A Common Fallacy in Quantum Mechanics

Why Delayed Choice Experiments do NOT imply Retrocausality

David Ellerman

UCR

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Overview: Separation Fallacy

- There is a common fallacy, here called the *separation fallacy*, that misinterprets as a measurement certain types of separation as in:
  - double-slit experiments,
  - which-way interferometer experiments,
  - polarization analyzer experiments,
  - Stern-Gerlach experiments, and
  - quantum eraser experiments.
- It is the separation fallacy that leads not only to flawed textbook accounts of these experiments but to flawed inferences about retrocausality in the context of "delayed choice" versions of separation experiments.
- Certain later interventions can show that the separation was not a measurement, so the flawed argument is that by not making or making the later intervention, one "retrocauses" either a measurement or not at the separation.

Flawed retrocausality reasoning: I

- In each experiment, given an incoming quantum particle, the apparatus creates an entangled superposition of certain eigenstates (the "separation").
- Detectors can be placed in certain positions so that when the evolving superposition state is finally projected or collapsed by the detectors, then only one of the eigenstates can register at each detector.
- The *separation fallacy* is the misinterpretation of these detections as showing that the particle had collapsed to an eigenstate at the separation apparatus, not at the later detector.

Flawed retrocausality reasoning: II

- But if the detectors were suddenly removed while the particle was in the apparatus, then the superposition would continue to evolve and have distinctive effects (e.g., interference patterns in the two-slit experiment).
- Then it *seems* that by the delayed choice to insert or remove the appropriately positioned detectors, one can *retrocause* either a collapse to an eigenstate or not at the particle's entrance into the separation apparatus.
- The separation fallacy is remedied by:
    - taking superposition seriously, i.e., by seeing that the separation apparatus created an entangled *superposition* state of the alternatives (regardless of what happens later) which evolves until a measurement is taken, and

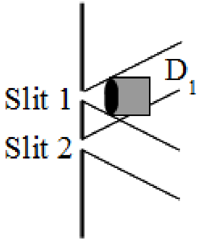
Flawed retrocausality reasoning: III

- taking into account the role of detector placement, i.e., by seeing that if a suitably positioned detector can detect only one collapsed eigenstate, then it does not mean that the particle was *already* in that eigenstate prior to the measurement (e.g., it does not mean that the particle "went through only one slit").

- The separation fallacy will be first illustrated in a non-technical manner for the first four experiments. Then the lessons will be applied in a more technical discussion of quantum eraser experiments–where, due to the separation fallacy, incorrect inferences about retrocausality have been rampant.

Double-Slit Experiment: I

- In the usual double-slit setup, suppose a detector D_1 is placed a finite distance after one slit but close enough so a particle "going through the other slit" cannot reach the detector.



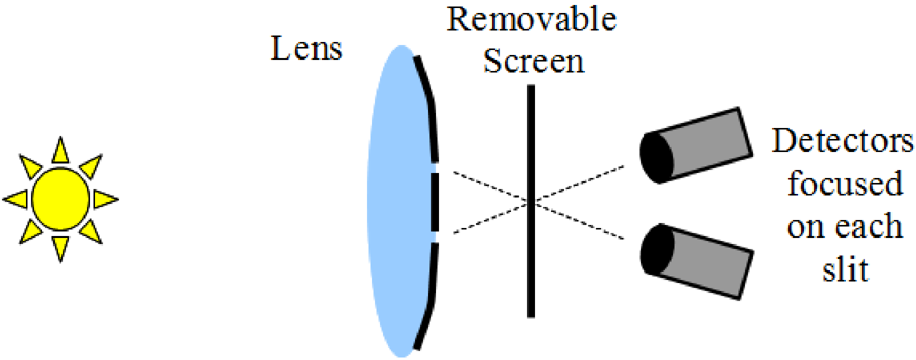
- Then it is commonly said that a hit at the detector records the particle "going through slit 1."

