10^9 yr.[3]

interpretation to the definition of the critical density was inferred. If gravity is an emergent phenomenon that has its origins tied to the characteristic length of the Hubble sphere imposed on or coupled with vacuum energy, and if it is assumed that the gravitational coupling constant holds its measured value at nearly all scales down to the atomic regime, a microscopic parallel to the macroscopic thought experiment can be formulated. In this microscopic formulation, the characteristic length $ct_H = \Delta x_H$ of the Hubble sphere is replaced by a microscopic characteristic length $\Delta x_{\upsilon\_local}$ that is tied to the geometric constraints associated with a local concentration of baryonic matter ρ_{m_local},[c)] say in the form of a lattice structure associated with a solid crystalline structure. How does the quantum vacuum respond to the presence of this ordinary matter arrayed in solid form?[d)] If one were able to shrink down to an atomic sized person and be able to make quasiclassical measurements of the density of the vacuum energy field in the presence of the lattice structure present in the "ordinary matter," can a relationship between the ordinary matter density and vacuum energy be derived? With this objective in mind, Eq. (6) can be restated in the microscopic form relating the ordinary matter density to the corresponding microscopic characteristic length imposed on the quantum vacuum

$$4\pi\Delta x^2_{\upsilon\_local}\rho_{m\_local}c^2 = \frac{c^4}{G}. \tag{8}$$

Using Eqs. (6) and (8), a ratio between the cosmological state of the vacuum energy ρ_υ and ordinary matter density ρ_{m_local} can be shown to be inversely proportional to the ratio of the characteristic lengths squared

$$\begin{aligned} \frac{\dfrac{c^2}{4\pi G\rho_{m\_local}}}{\dfrac{c^2}{4\pi G\rho_\upsilon}} &= \frac{\Delta x^2_{\upsilon\_local}}{(ct_H)^2} = \frac{\Delta x^2_{\upsilon\_local}}{\Delta x^2_H}, \\ \frac{\rho_\upsilon}{\rho_{m\_local}} &= \frac{\Delta x^2_{\upsilon\_local}}{\Delta x^2_H}. \end{aligned} \tag{9}$$

Here, $\Delta x_{\upsilon\_local}$ represents the characteristic length or positional uncertainty of a vacuum fluctuation in the presence of an ordinary matter density, and Δx_H is the characteristic length or positional uncertainty of a vacuum fluctuation resulting from the cosmological causal horizon discussed in the development of Eq. (6). The Heisenberg Uncertainty Principle can be used to determine the ramifications of this reduction in positional uncertainty on the local vacuum energy density

$$\begin{aligned} &\Delta t\Delta E \geq \frac{\hbar}{2} \overset{yields}{\longrightarrow} \Delta x = \frac{\hbar c}{2\Delta E}, \\ &\Delta x = \frac{\hbar}{2mc}. \end{aligned} \tag{10}$$

Using the generic relationship in Eq. (10) stemming from the Heisenberg uncertainty principle, a second relationship between the cosmological state of the vacuum and the local state of the vacuum in the presence of an ordinary matter density can be shown to be

$$\frac{\Delta x_{\upsilon\_local}}{\Delta x_H} = \frac{\dfrac{\hbar}{m_{\upsilon\_local}c}}{\dfrac{\hbar}{m_H c}} = \frac{m_H}{m_{\upsilon\_local}}$$

multiply by unit volume over unit volume $\hat{V}/\hat{V}$

$$\begin{aligned} \frac{\Delta x_{\upsilon\_local}}{\Delta x_H} &= \frac{\dfrac{m_H}{\hat{V}}}{\dfrac{m_{\upsilon\_local}}{\hat{V}}} = \frac{\rho_\upsilon}{\rho_{\upsilon\_local}}, \\ \frac{\Delta x_{\upsilon\_local}}{\Delta x_H} &= \frac{\rho_\upsilon}{\rho_{\upsilon\_local}}. \end{aligned} \tag{11}$$

Equation (11) can be used with Eq. (9) to yield

$$\frac{\rho_\upsilon}{\rho_{m\_local}} = \frac{\rho^2_\upsilon}{\rho^2_{\upsilon\_local}}. \tag{12}$$

Equation (12) can be rearranged into the following form:

$$\rho_{\upsilon\_local} = \sqrt{\rho_{m\_local}\rho_\upsilon}. \tag{13}$$

Equation (13) suggests that a local density of ordinary matter will cause a perturbation to the local quantum vacuum state such that it is at a different density state when compared with the unperturbed cosmological vacuum energy state. This characteristic has some similarities to the Chameleon scenario discussed in the literature[7–20] which claims the existence of a "chameleon" field ϕ whose mass is dependent on the local matter density.

