Quantum time

IN PROCESS

LAST UPDATE 2010.04.28

David Finkelstein
School of Physics
Georgia Institute of Technology

Abstract

Classical infrastructures of gravity and the standard model are quantized, including space-time and the imaginary i. If one regards the standard-model vacuum and its space-time continuum as a large thin dome with classical continuous longitudinal dimensions and quantum transverse ones, it is resolved here into a truss dome of generalized spins. This quantization preserves the common Lie groups and conservation laws as accurately as experiment requires but bounds all variables. Its stat(e)vectors come from a typed exterior algebra $\mathcal{Q}$ of real spinors that merges the space-time and the Hilbert space of quantum theories. Its dynamics is specified by a history statvector in $\mathcal{Q}$, local in a simplicial sense, not differentially. The classical space-time manifold and the imaginary i arise in a singular organized limit as the history undergoes a "superconducting" condensation.

Keywords: Quantum gravity, quantum logic, quantum pragmatics, quantum set theory, quantum topology, standard model, superconducting vacuum

Contents

1 Infraquantization

1.1 Quantizing the infrastructure 2010.04.25

Nature seems to exhibit more dimensions in the small than in the large. In this respect it is a large thin dome of events, with some cosmic "longitudinal" dimensions, and some submicroscopic "transverse" ones. Its long dimensions support the orbital coordinates x^μ, p_μ of space-time position and momentum, and have cosmic extent. Its short dimensions, for example spins, are short in the absolute sense that their coordinates take on few values. Of these, the spins are longitudinal in the sense that they take part in rotations and boosts of the large dimensions. Isospins are transverse in the sense that they do not, but have their own transformations.

In working theories today, the dome is a continuum with quantum variables attached to each point. Its infinitude of degrees of freedom causes some significant physical computations to diverge, but a finite discrete framework like a lattice would generally violate important conservation and invariance laws.

Quantization is a way to repair divergent theories by simplifying their Lie algebras, originally the commutative ones. It has already eliminated infinities like the heat capacity of ovens and the ground energy of hydrogen atoms, without violating conservation of energy or momentum. Quantizing the infrastructure by simplifying its Lie algebras—"infraquantization"— similarly eliminates the remaining infinities.

It represents the dome as a finite network or complex composed of quantum simplices. Its elementary events, the atomistic elements of history, are now finite in number and quantum in nature, and connect into simplices with familiar continuous symmetries.

This unification of the long and short is of a Feynman-Penrose kind rather than a Kaluza kind, atomistic rather than continuous. It remodels the large dimensions after the fashion of the small rather than conversely. This process does not introduce manifold dimensions but removes them, replacing them by many spinlike variables.

Experiment today, however, seems rather consistent with manifold theories like those of the canonical commutation relations and general relativity. The main goal of this work is to find a crucial experiment for the quantum infrastructure.

The present infraquantization [hopefully] accounts for the huge anisotropy of the dome as one accounts for that of a soap bubble or a graphene: by the way atomistic elements organize themselves into a coherent whole. Unlike bubbles and graphene, however, the cosmic network does sit in a space-time but replaces the space-time, as well as the observed short dimensions. As a quantum system, the network dome is described statistically by a vector in a many-dimensional vector space of other possible statistical descriptions, not by a membrane in a larger classical space.

Such statistical vectors are often called "state vectors". This term came from a non-operational interpretation that doesn't work. We interpret them statistically and therefore call them "statvectors" or simply "statvectors" (§17).

The familiar axiomatization of quantum theory in Hilbert space omits an indispensable construct, quantum statistics. Since quantum theory is statistical, a one-quantum theory has little meaning. A quantum theory should be able to construct descriptions of beams of quanta out of descriptions of its quanta. Infraquantum statvectors are posited here to have a product representing the composition of quanta. They reside in a graded Lie algebra $\mathcal{Q}$ whose grade counts quanta, and whose monadics—first-grade elements—represent input or output processes for individual quanta. In the present infraquantization the multiplication is exterior, so a product can be interpreted as a simplex, and the addition is quantum superposition, so the simplex is a quantum simplex. To form a complex of quantum simplices, one does not add them but braces them and multiplies (§5). A quantum complex is a simplex of simplices, not a sum of simplices.

The thread that leads us through the maze of possible simplicial complex theories is a correspondence principle in the sense of Bohr. It is based like Bohr's on a Lie algebra homotopy that simplifies its Lie algebra, now of scale parameters other than $\hbar$. This makes it a special case of the theory of group deformation quantization. The general hypothesis that we adopt here is that basically simple symmetries are deformed to compound ones, including gauge symmetries, by *singular organizing limits* [16]. Examples:

- The group homotopies of Yang [48] and Segal [33] (§2.2).
- The group contraction of Inönü and Wigner [25].
- The symmetry rearrangement of Heisenberg, Umezawa [41], and Nambu-Goldstone boson theory.

An infraquantization must undeform these deformations, or *reform* them.

Standard quantum field theories assume that there is a fixed dynamical law, and that it is Poincaré invariant, unitary, and local. These seem to be features of a singular limit. Infraquantization changes them all, slightly in the usual experimental domain but eventually greatly.

Recall that the process of quantification converts a theory of an individual to a theory of an unspecified quantity of similar individuals, and qualitative yes-or-no predicates to quantitative how-many predicates. Here the individuals are quantum. Quantum quantification is discussed in §6.

In most developments of physical theory, quantization and gauging are separate and independent processes. Infraquantum theories carry out both by iterated quantification.

In a canonical theory, field operators of quantum annihilation and creation are functions on the domelike infrastructure, and so have a higher set-theoretic (Quine) *type* than the dome events. To deal with such type-distinctions between quantum systems, infraquantization replaces Hilbert space $\mathcal{H}$ and its dual Dual $\mathcal{H}$ by a finite-dimensional subalgebra ${}^F\mathcal{Q} \subset \mathcal{Q}$ of a typed exterior algebra $\mathcal{Q}$, defined in §5. ${}^F\mathcal{Q}$ replaces $\mathcal{H}$ and Minkowski space-time too, which return only as singular organized limits of $\mathcal{Q}$. ${}^F\mathcal{Q}$ also replaces Newton's differential calculus by a typed extension of Grassmann's algebraic differential calculus. Dynamical variables are finite matrices, hierarchically composed of smaller finite matrices. The vertices of its simplicial complexes are lower-type simplices, braced to serve as points.

The infrastructure of canonical quantum theories includes the complex plane $\mathbb{C}$ of probability amplitudes as well as a classical space-time manifold. While $\mathbb{C}$ is useful, it fails in structural stability and locality, two working principles that seem indispensable. This is reason enough to consider seriously whether a real quantum theory will not work better than a complex one.

It is proposed that the imaginary i of quantum theory is no more fundamental than that of Steinmetz's electric network theory. The Steinmetz i represents a quarter-wave phase shift of an external sine-wave generator, enabling us to replace d/dt by $i\omega$ in the network differential equations. It loses meaning if the external generator is significantly non-sinusoidal. Then one returns to the more generally valid second-order differential equations. It is proposed that the Heisenberg i likewise represents a symmetry of the ambience, and that in a less symmetric ambience the first-order differential equations of Heisenberg and Schrödinger must be replaced by one involving higher powers of ∂_t. The variable i [hopefully] becomes a Higgs field, imparting mass to otherwise massless gaugeons that do not annul it.

1.2 Outline 2010.04.25

§1 motivates and outlines the work.
§2 gives some background.
§3describes nature phenomenologically, as a dome.
§4 explains the quantization of the imaginary i of quantum theory.
§5 sets up a quantum set theory $\mathcal{Q}$ useful for infraquantization.
§6 reduces quantization to quantification, constructs the spin tree $\mathcal{Q}$, and interprets its negative probabilities.
§7 formulates infraquantum dynamical laws.

§8 relates gauging to quantification.
§9 discusses infraquantum space-times, the Umklapp problem and the chiral spinors of the standard model.
§10 formulates hypotheses for a microscopic quantum theory of the three fermion generations.
§11 injects the chemical concept of valence and the covalent bond for the binding of space-time on one type-level and of gaugeons on the next.
§12 composes the standard model gaugeons and gravitons out of four fermions bound by deep covalent exchanges.
§13 relates the main condensates of the standard model to those of the infraquantum dome.
§14 [hopefully] sets up a fermionic infraquantum dynamics that may suffice to control the gaugeons too.
§15 sums up consequences of this theory so far, with grateful acknowledgments.
§16 consists of appendices on general quantum theory and on Clifford and Fermi algebras, and a glossary.

1.3 Infinity in, infinity out revised 2010.04.25

In hindsight the earthquake warnings in classical mechanics are clear, precursors of the quantum theory to come. They can alert us to similar precursors in the canonical quantum theories of today.

The optical phenomenon of quantum superposition studied experimentally by Newton and at least theoretically by Malus was not taken seriously, since the evidence for the existence of photons was not. The earliest warnings to be heeded were the noxious infinities in oven heat capacity and in the hydrogen atom ground energy. Planck's quantum of action entered physics as a cut-off for the infinite heat capacity of an oven.

To be sure, the standard model of today is renormalizable, but each infinite renormalization constant renounces one finite prediction of the theory, and the renormalization process is non-local, involving Fourier transformations. Current renormalizations frustrate the prediction of the cosmological constant stress, for example, and canonical quantum gravity seems to require so many infinite renormalizations that no physical predictions remain. Since an infinity is never observed, let us refrain from injecting infinities into our theories in the first place. In the term of Pauli and Villars, let us perform a physical regularization, not a formal one.

Umezawa [41], for example, speaks of two languages in quantum field theories, "basic" and "phenomenological" (his quotation marks). The "basic" expresses Lagrangians and fields, and the "phenomenological" describes quantum annihilation and creation processes. Let us constrain ourselves to one language of quantum annihilation and creation processes, taking a phenomenological language as basic. The Lagrangian language is then merely a singular limit, badly defined, no more "basic" for field theory than Newtonian mechanics is for quantum mechanics.

The distinction between creation and annihilation, however, is defined relative to the ambience, including the arrow of the observer's time. To overide this distinction relativistically, let us call elementary creation and annihilation operations, or input and output operations, and their superpositions, collectively, *io* operations. Stats represent io operations. The most basic statvectors are monadics, which generate a graded algebra in which the monadics have grade 1. The monadics of a canonical quantum theory form a Heisenberg Lie algebra for bosons and a Fermi Clifford algebra for fermions. Collectively these algebras and their reforms, interpreted statistically, will be called*statalgebras*. The statvectors of the usual Hilbert-space formulation of a generic quantum theory are, in this view, elements of a statalgebra whose product is forgotten.

The infraquantum statvectors belong to a finite-dimensional algebra of io operations, an operational language. In what follows, it is a Clifford algebra ${}^F\mathcal{Q}$ with additional type-structure, and its statvectors concern quantum particles, space-time events, and sub-events of a lower type.

Quantum theories with finite-dimensional statalgebra (or Hilbert space) are termed *regular* after [7]. For example, a theory of a googol spins is regular, since its Hilbert space dimension is only $2^{(10^{100})}$, but not the theory of one linear harmonic oscillator, which has ∞ dimensions. To be more phenomenological than canonical quantum theories, let us assume regularity:

Physical quantum systems have regular theories. (1)

Theories with a statvector space of ∞ dimensions are then irregular, or *singular*. They are supposed here to arise in a singular limit, useful for replacing some hard sums by easier integrals.

Infinite-dimensional spaces are especially useful in the study of physical organization processes, for example the condensation from iron vapor of a magnetized crystal of one domain. It is imagined that the magnetic field of the crystal breaks the so(3) symmetry of the dynamics, the process often referred to as "symmetry breaking". More precisely, the condensation does not break this symmetry but rearranges it [41]. In the Nambu-Goldstone theory of symmetry rearrangement, the crystal is imagined to grow without bound, and the group is contracted. This incurs divergences. A regular theory of symmetry rearrangement does not go all the way to the limit of singular Nambu-Goldstone bosons, but stops at regular Nambu-Goldstone Palevons (§7.7 [hopefully]).

The regularity postulate encourages us to quantize until we reach a regular physical theory. Singular limits like the Poincaré group are less physical than their regular origins. Each quantization changes the working algebra, with consequences that may be small in the current experimental domain but ultimately become large.

A working physical theory must revise the structure of field theory at length scales much larger than the atom. If the Maxwell action is used, the zero-point energy of the electromagnetic field just for wavelengths larger than the atom suffices to curve the universe to fit into a sublunary sphere. The normal ordering that blocks this disaster is non-local from the viewpoint of field theory, yet natural within the io language. Infraquantization replaces the differential locality of field theory by a simplicial locality natural to $\mathcal{Q}$. In this sense the normal ordering is local and the field-theoretic action non-local.

1.4 Eliminating the center 2010.04.20

One portent of impending quantization is excessive commutativity, which results in structural instability: an arbitrarily small change in the structural tensor resulting in a non-isomorphic algebra [33]. Excessive commutativity is marked by a singular Killing form, with determinant 0. This is unstable because an arbitrarily small change in the structural tensor suffices to make the determinant non-zero, implying a non-isomorphic algebra.

Quantization reduces the center of the Lie algebra. Canonical quantization drastically reduced it to the one-dimensional algebra $\mathbb{C}$ of complex numbers, generated by the right-hand side of the canonical commutation relation

$$[p, q] = i, \quad [i, p] = 0, \quad [q, i] = 0, \tag{2}$$

among infinitesimal isometries of the quantum theory. Representing this i by a scalar matrix blocks regularity—to see this, take the trace of the equation—and is therefore responsible for all the infinities of present-day physics.

Call a system theory *simple* or *semi-simple* if all its basic Lie groups are simple or semi-simple, respectively, and otherwise *compound*. Among the basic groups is the automorphism group of the algebra of dunamical variables of the theory. Therefore only a regular quantum theory can be simple in this sense. Then its variables are all bounded, and its predictions finite. Therefore let us provisionally assume a simplification strategy [48, 33]:

Simplify all physical algebras by "small" homotopies. (3)

One of these algebras is that of the infinitesimal automorphisms of the algebra of operators, which must therefore become finite-dimensional. Therefore simplification implies regularity. It also provides a *correspondence principle*:

The homotopies (3) *correspond infraquantum constructs to singular.* (4)

The familiar correspondence principle with $\hbar \to 0$ is a special case.

1.5 Merging products 2010.04.20

Another portent of quantization in classical mechanics is a plurality of products. This was insistently called to my attention by A. Petersen and E. Grgin, and was pointed out by L. Flato and others. The variables of classical mechanics have two useful products, the ordinary numerical product pq and the Poisson Bracket $[p,q]_{\mathrm{P}}$ of dynamical variables. Canonical quantization fuses them into one non-commutative operator product involving a constant $\hbar$. They are re-separated by a power-series expansion in $\hbar$.

Space-time tangent vector fields have several useful products, such as the commutative scalar product $u{\cdot}v$, the anti-commutative exterior $u{\times}v$, the tensor product uv, and the Lie Bracket $[u,v]_{\mathrm{L}}$. The scalar and exterior products indeed merge cosily into one, a Clifford product, already used in the Dirac equation. The present infraquantization blends all these products into one exterior algebra product.

1.6 The sea of events 2010.04.22

Any fermionic statvector algebra (see Appendix 17.4) is also a Clifford algebra. The canonical Lie algebra of classical physics becomes a door to quantum mechanics when we identify it with the Bose statvector algebra of the constituent quanta. Indeed, it is often called the Heisenberg Lie algebra, as though Hamilton had nothing to do with it. Likewise one may use the Dirac Clifford algebra of spin as a door to the lower quantum levels, by identifying it with the Fermi algebra of *sub-events*, the quantum elements of quantum space-time events. We assume that the Dirac Clifford elements too include not only spin operators but also fermionic statvectors, now for sub-events. Sub-events too have a statvector algebra, for which the following notation is useful:

${}^{T}\mathcal{Q}$ is the subspace of $\mathcal{Q}$ of type T. The statvector algebras ${}^{E}\mathcal{Q}^1$ for events, ${}^{D}\mathcal{Q}^1$ for event differences, and ${}^{C}\mathcal{Q}^1$ for cell events, are constructed in §5.

Let us make the tentative assumption that *the space-time tangent bundle is a classical organized limit of a Fermi-Dirac system of infraquantum sub-events:*

$$\tan\mathcal{M} \prec \text{Fermi}\,{}^{E}\mathcal{Q}^1; \quad {}^{E}\mathcal{Q}^1 = \text{ext}({}^{D}\mathcal{Q}^1) = (\text{ext})^2({}^{C}\mathcal{Q}^1). \tag{5}$$

The term "vacuum" recalls the void of Newton, while classical field theory recalls the plenum of Descartes. A quantum infrastructure, however, is neither vacuum nor plenum, but may be called a "plexus": a structure described by a finitude of topological connections, here membership (ι) relations between quantum simplices of several types.

A classical crystal has a discrete symmetry group. The quanta have a continuous one, however, currently including the Poincaré group and the standard-model unitary groups. If the ambient medium is a crystal, as Newton and Fresnel declared the ether to be, it must be a quantum and relativistic crystal, a synthesis of the continuous and discrete. We model it as a quantum plexus. Since this plexus does not collapse at once, even when it is cold, its elements presumably obey Fermi statistics. They then may form a Fermi *event sea* . This is our ambience. It [hopefully] becomes the standard-model vacuum in a singular organized limit.

The hypothetical Dirac sea is made of particles with charge and mass, resulting in infinite charge and mass density. We do not assign charge and mass to events of the plexus. Charge and mass are, however, sources of curvatures of different kinds. They are not properties of individual events but describe defects in the organization of many events, like the Burgers(-Volterra) vector of crystal physics. Events carry charge and mass no more than a gas atom carries a Burgers vector.

1.7 Atoms of time and energy 2010.04.14

While Aristotle assumed infinitely many infinitely small instants of time, without beginning or end, and infinitely many points of space, this leads to noxious divergences already mentioned. To avoid infinities, let us seek meanings for the constructs of space and time that are more operational.

Einstein associated a space-time point, or event, with a smallest possible occurrence, for example a collision of two small bodies. The standard model and general relativity still use this classical event construct, but it is obsolete. Today we analyze the collision of the smallest bodies into supposedly quantum input-output operations represented in standard theories by arrows in a Feynman diagram. Unlike the classical ideal event of Einstein, which has only space-time position coordinates, physical operons also manifest space-time momentum, spin, hypercharge, isospin, color, and a generation number. These are all quantum variables in the standard model, coordinates in a quantum space.

Fermion theories can be finite-dimensional and bosonic ones cannot. Simplicity therefore seems to favor fermionic operons over bosonic. To be sure, Palev has reformed the bosonic statistics [29] and Palev statistics can be finite-dimensional; but a pair of fermions obey Palev statistics exactly. Let us therefore attempt to express all quanta as aggregates of fermions.

A fermionic statvector is a monadic—first-grade element—of an exterior algebra. Let us therefore call the operon it represents a *monad*, with apology to Leibniz and Aristotle. What a monad creates or annihilates can then be called a *monon* for the nonce. In brief: Monadics are mathematical vectors, that represent physical monads, that input or output monons; and analogously for g-adics, tensors of grade g. In the present application to quantum history, a monad is a single event of history. What it inputs or outputs depends on its type.

Leptons and quarks are presumably global defects, with strength measured by the mass and charge they carry. We nevertheless explore the possibility that they can be associated with individual monons, as global defects like disclinations and dislocations can sometimes be attached to single atoms of a crystal.

We draw our statvectors from a typed exterior algebra $\mathcal{Q}$ (§6.2). $\mathcal{Q}$ replaces both differential manifold and Hilbert space as a mathematical framework for the theory of the quantum plexus. The space-time and momentum-energy spaces of special relativity, and the Hilbert space of quantum theory, all merge into subspaces of $\mathcal{Q}$ and return as singular organized limits.

The typical canonical commutation relation (2), suitable for a particle on a line or a boson mode, is a singular organized limit of the regular commutation relations

$$[\widehat{q},\widehat{p}] = \widehat{\imath}, \quad [\widehat{\imath},\widehat{q}] = \mathsf{Q}^2\widehat{p}, \quad [\widehat{p},\widehat{\imath}] = \mathsf{P}^2\widehat{q} \tag{6}$$

in which $\mathsf{Q},\mathsf{P} > 0$ are small structural constants like $\hbar$. Rescaled, these relations define skew-hermitian so(3) rotation generators

$$L_x = \frac{q}{\mathsf{Q}}, \quad L_y = \frac{p}{\mathsf{P}}, \quad L_z = \frac{\widehat{\imath}}{\mathsf{QP}}, \quad \vec{L}\times\vec{L} = \vec{L}. \tag{7}$$

If we introduce an auxiliary external imaginary constant i for the moment, we may use it to construct hermitian operators iL_x, iL_y, iL_z, dimensionless angular momenta that can also be regarded as occupation numbers, positive or negative, counting angular-momentum quanta of size 1, "rotons", around the three axes. Then Q is the quantum of q, P is the quantum of p, and QP is the quantum of $\widehat{\imath}$. The rotator algebra (6) approaches the oscillator algebra (2) as the physical coefficients $\mathsf{Q},\mathsf{P} \to 0$, provided that there are enough rotons of all three kinds to imitate a continuum, and the vast majority are z-rotons:

$$\mathrm{so}(3;\mathbb{R}) \rightarrowtail h(1,\mathbb{R}). \tag{8}$$

This Lie algebra can be applied to time and energy too, if we take $q = t$ and $p = E$. Their quantum units are designated by X and E. A sub-event possessing one X of duration may be called a chronon To recover the complex quantum theory in a singular limit, one goes to an aggregate of such oscillators with a cumulative $\widehat{\imath} \approx i$.

The simple commutation relations (6) have been applied to particle coordinates ([48], [43],[27], and others) and to Bose statistics [29].

The symplectic group $\mathrm{Sp}(2,\mathbb{R}) = \mathrm{SL}(2,\mathbb{R})$, mixing q and p and fixing i, is a symmetry of (2), but not of (6). It is stable but infraquantization does not preserve it. Infraquantization regularizes the commutation relations, not their symmetry.

To recover the canonical quantum theory from such a simple theory, we must perform a homotopy (3) of the group or Lie algebra, here letting $\mathsf{Q}, \mathsf{P} \to 0$. This centralizes the variable $\widehat{\imath} \succ i$. To justify (6) we must find $\widehat{\imath}$ in experiment.

1.8 Asymptotic freedom? 2010.04.16

A symmetry between x^μ and p^μ can remain in the limit $\mathsf{X}, \mathsf{E} \to \infty$. While fields depend on x^μ alone, not both x^μ and p^μ, after a Fourier transformation they depend on p^μ alone; this does not break the $x \leftrightarrow p$ symmetry. But canonical field interactions are local in x. They are not usually assumed to be local in p. Thus canonical kinematics has $x \leftrightarrow p$ symmetry but not the canonical dynamics. We have to consider whether the dome organization can rearrange the $x \leftrightarrow p$ symmetry in the way that graphene formation rearranges the $x \leftrightarrow z$ symmetry, if the z axis is normal to the graphene plane and the x axis is parallel.

One way to express such a symmetry rerarrangement would be to have $\mathsf{X}/\mathsf{E} \to \infty$ as $\mathsf{X},\ \mathsf{E} \to 0$. This would mean that in ordinary experiments we deal with many fewer chronons than ergons.

Interactions weaken when interactants differ greatly in position. Therefore, according to $x \leftrightarrow p$ symmetry, they should weaken when the interactants differ greatly in momentum. This suggests asymptotic freedom, though the connection seems far-fetched: Asymptotic freedom has been understood as a consequence of shielding arising from the specific nature of the non-abelian color gauge group, an expression of locality in x alone. The argument given here is based on locality in both x and p. It does not seem to approach the non-abelian argument in the singular limit.

Since we assume that the regular variables have finite matrix representations, the matrices for $\widehat{p}$ and $\widehat{q}$ must grow very large on the way to the singular organized limit, so that they can pass for continuous variables. We accomplish this by quantifying over sub-events. Then the cumulative $\widehat{\imath}$ can polarize, analogous to spontaneous magnetization, allowing us to approximate $\widehat{\imath}$ by a local constant i. This organization restricts the quasi-continuous variables $\widehat{p}$ and $\widehat{q}$

to small values but leaves them variable. The singular organized limit shrinks the three-dimensional $p, q, \widehat{\imath}$-space to a two-dimensional p, q-phase space.

Let us refer to such a combined process of singular homotopy and organization as a *singular organized limit* or LIMO . For example, we write schematically

$$\widehat{q} \succ q = \text{LIMO } \widehat{q}, \quad \widehat{p} \succ p = \text{LIMO } \widehat{p}, \quad \widehat{\imath} \succ hi = \text{LIMO } r. \tag{9}$$

In particular, the compound canonical commutation relations and Bose statistics are singular organized limits of regular Lie algebras.

2 Prior studies

2.1 Space-time quantization 2010.04.14

The oldest surviving mathematical theory of physical space is Euclid's. Mechanics enlarged Euclidean space to phase space, and special relativity to space-time and its tangent bundle. The Euclidean group ISO(3) of space, the Galilean group and the Poincaré group ISO(3, 1) of space-time, and the canonical commutation relations are compound and must be simplified, according to (3). We recapitulate some relevant history of this project, with apologies to authors undoubtedly overlooked.

Historic classical simplifications of Euclid include the classical spherical, hyperbolic, de Sitter, and conformal geometries. Differential geometry and general relativity, however, introduced a new kind of classical non-simplicity, with the diffeomorphism group Diff.

Quantum spaces form a newer line of simplified spaces that branched off the line of classical phase-space geometries in 1924, when the quantum theory was invented, and further In 1930, when a quantum space-time was proposed in order to avoid infinities [47, 2].

R. P. Feynman, as graduate student, quantized space-time by replacing commuting coordinate differentials dx^m with Dirac spin operators γ^μ as in (11) [17]. He interrupted this work to study the Lamb shift at Bethe's behest but never completely dropped it.

Oppenheimer carried the idea of quantized space-time from Heisenberg to Snyder, who then quantized space as well as phase space [35, 47]. The Snyder algebra is still compound, however. C. N. Yang replaced it by the algebra so(5, 1) precisely to make it simple [48]. Where Feynman had quantized four-dimensional space-time, Yang quantized eight-dimensional phase space, the cotangent bundle of space-time. Conformal so(5, 1) acts on a classical space-time of four dimensions, but Yang so(5, 1) acts on a quantum phase space of

15 dimensions. It uses quantum constants of speed c, action h, time X, and energy E, to make the orbital variables dimensionless.

On another track, Segal [33], attempting to understand and forecast the evolution of quantum physics, argued on Darwinian grounds that physics is evolving toward simple Lie algebras: A compound Lie algebra has a singular Killing form, and an arbitrarily small change in its structure tensor makes it non-singular. As measurements of the structure constants improve, therefore, a compound Lie algebra has survival probability 0 relative to its semi-simple neighbors, which outnumber it ∞ to 1. Natural selection favors simple Lie algebras.

Since the isometry group of the statvector space of a quantum theory is one of these Lie algebras, simplicity in the sense of Segal implies regularity in the sense of Bopp and Haag [7]:

$$\textit{A simple quantum theory is regular.} \tag{10}$$

Gerstenhaber, influenced by Segal, described homologically a rich terrain of Lie algebras connected by homotopies, such as *contractions* [25], that carry groups out of stable valleys of simplicity, to ridges between the valleys, and up to singular peaks [23]. According to the simplicity principle, physics is currently a glacier flowing down the simplicity-gradient to valleys in Gerstenhaber land. The Galileo Lie algebra is on a ridge between the valleys of Lorentz $\mathrm{so}(3,1)$ and orthogonal group $\mathrm{so}(4)$.