Double-Slit Experiment: II

- But this is wrong; the particle is in a superposition state, which might be represented as $|S1\rangle + |S2\rangle$, until the detector induces the collapse to an eigenstate.
- The story is about *detector placement*, not going through only one slit. With this placement of the detector, it will only record a hit when the collapse is to $|S1\rangle$.
- If the detector were suddenly removed after the particle traversed the slits but before encountering the detector, then the particle would continue and show the interference effects of its superposition state.

Double-Slit Experiment: III

- With the incorrect inference that a detector hit means "the particle went through slit 1," the delayed choice of removing the detector or not would seem to retrocause the particle to "go through both slits" or "go through only one slit."
- In Wheeler's more elaborate version of this delayed choice double-slit experiment, the detector or detectors are again placed and focused so as to record only one part of the superposition $|S1\rangle + |S2\rangle$ when it collapses.



Wheeler's delayed choice 2-slit setup

- Then the delayed choice is to remove the screen or not after a particle has traversed the slits.

Double-Slit Experiment: V

- By erroneously inferring that a hit at one detector means the particle went through the corresponding slit, it seems again that one can retrocause the particle to:
  - "go through both slits" (screen left in place), or
  - "go through only one slit" (removing screen and getting hit at only one detector).
- This form of the separation fallacy is unfortunately rather common in the literature. For instance, here is Anton Zeilinger:

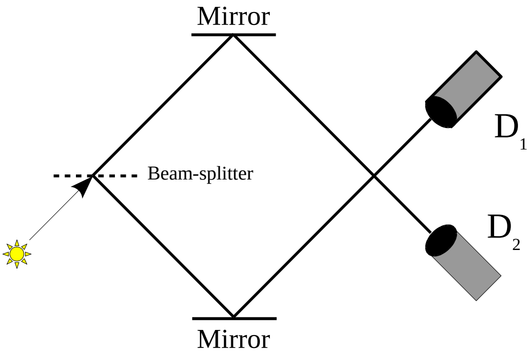
Double-Slit Experiment: VI

"We decide, by choosing the measuring device, which phenomenon can become reality and which one cannot. Wheeler explicates this by example of the well-known case of a quasar, of which we can see two pictures through the gravity lens action of a galaxy that lies between the quasar and ourselves. By choosing which instrument to use for observing the light coming from that quasar, we can decide here and now whether the quantum phenomenon in which the photons take part is interference of amplitudes passing on both side of the galaxy or whether we determine the path the photon took on one or the other side of the galaxy."

Which-way interferometer experiments: I

- Consider a Mach-Zehnder-style interferometer with only one beam-splitter (e.g., half-silvered mirror) at the photon source which creates the photon superposition: $|T1\rangle + |R1\rangle$ (which stand for "Transmit" to the upper arm or "Reflect" into the lower arm at the first beam-splitter).

Which-way interferometer experiments: II

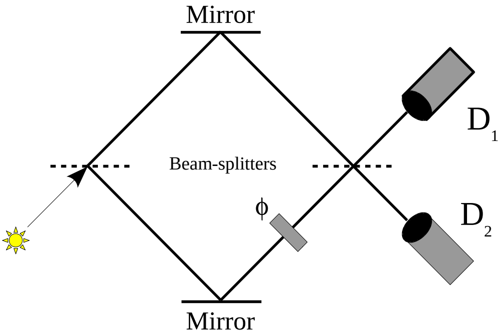


- If detector D_1 gets a hit, then it is said that "the photon took the lower arm."
- If detector D_2 gets a hit, then it is said that "the photon took the upper arm."

Which-way interferometer experiments: III

- But again, this is wrong. It is about detector placement so that when the superposition $|T1\rangle + |R1\rangle$ collapses, it will only be recorded at one detector. Thus the detectors were NOT recording "which-way information" since the photon was in a superposition prior to the detections.
- When a second beam-splitter (and phase-shifter) is inserted, then each detector will record an interference pattern so it is said that the (non-existent) "which-way information" was erased and "the photon took both arms."