IV. CALCULATING THE BOHR RADIUS

Equation (13) can be used to evaluate the state of the quantum vacuum in close proximity of the proton at the center of the hydrogen atom. The first step is to calculate a quasiclassical density $\rho_{m\_local} = (m_{proton})/(\frac{4}{3}\pi R^3_{proton})$ for the hydrogen nucleus. There are three options to use for the proton radius in this calculation. The first option is to use the charge radius of the proton which has a measured value of 0.88 fm (Ref. 21), the second is to use the Compton wavelength of the proton which is 1.32 fm, and the third option is to use the Fermi model, $R = R_0 A^{1/3}$ where $R_0 = 1.2$ fm and A is the atomic number yielding a radius of 1.2 fm for the hydrogen nucleus. Equation (13) can be used with this quasiclassical ordinary matter density of the nucleus to determine a perturbed state of the quantum vacuum ρ_{υ_local} around the hydrogen nucleus. The question can then be asked how

[c)]It should be noted that counter to particle physics, in cosmological parlance, baryonic matter is always meant to include the electrons even though they are fermions, so baryonic matter here is meant to represent "ordinary" matter from the periodic table in all its forms. In order to avoid further confusion, the concept of cosmological baryonic matter as just described will be referred to as "ordinary" matter.

[d)]This line of logic is not meant to exclude liquid or gas forms, but focuses on the solid form for brevity and clarity.

TABLE I. Predicted local vacuum density and Bohr radius.

	Proton radius								
	0.88×10^{-15} m			1.20×10^{-15} m			1.32×10^{-15} m		
Density parameter Ω	ρ_{v_local} (kg m^{-3})	r (m)	Error (%)	ρ_{v_local} (kg m^{-3})	r (m)	Error (%)	ρ_{v_local} (kg m^{-3})	r (m)	Error (%)
1.000	7.11×10^{-5}	4.33×10^{-11}	−18.2%	4.47×10^{-5}	5.06×10^{-11}	−4.4%	3.87×10^{-5}	5.30×10^{-11}	0.2%
0.692	5.92×10^{-5}	4.61×10^{-11}	−13.0%	3.72×10^{-5}	5.38×10^{-11}	1.6%	3.22×10^{-5}	5.64×10^{-11}	6.6%
0.667	5.81×10^{-5}	4.63×10^{-11}	−12.4%	3.65×10^{-5}	5.41×10^{-11}	2.2%	3.16×10^{-5}	5.67×10^{-11}	7.2%

much volume of this perturbed state of the quantum vacuum is needed to have the equivalent energy value as the ground state of hydrogen. The ground state of the hydrogen atom is −13.6 eV (2.18×10^{-18} J) which can be classically thought of as the sum of both the potential energy and kinetic energy for the electron in this orbit. Determining the radius of the bubble of perturbed quantum vacuum necessary to achieve this magnitude of energy is a simple calculation

$$r = \left(\frac{E}{\rho_{\upsilon\_local} c^2 \frac{4}{3}\pi}\right)^{\frac{1}{3}}. \tag{14}$$

Table I shows the predicted radius for several combinations of input parameters. The cosmological vacuum energy density parameter is ranged across three values: $\Omega = 0.667$, 0.692, and 1.000. The radius of the proton is ranged across the three values just discussed: 0.88, 1.2, and 1.32 fm. For each combination of input parameters, the table provides a predicted radius using Eq. (14), and provides the percent error to the accepted value of the Bohr Radius, $a_0 = 5.29 \times 10^{-11}$ m.[21] Consideration of the findings presented in the table shows that the closest prediction for the Bohr radius from Eq. (14) is for $\Omega = 1$, and using the Compton wavelength for the proton to calculate the quasi-classical ordinary matter density, ρ_{m_local} yielding a predicted Bohr radius within 0.2% of the accepted value. The next closest prediction for the Bohr radius is for $\Omega = 0.692$, and using the Rutherford radius from the Fermi model for the proton quasiclassical ordinary matter density yielding a predicted Bohr radius within 1.6% of the accepted value. This assessment has identified a unique connection between the value of the Bohr radius and cosmological critical density not previously identified in the literature. To be explicit, Eq. (14) can be expanded into the form shown in Eq. (15) which only has the cosmological density value appearing once in the expression precluding the presence of a tautology in the logic of the calculation. No simplification is performed so that the expanded terms are still *discrete* and can be more easily identified. The more familiar form for the calculation of the Bohr radius a_0 is provided for comparison. The term Z is the atomic number, n is the principal quantum number (both of which are set to 1 for the ground state of hydrogen), m_e is the mass of the electron, q is the elementary charge, ε_0 is the vacuum permittivity, and h is the Planck constant. The remaining terms have already been discussed