Group contraction [25] and deformation quantization [4] are also homotopy-based theories of the evolution of physics. They do not heed the simplicity principle but may rest on classical space-times.

R. Penrose, as a graduate student, quantized the Euclidean 2-sphere by representing its points as directions of sums of many Pauli spins [30]. His space quantization was extended to a space-time quantization [18].

Vilela Mendez, inspired by Gerstenhaber, rediscovered the Yang group [43] and proposed physical consequences. It has been rediscovered several times since.

The simple quantum spaces (11) of Penrose [30], Feynman [17], Segal [33], and Yang [48], encountered in that order, greatly influenced the present study. Feynman and Penrose both analyze coordinates into a finitude of spins, but not the conjugate momenta, it seems. Yang quantizes momentum too, replacing canonical commutation relation like (2) with spin-like commutation relations like (6).

Segal posed and explained the simplicity principle and rediscovered the Yang space. The phase space of special relativity was later quantized into half-spins with Fermi statistics [20].

Galiautdinov [22], Shiri-Garakani [34], Bayer ([5], and others have studied Yang-invariant physics in $\mathcal{Q}$. Baugh [3] represented the Yang so$(5,1)$ Lie algebra in sl$(6\mathbb{R})$ much as we do here.

2.2 Feynman, Yang, and Penrose quantum spaces

2010.04.20

Let us compare the quantized orbital variables of the simple spaces mentioned, using *ch*EX units. Here $\widehat{x}$ is a quantized x; $k \in 3$; $m \in 4$; and δ indicates a finite difference to be summed later:

$$\begin{array}{llllll} \text{Feynman [17]} & \delta\widehat{x}^m & \sim & \gamma^m, & \delta p_m & = & ? \\ \text{Yang [48]} & \widehat{x}^m & \sim & i(g^{mn}\eta^5\partial_n - \eta^m\partial_5), & \widehat{p}_m & \sim & i(\eta_6\partial_m - \eta_m\partial_6). \\ \text{Penrose [30]} & \delta\widehat{x}^k & \sim & \sigma^k, & \delta\widehat{p}_m & = & ? \\ \text{Present} & \delta\widehat{x}^m & \sim & \gamma^{m5}, & \delta\widehat{p}_m & \sim & \gamma_{m6}. \end{array} \tag{11}$$

The Penrose and Feynman quantum spaces still assume an absolute space or space-time, with coordinates corresponding to x^μ but not p_μ. Simplicity led Yang to relativize space-time within a larger quantum phase space, *Yang space*, including momenta, boosts and angular momenta, as well as a complex plane.

These spaces are inadequate today. Standard model operons carry a hypercharge y, a generation variable Γ, three isospin variables τ^k, four space-time position variables x^m, four momentum-energy variables p_m, four Dirac spin variables γ^m, and eight color charges χ^c. In addition there are bosonic fields: 12 gauge 4-vectors Γ_A^m, a Higgs field ϕ, and gravity. Infraquantizations must fit all these variables and their commutation relations into the operator algebra Lin $\mathcal{Q}$ (§14).

Yang, like Snyder, represented his algebra by differential operators on an infinite-dimensional function space, as shown in (11). Simplicity in the present sense requires a finite-dimensional representation like Feynman's and Penrose's. This is the bottom line of (11). These finite-dimensional representations must have indefinite metrics, and so do not fit into Hilbert space. A physical interpretation of indefinite metrics and negative probability was indicated by Dirac [13].

2.3 The reform of the Heisenberg-Poincaré algebra

The commutation relations of the Yang generators $L_{m'm} = -L_{mm'} \in$ so$(6-n,n)$, acting on a statvector space $6\mathbb{R}$ with metric tensor $g_{m'm}$, are

$$[L_{m'''m''}, L_{m'm}] = g_{m'''m'}L_{m''m} - g_{m''m'}L_{m'''m} + g_{m''m}L_{m'''m'} - g_{m'''m}L_{m''m'}, \tag{12}$$

with $m, m', m", m''' \in 6$.

DO: *Check signs.*

The commutation relations of the Heisenberg-Poincaré Lie algebra of space-time position x^μ, momntum-energy p^μ, and Lorentz generator $L_{\mu'\mu}$, are

$$\begin{aligned}
[L_{\mu'''\mu''}, L_{\mu'\mu}] &= g_{\mu'''\mu'} L_{\mu''\mu} - g_{\mu''\mu'} L_{\mu'''\mu} + g_{\mu''\mu} L_{\mu'''\mu'} - g_{\mu'''\mu} L_{\mu''\mu'}, \\
[L_{\mu''\mu'}, x_\mu] &= g_{\mu'\mu} x_{\mu''} - g_{\mu''\mu} x_{\mu'}, \\
[L_{\mu''\mu'}, p_\mu] &= g_{\mu'\mu} p_{\mu''} - g_{\mu''\mu} p_{\mu'}, \\
[L_{\mu''\mu'}, i] &= 0, \\
[x_{\mu'}, p_\mu] &= i g_{\mu'\mu}, \\
[x_\mu, i] &= 0, \\
[p_\mu, i] &= 0.
\end{aligned} \tag{13}$$

with $\mu, \mu', \mu", \mu''' \in 4$ and $\hbar = 1$.

To deform (12) $\rightarrowtail$ (13), one considers a representation of (12) on a statvector space $V = 2(2\mathsf{N}+1)\mathbb{R}$ with large dimension $2(2\mathsf{N}+1)$, so that the spectra of the $L_{m'm}$ are quasicontinuous, and focuses attention on a polarized sector $V_{\rm pol} \subset V$, supposed to represent ordinary experience, in which $|L_{65}|$ is close to its maximum eigenvalue N and the other components of $L_{\mu'\mu}$ are much smaller, though still quasicontinuous. Let us set

$$\begin{aligned}
x_\mu &= \mathsf{X} L_{\mu 5}, \\
p_\mu &= \mathsf{E} L_{\mu 6}, \\
\widehat{\imath} &= \mathsf{EX} L_{65}.
\end{aligned} \tag{14}$$

Then X is the quantum of x^μ, a natural unit of time; call it the *chron*. E is the quantum of p^μ, a natural unit of energy; N is the maximum magnitude of any component of $L_{m'm}$ in this representation, and in that sense is the maximum number of rotons; and $\widehat{\imath}^2 \approx -1$ in $V_{\rm pol}$. In the limit

$$\mathsf{E}, \mathsf{X} \to 0, \quad \mathsf{N} \to \infty, \quad \text{with } \mathsf{NEX} = 1, \tag{15}$$

the Heisenberg-Poincaré relations (13) follow.

The Casimir operator for Yang $\mathrm{so}(6-n, n)$ is then

$$L^{m'm} L_{m'm} = L^{\mu'\mu} L_{\mu'\mu} + \frac{\widehat{x}^\mu \widehat{x}_\mu}{\mathsf{X}^2} + \frac{\widehat{p}^\mu \widehat{p}_\mu}{\mathsf{E}^2} + g^{66} g^{55} \mathsf{N}^2 (\widehat{\imath})^2. \tag{16}$$

This approximates the square of the mass in E units if $\mathsf{X} \gg \mathsf{E}$ subject to (15).

DO: *Classify representations of* $\mathrm{so}(6-n, n)$ *as Wigner classified representations of the Poincaré group.*

DO: Find a correspondence between the two families of representations. Note that the dome breaks $\mathrm{so}(6-n, n)$, while the vacuum is invariant under $\mathrm{iso}(3, 1)$ and that Wigner uses Heisenberg statvectors while infraquantization uses history statvectors.

2.4 The reform of Bose statistics revised 2010.04.17

The commutation relations for Bose statistics define a canonical Lie algebra, and so require reform. Palev [29], also explicitly seeking simplicity, reformed the canonical algebra of Bose statistics to a simple Lie algebra, such as so(N).

Pairs of $\mathcal{Q}$ fermions do not exactly obey Bose statistics. It is easy to show that they obey a Palev statistics [29] defining an so(N) Lie algebra of which the canonical Lie algebra of Bose statistics is a singular organized limit. In infraquantum theories, therefore, it is natural to represent all empirical bosons as even polyads, with fermionic hard cores. It is understood that this hard core must show up in high-energy photon-photon collisions.

2.5 The Umklapp problem 2010.04.14

When Heisenberg proposed to quantize space-time, Pauli pointed out the bounce problem (*Umklapp Problem*) [47], and Feynman pointed it out to me:

The canonical commutation relations imply that the canonical conjugate to a circular variable is a lattice variable, one with a discrete uniformly spaced spectrum. For example, the conjugate to an angle $\theta \cong \theta + 2\pi$ is an angular momentum $L \doteq nh$. Conversely, if the space coordinates are lattice variables, $x = n\mathsf{X}$, then the conjugate momenta are circular, $p \cong p + 2\pi n\mathsf{E}$. Then the high momentum $p = \pi n\mathsf{E}$ in one direction would be indistinguishable from an equally high momentum $-p$ in the reverse direction. The particle can bounce from $p \to -p$ with no applied force. This bounce is well known for phonons in crystals. There the phonon bounces off the crystal, which accepts the lost momentum. But particles are not observed to bounce off the vacuum. Therefore the ambient event space is probably not a lattice of that kind.

Infraquantization replaces the canonical commutation relations with spin commutation relations, not a lattice, though each component of total spin has a spectrum like the coordinates of a lattice. This replaces the bounce by a bound. While the Umklapp momentum for spacing X is $\hbar/\mathsf{X}$, the bound in plexus momentum is another quantum constant $\mathsf{NE}/c\hbar \gg \hbar/\mathsf{X}$. But spin "up", $L_{m6} = \mathsf{N}$, is not identified with spin "down", $L_{m6} = -\mathsf{N}$, so there is no bounce. An infraquantum space-time has no Umklapp problem.

Moreover, to attain the bound $L_{m6} = \mathsf{N}$, all N spins in the sum for p_m must align in the $+m6$ direction. Since $[L_{56}, L_{m6}] \sim L_{m5}$, this alignment is complementary to the dome alignment of L_{56} that organizes $\hat{\imath} \twoheadrightarrow i$, unless $\langle L_{m5} \rangle \doteq 0$.

3 The cosmic dome 2010.04.08

The plenum of canonical field theory emerges from a plexus of fermionic subevents as a singular organized limit. We do not use this limit here since it diverges. Instead we study a symmetry rearrangement [41] that preserves the full Yang symmetry for the dome condensate. This keeps some dimensions of the statvector space that are usually thrown away, but the total number is still finite, while for the singular limit it is infinite.

The standard model and the semiclassical theory of gravity describe a dynamically variable cosmic dome. The continuous space-time coordinates of field theory represent its longitudinal dimensions. The unitary charges represent its transverse dimensions. A quantum spin of the Lorentz group is attached to each operon, and can be considered longitudinal but short, quantum, and regular; in contrast to the orbital variables, which are longitudinal and long, and presently singular, still undergoing the singular diffeomorphism group of general relativity. Chamseddine, Connes, and Marcolli [11] work with such a singular dome.

The $\mathcal{Q}$ language of §5 enables one to resolve this dome into a space truss of variable topological structure, a typed simplicial complex, with similar spin variables for both longitudinal and transverse struts, and no continuous variables.

The Yang SO(6) group (11) implies that in a singular organized limit $\widehat{\imath} \succ\!\!\!\!\rightarrow i$ there is a symplectic symmetry between position x^μ and momentum p_μ. Our ambient dome reduces the Yang symmetry. Its variables x^μ vary over ranges of cosmological magnitude, while momenta p_μ, ordinarily undefined for the empty manifold, may be taken to be 0 in the ambient "vacuum".

Something similar occurs for a bubble or a graphene flake. Organized into a graphene, carbon atoms collectively define a $2 + 1$-dimensional quasi-manifold. The manifold (x,y,t) coordinates range over macrocosmic ranges defined by the graphene. The carbon momenta obey the constraint $p_x, p_y \approx 0$ in the manifold coordinate system due to the same binding forces that define the quasi-manifold (x, y, t) coordinate systems. One cannot carry such results from a graphene to a quantum plexus because graphene theory assumes a background space-time manifold; but they are suggestive.

It is natural to enlarge the Yang $\mathrm{so}(3, 3)$ Lie algebra with the unitary charges. In the standard model each operon also has a Lorentz spin, here termed longitudinal, and there is no symmetry between the longitudinal and transverse spins. We suppose that such an extended symmetry exists for the kinematics of the individual operon, and is badly broken as a symmetry by the organization of the dome. Unitary charges remain after the dome organization, as spins transverse to the dome, along its smallest dimensions. Lorentz spins, similarly, survive as short longitudinal variables, of magnitude $\sim \mathsf{X}$. Orbital dimensions (x, p) are longitudinal and long.

Standard theories, to be sure, assume that the proper times between physical operons depend solely on their orbital variables and not on their spins or charges, longitudinal or transverse. Spins and charges are usually given space-time length 0.

This assumption makes the chronometric form singular. Therefore the theory is structurally unstable. There is a natural invariant metrical form for a spin plexus, and it defines non-zero proper times for longitudinal and transverse spins, though they seem to be too small to be detected yet. Then a change in a longitudinal spin generally changes the proper time between two operons, while a change in a transverse (unitary) spin does not.

The plexus then resembles a truss dome or graphene that is as wide and old as the cosmos but too thin to measure as yet. In an infraquantum theory the dome is a typed simplical complex, whose vertices are generated by the membership operation ι and multiplicative composition $\vee$.

The imaginary spin dyad $\widehat{\imath} \rightarrowtail i$ should be considered transverse but long, since it is macrocosmic, involving many cells, but polarized and frozen. If the longitudinal dimensions are regarded as angular or azimuthal, the transverse dimension of $\widehat{\imath}$ is radial. We are to narrate the the structure and excitations of the dome within $\mathcal{Q}$ as parsimoniously as possible.

In such theories, particles are propagating excitations of the dome. They may define representations, not of the unstable Poincaré group, as Wigner proposed, but of the symmetry Lie group $G[E]$ of the dome. The possible Yang groups $\mathrm{SO}(5,1), \mathrm{SO}(4,2), \mathrm{SO}(3,3)$, are simple but mix space-time, momentum, and complex-plane dimensions, and so are broken by the dome. One of them may still serve as group $G[C]$ of a generic cell of the dome, on a lower type-level $C < E$ that includes the standard model unitary charges. Then the casimir (operator) of the Yang Lie algebra includes and unifies casimirs of the Lorentz group, the complex-phase group $\mathrm{U}(1)$, and the infraquantized proper time and proper mass. The casimir of the cell Lie algebra similarly unifies the casimirs of the Yang group and the unitary charges.

The physical constants entering into the Feynman or Yang groups are the speed and action units c, h of earlier groups, elementary time and energy units X and E with $\mathsf{XE} = h$, and a cosmological integer N. Under either group, time is just energy measured in huge units. The vacuum is supposed to reduce the Yang casimir to the Minkowski form in an singular organized limit $\mathsf{X}, \mathsf{E} \to 0, \mathsf{N} \to \infty$.

Infraquantization iterates Fermi quantification, which is represented by the functor ext, forming the exterior algebra of a statvector space. Quantization, general relativization, and gauging then become special cases of quantification.

Multiple Bose quantification has also been proposed [40, 46].

Nowadays there seem to be three variables with important non-zero vacuum values:

- Gravity's $g_{\mu\nu}$ breaks space-time sl($4\mathbb{R}$).
- Higgs h_i breaks weak su(2).
- The quantized imaginary $\widehat{\imath}$ breaks complex-plane sl($2\mathbb{R}$) and time reversal $T \in \mathrm{SL}(2\mathbb{R})$.

Let us designate the regional mean values by $\overline{g}, \overline{h}, \overline{\imath}$. There may be close functional relations among these three varables. Both i and $g_{\nu\mu}$ distinguish time from space in their way; i responds to time reversal but not to space-reflection, while $g_{\nu\mu}$ responds to neither. Particle masses may derive from couplings to h, but are defined as couplings to g.

It has been supposed that $\overline{g}$ and $\overline{h}$ are statistical parameters of the vacuum statvector of the system ([38], for $\overline{h}$). The global concept of a vacuum statvector, however, depends on the assumption that the dynamics is homogeneous under time translation. This assumption may not work for time-shifts that are too small or too large. In an infraquantum theory it may be impossible in principle to carry out time translations by 10^{100} s or 10^{-100} s, as assumed in canonical theories.

More conservatively, let us replace "vacuum" by a regional description of the dome, which is mostly metasystem, and has only approximate symmetries, to be determined experimentally.

The ambient $\overline{g}, \overline{h}, \overline{\imath}$ values are statistical parameters of the dome. (17)

The hypothetical underlying quantum variables g, h, i may depend on spin variables of the plexus that we ignore when we study its small excitations. They may be slightly modulated by the system and the experimenter, and may vanish completely in a local dome melt-down. The ambient means are closer to experiment than the underlying quantum variables, which are still highly theoretical.

3.1 Infraquantum relativity 2010.04.09

The special relativity group $\mathrm{ISO}(3,1)$ reduces the observer to but 10 degrees of freedom, like a speck of crystal adrift in Minkowski space-time $\mathcal{M}$. This group is simplistic but not simple.

General relativity over-compensates. While it works empirically in the classical macrocosmic realm, its gauge group Diff represents observers with a continuous infinity of degrees of freedom, as if they were infinitely elastic organisms, cosmic super-squids, unaffected by gravity or distortion, crossing light-cones freely in both directions. General relativity is even more singular than special relativity. But it is also points toward simplicity, in that it eliminates the toxic commutativity of the covariant or kinetic momentum components. And it inspired the Yang-Mills theory of gauge and thus the standard model.

In standard quantum theory there is a unitary relativity group $\mathrm{U}(\infty)$ relating different frames in Hilbert space. Usually the quantization of gravity is taken to require representing Diff within $\mathrm{U}(\infty)$.

Let us seek a still better form of relativity, with observer somewhere between infinitesimal speck and infinite squid, that supports an infraquantum regularization of the standard model. The strategy is to morph our theories from the language of $\mathcal{H}$ and $\mathcal{M}$ to that of some finite-dimensional "field type" ${}^F\mathcal{Q} \subset \mathcal{Q}$, without changing them drastically. This is the project of an infraquantum relativity, with memorandum

$$\text{Infraquantization} : \mathcal{H}, \mathcal{M} \mapsto^F \mathcal{Q}. \tag{18}$$

Simplicity requires an infraquantum relativity to unify all the variables of the standard model, and to account for their separation in the standard model by a dynamical organization.

A regular quantum theory in the sense of (1) has to renounce either unitarity or Lorentz invariance, since the non-compact Lorentz group will not fit into a compact unitary group.

For example, the regular space $\mathcal{D} \sim 4\mathbb{R}$ of real Majorana Dirac spinors is Lorentz invariant but has no invariant definite metric form. Instead it has an invariant indefinite bilinear form, the Pauli form often designated by β, and a cone of definite non-invariant Hilbert forms h, associated with possible time axes.

Since both definiteness and Lorentz invariance accord well with experiment, this is a significant conceptual problem. It is taken up in §6.5

Infraquantization incurs a heavy start-up obligation: to reconstruct the experimental g, h, and i, as singular organized limits of their infraquantizations, $\widehat{\mathsf{g}} \rightarrowtail \mathsf{g}$, $\widehat{\mathsf{h}} \rightarrowtail \mathsf{h}$, and $\widehat{i} \rightarrowtail i$.

Yang space at last eliminates the problem of large zero-point energy. The commutation relations of the Yang space variables are those of a Lie algebra. They have the generic form $[L, L] = cL$ instead of $[L, L] = i$. Unlike the canonical commutation relations, the Yang relations are satisfied by $L \equiv 0$. The zero-point energy is zero in Yang space, or in any simple Lie-algebra space. The singular organized limit of the condensate evidently does not attain the zero-point of energy, since its commutation relations are canonical. The condensate is indeed important for gentle measurements on large blocks of the plexus, but it is disorganized at the zero point of energy.

4 The imaginary quantum i 2010.04.20

Electric circuit theory is a real theory. Steinmetz introduced i into it replace trigonometric functions like $\cos \omega t$ by exponentials like $e^{i\omega t}$, which are easier to combine. Steinmetz's i represents a $\pi/2$ rotation in the plane of any variable q and its time-derivative $\dot{q}$. It enables us to factor the second-order differential equations of circuits into two first-order equations. Its utility does not cause engineers to doubt that circuit theory is basically real.

The imaginary i was introduced into quantum mechanics to permit first-order differential equations, those of Heisenberg and Schrödinger. Hamiltonian equations are indeed first order but Lagrangian equations need not be, and history quantum theory is related more directly to the Lagrangian than to the Hamiltonian.

We must consider the possibility that the quantum physicist's i is no more physical than the electrical engineer's. The meter readings of both are real numbers. Is it possible that i is really imaginary?

If so, then the now standard postulations of a Hilbert space over $\mathbb{C}$ and a Heisenberg dynamical equation

$$\frac{dq}{dt} := [\frac{d}{dt}, q] = \frac{i}{\hbar}[H, q], \tag{19}$$

are not in the best possible agreement with practice. They proceed as if a unique central i had physical meaning.

Most of the rules of quantum kinematics work equally well with statvector spaces over the real field $\mathbb{R}$, the complex field $\mathbb{C}$, or the quaternion field $\mathbb{H}$. The absence of a bilinear tensor product seems to exclude $\mathbf{H}$ for practical purposes. Stückelberg formed a quantum theory over $\mathbb{R}$. A one-dimensional projector of $\mathcal{H}(\mathbb{C})$ is a two-dimensional projector in $\mathcal{H}(\mathbb{R})$. A statvector $\psi \in \mathcal{H}(\mathbb{C})$ becomes a random statvector $e^{i\theta}\psi \in \mathcal{H}(\mathbb{R})$ with a random phase angle θ and a constant operator i, $i^2 = -1$. There are strong experimental arguments for $\mathcal{H}(\mathbb{C})$ over $\mathcal{H}(\mathbb{R})$: the phenomenon of elliptical polarization, the connection between symmetry and conservation, the uncertainty relation [37] and the correspondence between classical Poisson Brackets and quantum commutators, all depend on i.

A quantum superposition of two beams is a third beam that is experimentally recognized by the fact that it passes—with probability 1, this means—through any filter that passes both beams. Experimentally, the superpositions of two statvectors seem to form a two-real-parameter family of statvectors. This indicates $\mathbb{C}$; a one-parameter family implies $\mathbb{R}$, a four-parameter family $\mathbb{H}$. If there are violations of i symmetry in nature, they hide even better than violations of parity and time symmetry did.

As long as time is continuous, moreover, any discrete variable that is continuous must be constant. The choice between i and $-i$ could propagate throughout the connected universe in this way. But here we consider spacetimes that are quantum, not continuous, and cannot use that argument. And as an operator with $i^2 = -1$, i has a continuous infinity of possibilities, not just two.

Another difficulty of principle with an absolute i is that the centrality assumption $[i, o] = 0$ is structurally unstable. A stabler condition is that the commutator Lie algebra of the operator i and present observables o is simple, but the commutators with i are small. This instability, however, seems to lead to no infinities, and could be tolerated. We can muster no hard evidence that i is not an absolute.

Nevertheless it moves: Assuming a fixed i throughout the cosmos violates Einstein locality, one of the most fruitful principles of the last century of physics.

Therefore we continue the construction of a real quantum theory with a quantized i. This includes replacing the Heisenberg Lie algebras by orthogonal Lie algebras, which replaces Bose statistics by Palev statistics (§2.4). As (6) illustrates, simplification may require a quantized imaginary $\widehat{\imath} \succ\!\!\!- i$.

One especially simple way to quantize i is to imbed it in the quaternions, where it is no longer central. This idea was put forward by Yang at a Rochester Conference in 1959. The proper framework for it seems to be a real quantum theory with an SO(3) gauge group and a dynamical i-field replacing the constant i, defining the electric axis in electroweak isospin space. (One first effort explored a quaternion quantum theory, which seems to be less relevant to particle physics.) The quantized i-field of the so(3) gauge theory then turns out to be a Higgs field [39, 1]. This is the closest we come so far to a phenomenological justification for quantizing i.

More generally, to correspond with the standard complex theory, a real quantum theory will have an infraquantized imaginary $\widehat{\imath}$ in its operator algebra, with a correspondent $\mathbf{i} \prec\!\!\!- \widehat{\imath}$, possibly depending on the reference frame, as with the relativization of time. Yang's $\widehat{\imath} = L_{65}$ [48], suitably normalized, is such an operator.

$\widehat{\imath}$, the quantized i, has a universal direct coupling to all the other basic fields, appearing as a multiplier for their action operators (§9.3); unlike the Higgs field, which is not assumed to couple directly to the strong interactions.

The quantization $\widehat{\imath} \sim \Sigma^2\gamma^{65}$ also recalls the proposal $i = \gamma^{4321}$ of Hestenes

[24].

$\widehat{\imath}$ enters into the quantum skew-action operator with any action operator δA:

$$\delta\Phi = \widehat{\imath}\,\delta A/h. \tag{20}$$

Like angular momentum and energy, $\widehat{\imath}$ is a sum of contributions from every system, including the metasystem. Let us hypothesize an organization akin to polarization that centralizes ("superselects") the resultant $\widehat{\imath}$ and results in the imaginary unit of complex quantum theory: $\widehat{\imath} \succ\!\!\!\rightarrow i$.

The Heisenberg relation $[x^m, p_m] = \hbar i$ returns in a singular organized limit but cannot hold for one cell, where the Yang relation is more accurate. Therefore canonical quantum theory grossly overestimates zero-point energies in the small, and the vacuum energy density. The coordinates, momenta, and $\widehat{\imath}$ are now cumulated spins, of finite spectra. Measuring a cumulated spin does not take indefinitely great momentum-energy in the way that measuring a position coordinate is supposed to. It may merely incur a space-time meltdown.

Each subsystem contributes to $\widehat{\imath}$ as it does to the total angular momentum tensor. When the system is gauged for the sake of locality, $\widehat{\imath}$ acts as a Stückelberg or Higgs field in giving mass to some components of the gauge field; namely, those in the plane normal to $\widehat{\imath}$. $\widehat{\imath}$ thus serves to define the electric direction in isospin space. First noted for the quaternions, this feature of the quantized imaginary generalizes to any Clifford algebra, including $\mathcal{Q}$, and to any gauge field.

An absolute i seems almost indispensable today because it was assumed quite early that basic quantum equations like the Heisenberg Equation of Motion have to be of first order in ∂_t, a formally skew-hermitian operator on functions of time, and the Hamiltonian provided by classical mechanics is hermitian. In Dirac's development, the Equation of Motion has to be first-order because by definition a state gives complete information, and so should determine its own future uniquely. The development equation for the state must then contain no derivatives higher than the first.

But a statvector is not a state. It does not give complete information, only maximal information, almost all statistical. There is no compelling reason why a statvector should determine its own development. The concept of history is more general than the concept of a first-order differential equation. It puts the construct of Lagrangian before that of Hamiltonian, and Lagrange's theory is not partial to first order equations. In a quantum history theory the basic dynamical equations of quantum physics could be of higher degree than the first. Then one could reduce them to the first degree by introducing new variables and an i, but this could be done in many ways, and the differences among them need have no physical significance.