Which-way interferometer experiments: IV



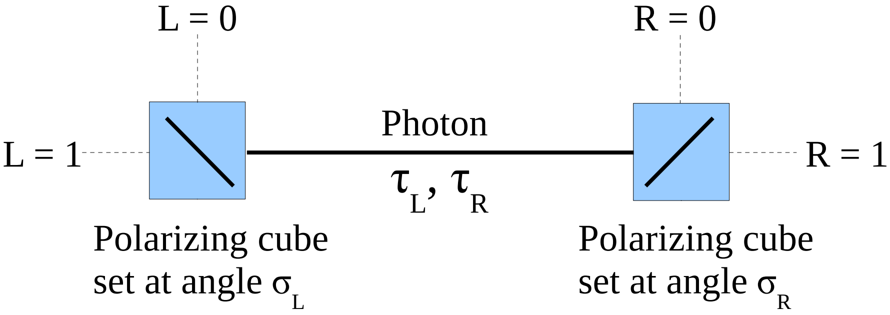
Which-way interferometer experiments: V

- Without the second beam-splitter, the incorrect inference that the detectors record "which-way information" (when in fact the photon was always in the superposition), makes it seem that one can retrocause the photon to "go through both arms" or only "go through one arm" by the delayed choice to insert the insert or not insert the second beam-splitter.
- All the "talk" in the literature about "which-way information" and "erasing which-way information" are illustrations of the separation fallacy in the context of the Mach-Zehnder interferometer.

Huw Price retrocausality argument: I

- At a recent UCSD conference, Huw Price presented a new retrocausality argument. Although not a delayed choice argument, it commits the same separation fallacy involved in the interferometer experiment of assuming that hits at one or another appropriately placed detectors gave which-way information about the photon discretely going through one arm or the other (instead of being in a superposition state prior to detection).
- The Price setup is pictured below.

Huw Price retrocausality argument: II



- The argument will be described (leaving out many details not central to the conclusions).
- Think of person on the left, Lena, controlling the angle σ_L on the polarizing cube on the left.

Huw Price retrocausality argument: III

- Think of a demon controlling the inputs at $L = 1$ and $L = 0$ but where the demon is restricted to discrete (non-superposition) inputs of a photon in one of the channels with a certain probability.
- Even if the demon knows Lena's setting σ_L ahead of time, Lena can still set σ_L to essentially determine the resulting output polarization τ_L, regardless of the demon's discrete inputs.
- Then we assume time symmetry on the right and further assume that there is discreteness in the outputs, i.e., a photon probabilistically either reflects or transmits at the polarizing cube on the right (but no superposition).

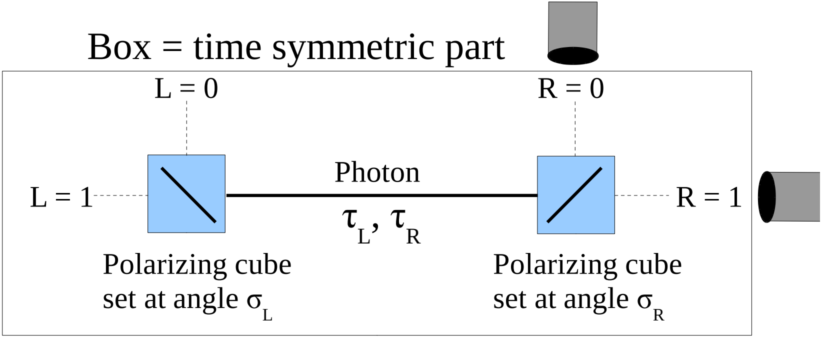
Huw Price retrocausality argument: IV

- With a further unproblematic assumption of realism, Price argues that these three assumptions, Time Symmetry, Realism, and Discreteness, imply that a person on the right, Rena, by changing the polarizer angle σ_R can determine the incoming polarization τ_R even though that is earlier in time that the separation at the right-hand polarizing cube. It is symmetric to the left-hand case where even allowing the demon to change his discrete probabilistic inputs knowing ahead of time Lena's setting for σ_L, Lena can still set σ_L to determine τ_L.
- On the right-hand side, only certain settings of σ_R and τ_R are compatible with discrete outputs, so given Discreteness, Rena by setting σ_R seem to retrocause τ_R to a setting compatible with discrete outputs.