$$r = \left(\frac{\dfrac{Z^2 m_e q^4}{8n^2 h^2 \varepsilon_0^2}}{c^2 \dfrac{4}{3}\pi\left[\left(\dfrac{m_{proton}}{\frac{4}{3}\pi R_{proton}^3}\right)\left(\dfrac{3\Omega H_0^2}{8\pi G}\right)\right]^{\frac{1}{2}}}\right)^{\frac{1}{3}}. \tag{15}$$

The above derived form can be compared with the more familiar form of the Bohr radius

$$a_0 = \frac{h^2 \varepsilon_0}{\pi m_e q^2}. \tag{16}$$

V. ELECTRON MASS

As another check on the validity of Eq. (13), Eq. (14) will be used to derive a predicted mass for the electron. Given Eq. (14) as a starting point, it can be rearranged into the following form:

$$a_0^3 = \frac{E}{\rho_{\upsilon\_local} c^2 \frac{4}{3}\pi} \quad \text{then} \quad a_0^3 = \frac{\frac{1}{2} m_e c^2 \alpha^2}{\rho_{\upsilon\_local} c^2 \frac{4}{3}\pi},$$
$$\frac{4}{3}\pi a_0^3 \rho_{\upsilon\_local} c^2 = \frac{1}{2} m_e c^2 \alpha^2. \tag{17}$$

In this equation, α is the fine structure constant.[e] Upon further consideration, Eq. (17) has the appearance of being the volumetric integral of the perturbed quantum vacuum state over the spherical volume defined by the Bohr radius. This equation can be rearranged to solve for the electron mass, and using the predicted value for the vacuum state around the hydrogen nucleus of ρ_{υ_local} of 3.87×10^{-5} kg m^{-3} yields a predicted mass for the electron of 9.01×10^{-31} kg which is within 1% of the measured value

$$m_e = \frac{\frac{8}{3}\pi a_0^3 \rho_{\upsilon\_local} c^2}{(c/137)^2} = 9.01 \times 10^{-31}\,\text{kg}. \tag{18}$$

[e]The famous fine structure constant α characterizes the tiny perturbations to the Bohr energies resulting from relativistic corrections and spin orbit coupling. It is a dimensionless number and has the same numerical value in any system of units.

In the domain of physics, no model to date can derive the mass of the electron from first principles. Although the work of Wilczek in the area of QCD has been used to derive mass values for quarks and gluons and the resultant particles made from them (e.g., proton, neutron, etc.), it does not address the origin of the mass of the electron. In his own words, Wilczek states, "We have achieved a beautiful and profound understanding of the origin of most of the mass of ordinary matter, but not of all of it. The value of the electron mass, in particular, remains deeply mysterious."[22]

A. Electrostatic force vignette

A short aside is presented to illustrate scale symmetry between the macroscopic cosmological consideration and the microscopic treatment of the hydrogen atom. Given Eq. (17) as a starting point, it can now be rearranged into the following interim form:

$$4\pi a_0^2 \frac{2}{3}\rho_{\upsilon\_local}c^2 = \frac{m_e c^2 \alpha^2}{a_0}. \tag{19}$$

The following identities will be used to reconfigure the right hand side of the above equation:

$$a_0 \equiv \frac{\hbar}{m_e c\alpha}, \quad \alpha \equiv \frac{q^2}{4\pi\varepsilon_0 \hbar c}. \tag{20}$$

Multiply the right side of Eq. (19) by a_0/a_0 using the above identity

$$4\pi a_0^2 \frac{2}{3}\rho_{\upsilon\_local}c^2 = \frac{m_e c^2 \alpha^2}{a_0}\frac{1}{a_0}\frac{\hbar}{m_e c\alpha}, \tag{21}$$

then simplify to yield

$$4\pi a_0^2 \frac{2}{3}\rho_{\upsilon\_local}c^2 = \frac{c\hbar}{a_0^2}\alpha. \tag{22}$$