The imaginary is then needed mainly for *correspondence*: between the generator of ∂_t and the Hamiltonian; between commutators and Poisson Brackets; and so forth. But then it may be a product of the classical limit, like correspondence itself.

In the existing complex quantum theory, i is central and therefore strictly conserved. This has non-trivial experimental consequences. Since this centrality assumption is not local, let us assume that i is only approximately central, like the coordinates of a baseball or the components of the metric tensor, as a consequence of a singular organizing classical limit. Then all singlets of the complex quantum theory must in principle be resolved into doublets when the organization of i is broken and the underlying real theory peeps through.

Absent a fixed i, we must give up the Heisenberg first-order form of dynamics (19). In a quantum history theory, Lagrange's equations of motion are more natural than Hamilton's and are typically real second-order equations. For example, Lagrange's equation for a quantum oscillator takes the valid form

$$\frac{d}{dt}\frac{dq}{dt} + \omega^2 q : (\triangle\widehat{\partial}_t)^2 q + \omega^2 q = 0 \tag{21}$$

(using the commutator symbol $\triangle$ of §17.4).

4.1 Time-slicing 2010.04.24

In canonical quantum theories assume time coordinates from the start. The history statvector space $\mathcal{V}$ of a canonical quantum theory is then a tensor product of time slices $\mathcal{V}_t$

$$\mathcal{V} = \bigotimes_t \mathcal{V}_t \tag{22}$$

The canonical form form for a history dynamics statvector is a tensor product

$$\mathbf{E} = U_{T,T-1} \otimes U_{T-1,T-2} \otimes \ldots \otimes U_{-T+1,-T} \tag{23}$$

in which each time t appears twice, for input and output.

If an infraquantum theory an event may still have a time coordinate operator $\widehat{t}$. Then its statvector space is a direct sum of $\widehat{t}$-eigenspaces, and its exterior algebra is an exterior product of time slices. Several such slices may be needed to determine the rest of the history statvector, subject to the dynamics. For example a second-order theory can relate two final slices $t, t-1$ to two earlier ones $t-2, t-3$. A history quantum theory of order o is one whose dynamical statvector can be factored into sub-complexes of

An example of a covariant Killing form of Lagrange's equations of degree 2 in $\overline{\partial_t}$ is

$$\triangle L_{m'm} \triangle L^{m'm} q = M^2 q \tag{24}$$

where $L_{m'm}$ is the tensor of generators of a supposed symmetry group, like the Yang Lie algebra (12), and M^2 is a squared-rest-mass, or an operator generalization.

In circuit theory one might define i as the operator

$$i := |\partial_t|^{-1}\partial_t. \tag{25}$$

This could work for the quantum theory too, but it does not look relativistically invariant. Theories like the Yang theory provide a covariant quantized tensor that includes i as one component. Such theories have a tensor $L_{m'm}$ of Lie-algebra symmetry generators, with a Lorentz subalgebra. The familiar translation generators ∂_μ are approximations to some of its components. $Lm'm$ is supposed to have a non-zero vacuum expectation $\overline{L}_{m'm}$ dominated in an adapted frame by one enormous component, so that under ambient conditions

$$L_{m'm} = \overline{L}_{m'm} + \delta L_{m'm}, \quad \delta L_{m'm} \ll \overline{L}_{m'm} \tag{26}$$

For a covariant formulation, let the dominant component be $\theta^{m'm}\overline{L}_{m'm}$. Then

$$\widehat{\imath} := \frac{\theta^{m'm}\, L_{m'm}}{|\theta^{m'm}\, \overline{L}_{m'm}|}. \tag{27}$$

—compare (25).

To account for the fact that the i-trick works so spectacularly well, we should return to Einstein once again:

$$\textit{The dynamical law is of second order in the } L_{m'm}. \tag{28}$$

Possibly this is the first approximation in an unending progression of better ones with higher orders.

5 Quantum sets 2010.04.25

In the standard quantum theories, quantum variables are functions of time, or space-time. They therefore belong to a higher membership level (= type, rank, ...) than the time variable. Quantizing time therefore quantizes a deeper level than canonical quantization, and by correspondence calls for a *typed quantum theory*, the simplest being a quantum set theory.

Classical sets are set as in stone. Quantum sets, however, are more fluid, and are changed markedly by perception, in a way delimited by commutation rules.

Let us begin with founded quantum sets. The foundation is some quantum system with statvector space $\mathcal{F}$, a finite-dimensional vector space, here taken over $\mathbb{R}$, the modification for complex vector spaces being trivial. The functor ext, by definition, converts the statvector space $\mathcal{F}$ to the exterior algebra $\mathcal{Q}^1(\mathcal{F}) := \operatorname{ext}\mathcal{F}$, said to be the statvector space for the quantum set of type 1 over $\mathcal{F}$. Thus a quantum set is simply an aggregate with Fermi statistics.

We apply ext T times to form the statvector space

$$\mathcal{Q}^T(\mathcal{F}) := (\text{ext})^T\mathcal{F} \tag{29}$$

for quantum sets of type T over $\mathcal{F}$. Since $(\text{ext})^0 := \text{Id}$, $\mathcal{F}$ itself is the statvector space for type 0. $\mathbf{R}$, the statvector space for the empty set only, can be regarded as the statvector space of type -1.

The types form a nested sequence. The limit $T \to \infty$, which is merely their union, is the statvector space for the generic quantum set founded on $\mathcal{F}$:

$$\mathcal{Q}(\mathcal{F}) := \underset{T\to\infty}{\text{Limo}}\ {}^T\mathcal{Q}(\mathcal{F}) = \mathbb{U}_T{}^T\mathcal{Q}(\mathcal{F}) \tag{30}$$

We write simply $\mathcal{Q}$ for the quantum sets without foundation, where $\mathcal{F} = \emptyset$.

We build on Quine's concept of type here. Quantum type is represented by a linear operator Type whose spectrum is $\mathbb{N}$ (including 0). To say that a set has type T is to say that the statvectors x for that set obey $\text{Type}\, x = Tx$.

$$\begin{aligned}&\text{If } x \text{ has type } T \text{ then } \iota x = \{x\} \text{ has type } T+1.\\ &\text{The null set 1 has type 0.}\\ &\text{If Type}\, x = Tx \text{ and Type}\, x' = T'x' \text{ then Type}(x'x) = \sup(T', T)(x'x)\end{aligned} \tag{31}$$

The statvectors of type T form a subspace ${}^T\mathcal{Q} \subset \mathcal{Q}$ composed of eigenvectors of the operator Type. Those of type $\leq T$ form an exterior subalgebra of ${}^T\mathcal{Q}$ written

$${}^{\leq T}\mathcal{Q} := \bigoplus_{T' \leq T} {}^{T'}\mathcal{Q} \subset \mathcal{Q}. \tag{32}$$

The set y whose element is x (and x alone) is usually written as $\{x\}$. Peano wrote it as

$$y = \iota x, \tag{33}$$

both in his theory of the natural numbers, where he called ιx the successor of x, and in his set theory. His functional notation is more convenient for iteration than braces, and we adopt it here, but continue to call the operation of ι *bracing*.

Leptons and quarks have significant structure, described by their spins and charges. Their statvector spaces are tensor products of statvector spaces associated with each of these properties. Tensor products are how parts are assembled into wholes in quantum theory; one may therefore call the spin of an electron a part of the electron. This does not imply that one can disassemble the electron into such parts. We may see a particle without a spin, but we never see a spin without a particle.

Such a structured but indivisible unit can be represented by a unit set = singleton = monad whose fine structure is in its *second members*, the members of its sole member.

We construct a typed quantum set algebra $\mathcal{Q}$ suitable for framing a finite quantum theory by enriching a classical set algebra $\mathcal{C}$ with quantum superposition so that it becomes an exterior algebra $\mathcal{Q}$. The exterior product corresponds to the disjoint union. The grade-g subspace of $\mathcal{Q}$ is written $\mathcal{Q}^g$. ι becomes a linear operator

$$\iota : \mathcal{Q} \mapsto \mathcal{Q}^1 \tag{34}$$

$\mathcal{Q}$ is a real exterior algebra over itself, and minimal in that respect:

$$\begin{array}{lllcl} (1) & & \mathcal{Q} & = & \mathrm{ext}\, \iota\text{'}\, \mathcal{Q}; \\ (2) & \text{If} & \mathcal{P} & = & \mathrm{ext}\, \mathcal{P}, \\ & \text{then} & \mathcal{P} & \supset & \mathcal{Q}. \end{array} \tag{35}$$

The $\mathcal{Q}$ grade corresponds to classical cardinality.

$\mathcal{Q}$ is inductively constructed by iterating the functor ext ι'. The bracing ι is inserted to separate the exterior product on ext ι' V from any product that might already be defined on V. This converts a vector space into its exterior algebra. The iteration can start with the almost trivial exterior algebra $\mathbb{R}$; or for that matter with the null set.

Several metrics on $\mathcal{Q}$ come to hand:

As a spinor space each level of $\mathcal{Q}$ has a natural *Pauli metric* (63).

Each level $\mathcal{Q}$ also has a natural*Hilbert metric*, defined inductively by assuming that ι preserves the metric, and treating each type level as the many-body statvector space over the previous.

As a Clifford algebra, $\mathcal{Q}$ has a natural *Clifford metric*, defined as preserved by ι and obeying the Clifford rule (43).

$\mathcal{Q}$ is then used to regularize standard quantum field theory and gravity by infraquantizing them. This heuristic process is called $\mathcal{Q}$ quantization.

Much as canonical quantization represents the canonical Lie algebra of a classical theory isomorphically in the quasi-Lie algebra su(∞) of Hilbert space $\mathcal{H}$ (Appendix 17), $\mathcal{Q}$ quantization represents the Lie algebras of the system in a (finite dimensional) Lie subalgebra of sl($\mathcal{Q}$), isomorphically where possible, approximately where not.

5.1 Spin-statistics equality

A useful clue to this quantization is the observed *spin-statistics* equality

$$W = X; \tag{36}$$

W being the *spin* parity, a homotopy that rotates the system continuously through 2π. $W \doteq +1$ for even spin, -1 for odd (in units of $h/2$). And X being the *statistics* parity, interchanging two statvectors of the system: $X\psi\phi = \phi\psi X$. $X \doteq +1$ for even (Bose) statistics, -1 for odd (Fermi).

Since monadics $x \in \mathcal{Q}^1$, the first grade of $\mathcal{Q}$, obey $x^2 = 0$, the monads they describe statistically have fermionic statistics, $X \equiv -1$. We may infer from (36) that monadics are to serve as spinors, with $W = -1$.

$$\textit{Each level } {}^{\leq T}\mathcal{Q} \textit{ is a spinor space.} \tag{37}$$

The quadratic space $\mathcal{W}[T]$ of its orthogonal group $\mathrm{SO}(\mathcal{W}[T])$ will be defined (in (53)). All bosons must be pseudo-bosons composed of fermions in $\mathcal{Q}$ theory, obeying a Palev statistics [29], which is tautologically simple, with a Lie algebra such as $\mathrm{so}(n+2)$ instead of the canonical Lie algebra $\mathrm{H}(n)$. The $\mathcal{Q}$ operators for orbital variables (§8), isospin, color, hypercharge, generation, gaugeons, and the higgs are in the Lie algebra $\mathrm{sl}({}^{E}\mathcal{Q}) \subset \mathrm{sl}(\mathcal{Q})$ (not in $\mathcal{Q}$) and sums of atomic terms of a cellular level $\mathrm{sl}({}^{C}\mathcal{Q}) \subset \mathrm{sl}(E)$

The typed exterior algebra has a long beard [19, 21]. The present work uses a simplicity principle (3) and a correspondence principle (4) lacking in those first efforts.

One disturbing feature of the ι operator is that it forgets the statistics of its operand. Whether x has odd or even statistics, ιx has odd statistics.

Fortunately we encounter a similar anomaly daily in particle physics. The statistics of a particle forgets all its parts except its spin—for example, its isospin, color, and flavor.

Again, in the standard constriction of a spinor space as the exterior algebra over a semivector space, both odd-grade and even-grade exterior products constitute spinors and are subject to Fermi statistics.

We may represent this feature of nature by a difference in type. The properties of a particle that do not matter for its statistics are internal in the sense that they act on the contents of the outermost ι defining a particle statvector. They operate on a lower type than spin.

In the topological theory of quantum statistics, one expresses an exchange of a pair in terms of a macroscopic continuous rotation of the pair through π. In this infraquantum theory, the explanatory direction is reversed. A macroscopic rotation of a spin is composed of microscopic input-output operations on the semivectorial parts of the spin. For spin 1/2, the the statvectors are the factors γ^μ in $\gamma^{\mu'\mu} = [\gamma^{\mu'}, \gamma^\mu]/2$.

To incorporate the spin-statistics equation, we assume:

All quantum histories are assembled from monads described by spinors. (38)

5.2 Numbering the sets

The *sequences* of N 2-valued objects and the *combinations* of any number of N-valued objects are both 2^N in number. This suggests that the combinations in $\mathcal{C}$ might form a natural sequence and have natural identification numbers or *indices*.

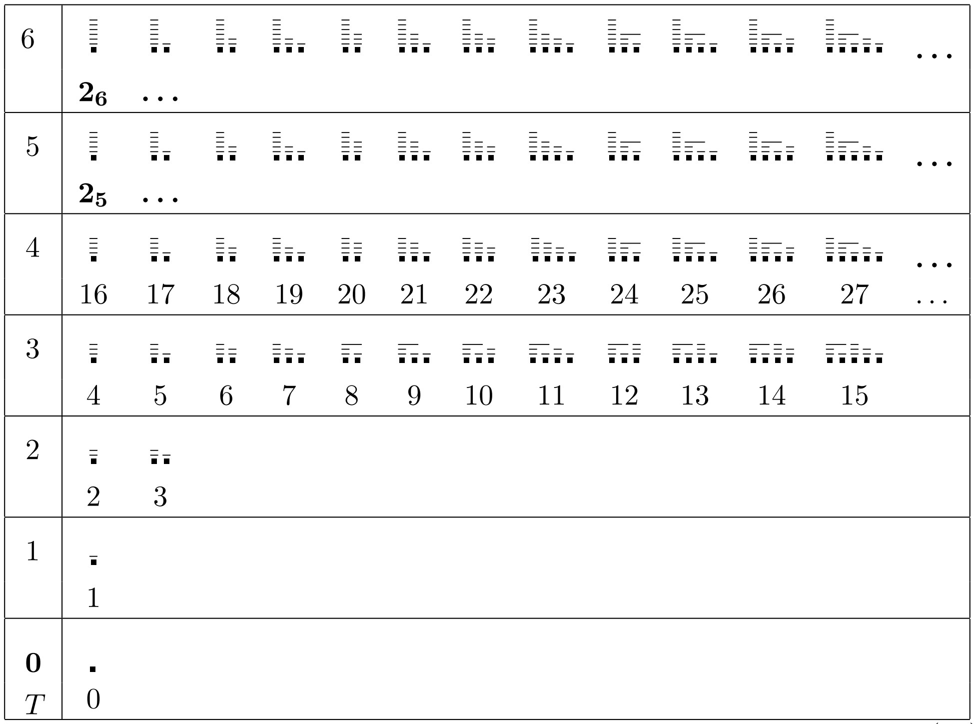
Indeed, each set $s \in \mathcal{C}$ can be regarded as a positional notation for a natural number $\mathrm{N}s$, the index of s, in a base that grows hyper-exponentially with position:

$$\mathrm{N}s = \sum_n a_n \, 2_n, \quad 0 \le a_n < b_n, \quad 2_n := 2^{(2_{n-1})}, \quad 2_0 := 1. \tag{39}$$

(Compare binary numbers $N = \sum_n a_n \, 2^n$.)

Multiplying disjoint sets adds their indices, which are thus logarithms. The successor of the set indexed with N is the set indexed with 2^N.

(40) lists some basic polyadics of $\mathcal{C}$ and $\mathcal{Q}$ with their numbers. Type 4 would overflow the page; its monadics are listed in (41). Type 5 in Planck-size type would fill the known universe many times over.



(40)

Table. Some polyadics of type $T \leq 6$. All occurences of "▪" in one polyadic represent the same empty set 1, reduplicated to simplify the graphics.

L	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\mathbf{1}^{(2^L)}$																

(41)

Table. The 16 basic monadics of type 4.

5.3 Reversible infraquantum theory

In ordinary quantum usage, statvectors in some statvector space V represent input processes or system sources, dual statvectors represent output processes or system targets, and system transformations are represented by linear operators in $V \otimes \mathrm{Dual}\, V = \mathrm{Lin}\, V$.

The practice of set theorists, however, is different. They have arbitrarily agreed to represent a map or function $y = f(x)$ of sets as a set itself, one of ordered pairs (y, x), and also to represent an ordered pair (y, x) as a set, one of the form $\{\{y, x\}x\}$; or something of that sort. They represent both sets and their transformations in the same space $\mathcal{C}$ of sets.

This conflicts with the standard quantum division of labor between V and $\mathrm{Lin}\, V$, which is the distinction between an algebra of operators and its defining representation space. The conflict is less than it seems. The operators $V \to V$ belong to $\mathrm{Lin}\, V \cong V \otimes DualV$, which is the statvector space of a single pair, namely of a system and a dual system. The classical theory requires a set of pairs, rather than just one, to represent transformations of the system because it drops the quantum requirement of coherence.

This standard quantum practice also conceals a major continuous symmetry of fermionic kinematics, one that coherently mixes annihilation and creation operators, as in Bogoliuboff transformations. If fermion input statvectors form a real vector space V, its Fermi-Dirac anti-commutation relations are invariant under so(W), where $W = \mathrm{Dup}\, V \supset V$ is defined in (53) and has the duplex quadratic form (54). This symmetry of the Clifford algebra $\mathrm{Cliff}(W) = \mathrm{Fermi}\, V$ is not induced by a linear symmetry of V but mixes V and $\mathrm{Dual}\, V$.

Thermodynamic irreversibility is built into ordinary quantum theory as this absolute distinction between input and output statvectors. It is inescapable for experimenters like ourselves, but robotic nano-experimenters can be more nearly reversible, and might warrant a reversible infraquantum theory.

One way to set up a reversible infraquantum theory, with no absolute separation between input and output statvectors, is to imbed $\mathcal{Q}$, Dual $\mathcal{Q}$, and Lin $\mathcal{Q}$ all within $\mathcal{Q}$. One can then distinguish between input and output statvectors by the sign of the energy change: positive for input, negative for output. In a frame where $iE = \partial_t \prec L_{46}$ and $i \prec L_{56}/\mathsf{N}$, a natural approximate energy is

$$E \approx \frac{\{L_{46}, L_{45}\}}{\mathsf{N}}. \tag{42}$$

5.4 Digression on Minkowskian forms

This section is not used in this work. It refers to an interpretation of monadics in $\mathcal{Q}$ as space-time vectors rather than spinors.

Proposition 1 *The Clifford rule*

$$\|x\| = \mathsf{n}xx = \mathrm{Grade}^0\, x \sqcup x \tag{43}$$

and the isometry rule

$$\|\iota\, x\| = \|x\| \tag{44}$$

define a quadratic form on $\mathcal{Q}$, associated with a bilinear form

$$\mathsf{n}yx = \frac{1}{2}\,\mathrm{Grade}^0\{y \sqcup x + x \sqcup y\}, \tag{45}$$

the Clifford metric form. *Each entry e in* (40) *has norm $\|\mathsf{e}\| = \pm 1$. For all N, the quartet $\{\mathbf{1}^{4N}, \mathbf{1}^{4N+1}, \mathbf{1}^{4N+2}, \mathbf{1}^{4N+3}\}$ has a Minkowskian signature with respect to n: three elements of one sign and one of the other. $^2\mathcal{Q} \cong \mathcal{M}$, the Minkowski 4-space. n is asymptotically neutral in the sense that the signature of n on $^T\mathcal{Q}$ is $\sqrt{2_T} \sim o(2_T)$.*

The bilinear extension of $\widehat{\mathsf{n}}$ to $\mathcal{Q}$ is also designated by n; it is not multiplicative, of course.

If norms were assigned values ± 1 independently at random, the signature of the norm would have expectation 0 and its standard deviation would be $\sqrt{2_T}$, in asymptotic agreement with the actual values for n. The Clifford form n is considered because its invariance group is much larger than the invariance group $\mathrm{SO}(^T\mathcal{Q}^1)$ of the Hilbert form on $^T\mathcal{Q}$. But is it of any use?

6 Quantization as quantification

It is well known that "second quantization" is not a quantization proper but a quantification, passing from one-quantum to many-quantum properties. Here we reformulate canonical quantization too in terms of quantification, and use it as a pattern for infraquantization. Canonical quantization in a Hilbert space $\mathcal{H}$ will be referred to as $\mathcal{H}$ quantization. It has two steps:

1. *Atomize*. Select a canonical Lie algebra V of basic system variables.
2. *Quantify*. Form a linear algebra $\mathcal{H}$ generated by V.

Step 1 amounts to guessing the atom of the quantized system. It passes from the classical system to a quantum atom. Step 2 clones that atom many times using Bose statistics.

For a linear harmonic oscillator, for example, V is spanned by three vectors q, p, i, and is also the statvector space of one phonon. The quantized oscillator is then a bosonic assembly of such phonons.

The corresponding steps of infraquantization are:

1 *Atomize.* Select a Lie algebra V of system variables. (46)

2 *Quantify* Form the algebra generated by V. (47)

One extends canonical quantization to allow graded Lie algebras, such as exterior algebras, as well as Lie algebras proper, in order to construct variables without classical correspondents, like spins and fermion annihilators, as well as those that have classical correspondents. We will assume that V and its algebra are finite-dimensional subalgebras of $\mathcal{Q}$.

The functor ext quantifies, quantizes, and gauges. (48)

It quantifies when it converts a one-quantum statvector space to a many-quantum one. It quantizes when it converts the quantum atom inferred from the classical Poisson Bracket to the quantized system. It gauges when it converts one global Lie algebra to many local ones.

A coordinate system (x^1, x^2, x^3, x^4) orders space-time events into a row of rows of rows of rows, like the continuum limit of a cubical lattice in four dimensions. Rows, however, have no continuous symmetries, besides those already present in their elements, while space-time exhibits such symmetries experimentally, as through conservation of momentum-energy. By contrast, a quantum g-simplex over a statvector space V has a simple group, a representation of $\mathrm{SL}(V)$ by its action on $\mathrm{Grade}_g \operatorname{ext} V$.

Every simple Lie algebra can be so represented. (49)

In the classical theory of space-time, events are ultimately small, fields are macrocosmic in extent, and there is no organization of space-time of an intermediate size scale. Therefore let us suppose that an infraquantum theory has an event type $\mathcal{Q}[E] \subset \mathcal{Q}$, and a field type ${}^F\mathcal{Q}$, with no type between them. An event statvector is then a monadic of level E [see (41)] in 1-1 correspondence with the polyadics of ${}^{E-1}\mathcal{Q}$

and

$$\mathrm{Dim}\,{}^{E-1}\mathcal{Q} \text{ is microcosmic but } \mathrm{Dim}\,{}^{E}\mathcal{Q} \text{ is cosmological.} \tag{50}$$

Therefore $\mathrm{Dim}\,{}^{E-1}\mathcal{Q}$ must be so large that we can leap from ${}^{E-1}\mathcal{Q}$ to the macrocosmic in a single quantification. This is the *cosmological leap*, from microcosm to macrocosm.

If N is the number of possible quantum monads in a system history, the dimension of the level E monadics must obey

$$\mathrm{Dim}\,{}^{E}\mathcal{Q}^1 = 2_{E-1} \geq N \tag{51}$$

$E = 5$ would imply $N \leq 2_4 = (2^{16})$, too small for a quasi-continuum of events; while $E = 6$ implies $N \leq 2_5 = 2^{64\mathrm{K}}$, much more than needed ($\mathrm{K} := 1024$). Then the preceding level $E-1$ has monadic dimension 64K, and its characteristic length $64\mathrm{K}\,\mathsf{X}$ may well be microcosmic. Level $E = 6$ has $2_6 = 2^{64\mathrm{K}}$ distinct monads. The Poincaré group can be approximated within present experimental error, though non-uniformly, by a subgroup of $\mathrm{SL}({}^6\mathcal{Q})$ but not $\mathrm{SL}(\overset{5}{\mathcal{Q}})$. Let us therefore assume that a system history is a ${}^6\mathcal{Q}$ simplex (of simplices ... of simplices), or equivalently a monad of ${}^7\mathcal{Q}^1$.

The standard model groups can all be defined by their actions on about $16 = 2_4$ monads, in ${}^4\mathcal{Q}^1$. If there are N monads in history, a random simplex has $\mathsf{N}/2$ vertices on the average. Since $\mathsf{N}/2 \gg 16$, operons of current history are not random but are highly organized locally, into something like a thin truss dome or graphene, perhaps only one cell thick. The aspect ratio between the long and short dimensions of this dome is far greater than for a graphene. The dome also supports the particle spectrum, sharp bands of highly coherent transmission, and so is presumably crystalline, as Newton already inferred, though in a quantum sense that must be clarified.

Classical set-theoretic operations do not produce quantum sets; the classical tree of sets bears no quantum fruit. Canonical quantum theory therefore grafts a rootless quantum-dynamical stock onto the classical tree of types, just below the level of the quantum dynamical system. This graft must occur at an infinite type for the classical infrastructure to have continuous symmetry groups, and divergences ensue.

To describe simple systems, we replace the classical tree of sets by a quantum tree of simplices $\mathcal{Q}$ that is equally autogenous, but quantum to its roots and simple on every level. Unlike $\mathcal{C}$, the tree $\mathcal{Q}$ can bear quantum theories with continuous symmetries even on its lowest branches, which are all finite-dimensional.

A simplex described by a monadic is a monad. Its one vertex is a polyad of lower level. A collection of simplices is usually described in homological algebra by a formal sum of the simplices, but in $\mathcal{Q}$ addition $+$ is mere quantum superposition, not aggregation. A multiplicity of g $\mathcal{Q}$ simplices is represented by a g-adic, a higher-level simplex. A general simplicial complex is then merely a general simplex or polyad. To emphasize that we deal with a simplex of simplices, we sometimes call it a *meta-simplex*.

We may express the two main hypotheses as one:

$$\textit{The physical system history is a metasimplex.} \tag{52}$$

We write the type level of $\mathcal{Q}$ within which the history statvector is represented as ${}^{E}\mathcal{Q}$. Classical homology works with simplices of one level, that of its vertices. Infraquantum simplices of levels $1-6$ are convenient and apparently sufficient for field theory.