Huw Price retrocausality argument: V

- Price admits that the Discreteness assumption does not reflect the actual QM behavior at the right polarizing cube which would create a superposition.
- But then Price argues that the appropriate discreteness can be obtained "simply by placing photon detectors on the output channels." (draft paper dated Oct. 19, 2011).
- There seem to be two ways to interpret this. Since the measurement ("collapse of the wave packet") at the detectors is admittedly time-asymmetric, the detectors can be placed outside the part of the set-up that is time-symmetric.

Huw Price retrocausality argument: VI



Huw Price retrocausality argument: VII

- But then the mistake is like the one in the interferometer case where the hits at appropriately place detectors are misinterpreted as giving which-way information. In this case, it means misinterpreting the hits as showing that the photon, *prior* to the detection, was discretely either transmitted ($R = 1$) or reflected ($R = 0$) but not in superposition.
- Incidentally, under that assumption, one can easily construct a delayed-choice version of the retrocausality by treating the right-hand polarizing cube as the first beam-splitter in a which-way interferometer and then by the delayed choice of inserting (or not) the detectors into the two channels after a photon had traversed the cube or beam-splitter, one seemingly retrocauses the photon to go

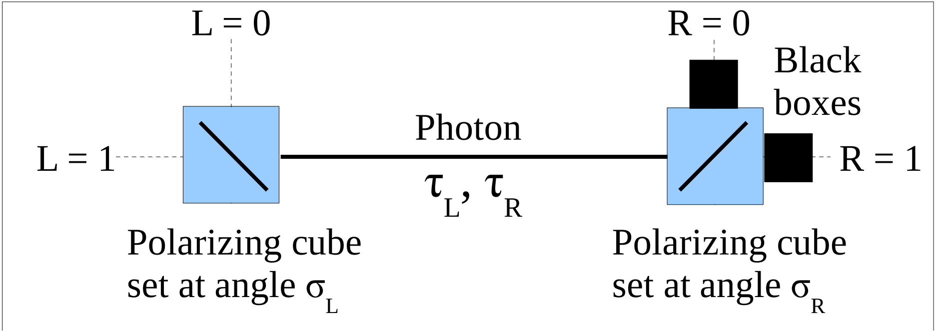
Huw Price retrocausality argument: VIII

one way or another, or to be in a superposition state ("go both ways") at that first splitter.

- The other alternative for Price is to assume "black boxes" placed inside the time-symmetric box that gives the discrete outputs.

Huw Price retrocausality argument: IX

Box = time symmetric part

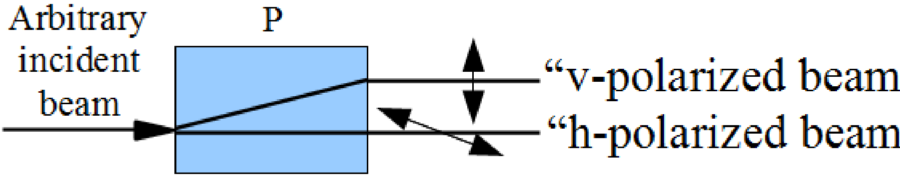


Huw Price retrocausality argument: X

- The problem is then that the superposition $\longrightarrow$ discreteness process has to be time symmetric and there is no such process in QM. An invertible description of that process would be tantamount to a solution to the measurement problem!
- Hence depending on the form that Price uses to get discreteness, it either stays within QM and commits the separation fallacy, or it goes outside of QM with a time-symmetric "measurement" box.