Plugging in the definition for the fine structure constant α produces the equation

$$4\pi a_0^2 \frac{2}{3}\rho_{\upsilon\_local}c^2 = \frac{c\hbar}{a_0^2}\frac{q^2}{4\pi\varepsilon_0 \hbar c}, \tag{23}$$

with some simplification yields

$$4\pi a_0^2 \frac{2}{3}\rho_{\upsilon\_local}c^2 = \frac{q^2}{4\pi\varepsilon_0 a_0^2}. \tag{24}$$

From the earlier cosmological consideration, recall that the derivation of the gravitational coupling constant included an integration of the vacuum energy over the Hubble sphere. This integration resulted in a relationship between the gravitational coupling constant and the vacuum energy. In the microscopic case around the hydrogen nucleus, Eq. (24) can be viewed as a similar integration of the perturbed quantum vacuum state over the surface area of the sphere defined by the Bohr radius that demonstrates a connection between the coupling constant of the electromagnetic field (unit of charge) and the quantum vacuum. It is also noted that the development of Eq. (24) has a factor of 2/3[f)] on the left side of the equation and it has been arranged in a similar position as was seen in the macroscopic development of the gravitational coupling constant. Evaluating both sides of the equation independently yields

$$\begin{aligned} 4\pi a_0^2 \frac{2}{3}\rho_{\upsilon\_local}c^2 &= 8.17\times 10^{-8}\,\text{N} \quad \text{and} \\ \frac{q^2}{4\pi\varepsilon_0 a_0^2} &= 8.24\times 10^{-8}\,\text{N}. \end{aligned} \tag{25}$$

B. Electrostatic field vignette

Using the following relationship for the electric field magnitude felt by the orbiting electron: $E = q/(4\pi\varepsilon_0 a_0^2)$, Eq. (24) can be rearranged into the following form:

$$\frac{1}{3}\rho_{\upsilon\_local}c^2 = \frac{\varepsilon_0}{2}\frac{q}{4\pi\varepsilon_0 a_0^2}\frac{q}{4\pi\varepsilon_0 a_0^2}. \tag{26}$$

Substituting the electric field and simplifying yields

$$\frac{1}{3}\rho_{\upsilon\_local}c^2 = \frac{\varepsilon_0 E^2}{2}. \tag{27}$$

This equation appears to be showing a relationship between the energy density of the electric field and the perturbed quantum vacuum state around the hydrogen nucleus. Evaluating both sides of the equation independently yields

$$\begin{aligned} \frac{1}{3}\rho_{\upsilon\_local}c^2 &= 1.16\times 10^{12}\ \text{Nm}^{-2} \quad \text{and} \\ \frac{\varepsilon_0 E^2}{2} &= 1.17\times 10^{12}\ \text{Nm}^{-2}. \end{aligned} \tag{28}$$

It should be noted that the significance of the 2/3 factor in the electrostatic force vignette and the 1/3 factor in the above calculation is not immediately clear. Both of these factors are integral parts of the equation that are simply a result of a convenient arrangement of terms that allowed the rearranging of Eq. (17) into a form allowing direct comparison with the value of the electrostatic force shown in Eq. (24), and into a direct comparison of the value of the energy density of the electric field shown in Eq. (27). In both cases, all remaining terms were collected on the side with the perturbed quantum vacuum state term. It could be speculated that a physical interpretation of these factors is that they indicate that part of the energy density of the perturbed quantum vacuum state around the hydrogen nucleus contributes to the electrostatic force, while the remaining portion contributes to the energy density of the electric field. The exact causative agent or physical rationale for the delineation of 2/3 and 1/3 factors, *if one exists*, is a matter for future consideration.

[f)]The factor of 2/3 was not arbitrarily added to the formulation of Eq. (24), rather it is part of the equation that has been isolated to a particular location to show the symmetry between this microscopic formulation around the hydrogen atom with the macroscopic cosmological formulation discussed earlier in the manuscript.