For any $v \in \mathcal{Q}$, ${}^{\mathsf{L}}v : \mathcal{Q} \to \mathcal{Q},\ u \mapsto v \vee u$ is the linear operator of left exterior multiplication by v; ${}^{\mathsf{R}}v : \mathcal{Q} \to \mathcal{Q},\ u \mapsto u \vee v$, that of right multiplication by v (Appendix 17.4). The duplex space $\mathcal{W}[T]$

$$\mathcal{W}[T] := \mathrm{Dup}^{\mathsf{L}}\, {}^{T}\mathcal{Q}^{1} \subset \mathrm{Lin}\, {}^{T}\mathcal{Q} \cong \mathrm{Fermi}\, {}^{T}\mathcal{Q}^{1} \cong \mathrm{Cliff}\, \mathcal{W}[T], \tag{53}$$

has the neutral quadratic form

$$\mathsf{Q} : \mathcal{W} \to \mathbb{R}, \quad \mathsf{Q}(w) = w^2. \tag{54}$$

This defines a Clifford algebra $\mathrm{Cliff}\, \mathcal{W}[T]$. Because ${}^{T}\mathcal{Q}$ is an exterior algebra over ${}^{T}\mathcal{Q}^{1} \cong {}^{T-1}\mathcal{Q}$, its operator algebra $\mathrm{Lin}\, {}^{T}\mathcal{Q}$, a Fermi algebra, is also the Clifford algebra $\mathrm{Cliff}\, \mathcal{W}[T]$. This Clifford structure includes the Dirac Clifford algebra of the γ^{μ} as level B. We therefore re-designate its first-grade generators as

$$\gamma^{w} \in \mathrm{Cliff}^{1}(\mathcal{W}). \tag{55}$$

(40, 41) tabulate some $\mathcal{Q}$ polyadics of level ≤ 5 and all the monadics of level 4.

Dirac interpreted $\gamma^{\nu\mu}$ as a Lorentz group generator, a spin component, and a dynamical variable of the single electron. Cartan interpreted γ^{μ} as a Lorentz reflection in the μ axis. Both assumed a background Minkowski space-time that $\mathcal{Q}$ lacks. We seek a root for spin that goes deeper than classical space-time.

Schur [32] and Frobenius use spinors to represent swaps, not spins. Quantum swaps are dyadics of a statvector algebra. Let us take statvectors as our primitives, exemplified by the Fermi Clifford algebra operators γ^{q}.

Then c, c' index cell vertex monadics $\mathbf{1}^{c}, \mathbf{1}^{c'} \in {}^{C}\mathcal{Q}^{1}$ of level C, which are exchanged by $\gamma_{c'c}$. If we confine the vertices to (say) positive annihilators $\mathbf{1}^{c}$, the swaps generate $\mathrm{SL}(2_C)$. If we allow negative annihilators $\mathbf{1}^{c}$ as well, that is, creators, and allow both symmetric or skew-symmetric swaps, corresponding to $(\gamma c'c)^2 = \pm 1$, then the swaps generate $\mathrm{SO}(2_C, 2_C)$. Then $L^{c'c}$ represents the swap $\gamma_{c'c}$ on $\mathrm{Dual}\,{}^E\mathcal{Q}^1$, of the event level E.

The statvector algebra of level T is ${}^T\mathcal{Q} = (\mathrm{ext})^T\,\mathbb{R}$. Any infraquantum operon is composed of lower level ones, in the way that sets and simplices are composed of lower level sets and simplices. $\mathcal{Q}$ has commuting operators Grade and Level, corresponding to classical cardinality and level. A g-adic is an element of $\mathcal{Q}$ of grade g, corresponding to a set of cardinality g. The operon it describes is called a g-ad or g-simplex.

6.1 Cellular locality 2010.04.25

Well before Einstein, Newton doubted his law of gravity on the basis of a locality principle that has become an indispensable guide for theoretical physics:

All action is by contact.

In the standard model, the theories of the space-time structure of special relativity and the complex-phase structure of quantum theory, remain global rather than local, and the theories of the hypercharge, electroweak, and strong interactions are local gauge theories built on this global structure. Classical locality requires the Lagrangian to couple only field variables at the same space-time point; not, say, at the same momentum-energy point in the Fourier-transform space; and with time derivatives of bounded order. Diffeomorphisms respect such locality, but the canonical group violates it. The conflicting algebras of general relativity and canonical quantum theory must blend into one in a simple theory, for they both work too well to be merely suppressed. Let us suppose that a reformed version of locality will survive infraquantization.

In a topological quantum theory like that developed here, two cells shall be defined to be in contact not by coordinate relations but by shared elements, simplicial vertices. This permits one to sharpen the locality principle to *cellular locality*:

The action swaps elements between cells in contact. (56)

Then the fundamental operations are not true spins, which assume an external quadratic classical space, but swaps, which do not. When Cartan represented rotations by their action on spinors [9], Schur and others had already represented swaps in that way [32].

Infraquantum theories are naturally local in this sense. One sees this as follows.

The action of gauge theories is built up from gauge differentiators $D_\mu(x)$. Its locality depends on the action having a low polynomial degree in the $D's$, since an infinite power series e^{iaD} is a no-local translation.

Infraquantization replaces these differentiators by orthogonal-group generators $L_{m'm}$, represented by dyadics $\gamma_{m'}\gamma_m \in \mathcal{Q}$. Their monadic factors γ_m are io operators, creating and annihilating vertices of the complex on which they act. While the D's are represented in an orthogonal group of huge dimension, they are formed by cumulation from the vertices of a simplex of low type with fewer than 16 vertices. Then the vertices m, m' being swapped belong to an image of this basic cell.

A product like $D_{m'''m''}D_{m'm}$ in the dynamical phase statvector would be non-local, if it coupled vertices m and m'' that are not in contact. But such products actually appear in the context of connecting tensors $\gamma^{m"m'}$ that locate m' and m" in the standard cell of ${}^C\mathcal{Q}$. They are then local in the cellular sense.

6.2 The spin tree 2010.04.25

$\mathcal{Q}$ is a geyser of spinor spaces. One sees them as follows.

The classic spinor constructions begin with a duplex vector space

$$\operatorname{Dup} V := V \oplus \operatorname{Dual} V =: W, \tag{57}$$

with its natural, neutral quadratic form and orthogonal group. In this context, elements of V and W are called semivectors and vectors, respectively. The spinors of $\mathrm{SO}(W)$ are then multi-semivectors, the multivectors over V. Their exterior product violates $\mathrm{SO}(W)$ and is ultimately forgotten in the standard quantum theory.

The *spin tree*, the central column of (58), is constructed inductively, by iterating ext, beginning with the trivial exterior algebra $\mathbb{R}$, of grade and level $\equiv 0$. *A spinor space of any level is the exterior algebra over the spinor space of the previous level.* We may readily reconstruct the orthogonal group of which this is a spinor space. The spinor space of level T is defined to be the vector space ${}^T\mathcal{Q} \cong {}^{T+1}\mathcal{Q}^1$. Its *semivector* space is the space $V[T-1] := {}^{T-1}\mathcal{Q}^1$, which is an isotropic (= null) subspace of the neutral vector space $\mathcal{W}[T-1]$ of (53), and $\mathrm{SO}(\mathcal{W}[T-1])$ is its orthogonal group [10, 8, 12].

Orthogonal groups and Clifford-Fermi algebras sit on each rung of the spin tree:

$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
		$\uparrow$ext		
3	Fermi $4\mathbb{R}$	$16\mathbb{R}$	$32\mathbb{R}$	$\mathrm{SO}(16,16)$
		$\uparrow$ext		
2	Fermi $2\mathbb{R}$	$4\mathbb{R}$	$8\mathbb{R}$	$\mathrm{SO}(4,4)$
		$\uparrow$ext		
1	Fermi $\mathbb{R}$	$2\mathbb{R}$	$4\mathbb{R}$	$\mathrm{SO}(2,2)$
		$\uparrow$ext		
0	Fermi 0	$\mathbb{R}$	0	1
T	$\mathrm{Lin}\,{}^T\mathcal{Q}$	${}^T\mathcal{Q}$	$\mathcal{W}[T]$	$\mathrm{SO}(\mathcal{W}[T])$
Level	Algebra	Spinors	Vectors	Group

(58)

The symmetry Lie algebra of ${}^T\mathcal{Q}$ as a vector space is written $\mathrm{sl}({}^T\mathcal{Q})$. The symmetry Lie algebra of ${}^T\mathcal{Q}$ as an exterior algebra is only $\mathrm{sl}(\mathcal{Q}[T-1]$, exponentially smaller. The monad coordinate operators of level T are elements of

$$\mathrm{Lin}\,{}^T\mathcal{Q} = \mathrm{Fermi}\,{}^{T-1}\mathcal{Q} = \mathrm{Cliff}\,\mathcal{W}[T-1], \quad \mathcal{W}[T] := \mathrm{Dup}\,{}^T\mathcal{Q}^1. \tag{59}$$

In classical set theory the power set of $\mathcal{X}$ is written as $2^{\mathcal{X}}$. Therefore we write the exterior algebra of a semivector space $\mathcal{V}$ as

$$\Psi = \mathbf{2}^{\mathcal{V}}. \tag{60}$$

The orthogonal group associated with this spinor space is $\mathrm{SO}(\mathcal{W})$ where $\mathcal{W} = \mathrm{Dup}\,\mathcal{V}$. If $\mathcal{V} \subset \mathcal{Q}$ then $\Psi \subset \mathcal{Q}$.

The distinction between spinors of $\mathrm{SL}(n)$ and vectors of $\mathrm{SO}(m)$ is relativized in $\mathcal{Q}$. Namely, one and the same space ${}^T\mathcal{Q}^1$ of any level T is a spinor space relative to the orthogonal group $\mathrm{SO}(\mathcal{W}[T-1]) = \mathrm{SO}(\mathrm{Dup}\,{}^{T-1}\mathcal{Q}^1)$ of the previous level $T-1$, and is a semivector space relative to the orthogonal group $\mathrm{SO}(\mathcal{W}[T])$ of its own level. Therefore the operator ι can be spinor-valued or vector-valued depending on context.

Because of their spinorial statvectors, let us represent leptons and quarks as monons. Their statvectors are monadics of the event type ${}^E\mathcal{Q}^1$. Then gaugeons and gravitons are tetrons, with statvectors in ${}^E\mathcal{Q}^4$.

6.3 Infraquantum Pauli adjoints

Like the Dirac spinors, each level T of the spin tree $\mathcal{Q}$ has an invariant bilinear form, its Pauli form (or Pauli adjoint)

$$\mathsf{p}[T] \equiv \mathsf{p} := \mathsf{p}_{q'q}\mathbf{e}^{q'}\mathbf{e}^{q} : {}^T\mathcal{Q} \to \mathrm{Dual}\,{}^T\mathcal{Q}. \tag{61}$$

The case $\mathsf{p} = \mathsf{p}^{[2]}$ for four-dimensional spinors, usually designated by $\mathsf{p} = \beta$, is special in being skew-symmetric; the higher Pauli forms can be made symmetric. This requires

$$\forall \omega \in \mathrm{so}({}^T\mathcal{Q}) : \quad \mathsf{p}^{[T]}_{q''q'}\omega^{q'}{}_q + \mathsf{p}^{[T]}_{q'q''}\omega^{q'}{}_q = 0. \tag{62}$$

In an orthonormal basis where the metric on $\mathrm{Dup}\,{}^T\mathcal{Q}$ is diagonal with diagonal elements ± 1, the matrix for $\mathsf{p}^{[T]}$ may be chosen to be a product of the matrices for the γ^q whose squares are negative:

$$\mathsf{p} = \prod_{(\gamma^m)^2=-1} \gamma^m \quad \text{in one orthonormal frame.} \tag{63}$$

p^2 is skew-symmetric, and $\mathsf{p}^{[T]}$ is symmetric for $T > 2$. The Pauli adjoints do not have a well-defined limit form for the entire space $\mathcal{Q}$. Since the levels and their groups nest, however, the γ's and p of one level serve as well for all lower levels.

$\mathsf{p} : \mathcal{Q} \to \mathrm{Dual}\,\mathcal{Q}$ generates an anti-automorphism of $\mathrm{Lin}\,\mathcal{Q}$ that effects a *total reversal*, interchanging creators $\mathsf{e}^q \in \mathcal{Q}$ and annihilators $\partial_{q*} \in \mathrm{Dual}\,\mathcal{Q}$:

$$\mathsf{T}(\mathsf{p}\mathsf{e}^q\mathsf{p}^{-1}) = \partial_{q*}, \quad \mathsf{T}(\mathsf{p}\mathsf{e}_{q*}\mathsf{p}^{-1}) = \mathsf{e}_{q*}. \tag{64}$$

It defines an invariant form of pseudo-expectation value $\mathrm{Av}\, A := \psi\mathsf{p}A\psi$ for any operator $A \in \mathrm{Lin}\,\mathcal{Q}$ that preserves transformation properties under $\mathrm{SO}^{[T]}$. For example, if A^m is a vector of operators under $\mathrm{SO}^{[T]}$ then $\mathrm{Av}\, A^m$ is a vector of scalars.

The Pauli form p is neutral, however, not positive definite. Square operators like $A = \psi \otimes \mathsf{p}\psi$ would have positive averages in a pre-relativistic theory but can have negative averages for $\psi \in \mathcal{Q}$.

The orthogonal group $\mathrm{SO}(4,4)$ may be a symmetry of a cell or of the dynamics statvector, but is not a symmetry of the ambient dome, which reduces $\mathrm{SO}(4,4)$ locally to the group .

6.4 Cumulation

A monadic of any level embraces parts of the level below. Lower-level operators induce higher-level ones; in particular, quantum spin operators induce classical orbital operators as follows.

$\mathcal{Q}$ theories represent the statvectors of simplices of simplices by multivectors of multivectors, polyadics of polyadics. Each polyadic in level $\mathcal{Q}[T+1]$ is a unique exterior polynomial in insertions of the basic polyadics of the previous level ${}^T\mathcal{Q}$, as in (40). Any transformation of one level thus extends naturally to monadics of the next level, and thence to polyadics. This defines a unique *exponentiation*

$$\Pi : \mathrm{SO}({}^T\mathcal{Q}^1) \to \mathrm{SO}({}^{T+1}\mathcal{Q}^1), \quad \mathrm{SO}(\mathcal{Q}^1) \to \mathrm{SO}(\mathcal{Q}^1) \tag{65}$$

representing each orthogonal Lie algebra within the next. Iteration results in an *nth exponential representation*

$$\Pi^n : \mathrm{SO}({}^T\mathcal{Q}^1) \to \mathrm{SO}({}^{T+n}\mathcal{Q}^1) \tag{66}$$

acting on multivectors, over multivectors, over ..., over ${}^T\mathcal{Q}$.

Cumulation Σ is the infinitesimal version of the exponentiation Π:

$$\Pi(1 + \lambda x) = 1 + \lambda \Sigma x + O(\lambda^2). \tag{67}$$

It occurs as θ in [26] for example. If x is an operator on $\mathcal{Q}$, and y is any $\mathcal{Q}$ simplex, $\Sigma x \cdot y$ sums x over the elements of y. It is often written in terms of AC operators ψ, ψ^{h} as $\psi^{\mathsf{h}} x \psi$. Here we may write, schematically,

$$\Sigma x := \iota x \frac{\partial}{\partial \iota_{\mathrm{o}}} \tag{68}$$

This replaces an outermost ("o") iota ι_{o} by ιx, and sums all such terms. Σ is a Lie homomorphism of the Lie algebra of each level in that of the next. $(\Sigma^n x)$ represents the operator $x \in \mathrm{Lin}({}^T\mathcal{Q})$ in $\mathrm{Lin}(\mathcal{Q}[T+n])$. If x is further subdivided, its parts no longer support the same Lie algebra.

Let us call x the *cumulandum* of Σx, and Σx the *cumulant* of x, and write $x = \delta y$ to mean that $y = \Sigma x$, and $x = \delta^n y$ to mean that $y = \Sigma^n x$. Thus when δ is defined, it is a left inverse of Σ.

6.5 Negative probability

Usually one uses unitary representations of (2). These are necessarily ∞-dimensional. For example, the statistical form for the history statvector of a Dirac spin-1/2 quantum is an integral

$$\|\psi\| = \int_{\mathcal{S}} (dx^{lmn} \psi^*(x) \beta(x) \gamma_{lmn}(x) \psi(x) \tag{69}$$

over an infinite spacelike subspace $\mathcal{S} \subset \mathcal{M}$.

Infraquantization must trim this infinite structure to fit into the finite bed ${}^F\mathcal{Q}$ without contradicting with the many experiments that endorse it. Regularity demands finite-dimensional representations. In conjunction with Lorentz invariance, this leads to probabilities outside the usual interval $[0, 1]$.

These have nothing to do with the so-called negative probabilities sometimes used to describe ordinary quantum interference of probability amplitudes.

Negative probabilities have a useful physical interpretation [13]. They are no more problematical than a negative bank deposit or a negative energy. The number is the change in population of the system, positive for input and negative for output. This permits us to use a single vector space V for both input and output operations, instead of V and $\mathrm{Dual}\, V$, and distinguish input from output statvectors in V by the signs of their norms.

The Dirac form $\|\psi\|$ of (69) as written is not definite. Its density is the product of a current vector $j_\mu(x)$ of the electron with a normal vector $n^\mu(x)$ to the surface $\mathcal{S}$. If both point everywhere into the future, $\|\psi\|$ is positive. But by the fermionic symmetry between input and output, such a statvector has a partner whose current vector points everywhere into the past and whose norm is negative. This is not considered a problem. We interpret such statvectors ψ of negative norm as describing an output rather than an input. Under further inspection of this example, especially of the electric charges, we see that if the input is an electron, the partner output is a positon. We adopt the Dirac rule in general [13]:

$$\|\psi\| \;>\; 0 \quad \text{for input,} \tag{70}$$

$$\|\psi\| \;<\; 0 \quad \text{for output.} \tag{71}$$

Let $\psi_+ \in V$ and $\phi_- \in \mathrm{Dual}\, V$ be input and output statvectors, respectively, with associated counters whose readings are n_+ and n_- The probability of a transition $\psi_1 \to \phi$ is the mean value of the increase in n_- per unit increase in n_+:

$$P = \mathrm{Av}\, \frac{\delta n_-}{\delta n_+}. \tag{72}$$

When a positive input count is followed on the average by a negative output count, the transition probability is negative. This indicates the conversion of some input quanta to anti-quanta. This is a meaningful description of a process, feasible or not. As a result probabilities can theoretically be negative or greater than unity.

This concept of number-sign may conflict with one determined by the arrow of time, which counts thermodynamically irreversible inputs and outputs as positive. But thermodynamics is a singular organized limit of a reversible theory as a particle number approaches ∞. Experiments carried out within a minute metasystem may not exhibit irreversibility, which is then manifest in the meta^{2}system.

The probability of a transition $\psi \to \psi'$ defined by two monadics depends on the Fermi sea in which the experiment is carried out. An experimenter can borrow from the sea; the resulting vacancy in the sea counts as -1 event. A transition that has negative probability is also one that would be impossible if the sea were empty.

From the one-quantum viewpoint the Fermi sea is part of the metasystem, the ambient environment, not the system. It has too many variables to be the system of any experimenter that the universe can accommodate. In the many-quantum or field-theoretic viewpoint, however, the system can include at least a droplet of the Dirac sea, described by a fiduciary polyadic. It then has entropy 0, and so is highly ordered, frozen stiff, like the ether of Newton and Fresnel, who may have wondered about how we and the planets move so freely through such a crystal.

In the standard theories the polyadic statvector $\mathbf{E}_{\text{Dirac}}$ of the Dirac sea is too divergent to be of much use. For example, its grade, the number of events in the history of the sea, is infinite. It is usually taken into account roughly by subtracting an infinite constant from the Hamiltonian that is said to make the vacuum energy expectation zero, although experimentally it is not exactly zero, and mathematically neither is $\infty - \infty$. Since the subtraction requires a Fourier transform, it is non-local as well as infinite. In infraquantum theories the grade of the event sea $\mathbf{E}$ is no longer infinite but merely too large to be counted as yet, and the vacuum energy is finite.

7 Dynamical law 2010.04.29

Canonical quantum theory uses a synchronous description, refering all transformations to one time, say $t = 0$. All its fields share the variables of that time-section. The theory represents a process P' at another time $t' \neq 0$ by the operator for the process P at $t = 0$ that has the effect of P' at t', assuming that the system undergoes a given development in the intervening time interval, with no external intervention. The development of the isolated system then causes its variables to depend explicitly on t while acting on statvectors defined at $t = 0$. This is the Heisenberg picture. One is then free to shift the time-dependence from variables to statvectors by a time-dependent unitary transformation, to a Schrödinger picture. This formulation is not relativistically invariant, but it reduces the number of independent variables infinitely, from all the $x(t)$ to one $x(0)$, making a finite atomic theory possible despite the infinitude of times.

7.1 History statvectors 2010.04.29

Dirac early on imagined a more general quantum theory based on paths or histories, that retained the degrees of freedom and the wide covariance of the Lagrangian theory. Its descriptions are em diachronic, while those of the canonical quantum theory are *synchronic*. Diachronic variables at different events vary independently, so diachronic theories describe a broader class of experimental situations. Dirac deduced the classical principle of stationary action metaphorically by corresponding classical paths to quantum paths of stationary phase. He sacrificed some mathematical meaning for this, speaking of sums over unsummably many classical paths.

Let us recall how the dynamics of a quantum system is summed up in a history statvector $\mathbf{D}$. Let $U(t'-t) = \exp[-iH(t'-t)]$ represent the dynamical development from time t to t' in a canonical quantum theory with Hamiltonian H. A Hilbert metric form h is assumed, and may be prefixed, superfixed, or infixed: $\mathsf{h}\psi = \psi^{\mathsf{h}} = \psi\mathsf{h}$. $U(t'-t)$ is unitary with respect to h. Then a discretized path from an initial time T_1 to a final time T_2 is represented by the dynamical statvector

$$\mathbf{D} := U(\delta t) \otimes \ldots \otimes U(\delta t) = \bigotimes_1^N U(\delta t), \quad \delta t := \frac{T_2 - T_1}{N}. \tag{73}$$

The Feynman path integral theory would describe this dynamics by the diagonalized product

$$D = U(q_n, q_{n-1})U(q_{n-1}, q_{n-2}) \ldots U(q_2, q_1), \tag{74}$$

a function of only n coordinates q. D is not a unitary invariant. It is designed to give only a space-time view of the development. The matrix element of $\mathbf{D}$ is a function of $2n$ coordinates q, however. This permits a history statvector $\mathbf{D}$ to be a unitary invariant and give a view of the development in any frame.

An experiment statvector of the dual form

$$\mathbf{E} = \psi(T_2)^{\mathsf{h}} \otimes E(n-1) \otimes \ldots \otimes E(1) \otimes \psi(T_1) \tag{75}$$

connects the atoms of $\mathbf{D}$ and provides input and output at the ends of the process. The two together define a path amplitude

$$\mathbf{D} \circ \mathbf{E} = \psi(T_2)^{\mathsf{h}} U(\delta t) E(T_2 - \delta t) \otimes \ldots \otimes E(T_1 + \delta T) U(\delta t) \psi(T_1). \tag{76}$$

In this example, a well-defined experimental operation $E(t)$ is carried out at each time $t_n = n\delta t$, but in principle superpositions of such products are also allowed, entangling them.

The limit as $N \to \infty$ may not exist. Infraquantum theory is finite-dimensional, and so has no such divergence problem, but it may still give absurdly large theoretical values for quantities that are experimentally small. The relativistic covariance of the canonical theory cannot be taken for granted, since the time slice is not a covariant construct. But we can assure that the infraquantum theory is covariant under a given simple group by building it with covariant processes from covariant atoms. To guarantee Lorentz invariance exactly, we choose the group of the infraquantum theory to include the Lorentz group. To approximate Poincar'e invariance, we choose the group of the infraquantum theory to have the Poincaré group as a singular organized limit, as the Yang group SO(3, 3) does.

Let us suppose for now that an infraquantum system history is maximally described by a history statvector $\mathbf{E}$ in a finite-dimensional subspace $V \subset \mathcal{Q}$. This means that V, and hence $\mathcal{Q}$, contains correspondents of both input and output statvectors of the canonical theory. A classical history is a singular organized limit.

The correspondence between inputs and outputs that make up an assured transition is now represented by a linear operator $\mathsf{h} : V \to V$, interchanging creators and annihilators, and reversing order of products. When $\phi = \mathsf{h}\psi$, this means that the transition $\psi \to \phi$ goes on every trial.

Canonical quantum dynamics permits us to measure the energy by measuring a Hamiltonian operator H constructed from canonical variables; perhaps by weighing the system. But the quantum energy can also be measured by measuring a frequency, as symbolized by the definition $E := i\hbar\partial_t$. This does not work in the classical limit $\hbar \to 0$, where for finite energy the frequency $\to \infty$. Nevertheless this conception of energy, based on frequency, seems more fundamental than the classical notion, based on work; but it is the frequency of a statvector, a statistical construct. The dynamical equation equates energy to a Hamiltonian, which can be measured on one system, but is not always the energy. Weighing the system seems to be a way to determine its energy non-statistically.

7.2 Infraquantum scattering 2010.04.21

To compute scattering amplitudes let us find history statvectors in $\mathcal{Q}$ representing the input and output phases of the scattering experiment.

Commonly a scattering statvector for a fermion of momentum k_m has the singular form

$$\psi(k) = e^{-ik\cdot x}\psi_0\chi \tag{77}$$

where the statvector χ provides values for the quantum numbers of simple groups like isospin and Lorentz spin, and ψ_0 is a singular "seed" statvector of momentum eigenvalue $p_m = 0$. $\psi(k)$ is made from ψ_0 by a translation in momentum space generated by ix. Metaphorically, ψ_0 is the principal statvector of a position basis.

Infraquantization need not modify χ but simply takes it to be a statvector in ${}^{C}\mathcal{Q}$.

Infraquantization plausibly replaces the exponential by a normalized orthogonal transformation:

$$\psi)(k) = e^{-ik\cdot x} \prec\!\!- N^{-1}\, e^{\mathsf{X}\, k^m L_{m5}} = \overline{\psi}(k). \tag{78}$$

Although the norm of the usual exponential is infinite, that of its reform is finite, and designated by N^2.

To deal with the "seed" statvector ψ_0, let us first replace it by a projector on momentum $\mathbf{p} = 0$, designated by $\delta(\mathbf{p})$. The probability operator for the incoming quantum is then

$$\rho(\mathbf{p}) = e^{-ik\triangle x} \cdot \delta(\mathbf{p}) \tag{79}$$

Specifying both the time and the energy of one event is impossible due to complementarity. One evades this problem in the canonical theory by allowing an infinite time to prepare the system, requiring only that the task be completed by a specified time t. The time t may then be continuously varied to define a sharp frequency. In this way time and the energy can both be known with arbitrarily small indeterminacy product $\Delta t \Delta E$. Canonical quantization corresponds a classical infinitesimal transformation to a quantum one by a Lie-algebra homomorphism, and then uses i to correspond observables to observables.

In an infraquantum theory, we do not have forever to prepare an energy, we cannot vary the time continuously, and we do not have an absolute i. Nevertheless a correspondence survives between such skew-hermitian operators, not hermitian operators. A skew-symmetric correspondent $\widehat{\partial}_t = \overline{E}$ is assumed to time-translation ∂_t, and a skew-symmetric correspondent $\bar{t}$ to energy translation it.

Since the correspondent $\widehat{\imath}$ of i is not central, we cannot form correspondents of t and E from those of $\bar{t}$ and $\overline{E}$ by simple multiplication. The factor-ordering problem is even greater for infraquantization than for canonical quantization.

If we have made a time operator $\widehat{t}$, we may define an instant by the projection operator P_{t_0} on a specific eigenvalue t_0 of the operator $\widehat{t}$. Since none of the spatial coordinates $\widehat{x}^k$ commute with $\widehat{t} = \widehat{x}^4$, P_{t_0} has little resemblance to the classical or canonical-quantum instants.