Polarization analyzers and loops: I

- Another common textbook example of the separation fallacy is the treatment of polarization analyzers such as calcite crystals that are *said* to create two orthogonally polarized beams, one in the upper channel and one in the lower channel, say $|v\rangle$ and $|h\rangle$ from an arbitrary incident beam.

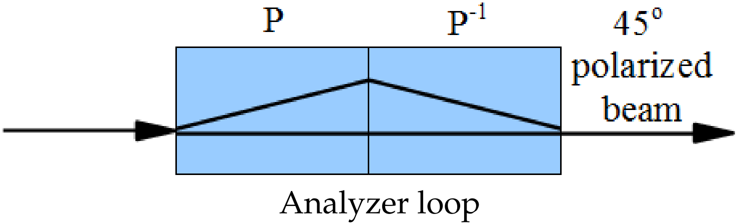


Polarization analyzers and loops: II

- But here again, this is wrong. What is created is an entangled state where vertically polarized is entangled with upper channel and horizontal polarization is entangled with the lower channel (symbolically $|v\rangle \otimes |U\rangle + |h\rangle \otimes |L\rangle$).
- This version of the separation fallacy is "sponsored" by the fact that if a polarization detector is placed in the upper channel, then it will only record vertically polarized photons–since the placement of that detector in the upper channel means that any hit is due to the entangled state collapsing to $|v\rangle \otimes |U\rangle$ and thus only shows v-polarization. And if a detector is placed in the lower channel, then it will similarly record only h-polarized photons.

Polarization analyzers and loops: III

- But that does NOT mean that the calcite crystal itself performed a measurement so that there were only v-polarized photons in the upper channel and h-polarized photons in the lower channel.
- Yet the description of the calcite crystal as creating two separate beams of orthogonally polarized photons is common in the literature.
- It is easy to show that this common description is wrong by appending a reversed polarization analyzer after the first one which will just reproduce the original beam–which could have been $+45°$ polarization.



- If the first calcite crystal had in fact performed a measurement producing only *v*-polarized photons in the upper channel and *h*-polarized photons in the lower channel, then the information about the incident beam would have been lost and thus could not have been reconstructed by the analyzer loop.

Polarization analyzers and loops: V

- In the delayed choice version of this experiment, the separation fallacy makes it seem like the delayed choice of not inserting or inserting the reversed calcite crystal P^{-1} would retrocause the first crystal to make a measurement or not.
- After giving the standard "measurement" description of the calcite crystal as creating two beams of orthogonally polarized photons, the Dicke and Wittke text is one of the few to realize that this can't be true!

Polarization analyzers and loops: VI

"The equipment [polarization analyzers] has been described in terms of devices which measure the polarization of a photon. Strictly speaking, this is not quite accurate....

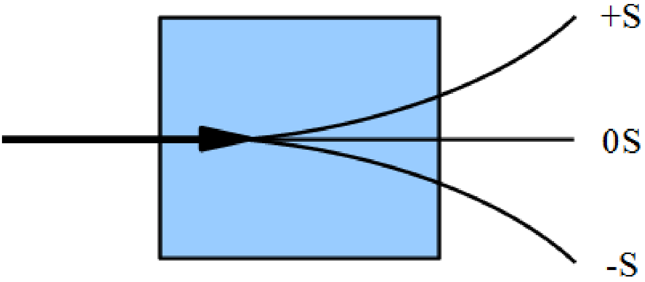
Stating it another way, although [when considered by itself] the polarization P completely destroyed the previous polarization Q [of the incident beam], making it impossible to predict the result of the outcome of a subsequent measurement of Q, in [the analyzer loop] the disturbance of the polarization which was effected by the box P is seen to be revocable: if the box P is combined with another box of the right type, the combination can be such as to leave the polarization Q unaffected....

- Finally their tortured description concludes:

Polarization analyzers and loops: VII

However, it should be noted that in this particular case [sic!], the first box P in [the first half of the analyzer loop] did not really measure the polarization of the photon: no determination was made of the channel ... which the photon followed in leaving the box P."