VI. DIRAC VACUUM

The predicted value of the local vacuum energy state from Eq. (13) suggests a negative pressure state around the hydrogen atom that is very different from the unperturbed cosmological vacuum energy state. A question to be posed is could this perturbed quantum vacuum state around the hydrogen atom be quasiclassically modeled as a Dirac vacuum? Said another way, can it be successfully modeled as a virtual electron-positron plasma using the tools of Magnetohydrodynamics (MHD)? Before this consideration of the hydrogen nucleus, it should be noted that in plasma physics, a boundary condition forms when there is a balance between magnetic pressure, $P_B = B^2/2\mu_0$, and plasma pressure, $P_P = n_e kT$. All the terms hold their normal physical meaning, except n_e is the number density of the electron-positron plasma, or rather the density, ρ, value divided by the mass of the electron, m_e. In nature when this scenario occurs, the plasma is held at bay by the adjoining magnetic field where the plasma pressure and magnetic pressure are in equilibrium. An observer would find a plasma on the one side with the temperature T and number density, n_e, while the other side would simply consist of the magnetic field B, and no plasma. The magnetic pressure around the hydrogen nucleus can be found by using the magnetic field generated by the orbiting electron with the speed of the orbiting electron given as αc. The magnetic field as perceived by the electron is given by the following relationship:

$$B = \frac{\mu_0 q V}{4\pi a_0^2} = 12.54 \text{ T}, \tag{29}$$

the magnetic pressure follows as:

$$\frac{B^2}{2\mu_0} = 6.26 \times 10^7 \text{ Nm}^{-2}. \tag{30}$$

The quasiclassical plasma pressure of the perturbed quantum vacuum state around the hydrogen nucleus can be calculated by converting the orbiting electron velocity to temperature using $\frac{1}{2}m_e v^2 = \frac{3}{2}kT$, and making the assumption that the virtual electron-positron plasma has the same effective temperature as the orbiting electron

$$P = n_e kT = \frac{\rho_{v\_local}}{m_e}\left(\frac{m_e(\alpha c)^2}{3}\right) = 6.18 \times 10^7 \text{ Nm}^{-2}. \tag{31}$$

The physical meaning of having the magnetic pressure and quasiclassical plasma pressure of the perturbed quantum vacuum state around the hydrogen nucleus in equilibrium may provide an alternate explanation for why the ground state of hydrogen is 13.6 eV.[g)] The ground state of hydrogen cannot take on any other value as the perturbed state of the quantum vacuum around the hydrogen nucleus is fixed, establishing the magnitude of the virtual plasma pressure, which then establishes the magnetic pressure value—and this in turn fixes the parameters of the orbiting electron in the ground state. In order for the ground state to be different, both the magnetic pressure and virtual plasma pressure would have to change to a new equilibrium value by changing some of the input parameters. This treatment also suggests that the tools of MHD can successfully be used to model the quasiclassical behavior of the quantum vacuum as a virtual electron-positron plasma (Dirac vacuum). This notion has recently been studied in significant detail.[23,h)]

VII. CONCLUSIONS

This body of work has shown a possible connection between the gravitational coupling constant and dark energy, or the quantum vacuum—specifically that gravity may be an emergent phenomenon, a long wavelength consequence of the quantum vacuum. The characteristics of the quantum vacuum were considered at both the cosmological scale, and then at the atomic scale which revealed some interesting scale symmetries. The atomic scale consideration yielded Eq. (13) that suggested a perturbed state of the quantum vacuum around the hydrogen nucleus of 3.9×10^{-5} kg m^{-3}, which might trigger a desire to consider the gravitational consequences of such a perturbed energy density state of the quantum vacuum. If gravity is an emergent phenomenon resulting from quantum vacuum fluctuations occurring out to the cosmological scale, then there would be no "gravitational" consequences at these microscopic scales. A slightly different logic approach to take is to simply use the ground state of hydrogen to establish the required vacuum state around the hydrogen nucleus to account for the 13.6 eV energy level. How much energy density needs to be in the spherical volume defined by the Bohr radius to sum up to 13.6 eV? This approach would yield the same energy density state predicted by Eq. (13). With this approach, the position could be taken that in the absence of Eq. (13), any physical model that does not predict a vacuum state of 3.9×10^{-5} kg m^{-3} around the hydrogen nucleus must account for the issue of how the energy density associated with the ground state of the hydrogen atom is distributed globally or locally. Equation (13) was also used to develop a predicted mass for the electron that is within 1% of the observed value. Equation (18) shows that the value of the electron mass can be traced back to the cosmological vacuum energy value coupled with the mass of the proton. Finally, it was shown that the perturbed state of the quantum vacuum around the hydrogen nucleus could be successfully modeled as a virtual electron-positron plasma, or Dirac vacuum, using the tools of MHD, which provided a possible phenomenological explanation for why the ground state of

[g)]The equation for the ground state of hydrogen can be found in any undergraduate quantum mechanics textbook, and for completeness is provided here: $E_1 = -[m/2\hbar^2 (q^2/4\pi\varepsilon_0)^2] = -13.6$ eV.