It is axiomatic in classical physics that all instants include the same infinite number of events. This is incompatible with regularity, which permits only a finite number of independent events in each time slice. Infraquantum theories typically describe histories with time-slices that grow and then shrink with time. This impairs the correspondence between infraquantum dynamics and the Hamiltonian dynamics of canonical theories. We assume that Hamiltonian dynamics applies to middle age, $t/\mathsf{NX} \to 0$ in the singular limit $\mathsf{NX} \to \infty$, $\mathsf{X} \to 0$. To extend dynamics to periods comparable to the total time NX, we must allow for information increase and decrease, for example by replacing some statvectors by probability operators.

Infraquantization admits a more pragmatic construct of dynamical law than classical physics. Classical physics traditionally took the "perspective of eternity", excluding Observer and Law from the physical universe, and imagining them to be fixed apriori instead. The Law was given much symmetry, the Observer none. Special relativity and quantum theory pluralized, naturalized, and activated the Observer, by accepting observers into the physical universe, relating us by a relativity group, and predicting that our determinations of a system variable changes some unobserved variable by a finite unpredictable amount.

In a typical quantum experiment, the experimenter inputs a beam of systems undergoing the experimental process. Therefore we may represent the metasystem for this purpose by a virtual population or reservoir of replicas of the system, and represent an input operation by a selection from this reservoir. Then the ideal quantum experimenter, while no longer infinite, is still exponentially larger than the system, which is therefore a logarithmically small part of the universe. A quantum theory does not require a universal ontology and a deterministic law; a manual of feasible experiments to be done in a rather coarsely specified ambience, and a statistical prediction of their outcomes will suffice. The dynamics of the system includes the influence of the mostly unobserved metasystem on the system undergoing experiment. This influence is sometimes expressed by external fields or sources. The ambience may have a finite temperature, but here we approximate it as cold, coherent in the quantum sense.

7.3 Infraquantum propagators 2010.04.21

Dynamical theories in classical space-time express vacuum autocorrelation amplitudes among any number of generating variables $x^1, \ldots, x^n$ by

$$\langle x^n \ldots x^1 \rangle_0 = \operatorname{Tr} x^n \ldots x^1 \mathbf{D} = \mathbf{E} \circ \mathbf{D}, \quad \mathbf{E} := x^n \ldots x^1. \tag{80}$$

These become propagators when the variables are monadic statvectors, commonly associated with specified space-time points. Let us retain this form in the infraquantum theory and merely reform **D**. **E** is now a statvector specifying an experiment and $\mathbf{E} \circ \mathbf{D}$ is the amplitude for **E** under the dynamical statvector **D**.

To reform the usual dynamics, let us absorb the usual imaginary factor i/h into the action A, which then becomes a dimensionless skew-symmetric real *skew-action operator* $\Phi \nrightarrow iA/h$ on history statvectors, representing an imaginary phase:

$$\mathbf{D} = e^{\Phi} \nrightarrow e^{iA/h} = e^{i\int (dx)L/h}, \quad \Phi \in \text{Lin}\, \mathcal{Q}[E]. \tag{81}$$

The history amplitude **D** is used to compute propagators as in (80). They in turn define the time-translation operator $W^{t'}{}_t$ connecting an input statvector for one time-slice t with that for a later time-slice t'. Choose independent variables st making up a complete set with time t, and basic eigenstatvectors e^{st} with the indicated eigenvalues. Then the transition amplitude can be written as

$$W^{s't'}{}_{st} := \text{Tr}\left[\overline{e}^{s't'}\, e_{st}\, \mathbf{D}\right]. \tag{82}$$

In the continuum theory there is supposed to be a limiting case $t' = t + dt$, of the form

$$w^{s'(t+dt)}{}_{st} = [\delta'(t) + \widetilde{H}(s', s, t)]dt. \tag{83}$$

This defines the skew-Hamiltonian operator $\widetilde{H}(t)$.

Today the term "vacuum" has been thoroughly relativized [41]. It stands for the part of the universe that is relegated to the metasystem, which is observed only coarsely. To escape the preconception of a uniquely defined, absolutely empty, physical vacuum, let us speak of an ambient medium or *ambience* rather than "the vacuum". The ambience enters the theory in the determination of the physical input/output statvectors and in the dynamical law of the system.

Sometimes the ambience is supposed to have some great symmetry, and sometimes a temperature. Such assumptions must be used with some discretion, since the metasystem includes the experimenter, who breaks all observable symmetries and is not in thermal equilibrium. The cosmos exhibits little symmetry, and the symmetry near the Big Bang is less than that of our ambience today. A vacuum symmetry is an approximation to the universe in the way that a tangent is an approximation to a curve. It extrapolates local conditions to the rest of the universe.

To study the quantum structure of space-time we we must include some of it in the system. We also must have a still greater place to stand, however; to satisfy the postulates of quantum kinematics, the metasystem must be exponentially larger than the system. So we must leave most of space-time structure in the metasystem.

If a quantum description of our metasystem would require a statvector space of (say) $10^{100} \sim 2^{300}$ dimensions, the largest system we can fully observe has only about 300 dimensions in its statvector space. Since $300^{1/4} \sim 4$, this would restrict the system to a microcosmic space-time cell only $4\mathsf{X}$ on each edge, imbedded in the ambient space-time. Like an ice-cube in a glacier, its quantum structure will then be mainly determined by its ambience.

Let us assume that the standard model spins and charges originate as spin-like operators on a monadic statvector space ${}^{C}\mathcal{Q}^1$ of some "cell type" C within the typed statvector algebra $\mathcal{Q}$ of §5. We require that $C = 4$ accommodates not only the Yang Lie algebra $\mathrm{so}(6-n, n)$, which reforms the Poincaré and canonical Lie algebras, but also the unitary charges of the standard model, which are already semisimple:

$$C = 4, \quad \mathrm{Dim}\,{}^{C}\mathcal{Q}^1 = 16; \quad E = 6, \quad \mathrm{Dim}\,{}^{E}\mathcal{Q}^1 = 2_5. \tag{84}$$

7.4 Dynamics as ambience 2010.04.21

Recognizing that the quantum system is minute compared to its ambience permits another reading of dynamical relations:

Dynamics is an autocorrelation within the system induced by the ambience. (85)

The dynamical law for any small part S of the universe is assumed to be a history statvector $\mathbf{D}$ for S that serves as statistical surrogate for the rest of the universe and assigns amplitudes to experimental statvectors $\mathbf{E}$ describing a coherent experiment. More generally, a probability operator for the history could appear instead of either statvector $\mathbf{D}$ or $\mathbf{E}$, and gives less information than a statvector.

In a classical framework such a theory of dynamics begins an infinite regression. It seems that to describe how the ambience acts on the system requires an extension of the dynamics of the system to the ambience. The principle (85) seems to call for a dynamics rather than define one.

In a quantum theory, however, the system is described by statvectors that represent operations of the metasystem, and therefore of the ambience, on the system. Such statvectors also describe the metasystem, by its operation on the system. The theory predicts the probability amplitude for a certain action of the metaystem on the system to be the contraction of the experimental statvector $\mathbf{E}$ with the dynamical statvector $\mathbf{D}$. No additional dynamical theory is required.

7.5 Organizational entropy 2010.04.23

The existing concept of organization is based on the concept of Hamiltonian operator. It concerns ground statvectors, eigenstatvectors of minimum Hamiltonian eigenvalue. The Hamiltonian generates the development from one time slice to another determined by the history amplitude. In a canonical theory this is assumed to be a unitary transformation. This builds in a singular theory of time, and may badly break Poincaré invariance. The usual condensation theory is also based on symmetry rearrangement, the contraction of symmetry groups to Nambu-Goldstone groups, which are again usually singular. Both of these theoretical strategies incur infinities.

In the present project, it would seem absurd to contract a regular symmetry to a singular one so that we then have to of de-contract it. Here we [hopefully] base a theory of organization on a regular diachronic dynamical statvector instead of a singular Hamiltonian, and on rearranged rather than broken symmetry.

The concept of organization useful for superfluids is off-diagonal long-range order in a ground statvector of the Hamiltonian. This presents conceptual problems for an infraquantum theory. The Hamiltonian construct requires a time variable that the real infraquantum theory lacks, except in just the singular organized limit we are trying to formulate.

We can measure organization within the framework of history statvectors, first canonical and then infraquantum, without singling out a time variable.

For a given dynamical statvector $\mathbf{D}$ differences in the system are represented by differences in the experimental statvector $\widehat{\mathbf{E}}$. Systems are represented by experimental statvectors confined to one instant of time. The same system may be represented by a statvector at a different instant of time. Then the two $\mathbf{E}$ statvectors are connected by a time-development operator defined by $\mathbf{D}$.

Let us study how to analyze an infraquantum system into time slices related by a time-development. If the history statvector space is ${}^E\mathcal{Q}$, with projection operator $P[E]$, the first problem is to analyze $P[E]$ into a product of commuting time-projections

$$P[E] = \prod P_t[E], \quad [P_t[E], P_{t'}[E]] = 0 \tag{86}$$

in a way compatible with the dynamical statvector $\mathbf{D}$. Let us assume that this defines a factorization of ${}^E\mathcal{Q}$ into factor spaces $\mathcal{Q}_t[E]$:

$$ {}^E\mathcal{Q} = \bigvee_t \mathcal{Q}_t[E] \tag{87}$$

Each space has its own operator algebra and trace operation Tr_t. Given projectors P and P_t for the history and one instant t, the marginal distribution $\rho_{\mathrm{not}\ t}$ for the rest of history is defined by

$$\rho_{\mathrm{not}\ t} = \mathrm{Tr}_t\, P P_t \tag{88}$$

One defining property of an instant t is that when P_t is one-dimensional, so is $\rho_{\mathrm{not}\ t}$.

Typically the entropy of a one-quantum marginal probability operator grows without bound with n, the number of quanta, and D, the dimensionality of the one-boson statvector space. One well-known invariant measure of condensation is a marginal entropy that remains finite and bounded in this limit.

The prototype is a Bose condensation into a single statvector one-boson statvector ϕ. Some condensates are approximately represented by a tensor power of the form

$$\Phi = \overbrace{\phi \vee \ldots \vee \phi}^{n \text{ factors}} = \bigvee_1^n \phi. \tag{89}$$

Φ describes a union of n bosons all associated with the same statvector ϕ. The associated one-boson marginal probability operator is

$$\rho := \mathrm{Tr}_{n-1}\, \Phi \mathsf{h} \Phi = \phi \mathsf{h} \phi. \tag{90}$$

Its entropy is

$$S = -\,\mathrm{Tr}\, \rho \ln \rho \tag{91}$$

in units of k. This is 0, while the marginal entropy of a random aggregate is $\ln D$, infinitely greater in the limit of large D.

The entropy of the marginal one-quantum probability operator is defined for the diachronic quantum theory as simply as for the synchronic, thanks to the finiteness of the theory. In an infraquantum theory the assembly is carried out by ext, the quantum correspondent of power-set formation or set-exponentiation. The trace is over all histories, that is, over all simplicial complexes, which are finite in number.

In some cases of physical interest, the condensation is not of individual quanta but of Cooper pairs, or tetrads, or of polyads of some even grade g that is held fixed as $n, D \to \infty$. Then the condensation shows up as a polyad entropy that is bounded as $n, D \to \infty$.

7.6 Infraquantum ground statvector

The ground statvector ψ is usually defined by $H\psi = E_{\min}\psi$ where $E_{\min}$ is the minimum eigenvalue of the Hamiltonian H. While there is no Hamiltonian in an infraquantum theory, there is the propagator $\widehat{W}$ between two adjacent time-slices. This is given by (82) with the infraquantized dynamical statvector $\widehat{\mathbf{D}}$ for $\mathbf{D}$. (82) includes a kinematic transformation $t \to t'$ besides the induced dynamical transformation $s \to s'$. There seems to be no reason in an infraquantum theory, which has a first time and a last, for $\widehat{W}$ to define a one-parameter group of time translations. The existence of such a group is likely a consequence of organization and limited to a middle range of temporal resolution, much longer than X and much shorter than the age of the universe. The infinitesimal generator of this limiting group is the Hamiltonian operator of the time slice.

The Hamiltonian mode of description singles out a rest frame and a time variable. This is harmless when the condensate also defines a rest frame, as does a superconducting crystal. But our goal is to describe the ambient "vacuum" condensate, which is Lorentz covariant. A description that is transparently Lorentz covariant will sometimes be more useful than one which masks this covariance.

We must describe how the diachronic dynamics $\mathbf{D}$ enters into a condensate statvector $\mathbf{E}$. We review the canonical theory to guide the infraquantum one.

In the canonical theory, an input process ψ is carried out over a long interval in the remote past, $-T \geq t > -\infty$, and its totally time-reversed output process $\psi^{\mathbf{T}}$ in the remote future, $\infty > t \geq T$. If ψ is an energy eigenstatvector, a shift δt in the time between ψ and $\psi^{\mathbf{T}}$ changes the history amplitude by a phase-shifting factor $e^{\pm i\omega\delta t}$, and $\hbar\omega$ is the energy eigenvalue of the system.

The infraquantum theory has an operator corresponding to the time shifting operator ∂_t, namely the Yang skew-energy $L_{64} =: \widehat{\partial_t}$. And one corresponding to the i of the phase-shifting factor, namely $\widehat{\imath} = L_{65}/\mathsf{N}$. However $\widehat{\imath}$ is not central until an appropriate condensation occurs and a singular organized limit is taken.

It seems that the usual energy construct requires a limit process $T \to \infty$ for its definition. If we copy this in the infraquantum theory, we invalidate the guarantee of finiteness; the limit may diverge. Instead we choose an appropriate finite time interval for the experiment such as

$$\sqrt{\mathsf{NX}} > t > -\sqrt{\mathsf{NX}}. \tag{92}$$

This is short enough to be far from the beginning and end of time, and so approximate translational invariance. It is long enough to include a great many X's and so approximate continuity.

7.7 Infraquantum organization

7.8 Unbreaking symmetry

The problem is to describe Nambu-Goldstone symmetry transformations on a condensate without breaking the simple symmetry of the pre-condensate. Let us tackle the example of an organization of many spins 1/2.

Symbols: The individual spin vector is $\mathbf{s}$. The statvectors for an individual spin are spinors ψ, ψ^{h}. The collective spin is $\mathbf{S} = \psi^{\mathsf{h}}\, \mathbf{s}\, \psi$. For a set of N spins with statvector Ψ, the mean value of the collective spin is $\overline{\mathbf{S}} := \Psi^{\mathsf{h}}\, \mathbf{S}\, \Psi$. This is normalized to unit length to define the mean spin direction

$$\overline{\mathbf{e}} := \frac{\Psi^{\mathsf{h}}\, \mathbf{S}\, \Psi}{|\Psi^{\mathsf{h}}\, \mathbf{S}\, \Psi|}. \tag{93}$$

Therefore the collective spin about the mean collective spin is

$$\mathbf{S} \cdot \overline{\mathbf{S}} := \mathbf{S} \cdot \Psi^{\mathsf{h}}\, \mathbf{S}\, \Psi. \tag{94}$$

It is plausible to identify $\overline{\mathbf{s}}$ with the generator of the unbroken symmetry. This leaves two Nambu-Goldstone symmetries, usually approximated by two translations of the NG boson, a singular limit of the regular symmetry we seek. More accurately they are two rotations about axes normal to $\mathbf{S}$. They can be approximated well by translations because they are usually restricted to small rotations of a long vector. Rotating one spin $\mathbf{s}$ through a full 2π turns S only through $2\pi/\mathsf{N}$. It is only when we unthinkingly use the translational approximation for large rotations that infinities creep in.

The NG rotations are generated by two components of $\mathbf{S}$ orthogonal to $\overline{\mathbf{S}}$; let us designate them by $\overline{\mathbf{S}}_1$ and $\overline{\mathbf{S}}_2$.

The three generators $\overline{\mathbf{S}}_1, \overline{\mathbf{S}}_2, \overline{\mathbf{S}}$ have well-defined physical meaning for the ensemble of ensembles of spins statistically described by the statvector Ψ. They do not have the usual form of observables. They are certainly not observables of one spin, since they involve many. And they are not observables of the collective, since they involve the statvectors Ψ, Ψ^{h} of the collective. They may be interpreted as joint observables of one spin and one collective of spins.

8 Gauging as quantification

The Yang simplification adjoins dimensions to the usual (x, y, z, t). A Yang-space point y, relative to any frame, carries a space-time point x, a cotangent vector p, an element a of an so(n), and an infinitesimal complex-plane rotation

$$\begin{bmatrix} 0 & -\phi \\ \phi & 0 \end{bmatrix} \in \mathrm{so}(2), \tag{95}$$

in the array

$$y = \left[\begin{array}{cc|ccc} 0 & -\phi & -x^1 & \dots & -x^n \\ \phi & 0 & -p^1 & \dots & -p^n \\ \hline x^1 & p^1 & 0 & \dots & a^{n1} \\ \vdots & \vdots & a^{12} & \dots & a^{n2} \\ \vdots & \vdots & \vdots & & \vdots \\ x^n & p^n & a^{1n} & \dots & 0 \end{array}\right]. \tag{96}$$

Thus y includes the elements of a gauge connection, which associates a Lie algebra element a with a direction p (or its dual) at a point x. This suggests that a singular quantum gauge field might actually be a limo of a regular sea of events, taken from a quantum space like a Yang space with an appropriate orthogonal group.

The spin group represented within the Clifford algebra Cliff $\mathcal{W}[4]$ of level 4 is the double covering Cov SO$(4,4)$. This contains no SO$(5,1)$ subgroup. Instead let us adopt SO$(3,3) \subset$ SO$(4,4)$ for the Yang group for now. SO$(4,2)$ would also fit into SO$(4,4)$. The spinors of Yang SO$(3,3)$ with 8 real dimensions fit into the 16 dimensions of ${}^4\mathcal{Q}^1$, but not into the monadics of a lower level.

$\mathcal{Q}$ grade counts vertices of the cell, $\mathcal{Q}$ level counts nested iotas, and the basic $\mathcal{Q}$ dynamical operators $L^C \in \mathrm{so}(\mathcal{W}[C])$ of the cell level C count components of generalized angular momentum in units of the roots of this Lie algebra. The eight spin-like atoms of orbital angular momentum $\widehat{x}^m$ and $\widehat{p}_m$ ($m \in 6$), and the quantized imaginary $\widehat{\imath}$, are among the 15 generators γ^C of a Yang so$(3,3)$ Lie algebra, forming a Lie subalgebra of the 120-dimensional Lie algebra so$(\mathcal{W}[C] \subset \mathrm{Lin}\, {}^4\mathcal{Q}^1 \cong \mathrm{Cliff}\, \mathcal{W}[C]$ of level C

The unitary charges of the standard model can be represented using another 6 real dimensions and the quantized imaginary $\widehat{\imath}$ already constructed. This still fits into the 16-dimensional spinor space ${}^C\mathcal{Q}^1$ with $C = 4$.

The matrices of so$(\mathcal{W}[C])$ are too small to usefully approximate our usual orbital variables. They best represent spins, the "atoms" of the orbital variables. The orbital variables are cumulants of such spins. A Yang so$(3,3)$ of level C is faithfully represented on level E by second cumulants of its generators L^C (§6.4). The L^C in one $\mathcal{Q}$ frame are, up to constant multipliers, the operators

$$\begin{array}{lcl} \delta\widehat{x}^m & = & \mathsf{X}\,\gamma^{m5}, \quad m,n = 1,2,3,4, \\ \delta\widehat{p}_m & = & \mathsf{E}\,\gamma_{m6}, \\ \delta\widehat{\imath} & = & \mathsf{N}^{-1}\,\gamma^{65}, \\ \delta\widehat{L}_{nm} & = & h\gamma_{nm}, \\ \text{as}\quad \mathrm{h}(4) & \prec & \mathrm{so}(3,3) \prec \mathrm{sl}(6) \end{array} \tag{97}$$

The infraquantized imaginary $\widehat{\imath}$ is normalized to unit magnitude with a factor N^{-1}. To form macrocosmic monad coordinates, we must cumulate these atomic cell variables at least twice, to reach at least level 6, with $2_6 = 2^{(2^{16})}$ points, ample for a quasi-continuum.

The statvectors for the atoms of momentum do not commute. Neither do their cumulants, the quantified momenta, which correspond to infinitesimal translations. This quantum non-commutativity survives into general relativity as part of the curvature, perhaps including a cosmologically constant part.

Let us posit spontaneous polarizations of $\widehat{\imath} = \Sigma^2\gamma^{65} \succ\!\!\!\rightarrow i$ and of a Pauli form $\beta c' c = -\beta_{cc'}$ that makes γ^{4321} β-symmetric:

$$\beta : \gamma^{4321} \mapsto \beta\gamma^{4321}\beta^{-1} = \gamma^{4321\,\mathsf{T}}. \tag{98}$$

The form β is a special case of the Pauli metric β^T of level T, for which see (63). β singles out the first-grade γ^c: they are the elements of $\operatorname{Lin}{}^4\mathcal{Q}^1$ that anticommute with γ^{4321} and are β-skewsymmetric. These γ^c in turn define the Minkowski space-time, as a singular organized limit of cumulants of second-grade products $\gamma^{c'c}$. Thus the Pauli form determines the Minkowski form and space-time in this context.

While de Sitter relativity can be a useful approximation for patches of physical space-time the Yang $\mathrm{SO}(3,3)$ relativity is badly broken by the ambient dome, which is anisotropic, but it can still apply to a quantum cell of the dome.

8.1 Gauge in infraquantum space-time

it has been proposed that the Higgs field, like the BCS pair statvector, is an order parameter of a bosonic condensate. This is a further development of the metaphor of the superconducting vacuum. Usually it is supposed that the condensate fermions move in classical space-time and interact through gauge forces; for example, in the technicolor theories of Weinberg and Susskind [45, 38]. In an infraquantum theory the need for a vacuum condensate is even greater, to account for the vacuum means

- $\overline{g}$, the gravitational field
- $\overline{\phi}$, the Higgs field
- $i \leftarrow\!\!\!\prec \widehat{\imath}$, the quantum imaginary
- x^μ, the quasi-continuum event coordinates,

as order parameters. Here we assume that the ambient dome forms the superconducting vacuum quasi-continuum and the gaugeon fields in a singular organized limit.

We must then transfer the main ideas of Bardeen-Cooper-Schrieffer superconductivity theory from classical Minkowski space-time to an infraquantized space-time like Yang space. Many of the necessary ingredients for a gauge theory of the Yang-Mills kind are already present in the Yang space (96. A gauge theory associates a Lie algebra element $a(x, v)$ with each infinitesimal tangent vector v at each point x of space-time, depending linearly on v. $\delta u = a(x, v)u$ represents the change in a vector u due to transport of u from x to $x + v$.

Points of Yang space, however, have even statistics, wrong for a Fermi sea. Therefore we take for the statvector space of a fermionic sub-event the spinor space $\mathbb{R}^8$ underlying the Yang so$(3, 3)$ space, with odd statistics. The sub-event variables fulfilling the Yang commutation relations are then the dyadic spin components $\gamma^{c'c}$ of this event. To form an event of Yang space, many such spinorial sub-events have to pair off and condense, like Cooper pairs.

This raises the question of how the fermion interactions are to be described, before and after the organization of the ambient dome. Since the coupling of events is to have as its singular organized limit the dynamics of the standard model and gravity, our first approach is to infraquantize that dynamics.

It is parsimonious to surmise that all arise in a singular organized limit from one bosonic condensation of the plexus, with one critical temperature T_c, and can all be expressed in terms of one order parameter; rather than a sequence of condensations with distinct critical temperatures $T_{\mathrm{c}\,n}$, one for each of these vacuum mean values.

In support of the single-condensate notion, we first note that

Proposition 2 *In a Yang space* (11) *the singular organized limit of* $\widehat{\imath}$ *uniquely determines the Minkowski metric.*

Proof The Clifford algebra associated with Yang so(6) has six monadic generators γ^y $(y = 1, \ldots, 6)$. The Clifford element defining the quantized imaginary $\widehat{\imath}$ is γ^{65} in the Yang frame. To be specific let us assume that

$$(\gamma^4)^2 = (\gamma^5)^2 = (\gamma^6)^2 = -1, \tag{99}$$

so that the Yang group is SO$(3, 3)$. Then the Clifford complement of γ^{65} is $\gamma^\top\gamma^{65}$ where $\gamma^\top := -\gamma^{654321}$. Clearly

$$\gamma^\top\gamma^{65} = \gamma^{4321} \tag{100}$$

is the top element of the Minkowski Clifford algebra (not to be designated here by γ^5 for obvious reasons). Thus $\widehat{\imath}$ determines γ^{4321}.

But γ^{4321} in turn determines the subspace of Minkowskian generators γ^μ $(\mu = 1, 2, 3, 4)$. They are the monadic generators among the γ^y that anticommute with γ^{4321}.

As usual, the γ^μ in turn determine the Minkowski metric by their anticommutators:

$$\{\gamma^{\mu'}, \gamma^\mu\} = 2g^{\mu'\mu}. \quad \square \tag{101}$$

To develop this proposal into a microscopic theory, we must propose a trial statvector analogous to the Cooper pair of superconductivity. The classical Dirac vector field $\gamma^\mu(x)$ is not one operator but four. We may without loss of generality regard it as a surrogate for a more accurate, though more singular, two-point dyadic $\gamma(x', x) \approx \gamma^\mu(x)\delta x^\mu$, where $\delta x := x' - x$. This in turn is supposed to be the ambient mean of a dyadic

$$\widehat{\gamma} \in \mathrm{Cliff}^2\, \mathcal{W}[E] \tag{102}$$

of higher type, containing events as well as spins. $\widehat{\gamma}$ is second-grade in Cliff $\mathcal{W}$, and therefore fourth-grade in Dup $\mathcal{Q}$, and may be diagramed as an X. The relevance of two-point fields was pointed out by Einstein and Mayer [14, 15].

The angular momentum operator $\gamma^{c'c}$ is then a concomitant of $\widehat{\gamma}$. It provides the $\mathcal{Q}$ representation (97) of the Yang SO(3,3) group that reforms the canonical Lie algebra $\mathrm{h}(4) \prec\!\!-\, \mathrm{sl}(R)$. It thus provides each frame with an infraquantized imaginary $\widehat{\imath}$, a local variant of the global constant i of complex quantum theory, in the way that general relativity localized the global constant g_{nm} of special relativity to the local form $\mathsf{g}_{nm}(x)$.

9 Infraquantum space-times

9.1 The minimum black hole

Infraquantization modifies the usual estimate of the minimum black hole size. To review the standard heuristic argument, consider a body of mass M localized within a ball of radius R for a time much greater than R/c. Let us assume for the moment that Newton's law of gravity is still approximately valid for lengths as small as R. If the body is not to disappear into a black hole, it should be able to reflect light. The kinetic energy mc^2 of a photon at its boundary R must exceed the photon's gravitational binding energy $-V = GmM/R$:

$$mc^2 \gtrsim -V = \frac{GMm}{R}, \tag{103}$$

where G is the (unrationalized) gravitational constant.

If the Heisenberg determinacy limit holds for lengths as small as r, the body has a root-mean-square momentum

$$Mc \gtrsim \frac{h}{R}. \tag{104}$$

(103) and (104) imply the Planck-length bound,

$$r \gtrsim \sqrt{\frac{2hG}{c^3}} =: R_{\mathrm{P}}. \tag{105}$$

Newton's law of gravity presumably does not hold on the scale of X. Its error could be enormous if, as infraquantization suggests, there is a short-range fermionic hard core of repulsion within the long-range attraction of gravity.