- The Stern-Gerlach experiment is like the calcite crystal case except that it is spin rather than polarization that is misdescribed in the usual treatment.

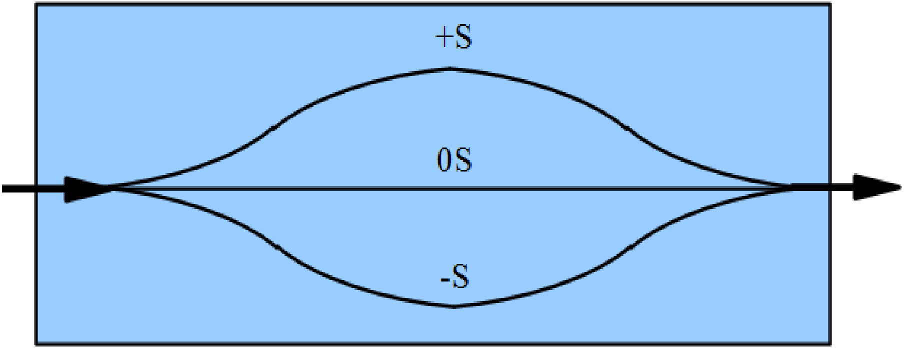


Stern-Gerlach apparatus T

Stern-Gerlach experiments: II

- And again, the fallacy is revealed by considering the Stern-Gerlach analogue of an analyzer loop that passes through the spin state of the incident particle.
- The idea of a Stern-Gerlach loop seems to have been first broached by David Bohm and was later used by Eugene Wigner. One of the few texts to consider such a Stern-Gerlach analyzer loop is *The Feynman Lectures on Physics: Quantum Mechanics (Vol. III)* where it is called a "modified Stern-Gerlach apparatus."

Stern-Gerlach experiments: III



Stern-Gerlach loop

- Ordinarily texts represent the Stern-Gerlach apparatus T as a measurement that projects the particles into spin eigenstates denoted by, say, $+S, 0S, -S$.

Stern-Gerlach experiments: IV

- But, as in our other examples, the S-G apparatus does not project the particles to eigenstates. Instead it creates an entangled superposition state, such as:

$$|+S\rangle \otimes |U\rangle + |0S\rangle \otimes |M\rangle + |-S\rangle \otimes |L\rangle.$$

- With a detector in a certain channel, then as the detector causes the collapse, the detector will only see particles of one spin state.
- Alternatively if the collapse is caused by placing blocking masks over two of the beams, then the particles in the third beam will all be those that have collapsed to the same spin eigenstate.

Stern-Gerlach experiments: V

- It is the detectors or blocking masks that cause the collapse or projection to eigenstates, not the prior separation apparatus T.
- As Feynman puts it:

"Some people would say that in the filtering by T we have 'lost the information' about the previous state $(+S)$ because we have 'disturbed' the atoms when we separated them into three beams in the apparatus T. But that is not true. The past information is not lost by the separation into three beams, but by the blocking masks that are put in. . . ."

Separation fallacy redux: I

- We have seen the same fallacy of interpretation in:
  - Two-slit experiments,
  - "Which-way" interferometer experiments,
  - Polarization analyzers, and
  - Stern-Gerlach experiments.
- The common element in all the cases is that there is some 'separation' apparatus that puts a particle into a certain superposition of spatially-entangled eigenstates.
- When an *appropriately positioned* detector induces a collapse to an eigenstate, then the detector will only register one of the eigenstates.