[h)]This paper explored the idea of a "natural" vacuum as opposed to immutable, non-degradable vacuum for all principal quantum numbers and showed consistency with observation at the level of Bohr theory. A comparison with the Casimir force per unit area was made, and an explicit function for the spatial variation of the vacuum density around the atomic nucleus was derived. This explicit function was then numerically modeled using the industry multi-physics tool, COMSOL®, and the eigenfrequencies for the $n = 1$ to $n = 7$ states were found and compared to expectation.

hydrogen is exactly 13.6 eV. This is due to the fact that the magnetic pressure generated by the orbiting electron is in equilibrium with the quasiclassical quantum vacuum plasma pressure around the hydrogen nucleus. In summary, the equation that predicts the perturbation of the quantum vacuum in the presence of a local concentration of ordinary matter can be consistently and successfully applied to derive formulations for well known and accepted constants, and provides constructs that can be successfully used to model the quasiclassical characteristics of the quantum vacuum.

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[1]B. Carroll, and D. Ostlie, *An Introduction to Modern Astrophysics* (Addison Wesley, New York, 1996).
[2]Planck Collaboration, "Planck 2015 results. XIII. Cosmological parameters 123," e-print arXiv:1502.01589v2 [astro-ph.CO].
[3]C. L. Bennett, D. Larson, J. L. Weiland, N. Jarosik, G. Hinshaw, N. Odegard, K. M. Smith, R. S. Hill, B. Gold, M. Halpern, E. Komatsu, M. R. Nolta, L. Page, D. N. Spergel, E. Wollack, J. Dunkley, A. Kogut, M. Limon, S. S. Meyer, G. S. Tucker, and E. L. Wright, Astrophys. J. Suppl. Ser. **208**, 20 (2013).
[4]M. Visser, Mod. Phys. Lett. A **17**, 977 (2002).
[5]H. E. Puthoff, Phys. Rev. A **39**, 2333 (1989).
[6]M. Novello, M. Visser, and G. Volovik, *Artificial Black Holes* (World Scientific, Hackensack, NJ, 2002).
[7]J. Khoury and A. Weltman, Phys. Rev. D **69**, 044026 (2004).
[8]J. Martin, Mod. Phys. Lett. A **23**, 1252 (2008).
[9]K. Hinterbichler, J. Khoury, and H. Nastase, J. High Energy Phys. **1103**, 61 (2011).
[10]H. Farajollahi and A. Salehi, J. Cosmol. Astropart. Phys. **1107**, 036 (2011).
[11]R. Pourhasan, N. Afshordi, R. B. Mann, and A. C. Davis, J. Cosmol. Astropart. Phys. **1112**, 005 (2011).
[12]S. Chattopadhyay and U. Debnath, Int. J. Mod. Phys. D **20**, 1135 (2011).
[13]J. Clampitt, B. Jain, and J. Khoury, J. Cosmol. Astropart. Phys. **1201**, 030 (2012).
[14]P. Burikham and S. Panpanich, Int. J. Mod. Phys. D **21**, 1250041 (2012).
[15]J. Noller, J. Cosmol. Astropart. Phys. **1207**, 013 (2012).
[16]A. Cabré, V. Vikram, G. B. Zhao, B. Jain, and K. Koyama, J. Cosmol. Astropart. Phys. **1207**, 034 (2012).
[17]D. Blas, M. M. Ivanov, and S. Sibiryakov, J. Cosmol. Astropart. Phys. **1210**, 057 (2012).
[18]P. Brax, A. C. Davis, B. Li, H. A. Winther, and G. B. Zhao, J. Cosmol. Astropart. Phys. **1304**, 029 (2013).
[19]P. Brax, A.-C. Davis, and J. Sakstein, J. Cosmol. Astropart. Phys. **1310**, 007 (2013).
[20]P. Brax and A. Upadhye, J. Cosmol. Astropart. Phys. **1402**, 018 (2014).
[21]The NIST Reference on Constants, Units, and Uncertainty, 2014, available at: http://physics.nist.gov/cuu/Constants/
[22]F. Wilczek, "The origin of mass," 2013, available at: http://www.frank-wilczek.com/Wilczek_Easy_Pieces/342_Origin_of_Mass.pdf
[23]H. White, J. Vera, P. Bailey, P. March, T. Lawrence, A. Sylvester, and D. Brady, J. Mod. Phys. **6**, 1308 (2015).