The following crude argument uses the Newton law anyway, so it does not give an estimate but merely invalidates one of the usual arguments that the observed elementary particles cannot be quantum black holes, and so reopens the question.

The Yang indeterminacy relation implies not (104) but

$$r \gtrsim \frac{h}{2mc} |\langle \widehat{\imath} \rangle|. \tag{106}$$

Then, still assuming Newton's law of gravity,

$$r \gtrsim \sqrt{\frac{2hG}{c^3}} |\langle \widehat{\imath} \rangle| = |\langle \widehat{\imath} \rangle| R_{\mathrm{P}}. \tag{107}$$

Since $\widehat{\imath}$ is the sum of an even number of spins, its spectrum includes 0. If $\langle \widehat{\imath} \rangle \to 0$, the black hole radius and mass can approach 0. Quantum black holes much lighter than the Planck mass become possible below the time scale of the ambient dome organization.

Suppose now that there is a potential energy minimum V instead of the unbounded Newtonian potential $-GmM/R$, but equivalence is still approximately valid. Then V is proportional to both masses:

$$V \approx -\frac{GmM}{R_0} \tag{108}$$

with a new physical constant R_0. Then instead of (103) one has

$$mc^2 \gtrsim -V = \frac{GMm}{R_0}, \quad M \lesssim \frac{R_0 c^2}{G}. \tag{109}$$

9.2 Infraquantum spin-orbit analysis

Let us agree provisionally that fermionic statvectors have the data structure

$$\Psi := \overline{\text{spin}\ \overline{\text{time}\ \overline{\text{charge}\ \overline{\text{field}}}}} \tag{110}$$

For example, the monadic statvector

$$\overline{\vdots\,\vdots\,\vdots\,\vdots\,\vdots} \in {}^6\mathcal{Q}^1 \tag{111}$$

has the Lorentz spin part $[\vdots\,\vdots] \in \mathcal{Q}[3]$, the unitary charge part $[\overline{\vdots\,\vdots}] \in \mathcal{Q}[4]$, and the orbital part $\left[\,\vdots\,\right] \in \mathcal{Q}[5]$.

This decomposition of $\mathcal{Q}[6]$ raises a significant question of covariance. The standard decomposition into spin, unitary and orbital variables is invariant under the standard model symmetry group $\mathrm{SO}(2\times 3)\times \mathrm{ISO}(3,1)$ and under its gauged form. The corresponding $\mathcal{Q}$ decomposition, however, is not invariant under $\mathrm{SO}(\mathcal{Q}[4])$ or $\mathrm{SO}({}^6\mathcal{Q}^1)$, natural candidates for $\mathcal{Q}$ gauge groups. It is understood that the ambient condensate breaks the symmetry and defines the decomposition in question.

Since the orbital variables are sums of many spin variables, they commute with one of the spin variables only approximately. In extreme conditions this may have experimental consequences. The total angular momentum too is the sum of orbital and spin parts, with a scale factor h, and this results in a conspicuous spin splitting of angular momentum levels. In a simple theory the total momentum coordinate L^{m6} too is the sum of orbital and spin parts and should exhibit a spin splitting. This spin splitting will be harder to measure, however, because under ordinary conditions its relative scale is much smaller and the spectral lines of momentum are not as sharp as those of angular momentum.

9.3 Infraquantum chirality

The condensation that produces the central i also reduces general spinors of the Yang group to chiral spinors of the Lorentz group. We may see this as follows.

The chirality of a fermion in the standard quantum theory is an operator at the top of its 16-dimensional Dirac Clifford algebra,

$$i\gamma^{4321} := i\gamma^4\gamma^3\gamma^2\gamma^1 \doteq \pm 1 =: i\gamma^\top \doteq \pm 1. \tag{112}$$

Let us adopt the convention $i\gamma^{4321} \doteq 1$ for a left-handed electron, the kind with isospin 1/2, and -1 for a right-handed electron, the kind with isospin 0. (Or is it the other way around? No matter.)

The Yang SO(3, 3) group has a spinor space $\cong 8\mathbb{R}$. Its pseudoscalar volume element $\gamma^\top := \gamma^{654321} = \gamma^{65}\gamma^{4321}$ commutes with so(3, 3) transformations $\gamma^{y'y}$ $(y, y' = 1, \ldots, 6)$ and has eigenvalues $\gamma^\top \doteq \pm 1$. It reduces the spinor space to two eigenspaces $\cong 4\mathbb{R}$ with $\gamma^{4321} = \mp\gamma^{65} = \mp\gamma\widehat{\imath}$. These are therefore chiral spinors. That is, when Yang SO(3, 3) is reduced to the Lorenz group, the Yang spinors are reduced to chiral Lorentz spinors.

The atoms of the quantized orbital operators may be represented by $\delta\widehat{x}^m \sim \gamma^m$, $\delta\widehat{p}_m = \gamma^{4321}\gamma_m$, as was suggested also by Marks [28].

Present experience, where the canonical relations work, is with a part of the spectrum of $|\widehat{\imath}| = +\sqrt{-Qi^2}$ so near to the maximum value N as to be indistinguishable from it. Yet this narrow band must have a multiplicity that passes today for infinite. For example the band

$$1 - \mathsf{N}^{-1/2} < |\widehat{\imath}| \leq 1 \tag{113}$$

is narrow and crowded, with width $\mathsf{N}^{-1/2} \to 0$ and multiplicity $\mathrm{O}(\sqrt{N}) \to \infty$.

In the singular organized limit $\widehat{\imath} \to i$, $E \to 0$, and (supposedly) $(\widehat{\imath})^2 \to -1$, the classical time polyad and the canonical commutation relations are to emerge. We must suppose that

$$\mathsf{XEN} = \hbar, \quad \mathsf{N} \gg 1, \quad h/\mathsf{X} \gg 1\,\mathrm{TeV}, \quad \frac{h}{\mathsf{NX}} \approx 0, \tag{114}$$

in the sense that h/NX is presently not resolvable from 0.

9.4 The infraquantum dome

In the standard model physical lepton and quark annihilators corresponding to $\mathcal{Q}$ monads carry

- the classical four space-time coordinates, which have a large spectrum and vary greatly over the dome,
- four momentum-energy coordinates, which have a large spectrum but are very small over the dome, and
- several spin and charge coordinates, which have a small discrete spectrum and no detectible space-time extent.
- a generation index $\Gamma = 1, 2, 3$.

Kaluza-Klein theory created a compactification problem: What energies curve the gauge dimensions into small loops? $\mathcal{Q}$ theories have no such problem, having no such loops. Instead they have a difficult organizational problem: the self-organization of its spin-struts into the bubble dome on which we live. Canonical quantum fields and then classical fields are to arise as successive singular organized limits of the excitations of this dome. The particle spectrum is to inform us about the fine structure of the dome as the Brillouin zones of a crystal inform us about the fine structure of the crystal.

Infraquantization quantizes the orbital variables left classical in the standard model and S_{CCM}, using similar atomic quantum elements for orbital, spin, and charge variables. Flipping the spin of an electron annihilator is supposed to change the proper time between it and another electron annihilator, possibly by too little to resolve with present experimental space-time resolution.

Let us assume that the dome has a simple Lie algebra

$$a_{\mathrm{dome}} \subset \mathrm{so}(3,3)_{\mathrm{Yang}} \subset {}^{E}\mathcal{Q}^{2} \tag{115}$$

to be found.

The Poincaré relativity group does not fit into the groups of ${}^{E}\mathcal{Q}$. It is a singular approximation to the Yang relativity group SO(3, 3) at the cellular level.

DO: Show this.

Since the Poincaré Lie algebra is a reduction of diff, the Yang Lie algebra in turn is assumed to be a reduction of a higher-level relativity gauge Lie algebra suitable for quantum gravity and the other gauge forces too:

$$\mathrm{diff} \prec \widehat{\mathrm{diff}} = \mathrm{sl}({}^{E}\mathcal{Q}) \subset \mathrm{sl}(\mathcal{Q}). \tag{116}$$

Level 5 supports $\mathrm{sl}(2^{16})$ on its first grade. The second-cumulant $\Sigma^2\,\mathrm{sl}(2^{16})$ then represents the reformed coordinate and momentum operators $\Pi^2\, L_{E}^{E'} \in {}^{5}\mathcal{Q}^{2}$ with quasi-continuous but finite spectra.

10 Generations

Empirically, the fermion spectrum is unexpectedly divided into three generations differing only in their masses, their coupling to gravity. Properly understood, this must tell us something equally peculiar about the fine structure of the quantum plexus.

The division into generations is not quite unique. One basis diagonalizes the fermion mass operator M, another the weak charge operator Q_{W}, another the coupling coefficient g_{H} of the Yukawa coupling to the Higgs field. The unitary transformation from the charge basis to the mass basis is *fermion mixing*. It reduces to a lepton-mixing part U_{W}^{M}, the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) transformation, and a quark-mixing part U_{S}^{M}, the CKM (Cabibbo-Kobayashi-Maskawa) transformation.

The generation number Γ is often described as a radial or principal quantum number of the particles, in analogy to the radial variable r or principal quantum number n of atomic physics, because Γ commutes with the groups of the standard model, whose parameters are likened to angles, as n commutes with angular momentum. On the other hand, there are remarkable violations of this analogy in the relation of Γ to multiplicities and to couplings. The multiplicities of the standard model group representations are independent of Γ, while those of the atomic rotation group depend markedly on n, namely quadratically. For example, the multiplicity M_n of hydrogen shells changes from $M_1 = 2$ for the K shell to $M_2 = 6$ for the L shell. There seems to be a unitary operator connecting the generations, but not the shells of hydrogen.

In the limit of very large radius $n \to \infty$, however, the fractional change from one hydrogenic radial shell to the next becomes small:

$$\frac{M_{n+1} - M_n}{M_n} \to 0. \tag{117}$$

As $n \to \infty$, neighboring shells of hydrogen then become approximately isomorphic, in that their differences are small fractions of the total. Γ still resembles a radial variable, but a very large one, compared to the radial distance between shells. The generations are shells that are flat and isomorphic on the scale of particle physics.

This fits the dome model reasonably well, for the dome has a cosmic radius and a subparticle thickness, and so its strata can be practically flat. On the other hand, instead of one transverse dimension the dome has one for every standard-model charge.

What singles out the generation coordinate Γ? And why do fermion masses, to a crude approximation, seem to vary geometrically with Γ? Roughly speaking,

$$M(\Gamma) \sim M(0)e^{\Lambda\Gamma}, \tag{118}$$

with a large inter-generational mass ratio $e^{\Lambda} \sim 10^{2\pm1}$. No radial variable of elementary quantum mechanics effects the particle energy so powerfully. Perhaps this is a clue.

Some speculations are obvious in an infraquantum perspective. Though still poorly founded, they might be worth future study:

The Higgs field singles out one direction in the dome at each event, and determines some gaugeon masses. Parsimony suggests that Γ indexes eigenvalues of the quantized Higgs field. In the infraquantum theory, correspondingly, it is conceivable that Γ indexes the spectrum of $(\widehat{\imath})^2$.

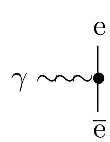
One familiar exponential process in superconductors is shielding or penetration. In a boundary layer the electric vector potential falls off as $A \sim e^{-\mu\xi}$ with penetration ξ into the superconductor. Here μ is a measure of photon rest mass $h\mu/c$ within the superconductor. In a bulk superconductor the skin depth λ is a small part of the bulk, and the mass shift of the Cooper pair is a small part of its mass, so superconducting generations, with various values for the Cooper-pair mass, have not been observed, as far as I know. If this analogy has some validity, generations of Cooper pairs would be seen more clearly in a superconducting film or graphene several atoms thick. What is the mass of Cooper pairs in the boundary layer of a superconductor? Let us return to the more exact considerations necessary to solidify such conjectures.]

11 Particle valence

11.1 Standard model valences

In this section we use a terminology for the standard model taken from chemistry. Each quantum of the standard model has a single-particle Hilbert space that is a product of several factor spaces on which act orbital, spin, and charge operators. The factor spaces provide the indices on the statvectors of the particle, describe formal parts of the particle, and are transported by the gauge connections, so they have some semblance of physical existence. Yet they are not quanta in space-time, which is only one of these parts. We may call these phenomenological parts of the quanta *valences*. In an infraquantum theory valences arise naturally as entities of lower type than particles and space-time.

What is known of the valences of the quantum particles today is summed up in the Feynman vertices of the standard model, such as the electron-photon vertex



$$\text{or} \quad \gamma \to \mathrm{e} + \bar{\mathrm{e}}. \tag{119}$$

In an infraquantum theory this must be a singular organized limit of the actual process. Photons are at least dyadic, and should be represented by at least two lines More generally, $\mathcal{Q}$ bosons have statvectors of even grade, and are composed of an even number of fermions. Furthermore, the components of the momentum do not commute, and cannot all be assigned eigenvalues at once. De Broglie and Feynman tried assembling photons and gravitons (respectively) from fermions, without positive results. They worked in a classical space-time with quanta, subject to the Heisenberg indeterminacy principle, which requires strong forces for close binding.

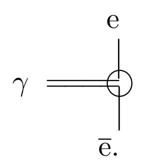
But $\mathcal{Q}$ weakens the Heisenberg indeterminacy principle at small range, and simultaneously provides another way to bind monads into polyads: covalent bonding below the event level. Polyadics cannot be bound with an ι since succession results in a monadic, but can be bound by sharing second, third, or n-th members of lower type, which may be valences. Let us call this phenomenon *deep covalent bonding*.

It results in pseudo-bosons with hard fermionic cores. The dyadics

$$\begin{aligned} \psi &= \overline{a}\,\overline{\overline{b}c} \pm \overline{a\overline{b}}\,\overline{c} &=:& \bigcirc\circledcirc \pm \circledcirc\bigcirc, \\ \phi &= \overline{a\overline{b}}\,\overline{\overline{c}d} \pm \overline{a\overline{c}}\,\overline{\overline{b}d} &=:& \odot\circledcirc \pm \circledcirc\odot, \end{aligned} \tag{120}$$

are simple examples of deep covalent bonds within $\mathcal{Q}$. In the example ψ, the leftmost circle represents $\overline{a}$, the inner circle $\overline{b}$, and the rightmost $\overline{c}$.

One candidate for the photon of (119) under $\mathcal{Q}$ resolution might be



$$\gamma \to \mathrm{e} + \mathrm{e}^- \tag{121}$$

in which the horizontal lines represent two electron processes held together by deep covalent bonds like those of (120). This model cannot be right. It would not account for direct photon emission by particles other than the electron.

The standard model gauge group itself suggests a more complex structure for the leptons, and a simpler structure for the gaugeons. In the standard model kinematics, the chiral lepton and quark and the gaugeon have tensor-product statvector spaces, which can also be written as exterior products:

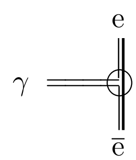
$$\begin{aligned} \text{Lepton} &= \text{Orbit} \otimes \text{Spin} \otimes \text{Isospin} = L^2(\mathbb{R}^4) \otimes 2\mathbb{C} \otimes 2\mathbb{C}, \\ \text{Quark} &= \text{Orbit} \otimes \text{Spin} \otimes \text{Isospin} \otimes \text{Color} = L^2(\mathbb{R}^4) \otimes 2\mathbb{C} \otimes 2\mathbb{C} \otimes 3\mathbb{C}, \\ \text{Gaugeon} &= \text{Orbit} \otimes \text{Spin} \otimes \text{Valence} \otimes \text{DualValence}. \end{aligned} \tag{122}$$

This expresses the chiral (left-handed) lepton as a composite of a scalar particle, a spin, and an isospin; the quark has a color as well. The anti-chiral right-handed fermions lack the isospin valence. The gaugeon carries a Lie algebra element that is also a dual pair of valences.

(122) is not a constitutive hypothesis added to the standard model; it is parr of the standard model. These constituents are not yet physical particles, but valences for the three forces, generalizing those of chemistry. Let us develop this terminology further.

Label the three current kinds of force with G (gravitational), W (weak, electromagnetic, and hypercharge), and S (strong) for brevity. Fermions may carry approximately conserved charges of several kinds. The G charges are momentum-energy p^μ and spin $\gamma^{\nu\mu}$ ($\mu = 1, 2, 3, 4$), The W charges are weak isospin and hypercharge $w^i \in \mathrm{u}(2)$ ($i = 1, 2$). The S charges are colors $s^C \in \mathrm{SU}(3)$ ($C = 1, \ldots, 8$).

Each gaugeon of the standard model relates an orbital direction and a pair of valences (G, W, or S). The orbital direction can itself be identified with a pair of G valences, spins. Thus the gaugeon has a tetrad of valences of appropriate kinds. The diagram (121) is replaced by the finer structure



$$\gamma \to \mathrm{e} + \mathrm{e}^- \tag{123}$$

The electron line has been broken into a thinner line standing for an inner valence and a thicker line that carries everything else including the spin. The gaugeon line is now composed of two valence lines rather than two electron lines. It can interact with any fermion carrying an appropriate valence, not only with an electron.

11.2 $\mathcal{Q}$ valence

To form a $\mathcal{Q}$ correspondent of the valence concept, let us assume tentatively that each fermion is a product of its valences, held together by deep covalent bonding.

This raises the question of whether the dynamics can effect such binding. The relevant pre-reformation skew-action is the fermionic term Φ_{F} of (139).

According to the standard model, W and S gaugeons map external (orbital) Lie algebra elements into internal ones. Allowing two valences to define a Lie algebra element, they have the structure L^2T^2. According to canonically quantized general relativity, G gaugeons have the valence L^4. In a pre-organization theory, whose symmetry is not yet broken by the split into L and T statvectors, the three gaugeons merge into one.

At present, however, superselection rules separate the G, W, and S valences. Superpositions of different valences do not seem to occur. A superselection rule, like a selection rule, implies a symmetry, but also implies decoherence. Decoherence can be due to random disturbances that destroy phase relations between two terms in a direct sum. They occur, for example, when each of the terms in question is itself a sum of too many sub-terms for us to resolve their phases accurately. In $\mathcal{Q}$ dynamics, the large numbers involved seem to be the number of paths joining two events, and the number of steps along the transport path. Too many steps make the space-time coordinates central. It is possible that too many paths result in the superselection rules between G, S, and W.

The gauge Lie algebra of the standard model is a direct sum of the gauge Lie algebras of hypercharge Y, isospin I, and color C, of the kind

$$g_{\mathrm{S}} = \begin{bmatrix} \mathrm{u}(1)_Y & & \\ & \mathrm{su}(2)_I & \\ & & \mathrm{su}(3)_S \end{bmatrix} = u(1) \oplus \mathrm{su}(2) \oplus \mathrm{su}(3). \tag{124}$$

This is not the invariance Lie algebra of any quantum entity supposed to exist in the standard model. Its defining representation acts on a 6-dimensional direct-sum complex vector space

$$V_{\oplus} := \mathbb{C}(\mathrm{Y}) \oplus 2\mathbb{C}(\mathbb{I}) \oplus 3\mathbb{C}(\mathrm{C}) = 6\mathbb{C}. \tag{125}$$

This is the statvector space of a hypothetical quantum that can be a hypercharge Y, an isospin I, or a color C. No such quantum entity is believed to exist. For example, a quark has a Y, I, and C all at once, as indices on its statvectors. Its charges act on a 6-dimensional tensor-product space

$$V_{\otimes} := 2\mathbb{C}_{\mathrm{W}} \otimes 3\mathbb{C}_{\mathrm{C}} = 6\mathbb{C}, \tag{126}$$

not the direct sum (125). The quark theory is invariant under the *charge Lie group*

$$G_{\mathrm{Q}} := \mathrm{U}(1) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(3). \tag{127}$$

To infraquantize the three interactions involves representing these groups in $\mathrm{SO}({}^E\mathcal{Q}^1)$, along with the groups of general relativity.

12 Infraquantum fields

We express the usual construct of a spinorial quantum field over classical Minkowski space-time $\mathcal{M}$ operating on a Hilbert space $\mathcal{H}$ as a singular organized limit of a polyad with statvectors in ${}^E\mathcal{Q}$. The constructs of field space, space-time, and Hilbert space thus merge into the typed exterior algebra $\mathcal{Q}$.

The correspondence between the canonical and infraquantum field constructs is crucial for a $\mathcal{Q}$ theory. It arises from the dome structure of the ambient plexus as follows.

In a completely classical model with field values in a linear space $\mathcal{F}$ and space-time events in a finite set $\mathcal{X}$, the field state space is the power-set $\mathcal{F}^{\mathcal{X}}$. Its multiplicity is

$$|\mathcal{F}^{\mathcal{X}}| = |\mathcal{F}|^{|\mathcal{X}|}. \tag{128}$$

A Lagrangian dynamical theory then uses the dual linear space $\mathcal{F}'$ for the canonical conjugate of the $\mathcal{F}$ field. In the canonical quantization, $(\mathcal{F} \oplus \mathcal{F}')^{\mathcal{X}}$ is used as statvector space for one quantum. Its vectors represent input-output processes for the quantum.

When $\widehat{\mathcal{X}}$ is a quantum space, however, as in an infraquantum theory, there is no satisfactory power-set construct $(\mathcal{F} \oplus \mathcal{F}')^{\widehat{\mathcal{X}}}$. There is the binary exponential $\widehat{2}^{\widehat{\mathcal{X}}}$, but this is not a proper power space of $\widehat{2}$.

(The space of linear maps $\widehat{\mathcal{X}} \to \widehat{\mathcal{Y}}$ is a mathematically natural candidate for the needed power space, but this is merely the statvector space of one pair of one $\mathcal{X}$ quantum and one dual $\mathcal{Y}$ quantum, and is not the statvector space required. Its multiplicity is merely $|\widehat{\mathcal{Y}}|\,|\widehat{\mathcal{X}}|$.)

The binary exponential can be used, however, to make a quantum theory that becomes a canonical quantum field theory in a singular organized limit. Suppose that the dome organization reduces the space of E monadics ${}^{E}\mathcal{Q}^1$ as vector space into a tensor product of two vector spaces, a transverse part $\mathcal{Y}$ that is of small extent in the dome and has a single point 0 as classical limit, and a longitudinal part $\widehat{\mathcal{X}}$ that is large for the dome and has a classical limit $\overline{\mathcal{X}} \approx \mathcal{M}$:

$$ {}^{E}\mathcal{Q}^1 \approx \widehat{\mathcal{Y}} \vee \widehat{\mathcal{X}} \succ \widehat{\mathcal{Y}} \vee \overline{\mathcal{X}}. \tag{129} $$

Then the field statvector space factors according to

$$ {}^{E}\mathcal{Q} = \operatorname{ext} {}^{E}\mathcal{Q}^1 = \widehat{2}^{\,\widehat{\mathcal{Y}} \vee \widehat{\mathcal{X}}}. \tag{130} $$

In the singular organized limit

$$ \widehat{2}^{\,\widehat{\mathcal{Y}} \vee \widehat{\mathcal{X}}} \succ 2^{\widehat{\mathcal{Y}} \vee \overline{\mathcal{X}}} \cong (2^{\widehat{\mathcal{Y}}})^{\overline{\mathcal{X}}} := \widehat{\mathcal{Z}}^{\overline{\mathcal{X}}}, \tag{131} $$

a classical field emerges with quantum field variable $\widehat{\mathcal{Z}} := \widehat{2}^{\widehat{\mathcal{Y}}}$ and classical space-time $\overline{\mathcal{X}}$.

This limits us to field spaces that have logarithm spaces. It is convenient therefore that a spinor space $\Psi = \operatorname{ext} V$ has a logarithm, namely its semivector space V (§6.2).

Thus even though there is no natural $\mathcal{Q}$ construct of a quantum function on a quantum space-time, the dome organization naturally provides such a construct as a singular organized limit of a quantum set of monads; and the field so defined can be a spinor field, or any polyadic product of monadic spinor fields. It is so far merely plausible that this path can be retraced to recover the spinor field on classical space-time from the infraquantum theory in $\mathcal{Q}$.

12.1 $\mathcal{Q}$ gaugeons

We turn now to infraquantum gauge theory. The central object of the standard gauge theory, following Einstein and Weyl, is a gauge differentiator, a reformed version of the Lie partial differentiator ∂_μ. Three known gauges, gravitational (G), electroweak (W), and strong (S), combine without unification into a *grand gauge differentiator*

$$\mathcal{D}_\mu = \partial_\mu - \mathcal{G}_\mu(x) = \partial_\mu - \mathcal{G}^{\mathrm{G}}_\mu(x) - \mathcal{G}^{\mathrm{W}}_\mu(x) - \mathcal{G}^{\mathrm{S}}_\mu(x), \tag{132}$$

with grand vector potentials $\mathcal{G}_\mu$ that act on each tensor according to its nature and degree. We have absorbed the relevant coupling constants and current operators into the grand vector potentials $\mathcal{G}_\mu$. Then all the coupling constants appear as appropriate "fine-structure constants" α_{G}, α_{S}, α_{W} standing before the respective gaugeon skew-action operators, yet to be written.

The infraquantum correspondent of $\mathcal{D}_\mu$ is the tensor $L_{c'c}[E]$ of cumulated generators of the reformed simple gauge Lie algebra so$({}^C\mathcal{Q}^1)$. $L_{c'c}[E]$ also includes correspondents of the space-time coordinates, angular momenta, and the infraquantized imaginary $\widehat{\imath}$.

Since the kinetic energy of a test particle is part of D_m and the potential energy is part of $\mathcal{G}^m$, let us speak of the three terms in any gauge differentiator

$$D_\mu = \partial_\mu - \mathcal{G}_\mu \tag{133}$$

as kinetic (D) , total (∂), and potential ($\mathcal{G}$). The gauge invariant one is the kinetic, D_μ. Einstein's principle of local equivalence implies three *local equivalence conditions* on $\mathcal{D}_\mu$:

1. $\mathcal{D}_\mu$ respects the tetrad vectors γ^μ and so the metric tensor.
2. $\mathcal{D}_\mu$ is atorsional, respects the coordinate tangent vector fields $e_n(x)$.
3. $\mathcal{D}_\mu$ agrees with the Lie derivative ∂_μ on the scalars x^ν:

$$[D_\mu(x), e_n(x)] = 0 = [D_\mu, \gamma^\nu(x)], \quad [D_\nu(x), x^\mu] = \delta^\mu_\nu. \tag{134}$$

These conditions are all structurally unstable, and must be reformed in an infraquantum theory. It is sufficient for immediate physical purposes if they hold in a singular organized limit.

The full skew-action operator Φ for the standard-model with gravity involves a fermion annihilator $\psi(x)$, the Dirac spin operator $\gamma^\mu(x)$, the grand differentiator $\mathcal{D}_\mu(x)$, a Higgs isospinor scalar $\phi(x)$, and other spin-like variables. As each gaugeon was adduced, an action operator was invented to control it. Schematically,

$$\Phi = \overbrace{i\overline{\psi}\gamma\mathcal{D}\psi}^{\Phi^{\mathrm{F}}} + \overbrace{\alpha_{\mathrm{G}} i\gamma\gamma[\mathcal{D},\mathcal{D}]}^{\Phi^{\mathrm{G}}} + \overbrace{\alpha_{\mathrm{W,S}} i[\mathcal{D},\mathcal{D}]\gamma\gamma[\mathcal{D},\mathcal{D}]}^{\Phi^{\mathrm{WS}}} + \overbrace{i(\phi\mathcal{D}^2\phi - \mathsf{W}(\phi))}^{\Phi^{\mathrm{H}}}. \tag{135}$$

The fermion term is linear in $\mathcal{D}_\mu$. The gravitational skew-action operator $\Phi^{\rm G}$ is the term quadratic in $\mathcal{D}$, and involves only $D^{\rm G}_\mu$, the G part of D_μ. The electroweak term $\Phi^{\rm WS}$ is quartic in the grand differentiator D_μ. The standard-model Higgs term $\Phi^{\rm H}$ involves a colorless isospinor spinless scalar ϕ in a W-shaped potential, a source of G and W gaugeons but not S. Yang space provides an isovector scalar $\widehat{i} \succ i$ instead.