Separation fallacy redux: II

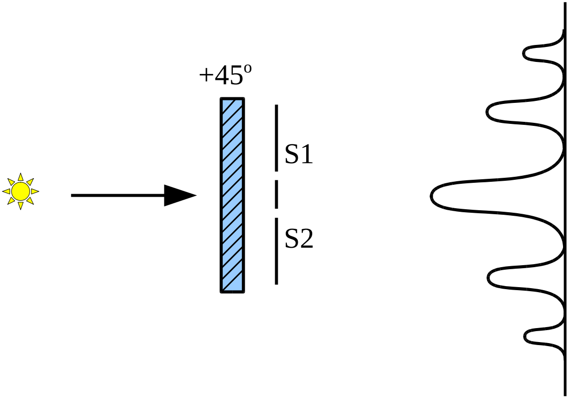
- The separation fallacy is that this is misinterpreted as showing that the particle was already in that eigenstate in that position as a result of the previous "separation."
- The quantum erasers are elaborated versions of these simpler experiments, and a similar separation fallacy arises in that context.

Quantum eraser example before markings: I

- Consider the setup of the two-slit experiment where the superposition state, $\frac{1}{\sqrt{2}}\left(|S1\rangle + |S2\rangle\right)$, evolves to show interference on the wall.
- If we put a $+45^\circ$ polarizer in front of the slits to control the incoming polarization, then we can represent the system after the polarizer as a tensor product with the second component giving the polarization state. The evolving state after the two slits is the superposition:

$$\frac{1}{\sqrt{2}}\left(|S1\rangle \otimes |45^\circ\rangle + |S2\rangle \otimes |45^\circ\rangle\right).$$

Quantum eraser example before markings: II



Interference pattern from two-slits

Insertion of H,V polarizers: I

- Then horizontal and vertical polarizers are inserted behind the $S1$ and $S2$ slits respectively.
- This will change the evolving state to: $\frac{1}{\sqrt{2}}\left(|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right)$ but since these new polarizers involve some measurements, not just unitary evolution, it may be helpful to go through the calculation in some detail.
- The state that "hits" the H, V polarizers is:

$$\frac{1}{\sqrt{2}}\left(|S1\rangle \otimes |45^\circ\rangle + |S2\rangle \otimes |45^\circ\rangle\right).$$

- The 45° polarization state can be resolved by inserting the identity operator $I = |H\rangle\langle H| + |V\rangle\langle V|$ to get:

Insertion of H,V polarizers: II

$$|45°\rangle = [|H\rangle\langle H| + |V\rangle\langle V|]\,|45°\rangle = \langle H|45°\rangle\,|H\rangle + \langle V|45°\rangle\,|V\rangle = \frac{1}{\sqrt{2}}[|H\rangle + |V\rangle].$$

- Substituting this for $|45°\rangle$, we have the state that hits the H, V polarizers as:

$$\begin{aligned}&\frac{1}{\sqrt{2}}\left(|S1\rangle \otimes |45°\rangle + |S2\rangle \otimes |45°\rangle\right)\\ &= \frac{1}{\sqrt{2}}\left(|S1\rangle \otimes \frac{1}{\sqrt{2}}[|H\rangle + |V\rangle] + |S2\rangle \otimes \frac{1}{\sqrt{2}}[|H\rangle + |V\rangle]\right)\\ &= \frac{1}{2}[|S1\rangle \otimes |H\rangle + |S1\rangle \otimes |V\rangle + |S2\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle]\end{aligned}$$

which can be regrouped in two parts as:

$$= \frac{1}{2}[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle] + \frac{1}{2}[|S1\rangle \otimes |V\rangle + |S2\rangle \otimes |H\rangle].$$

Insertion of H,V polarizers: III

- Then the H, V polarizers are making a (degenerate) measurement that give the first state $|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle$ with probability $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}$.
- The other state $|S1\rangle \otimes |V\rangle + |S2\rangle \otimes |H\rangle$ is obtained with the same probability, and it is blocked by the polarizers.
- Thus with probability $\frac{1}{2}$, the state that evolves is the state (after being normalized):