It has often been suggested, since Kaluza, that some of these gaugeons are expressible in terms of the others. In an infraquantum theory based on $\mathcal{Q}$, the gaugeons are associated with polyadics of even grade, composed of an even number of monadics, representing fermions. We should therefore explore the possibility that all the skew-action operators might be effective surrogates for the fermion one. Specifically, we have ordered the terms on the right-hand side of (135) so that each might be a manifestation of the ones preceding it, all ultimately flowing from the fermion skew-action operator, the only one that contains all the fields ordinarily considered to be independent.

12.2 The emergence of space-time

The standard model skew-actions are designed to couple gauge fields with their sources. None was designed to couple the events of nature into a space-time quasi-continuum. The dynamics that condenses the plexus into a plenum operates on a deeper level than can even be described in the standard model language. If we do not need a completely new dynamical theory to describe this, we need at least a reformation of the existing dynamics.

It seems that the organization of the space-time plenum requires the limits

$$\widehat{\imath} \succ i, \quad p_\mu, L_{\mu'\mu} \succ 0, \tag{136}$$

the first to recover the canonical commutation relations for orbital variables and field variables, the rest is to reduce the x, p-phase space to x-space-time. It is implicit that the variables x^μ, which commute in this limit, are not constrained; that the plenum is four-dimensional, not a lower-dimensional quasi-manifold like a wire or a bubble. In §13 we review how a condensate is described in a history-based quantum theory.

13 Infraquantum standard model

DO: Produce an ambient dome statvector.

DO: Make MISM (Minimal Infraquantum Standard Model), repressing the desire to unify or innovate.

13.1 Infraquantum fermions

Let us begin by infraquantizing the fermion skew-action operator $\Phi_{\rm F}$ of (139). Prior to infraquantization, but after a canonical quantization that includes gravity, the variables in $\Phi_{\rm F}$ represent tensors of the valences shown first in (122), rendered contravariant for convenience. We write x for the coordinates of a point in space-time, s for the spins-and-unitary-charges of the fermion annihilated by ψ, μ for a Minkowski vector index, f for the fermion quantum occupation numbers and g for gaugeon occupation numbers connected by ψ. Then simplification results in the valences shown on the right-hand sides of these equations. It replaces each Minkowski index and event μx of $\mathcal{D}_\mu$ by a pair of event indices $e'e$, suitably extended to represent the standard model internal unitary charges as well, and undergoing a split into spin and orbit in the limo. Let us write the resulting "cell" orthogonal group of the sub-event as SO[C]. its vector indices as c, c', and its $\mathcal{Q}$ spinor indices still as s, s'. In a canonical theory, ∂_μ and x^μ are canonically conjugate as single-event operators ($L^{e'e}$) on the event type level E. Probably $E = C + 2$ will suffice. While c need assume but 16 values or fewer, after infraquantization the spinor index s ranges over cosmologically values, perhaps $2_6 = 2^{64\mathrm{K}}$. Then covariance requires a corresponding conversion $\gamma^\mu(x) \prec\!\!- \gamma^{e'e}$. We designate fermion occupation quantum numbers by f, f', and infraquantum numbers by $\widehat{f}, \widehat{f'}$; and analogously for gaugeons g and $\widehat{g}$. In sum,

$$\begin{array}{rclcl}
\psi & = & (\psi^{xs|f'f}) & \prec & \psi^{e|\widehat{f}'\widehat{f}}, \\
\gamma & = & (\gamma^{x\mu|s's|g'g}) & \prec & \gamma^{e'e|\widehat{g}'\widehat{g}}, \\
\mathcal{D} & = & (\mathcal{D}^{x\mu|s's|g'g}) & \prec & L^{e'e|\widehat{g}'\widehat{g}}.
\end{array} \tag{137}$$

Let us also continue to suppose that all forces are ultimately exchange forces, differing in what is exchanged:

$$\textit{Gauge forces result from valence exchanges.} \tag{138}$$

In $\mathcal{Q}$ theory, "empty" space-time will also be supposed to be held together by exchange of spins at adjacent events. Let us suppose that this binding force is gravitational.

Since the monadics of $\mathcal{Q}$ have spin 1/2, we are back to a question of Feynman: Could a graviton be a composite of four fermions?

The graviton appears most nakedly inside the skew-action operator of a fermion, thus:

Write $\psi(x)$ for the fermion spinor operator field, and $\mathcal{D}_\mu(x)$ for the grand covariant differentiator. Write the Pauli adjoint of ψ as $\overline{\psi}$, defined so that $\overline{\psi}\psi$ is an invariant symmetric operator and $\overline{\psi}\gamma^{\mu'\mu}\psi$ is a skew-symmetric tensor of skew-symmetric operators. The Dirac spin operator field $\gamma^\mu(x)$ and the differentiator $\mathcal{D}_\mu$ can be regarded as field operators for gravitons. Then the usual skew-action operator coupling them is

$$\Phi^{\mathrm{F}} = i\int (dx)\,\overline{\psi}\gamma^\mu D_\mu \psi. \tag{139}$$

A $\mathcal{Q}$ reformation replaces:

- $\psi(x)$ by an operator $\mathsf{L}\psi$ where now $\psi \in {}^E\mathcal{Q}^1$.
- $i\int (dx)\,\overline{\psi}\dots\psi$ by a cumulation ΣL_{65} (§6.4).
- $D_\mu(x)$ by $L_{c'c}$.
- γ^μ (therefore) by $\gamma^{c'c}$.

Due to the hanging indices of L_{65} the result is not yet a tensor.

In classical gauge theory the gauge derivative coincides with the Lie derivative for coordinate functions:

$$[\mathcal{D}_\mu(x^\lambda), x^\nu] = [\partial_\mu, x^\nu] = \delta_\mu{}^\nu. \tag{140}$$

This is a singular condition on the theory. The canonical commutation relations between $\mathcal{D}$ and its conjugate field variables are even more singular, involving Dirac delta functions. As a result, the current gauge theories of gravity and the standard model are compound and singular, and so must be reformed and simplified. Infraquantum theory replaces both singular commutation relations by regular Lie algebraic commutation relations, of the event type and field type respectively.

Diff, the gauge group of gravity, can serve as a prototype for them all. (140) is a singular organized limit of the Lie algebraic relation

$$[L^{A''}, L^{A'}] = c^{A''A'}{}_A L^A \tag{141}$$

of some simple Lie algebra $\mathcal{A}$, here of operators on $\mathcal{Q}$, with constant structure tensor $c^{A''A'}{}_A$. (140) is diff-invariant but (141) is $\mathcal{A}$-invariant.

For definiteness, let $\mathcal{A} = \mathrm{so}({}^C\mathcal{Q}^1)$ be the orthogonal group of a cell level, represented by generators of the Clifford form $L^{c'c} = \gamma^{c'c}$. In an infraquantum theory, the gauge differentiator is a cumulation of $L^{c'c}$ from the cell level ${}^C\mathcal{Q}^1$) to the event level ${}^E\mathcal{Q}^1$. This can be represented by a single element of $\mathcal{Q}$, a contraction of the cell operator with the event operator, having the Dirac spin-orbit operator as a singular organized limit:

$$L := \gamma^{c'c}\Sigma^2\gamma_{c'c} \prec \gamma^m\mathcal{D}_m. \tag{142}$$

For gravity all four monads in the tetrad are longitudinal to the dome. For the other gauge interactions, two are transverse to the dome.

Spin and orbit variables undergo the same group ISO$(3,1)$ in special relativity, but not in general relativity. Unlike the vector representation, the spinor representation of so$(3,1)$ does not extend to one of sl$(4\mathbb{R})$, whose irreducible unitary double-valued representations are infinite-dimensional. Therefore, to general-relativize the construct of spinor, Cartan introduced a mobile frame, a field of tangent-vector quadruples $\gamma^0(x), \ldots, \gamma^3(x)$ with a fixed Minkowski inner product $\gamma_m(x)\cdot\gamma_n(x) = g_{nm}$. This defines a Lorentz group SO$(3,1;x)$ on each tangent space, but events transform under Diff. A Cartan vector-sextuple would appear in a theory of Yang so$(3,3)$.

The Dirac spin operators γ^μ have had several metamorphoses since their conception:

1. In gravity theory the Cartan vector-quadruple $\gamma^\mu(x)$ becomes the dynamical variable for gravity. The classical gravitational metric at x, described in the mobile frame, is then the tensor

$$g^{m'm}(x) := \frac{1}{2}\{\gamma^{m'}(x), \gamma^m(x)\}, \tag{143}$$

   which is arbitrarily fixed to be a constant Minkowskian form in the mobile frame.

2. In Dirac's theory of the single electron, the γ^μ become monadics of a Clifford algebra and represent physical quantum variables of the electron.

3. In the Yang space-time the coefficient γ^μ in the Dirac equation become a sector $\gamma^{\mu 6}$ of the generator γ^{nm} of Yang so$(3,3)$, contragredient to the momentum:

$$p_\mu \leftarrowtail L_{\mu 6}, \quad \gamma^\mu \leftarrowtail \gamma^{\mu 6}. \tag{144}$$

4. In $\mathcal{Q}$ the single-electron γ^μ become annihilators of type B, the predecessor of spin.

5. In standard quantum field theory the γ^μ are constant coefficients of the electron wave equation, not dynamical variables.

6. In canonical quantum gravity, the matrix elements of the $\gamma^\mu(x)$ become graviton annihilators.

7. $\mathcal{Q}$ gravity uses the cumulant spin operators

$$\gamma^{c'c}(E) := \Sigma^{E-C}\gamma^{c'c}(C). \tag{145}$$

   The Dirac operator becomes

$$\gamma^{c'c}(E)L_{c'c}(E) \rightarrowtail \gamma^\mu\partial_\mu \tag{146}$$

The $\mathcal{Q}$ operators $\gamma^{c'c}(E)$ act on the spinors $\psi \in \mathcal{Q}(E)$, which carry both spin and orbital information (§9.2). Therefore each γ^c carries information about two spins and two orbital variables, input and output. The $\mathcal{Q}$ gravitational metric corresponding to (143) is evidently the numerical tensor

$$g^{c'c} := \frac{1}{2}\{\gamma^{c'}, \gamma^c\}, \tag{147}$$

involving four spin and orbital variables. It will be symmetric in its spin variables if it is skew-symmetric in its orbital variables. It is simply the natural neutral metric on $\mathcal{W}(E)$, referred to a mobile frame. The variability of this metrical structure derives from the variability of the $\mathcal{Q}$ simplicial complex to which it is applied, in the way that a variable curved surface in a flat space of constant metric has a variable metrical structure.

The $\mathcal{Q}$ correspondent of the gauge covariant differentiator D_μ is the generator on the field level F induced by the cell Lie algebra generator L_C of level C. It requires $F - C$ iterations of cumulation Σ. The resulting correspondences are:

$$\begin{array}{rcl} \text{tangent space} & \prec & \mathcal{Q} \text{ simplex of level } C \\ \text{Cartan } n\text{-ad } \gamma^n(x) & \prec & \mathcal{Q}\ n\text{-adic in } {}^E\mathcal{Q}^n \\ \text{diff} & \prec & \text{sl}({}^E\mathcal{Q}^1) \\ D_\mu(x) & \prec & \Sigma^{E-C} L_C =: L_C[E] \\ K^{s'}_{sm'm} & \prec & [L_{C'}[E], L_C[E]] \end{array} \tag{148}$$

Because the cell map is an orthogonal transformation, the local equivalence principle is automatically satisfied.

A $\mathcal{Q}$ quantum theory has no supersymmetry between bosons and fermions. Even operons are composed of odd, and not conversely. The generating variables obey a Fermi statistics, not a Bose.

The atomistic analysis of the other gauge fields can be modeled on that of gravity. One adjoins to the six cellular ${}^C\mathcal{Q}$ dimensions required for Yang so$(5,1)$ as many more dimensions as needed for the unitary groups of the standard model, raising the cell level C if necessary. This enlarges the generator L_C of sl$({}^C\mathcal{Q}^1)$ accordingly. The induced generator $L_C^E := \Sigma^{F-C} L_C$ then includes all the coordinate and momentum variables of the full gauge field.

Near the singular organized limit, where one can speak of space-time points, we have supposed that the gravitational field variable $\widehat{\gamma}$ is not attached to one event but two (§8.1). This makes it possible to form a pseudo-boson from the basic fermions. The field tensor can be symmetric in its space-time indices because it is skew-symmetric in its space-time points, although these points are so closely bound that they presently pass for one point. All apparent bosons are hard-core pseudo-bosons in this model.

The four flat-space Dirac γ^m's of standard spin 1/2 theory have been assumed to be what remains of the cell operators $\gamma^w \in \mathcal{W}[C]$ after the dome organizes itself. The γ^w transform as so(3,1) vectors when ${}^4\mathcal{Q}^1$ transforms as a spinor space:

$$\Lambda\gamma^c\Lambda^{-1} = L^c{}_{c'}\gamma^{c'}, \quad \Lambda \in \mathrm{sl}({}^4\mathcal{Q}^1) = \mathrm{sl}(16\mathbb{R}),\ L \in \mathrm{so}(3,1). \tag{149}$$

Then $\mathcal{Q}$ gauging undoes the reduction

$$\mathrm{so}({}^4\mathcal{Q}^1) \otimes \mathrm{so}(2) \prec \mathrm{so}({}^6\mathcal{Q}^1). \tag{150}$$

The function $\gamma^m(x)$ is covariant under diff, so we assume that $\mathcal{Q}\gamma^m(x)$ is covariant under $\mathcal{Q}\,\mathrm{diff} = \mathrm{sl}({}^6\mathcal{Q}^1)$. Suspending the polarization that produces i and p, the γ^m merge into left multiplications by any basis element e^c of ${}^C\mathcal{Q}^1$, with $C = 4$ provisionally. Upon $\mathcal{Q}$ gauging, this becomes

$$\gamma^m(x) \prec {}^{\mathsf{L}}\mathsf{e}^e, \tag{151}$$

where the e^e are the basis elements of $\overset{5}{\mathcal{Q}}$.

13.2 Infraquantum fermion dynamics

In the canonical theory, the gravitational metric $g_{\mu'\mu}$ is constructed from γ and its canonical conjugate from $\mathcal{D}$, and they are connected by symplectic transformations. While $\mathcal{D}$ and x are canonically conjugate on the event level E, $\mathcal{D}$ and g are canonically conjugate as gravitational field operators on the higher field type level $F = E + 1$.

In the simple theory canonical conjugates unify into sectors of one tensor, let us suppose representing an orthogonal group, as $x, p, \widehat{\imath}$ do in (2). They re-separate in the singular organized limit. Therefore the infraquantum tensors L and γ are the same on each type level. For example on the event level,

$$L = (L^{e'e|g'g}) = (\gamma^{e'e|g'g}) = \gamma. \tag{152}$$

We will use γ rather than L to remind us that we are using its spinor representation. Then the core of the fermion skew-action operator simplifies to

$$\gamma \cdot \mathcal{D} \prec \gamma^{e'e|g''g'}\gamma^{ee'|g'g}. \tag{153}$$

When we supply the outermost fermion factors $\overline{\psi}\ldots\psi$ in the skew-action, it becomes

$$\Phi_{\mathrm{F}} \quad = \quad \int (dx) i\, \overline{\psi}\, \gamma \mathcal{D}\, \psi$$

$$
\begin{aligned}
&\prec && \widehat{\Phi}_{\mathrm{F}} \\
&\sim && \mathrm{Tr}\Big[\widehat{\psi}^{f"e"}{}_{f"''e"''c"''} L^{f"''e"''c"''}{}_{f"'e"'c"'} \\
& && \quad \times \gamma^{f"'e"'c"'}{}_{f"e"c"} \, L^{f"e"c''}{}_{f'e'c'} \;\; \widehat{\psi}^{f'e'c'}{}_{fe}\Big] \\
&\in && \mathrm{Lin}\,{}^{F}\mathcal{Q}.
\end{aligned} \tag{154}
$$

DO: : Complete this calculation.

13.3 Constructing the Higgs field

It has already been proposed that a quantized imaginary play the role of a Higgs field (§9.3). Here the quantized imaginary is one element of the Yang generator $L^{\cdot\cdot}$, and the differentiator D is another sector of L. Both appear in the quantization of Φ^{W}. This identification would absorb the H skew-action operator into the W. The factor i in Φ^{F} then becomes a $\widehat{\imath} \succ \eta$ in Yukawa interaction with the fermions.

13.4 Constructing the electroweak gaugeon field

Kaluza and deWitt showed in a classical context how an action linear in the curvature like Φ^{G} could absorb one quadratic in the curvature like Φ^{W}:

Suppose that some manifold dimensions of the gravitational G theory belong to a Lie algebra a. This was electric so(2ℝ) for Kaluza, and an arbitrary Lie algebra a for de Witt. If A indexes an a basis, the metric component g_{mA} is also a connection, coupling the transport direction m to a Lie algebra element A. If a is the appropriate sum of the W and S gauge algebras, the G skew-action operator Φ^{G} includes a quadratic one of the form Φ^{WS}.

The Kaluza-deWitt strategy works even better in the infraquantized theory: Where Kaluza had but one Lie-algebra dimension among four other manifold dimensions, the Yang-Segal simplification converts *all* the manifold dimensions to operators in a Yang Lie algebra. For example, ∂_m becomes one sector of a generalized angular momentum tensor $L_{c'c}$. $L_{c'c}$ is represented on a cellular level by a dyadic spin operator $\gamma^{c'c} =: \gamma^{C}$ in the Lie algebra of $\mathrm{Cliff}^2\,\mathcal{W}[C]$; and $L_{c'c}$ is the iterated cumulant $\Sigma^{E-C}\gamma^{c'c} \in \mathrm{Cliff}\,\mathcal{W}[E]$, the E level.

Then every component of the metric form $g^{C'C}$ has a Lie-algebra index C and admits a second interpretation as a connection amplitude. Let us suppose provisionally that the W skew-action operator is extracted from an extended G skew-action operator by such a $\mathcal{Q}$ version of the Kaluza-deWitt strategy.

13.5 Constructing the strong gaugeon field

DO: Do this.

13.6 Constructing the gravitational field

This brings us to another often-proposed unification, the last of this sequence: The representation of the graviton g as a fermionic tetrad $\gamma\gamma$ suggests reducing the gravitational G dynamics to the fermionic F by eliminating the fermion annihilators ψ from the term $\sim (\Phi^{\mathrm{F}})^2$ in $e^{\Phi^{\mathrm{F}}}$. One eliminate external lines by taking an ambient expectation value, internal ones by summation.

DO: Do it!

14 Infraquantum fermion dynamics

For practical reasons, $\mathcal{Q}$ dynamics divides into two realms:

1 *Pre-organization dynamics.* Dynamics of a logarithmically small, disorganized patch, relative to a metasystem supported by an organized ambient dome.

2 *Post-organization dynamics.* Small excitations in the organized dome, involving only a few defects, [hopefully] including the standard model quanta.

Let us assume that the grand skew-action operator Φ is formed by $\mathcal{Q}$-quantizing the fermionic skew-action operator Φ^{F} coupling the extended G gaugeon to the fermion. For the small-excitation theory we accept that the dome reduces the simple G gauge algebra to semisimple ones, including diff in a singular organized limit. Yang $\mathrm{so}(3,3)$ is not a symmetry algebra of the organized dome, but can still be a symmetry algebra of a cell and of the predome plexus, and can still be a dynamics algebra of the dome.

The emergence of the dome from a less ordered collection of operons is presumably a phase transition of the plexus, perhaps beyond our theoretical reach, but we might be able to at least formulate the problem. The sensible strategy, however, seems to be to infraquantize something known to work, like the Dirac equation in Minkowski space-time. Let us approach this problem starting from the standard, unregularized, gauged, fermionic dynamics statvector

$$\mathbf{D}_{\mathrm{F}} = \exp \int [d\psi d\overline{\psi}] e^{i\int (d^4x)\, [\overline{\psi}(x)\gamma^m \mathcal{D}_m(x)\psi(x) + \overline{\xi}(x)\psi(x) + \overline{\psi}(x)\xi(x)]} \tag{155}$$

The Higgs field has been left out for now because [hopefully] it is a cluster of $\mathcal{Q}$ fermions with deep covalent binding, arising from the infraquantized fermion dynamical statvector $\widehat{\mathbf{D}}_{\mathrm{F}}$.

In the standard theory $\mathbf{D}_{\mathrm{F}}$ describes a field of spin-1/2 free fermions propagating with the usual spin-orbit coupling γD between the spin of each fermion and its kinetic momentum. It includes no direct interaction between fermions. The fermion field operators act as a quantifier, summing over the individual fermions in the many-fermion field theory. All interactions between fermions arise from their coupling to the collective gauge fields of $\mathcal{D}$. To describe the propagation of the gaugeons requires gaugeon skew-action operators $\Phi_{G,W,S}$.

In the $\mathcal{Q}$ theory the fermion field operators still form a quantifier, summing over individual fermions. The gaugeons. however, have to be replaced by polyadics in the fermions, say of grade g, if only to fit their observed spin and statistics. The fermion skew-action operator then couples not two fermions, but $g+2$ fermions. It describes interaction as well as propagation. The gaugeon propagator is then no longer separate from the fermion propagator, but derives from it algebraically. It is then conceivable that Φ_{F} is the entire skew-action operator.

14.1 Composite gaugeons

For composite gaugeons (§14) we must choose polyadic expressions for the gaugeons that indeed bind and propagate as gaugeons. We proceed by trial and error.

DO: ...

We quantize (155) bit by bit, working outward from the center:

14.2 Dirac spin operators

$\gamma^m \prec$ cell operators $\gamma^C \in \mathrm{Lin}\,{}^C\mathcal{Q}^1$, generating a Yang so$(3,3)$.

DO: Express these gamma's in $\mathcal{Q}$ terms.

1 Differentiator

$$D_m \prec \Sigma^{E-C}\gamma^{c'c} \in {}^E\mathcal{Q}^1, \tag{156}$$

a cumulant kinetic energy-momentum operator.

14.3 Fermion annihilation operators

$\psi, \overline{\psi} \prec$ annihilation operators γ^E. Part of item (14.6).

14.4 Pauli form

β implicit in $\overline{\psi} \prec$ the Pauli form $\mathsf{p}[E]$ of ${}^E\mathcal{Q}$.

14.5 Integration over space-time

. This integral becomes a trace (operation) on the algebra generated by the coordinates. One possible $\mathcal{Q}$ correspondent is a trace on the subalgebra generated by the space-time atoms $\gamma^{5\mu}$ and represented on the event level ${}^E\mathcal{Q}$, an so$(3,2)$. (Not the isomorphic anti-deSitter so$(3,2)$, whose coordinates commute.) This breaks Yang so$(3,3)$, as does the dome and its excitation dynamics. A correspondent suitable for the structure problem is a trace on the event algebra of level ${}^E\mathcal{Q}$.

14.6 Sources $\xi, \overline{\xi}$.

Part of item (14.6).

An exterior integration $\int[d\psi d\overline{\psi}]$ over fermionic histories. This is part of a Fourier transform, a mere change of frame that can be omitted.

14.7 An imaginary factor

$i \prec \widehat{\imath} = \Sigma^2\gamma^{65}/\mathsf{N}$.

14.8 The orbital momentum

p_m (like x^m) $\prec \Sigma^2\gamma_{c'c} \in {}^4\mathcal{Q}^1$.

14.9 Dirac operator

A Dirac spin-orbit operator $D := \gamma^m D_m$.

The standard action couples singular organized limits of imaginary, spin, and orbital operons. We face choices for this operon coupling similar to the early choice among $SVTAP$ beta-decay couplings, modified today to exhibit maximal parity violation. In β decay theory, the possible grade of the coupled factors ranges from 0 for S to 4 for P. Here, assuming a Yang SO$(6-n,n)$ group, it ranges from 0 to 6. The $\mathcal{Q}$ action can be

$$A \;=\; \frac{h}{\mathsf{N}}\Sigma^2\gamma^{c}{}_{c''}(\text{imaginary}) \times \gamma^{c''}{}_{c'}(\text{spin}) \times \Sigma^2\gamma^{c'}{}_{c}(\text{orbit})$$

$$= \iota \left[\underbrace{\Sigma^2 \gamma^{c''}{}_{c'}}_{\text{imaginary}} \times \overbrace{\gamma^{c'}{}_{c}}^{\text{spin}} \times \underbrace{\Sigma^2 \gamma^{c}{}_{c''}}_{\text{orbit}} \right] \frac{\partial}{\partial \iota_{\rm o}} \tag{157}$$

with the indicated correspondences. A peels one outer ι from its operand, applies the bracketed coupling, and then restores the ι. The "imaginary" factor $\widehat{\imath}$ is needed for skew-symmetry, just as the standard Dirac action requires a factor i. Infraquantization incorporates gauging, and thus raises the grade of the Dirac propagator from the usual two to six, describing physical interaction as well as propagation. The $\mathcal{Q}$ Dirac equation is a specific topological relation between quantum operons, relating the edges of connected cells. The reduction $\gamma^{6\ldots 1} = \pm 1$ (§9.3) results in chiral operon spinors analogous to the chiral fermion spinors of the standard model.

The standard model algebra $s[\mathrm{u}(2)\times\mathrm{u}(3)]$ of charges becomes $s[\mathrm{u}(2)\times\mathrm{u}(4)]$. All the physical monads support one representation of this algebra. Black monads actually happen while colorful monads are individually only virtual. This is suggestively similar to the way timelike displacements actually happen while spacelike ones are virtual [31].

While every basic standard fermion has the same spin 1/2, they do not all have the same color representation, which acts on $1C$ for leptons and $3\mathbb{C}$ for quarks; nor the same isospin representation, which acts on $1\mathbb{C}$ for right-handed leptons and $2\mathbb{C}$ for quarks and left-handed leptons.

This part of the exploration has analyzed the standard singular description into atoms. This has to be worked out further for gauge variables. Then will come a largely synthetic task of a rather different character, reassembling the parts into a coherent whole.

15 Discussion

DO: ... Finish this section.

One possibility among many is that the generations represent three strata within the transverse structure of the dome. It is well known that particle masses represent shielding lengths, analogous to the Debye length of an electrolyte. In a continuum, shielding proceeds continuously, Is it conceivable that in a quantum plexus shielding proceeds in quantum jumps, the generations? Or is it possible that the difference between the generations can be expressed as a difference in type? I cannot formulate these questions algebraically yet.

15.1 Acknowledgements

I am indebted to James Baugh, Shlomit Ritz Finkelstein, Andry Galiautdinov, Dennis Marks, Zbigniew Oziewicz, Heinrich Saller, and Sarang Shah for recent helpful discussions and corrections.

16 Appendices

17 Quantum pragmatics 2010.04.28

In classical thought there is a clearcut distinction between basic theory and phenomenology, so deeply woven into our traditions that some try to find it in the quantum theory. It has ancient roots. Parmenides already divided knowledge, *episteme*, into *logos* and *doxa*, and logos, arrived at by reason, was deemed exact, while doxa, arrived at by observation and report, was approximate. In classical mechanics, the concept of the dynamical law that cannot be learned from one observation descends from the logos, and the system state, learned by observation only, harks back to the doxa. There is a categorical difference between the law and the system it governs. This theory of knowledge is also the core of the *perennial philosophy* [42].

Several contradictory formulations of the same quantum theory are in wide circulation. They may be broadly classed as either classical or operational (pragmatic, if one prefers a Greek root to a Latin one). It is termed classical here to theorize about reality, objects as they are in their essence, independently of observation, and operational to dispense with essences and speak instead of our operations, like perceptions and measurements, possible or actual. Operations can fail to commute; objects cannot.

A classical theory can take the universe as system. A quantum theory must leave most of the universe outside the system so that it can carry out observations on the system. The quantum law can then be outside the system but still inside the universe. The quantum theory thus eliminates the categorical difference between law and system. Both are represented as operators: operators like the Hamiltonian H for the law, and operators like io operations ψ for the system. §7 takes up the immanent law of quantum theory that regularizes the emmanent law of classical mechanics.

The present formulation is pragmatic and processual.

The quantum mode of reasoning could be called quantum logic, but many still assume that logic is a priori, and here quantum theory is understood as afortiori, so the term “quantum logic” could seem oxymoronic; and in any case it was preempted for a certain lattice algebra of little computational utility.

The quantum mode of reasoning is based on operations or actions and so is aptly termed *pragmatic* by Stapp [36]. Let us speak of quantum pragmatics rather than quantum logics, to avoid the misconceptions mentioned and emphasize the empirical nature of the study in question.

The *system* under study is taken here to be specified by how experimenters can produce it, act on it, and register it; by their input, throughflow, and output actions. The rest of nature, including the experimenter, instruments, and life support, constitute the *metasystem.*

Historically the metasystem was known only coarsely and described in natural language. Now it is possible to operate at least on a logarithmically small part of it with quantum precision. If this part can still serve as a metasystem for a still smaller system, then it may be considered to be a quantum metasystem. The possibility that such extensions of quantum theory might become necessary was anticipated by Bohr [6]. But then the rest of nature, including ourselves as experimenters on this metasystem, remains coarsely resolved, and can be regarded as meta-metasystem, or meta^{2}system.

In the present study, space-time coordinate variables are quantized. Space-time variables of canonical quantum field theory are not measured on the field system but on individual quanta, one type lower. Their quantization is therefore an infraquantization.

Let us reserve the term *state* as in classical mechanics for an observable property of the isolated system that determines all others. For an isolated system, knowing the state is complete knowledge of the system, by definition. When we sharply determine any property of a physical system, however, it seems that we change other properties of the same system, which are not being determined, greatly and uncontrollably. Therefore physical systems do not seem to have states. In quantum pragmatics a ray in a Hilbert space $\mathcal{H}$ provides a maximally informative—*sharp*—statistical description of a population of systems, certainly not a state. A ray is conveniently specified by one of its vectors, which therefore is also not a state.

Formulations of quantum theory using the term “state vector” often say that every quantum system has a state vector. This assumption seems meaningless or false, but is so widespread that the only practical way to evade it to drop the term. Here a non-zero vector in the ray describing a sharp input-output process is called a statistical vector or *statvector*. Statvectors and their duals represent sharply defined input and output quantum operations. The individual system does not “have” a statvector in the sense that it has a momentum, or in the sense that a classical system has a state. Statvectors also have a product, by which aggregates are formed from individuals.

The Hilbert space arises because physical filtration operations that are *crisp*—meaning $P^2 = P$—seem to form a projective geometry. The sharp projectors are its points or rays, the next sharpest are its lines, and so on. The incidence relation defining this projective geometry is easily expressed in terms of the inclusion relation $A \subset B$, and this is defined in terms of a statistical null relation $A \o B$ meaning that "every" beam that passes the A test entirely fails the B test, and conversely. Then

$$A \subset B \quad := \quad \forall C \; (C \o B \; \subset \; C \o A) \tag{158}$$

Two points fix a line, and two lines in a plane fix a point, just as in classical logic. But many points lie on every line, as in projective geometry, not classical logic. They are all called superpositions of two points that fix the line, and this is the superposition principle, the feature peculiar to quantum pragmatics and alien to classical logic. What is often called a quantum logic is a Galois lattice of the null relation.

To be quite explicit:

An individual quantum system has neither a state nor a statvector. (159)

The system has no state because quantum physics is radically indeterministic; no statvector, because an individual is not a beam or a statistical population, quantum or no.

What *can* have a unique statvector are the input, throughflow, and output phases I, II, III of an ideal quantum beam experiment. The transition metric form h of the Hilbert space maps each input statvector ψ to the unique output dual statvector $\mathsf{h}\psi$ of an experiment that results in a transition on every trial. If the statvector III and dual statvector I have unit h- norms, the transition probability amplitude and probability are

$$A = \mathrm{III} \cdot \mathrm{II} \cdot \mathrm{I}, \quad P = |A|^2. \tag{160}$$

An operator on $\mathcal{H}$, and now a statvector in $\mathcal{Q}$, is intended to represent an *operon*, a quantum physical process carried out on the syatem that respects quantum superposition relations among statvectors.

Canonical quantization represents the canonical Lie or quasi-Lie algebras of a classical theory in the Lie algebra su(∞) of bounded hermitian operators on Hilbert space $\mathcal{H}$. When von Neumann proposed Hilbert space and quantum logics (his plural) as the platform for elementary quantum mechanics and quantum field theory he recognized that it was non-relativistic, describing all space at one time, ignoring light lag $1/c$ [44]. Relativistic theories are field theories, and therefore use sets of sets, constructed for example by ι and $\vee$. For Boole, a class was a process, namely of selection, transforming one population to another. Von Neumann, in his doctoral thesis, also formulated classical set theory as a theory of transformations. His quantum logic was also a theory of transformations, namely projection operators, and he later mentioned "quantum set theory", but apparently never constructed one. The quantum set theory $\mathcal{Q}$ is a specially simple and limited quantization of classical set theory, replacing classical logic by a quantum pragmatics, and retaining the transformational interpretation of Boole and von Neumann.

In quantum theory it is especially important to distinguish mathematical objects from physical, since they are handled with different logics. In classical mechanics one may relate mathematical to physical objects by a generalized commutative diagram:

$$
\begin{array}{ccccc}
 & & \text{computation} & & \\
 & M_0 & \rightarrow & M_1 & \\
 \text{input} & \downarrow & & \downarrow & \text{output} \\
 \hdashline
 & P_0 & \rightarrow & P_1 & \\
 & & \text{dynamics} & &
\end{array}
\tag{161}
$$

Above the dotted line are mathematical operations on symbols, taking place in the metasystem; below, symbols for physical actions on the system under study. This interface exists for classical physics as well as for quantum physics, but is generally omitted from classical theories, which neglect the influence of measurement on the system and the limitations of measurement. If the computational transformation is expressed in a set algebra like $\mathcal{C}$, the roof of (161) now resembles a large truss assembled from ι struts by $\vee$ and resting on 1. $\mathcal{C}$ softens the unphysical conceptual wall between space-time and the other variables of physics by making both out of iotas.

For a quantum system, however, (161) fails on the level of the individual, which is generally unpredictable. For a valid dagram, the theory must confine itself to averages over many runs of each experiment. One does not merely accept or reject such a statistical theory but commonly sets up a confidence level for its acceptance, taking into account the pragmatic consequences of accepting or rejecting the theory, and one may revise the decision. Kolmogorov emphasized that probabilistic theories are not objective; this applies to quantum theories in particular, which have been called non-objective.

A metasystem can coherently produce and register a beam of systems only if the metasystem is exponentially larger than the system. The interface is therefore not as freely movable as von Neumann said [44]. We must put the interface where there are few active connections, so that we isolate the system to be studied rather than destroy it. Often the interface is located in vacuum.

The vacuum too is full of connections, but they may be inactive on the scale of current measurements. And we must leave enough metasystem outside the system to carry out the study.

17.1 The quantum i

Since the central imaginary i is the center of canonical quantum theories, it might help to understand its physical role.

The i of quantum theory is mainly important for the classical limit and the correspondence principle. The Dirac correspondence between classical Poisson brackets $[a, b]_{\rm P}$ and quantum commutators $[\widehat{a}, \widehat{b}]$ is

$$\widehat{[a, b]_{\rm P}} = \frac{i}{\hbar}[\widehat{a}, \widehat{b}]. \tag{162}$$

It converts the Hamiltonian dynamical equation into the Heisenberg dynamical equation

$$\frac{dq}{dt} = \frac{i}{\hbar}[H, q]. \tag{163}$$

For fermions this i can be cancelled by one in the Hamiltonian, though not for bosons.

In the present infraquantization, the imaginary i emerges in a singular organizing limit. This is consistent with its use in the correspondence relation (162), which also refers to such a limo.

17.2 The quantum cosmos 2010.04.24

If science is understood as the drawing of statistical inferences from a population of experiments, the question arises of how a science of cosmology is possible, since the cosmos is unique. When W. H. Pitts asked this question of Norbert Wiener in a lecture at M.I.T in the 1950's, the response was that the cosmos itself can be regarded as a population, of regions and eras. The question applies to a quantum cosmology as well; so does the answer.

On one hand, a statvector for the cosmos seems to make no sense on its face. An individual system has no statvector—(159)—and the cosmos, understood to embrace all that exists, is individual. We cannot know that we have sharply prepared ourselves, since such a preparation wipes out all memory. Therefore we can have no sharp input or output processes for the cosmos, which includes us. It is not clear that the cosmos can be regarded as a quantum system in the strict sense.

On the other hand, a hypothesized cosmic statvector generates reduced statvectors, or at least probability operators, for carefully selected subsystems of the cosmos, not including ourselves, that can have experimental meaning. The use of cosmic statvectors does not seem to be intrinsically more absurd than the use of cosmic states. One must not take it literally, and must exercise due caution for features peculiar to the quantum case, like non-commutativity.

17.3 Clifford algebras of $\mathcal{Q}$

A *Clifford algebra* C is a (linear associative) ring Poly(V) of polynomials $p(v, v', \dots)$ in the elements of a vector space V whose addition and multiplication extend those of V. The coefficients are called the scalars, 0-adics, or cenadics of C and form a commutative field and a central subring of C. The vectors of $C^1 = V \subset C$ are called the vectors or monadics of C. The sole additional axiom is the *Clifford rule*:

The square of every vector is a scalar.

This scalar is also called the *norm* of the vector:

$$\forall v \in C^1 : \quad v^2 = \|v\| =: v \cdot v \in C^0. \tag{164}$$

In the present study $C^0 = \mathbb{R}$. The polarization of the quadratic form $\|v\|$ is a bilinear form written $u \cdot v$, so that by definition $\|v\| = v \cdot v$.

An *exterior algebra* is a Clifford algebra with $\|v\| \equiv 0$. The *exterior algebra of a Clifford algebra* C with product $\sqcup$ is the exterior algebra $\breve{C}$ with the scalars and vectors of C, with product written as $\vee$. Then C and $\breve{C}$ have the same elements and

$$\forall u, v \in C^1 : \quad u \sqcup v = u \vee v + u \cdot v \tag{165}$$

$\breve{C}$ has a grade, which is also assigned to C. The g-grade subspace of C is written C^g. ite elements are called g-adics.

A classical algebra of finite sets defines a Clifford algebra over the binary field. The Clifford sum $x+y$ of two sets is a statistical mix with equal weights. The Clifford product $x \sqcup y$ is the symmetric union $x \cup y \setminus x \cap y$, so that Clifford's rule holds in the form $x \sqcup x = 1$.

In the primary interpretation of $\mathcal{Q}$, the monadics of $\mathcal{Q}^1 \subset \mathcal{Q}$ represent elementary acts of basic fermion annihilation or output, called monads. A fermion annihilated by a monad could be called a monon.

In the present application of $\mathcal{Q}$, leptons and quarks are monons. Polyadics represent composite operons, here including gaugeons and Higgs. Operons close to monads are generally modified by vacuum polarization with polyadic corrections, sometimes large but always finite.

$\mathcal{C}$ is a small part of set theory, the semigroup of all ancestrally finite sets without foundation, and is generated by a dyadic associative commutative multiplication operation $x \vee y$ ($\vee yx$ in prefix notation), a free monadic operation ιx, read *successor of* x, and the empty set 1, a constant. The classical product $\vee$ is the disjoint union, obeying $\iota x \vee \iota x = 0$. Peano's ι operation is better suited to an algebraic theory than Cantor's $\in$ relation. $\iota x = \{x\} = \overline{x}$ is the set whose only element is x. The familiar set notation $\{x, y, \dots, z\}$ is here an abbreviation for $\iota x \vee \iota y \vee \dots \vee \iota z$.

A quantum variant of ι serves as the microcosmic strut of which a quantum system history is constructed, an iota indeed. $0 \in \mathcal{C}$ serves as a space-holder, marking the absence of a meaningful expression; a back-formation from the quantum theory, where $0 \in \mathcal{Q}$ is the sole vector that defines no ray and no physical process, synonymous with the ∞ of C. S. Pierce and the OM (omega) of SETL.

$\mathcal{C}[T] \subset \mathcal{C}$ is the level-T part of $\mathcal{C}$, the subset of $\mathcal{C}$ whose sets can be written with 1, $\vee$ and $\leq T$ nested ι's. Levels nest: $T < T' :\subset: \mathcal{C}[T] \subset \mathcal{C}[T']$. Let $\mathcal{C}^g$ denote the subset of $\mathcal{C}$ of grade (:=cardinality) g, the g-ads, products of g monads. Let the (binary) exponential $\exp_2 X$ be the set of finite subsets of any set X. Then $\mathcal{C}$ is its own exponential: $\mathcal{C} = \exp_2 \mathcal{C}$. Define the *random (finite) set* as the mathematical object with state space $\mathcal{C}$.

Now for the quantum set or polyad. This is associated in quantum fashion with the vector space $\mathcal{Q}$. Vectors with the statistical interpretation of quantum theory are called statistical vectors or statvectors to distinguish them from states in the classical sense (Appendix 17). Any non-zero statvectors in $\mathcal{Q}$ statistically described a polyad.

$\mathcal{Q}$ is a real exterior algebra generated, by analogy with $\mathcal{C}$, by the associative exterior product $\vee$, an injective linear operator $\iota : \mathcal{Q} \to \mathcal{Q}$, the constant identity 1 for the empty set, and the constant 0 for the undefined. In an exterior algebra, monadics anti-commute.

Operations that in classical logic were considered mental or mathematical correspond in quantum pragmatics to physical operations. The $\mathcal{Q}$ monadics represent operations of individual quantum annihilation, monads. The $\mathcal{Q}$ product $b \vee a$ means doing a and then b; ιx means annihilating an annihilator. In the present application let us suppose that annihilations of first-generation leptons and quarks are near-monads. The higher generations are taken up in §10.

$\mathcal{Q}$ is the exterior algebra over itself:

$$\mathcal{Q} = \text{ext}\, \mathcal{Q}. \tag{166}$$

A certain sequence of rays in $\mathcal{Q}$ illustrated in (40) is isomorphic to $\mathcal{C}$. The levels ${}^T\mathcal{Q}$ and $\mathcal{Q}[\leq T]$ are defined analogously to $\mathcal{C}[T]$ and $\mathcal{C}[\leq T]$. The inclusive levels $\mathcal{Q}[\leq T]$ are nested exterior subalgebras of $\mathcal{Q}$.

$\mathcal{Q}$ has a natural Hilbert space metric, defined inductively:

- $\mathbb{R} = \mathcal{Q}[1]$ is a one-dimensional Hilbert space with norm $\mathsf{h}^0 xx = x^2$ for $x \in \mathbb{R} \subset \mathcal{Q}$.
- If $\mathcal{H}$ is a Hilbert space then $\operatorname{ext}\mathcal{H}$ has a unique natural Hilbert space form extending $\mathcal{H}$.
- Therefore $\mathcal{Q} \cong \text{LIMO}_{\,n\to\infty}(\text{ext})^n\mathbb{R}$ has a unique natural Hilbert space metrical form h extending h^0.

17.4 Glossary 2010.04.25

'	$F' S = \{F(s) \mid s \in S\}$; "the F's of S".
$a \prec A$	a is a singular organized limit of A, or Limo A. Informal.
$A \succ a$	$a \prec A$.
β	The Pauli adjoint for Dirac spinors, a bilinear spinor form.
$\mathcal{Q}x, \widehat{x}$	Infraquantization of x, with $\widehat{x} \succ x$.
$\mathbf{1}^q, \mathbf{1}^Q$	A basic monadic in $\mathcal{Q}$, as in (40).
γ^w	A basic Clifford generator in Cliff $\mathcal{W}$.
2_n	The second exponential of n. $2_0 := 1$, $2_n := 2^{(2_{n-1})}$.
Limo	Singular limit with organization.
$\mathrm{Lin}\, V$	The algebra of linear operators $V \to V$.
$\triangle$	$^{\mathsf{L}-\mathsf{R}}$. The adjoint representation.
$\mathrm{Dup}\, V$	duplex space[1] of V; $V \oplus \mathrm{Dual}\, V$ with quadratic form Q.
$\exp_2$	binary exponential[2]: $\exp_2 S := 2^S = \{\mathrm{poly} S \mid (\iota s)^2 = 0\}$.
$\mathrm{ext}\, V$	exterior algebra of V; $\mathrm{ext} = \mathcal{Q} \exp_2$.
$\mathrm{Fermi}\, V$	the Fermi algebra of V, generated by $\{^{\mathsf{L}}v, \partial_v \mid v \in V\}$.
Γ	a gravitational constant $\propto h/G$.
$\hbar$	rationalized Planck action constant $h/2\pi$.
$\mathrm{h}(n)$	canonical Lie algebra of n coordinates, n momenta, and i.
ι	successor,brace. $\iota\, q = \{q\} = \overline{q}$.
io	Creation or annihilation or superposition thereof.
$^{\mathsf{L}}, ^{\mathsf{R}}$	left, right exterior multiplication.
$L_{v'v}$	generators of the defining representation of $\mathrm{so}(V)$.
Limo $_{\mathrm{o}}$	Limit with organization. See (9).
metasimplex	Simplex of simplices of .... See (52).
monadic	first-grade statvector in $\mathcal{Q}$.
monad	physical process represented by a monadic.
$\circ$	$\phi \circ \psi$ = value of dual vector ϕ on vector ψ.
operon	quantum physical process represented by a linear operator.
p	Pauli adjoint for the highest level of $\mathcal{Q}$ in use.
polyadic	$\mathcal{Q}$ statvector of unspecified grade.
$\mathcal{Q}$	$\mathrm{ext}\, \mathcal{Q} = \widehat{2}^{\mathcal{Q}}$, the exterior algebra of simplex statvectors.
$\mathcal{Q}^g$	Grade g of $\mathcal{Q}$.
$^T\mathcal{Q}$	Level T of $\mathcal{Q}$; $\mathcal{Q}[0] = \mathbb{R}$.
Q	Quadratic form $\mathsf{Q}(v \oplus u) := u \circ v$, $v \oplus u \in \mathrm{Dup}\, V$.
$\mathrm{SO}(V)$	The special orthogonal group of the quadratic space V.
$\mathrm{SU}(V)$	The special unitary group of the Hermitian space V.
$\mathcal{W}[T]$	$\mathrm{Dup}\, {}^T\mathcal{Q}^1$. $\mathrm{Lin}\, {}^T\mathcal{Q} = \mathrm{Cliff}\, \mathcal{W}[T]$.

[1] The *antidouble space* of [31]

[2] Also called power set, and indeed a power of 2, but not of S.

References

[1] Stephen L. Adler. *Quaternionic quantum mechanics and quantum fields.* Oxford University Press, 1995.

[2] V. Ambarzumian and D. Iwanenko. Zur frage nach vermeidung des unendliche selbstrückwirkung des elektrons. *Zeitschrift für Physik*, 64:563–567, 1930.

[3] J. Baugh. *Regular Quantum Dynamics.* PhD thesis, School of Physics, Georgia Institute of Technology, 2004.

[4] F. Bayen, M. Flato, C. Fronsdal, A. Lichnerowicz, and D. Sternheimer. Quantum mechanics as a deformation of classical mechanics. *Letters in Mathematical Physics*, 1:521–530, 1977.

[5] Gregor W. Bayer. Theory of the cut-off. *Symmetry*, to appear, 2009.

[6] N. Bohr. Causality and complementarity. *Philosophy of Science*, 4:293–4, 1936.

[7] F. Bopp and R. Haag. Über die möglichkeit von spinmodellen. *Zeitschrift fr Naturforschung*, 5a:644, 1950.

[8] R. Brauer and H. Weyl. Spinors in n dimensions. *American Journal of Mathematics*, 57:425, 1935.

[9] E Cartan. Les groupes projectifs qui ne laissent invariante aucune multiplicit plane. *Bull. Soc. Math. France*, 41:53–96, 1913.

[10] E. Cartan. *Leçons sur la theorie des spineurs.* Hermann, Paris, 1938.

[11] A. H. Chamseddine, A. Connes, and M. Marcolli. Gravity and the standard model with neutrino mixing. *Adv. Theor. Math. Phys.*, 11:991–1089, 2007.

[12] C. Chevalley. *The construction and study of certain important algebras.* The Mathematical Society of Japan, Tokyo, 1955.

[13] P. A. M. Dirac. *Spinors in Hilbert Space.* Plenum, New York, 1974.

[14] A. Einstein and V. Bargmann. Bivector fields. *Annals of Mathematics*, 43:1–14, 1943.

[15] A. Einstein and V. Bargmann. Bivector fields II. *Annals of Mathematics*, 43:15–23, 1943.

[16] L. D. Faddeev. How we understand "quantization" a hundred years after Max Planck. *Physikalische Blätter*, 52:689, 1996.

[17] R. P. Feynman, 1941. Personal communication ca 1961. Feynman began this line of thought in about 1941, before his work on the Lamb shift, and may have published a similar formula.

[18] D. Finkelstein. Space-time code. *Physical Review*, 184:1261–1271, 1969.

[19] D. Finkelstein, J.M. Jauch, and D. Speiser. Notes on quaternion quantum mechanics I, II, III. Report CERN 59–7, 59–11, 59–17, CERN, Geneva, 1959.

[20] D. R. Finkelstein. Space–time structure in high energy interactions. In T. Gudehus, G. Kaiser, and A. Perlmutter, editors, *Coral Gables Conference on Fundamental Interactions at High Energy. Center of Theoretical Studies January 22—24, 1969. University of Miami*, pages 324–343, New York, 1969. Gordon and Breach.

[21] D. R. Finkelstein. *Quantum Relativity.* Springer, Heidelberg, 1996.

[22] A. Galiautdinov and D. R. Finkelstein. Chronon corrections to the Dirac equation. *JMP*, 43:4741, 2002.

[23] M. Gerstenhaber. On the deformation of rings and algebras. *Annals of Mathematics*, 32:472, 1964.

[24] D. Hestenes. *Space-time algebra.* Gordon and Breach, New York, 1966.

[25] E. Inönü and E.P. Wigner. On the contraction of groups and their representations. *Proceedings of the National Academy of Science*, 39:510–524, 1953.

[26] B. Kostant. Lie algebra cohomology and the generalized Borel–Weil theorem. *Annals of Mathematics*, 74:329–387, 1961.

[27] A. Kuzmich, N. P. Bigelow, and L. Mandel. *Europhysics Letters*, A 42:481, 1998.

[28] Dennis Marks, 2008. Private communication.

[29] T. D. Palev. Lie algebraical aspects of the quantum statistics. Unitary quantization (A-quantization). Preprint JINR E17-10550, Joint Institute for Nuclear Research, Dubna, 1977. hep-th/9705032.

[30] R. Penrose. Angular momentum: an approach to combinatorial space-time. In T. Bastin, editor, *Quantum Theory and Beyond*, pages 151–180. Cambridge University Press, Cambridge, 1971. Also available at http://math.ucr.edu/home/baez/penrose/. Penrose kindly shared some of this seminal work with me as his theory of 'mops', in 1960.

[31] H. Saller. *Operational Quantum Theory I. Nonrelativistic Structures.* Springer, New York, 2006.

[32] I. Schur. Über die Darstellung der symmetrischen und der alternierenden Gruppen durch gebrochene lineare substitutionen. *Journal für die reine und angewandte Mathematik*, 139:155–250, 1911. Translation: On the

representation of the symmetric and alternating groups by fractional linear substitutions. Transl. M-F Otto. *International Journal of Theoretical Physics* 40:413–458 (2001).

[33] I. E. Segal. A class of operator algebras which are determined by groups. *Duke Mathematical Journal*, 18:221–265, 1951. Especially §6A.

[34] M. Shiri-Garakani and D. R. Finkelstein. Finite quantum kinematics of the harmonic oscillator. *Journal of Mathematical Physics*, 2006.

[35] H.P. Snyder. Quantized space-time. *Physical Review*, 71:38, 1947.

[36] H. P. Stapp. The copenhagen interpretation. *American Journal of Physics*, 40:1098–1116, 1972.

[37] E. C. G. Stückelberg. Quantum theory in real Hilbert space. *Helvetica Physica Acta*, 33:727–752, 1960.

[38] L. Susskind. Dynamics of spontaneous symmetry breaking in the weinberg–salam theory. *Physical Review D*, 20:2619, 1979.

[39] M. Tavel, D. Finkelstein, and S. Schiminovich. Weak and electromagnetic interactions in quaternion quantum mechanics. *Bulletin of the American Physical Society*, 9:435, 1965.

[40] Armin Uhlmann. State vector and quantization in an over-all space-time view. *Wissenschaftliche Zeitschrift der Karl-Marx-Universität Leipzig, Mathematische-Naturwissenschaftliche Reihe*, 2/3:115–123, 1957/58.

[41] H. Umezawa. *Advanced field theory: micro, macro, and thermal physics.* American Institute of Physics, 1993.

[42] R. M. UNGER. *THE SELF AWAKENED: PRAGMATISM UNBOUND.* Harvard, 2007.

[43] R. Vilela-Mendes. Deformations, stable theories and fundamental constants. *Journal of Physics A*, 27:8091–8104, 1994.

[44] John von Neumann. Mathematische Begrundung der Quantenmechanik. *Nachr. Ges. Wiss. Göttingen*, pages 1–57, 1927. Also in J. von Neumann, Collected Works, volume 1.

[45] S Weinberg. Implications of dynamical symmetry breaking: An addendum. *Physical Review D*, 19:1277, 1979.

[46] C. F. v. Weizsäcker, E. Scheibe, and G. Sussmann. Komplementarität und Logik III. Mehrfache Quantelung. *Naturwissenschaften*, 13a:705, 1955.

[47] J. Wess. Gauge theories on noncommutative spacetime treated by the Seiberg-Witten method. In U. Carow-Watamura, Y. Maeda, and S. Watamura, editors, *Quantum Field Theory and Noncommutative Geometry*, page 177. Springer, Berlin, 2005.

[48] C. N. Yang. On quantized spacetime. *Physical Review*, 72:874, 1947.