$$\frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right].$$

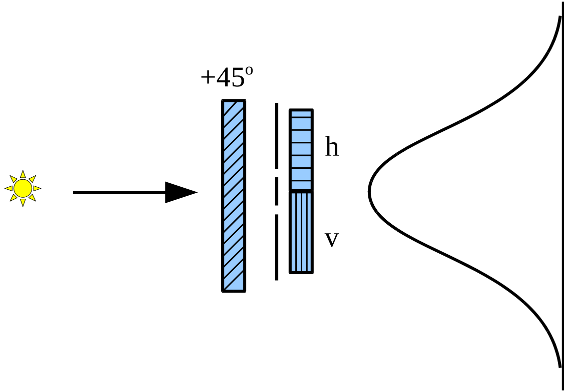
Interference removed by H,V polarizer markings: I

- If $P_{\Delta y}$ is the projection operator representing finding a particle in the region Δy along the wall, then that probability in the state $\frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right]$ is:

$$\frac{1}{2}\left\langle S1 \otimes H + S2 \otimes V | P_{\Delta y} \otimes I | S1 \otimes H + S2 \otimes V \right\rangle$$
$$= \frac{1}{2}\left\langle S1 \otimes H + S2 \otimes V | P_{\Delta y} S1 \otimes H + P_{\Delta y} S2 \otimes V \right\rangle$$
$$= \frac{1}{2}\left[\left\langle S1 \otimes H | P_{\Delta y} S1 \otimes H \right\rangle + \left\langle S1 \otimes H | P_{\Delta y} S2 \otimes V \right\rangle\right.$$
$$\left.+ \left\langle S2 \otimes V | P_{\Delta y} S1 \otimes H \right\rangle + \left\langle S2 \otimes V | P_{\Delta y} S2 \otimes V \right\rangle\right]$$
$$= \frac{1}{2}\left[\left\langle S1 | P_{\Delta y} S1 \right\rangle \langle H|H\rangle + \left\langle S1 | P_{\Delta y} S2 \right\rangle \langle H|V\rangle\right.$$
$$\left.+ \left\langle S2 | P_{\Delta y} S1 \right\rangle \langle V|H\rangle + \left\langle S2 | P_{\Delta y} S2 \right\rangle \langle V|V\rangle\right]$$
$$= \frac{1}{2}\left[\left\langle S1 | P_{\Delta y} S1 \right\rangle + \left\langle S2 | P_{\Delta y} S2 \right\rangle\right]$$

= average of separate slot probabilities.

Interference removed by H,V polarizer markings: II



Mush pattern with interference eliminated by which-way markings

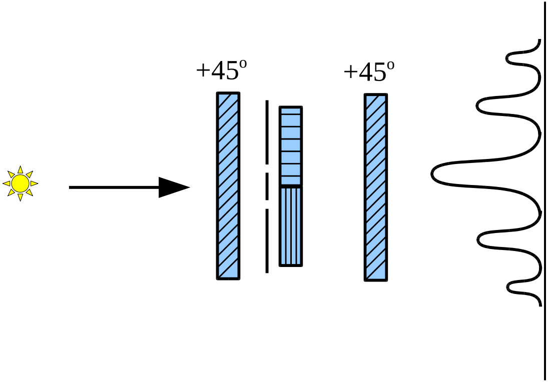
Interference removed by H,V polarizer markings: III

- The key step is how the orthogonal polarization markings decohered the state since $\langle H|V\rangle = 0 = \langle V|H\rangle$ and thus eliminated the interference between the $S1$ and $S2$ terms.
- The state-reduction occurs only when the evolved superposition state hits the far wall which measures the positional component (i.e., $P_{\Delta y}$) of the composite state and shows the non-interference pattern.

"Erasing" the markings: I

- The key point is that in spite of the bad terminology of "which-way" or "which-slit" information, the polarization markings do NOT create a half-half mixture of horizontally polarized photons going through slit 1 and vertically polarized photons going through slit 2. It creates the entangled *superposition* state $\frac{1}{\sqrt{2}}[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle]$.
- This can be verified by inserting a $+45°$ polarizer between the two-slit screen and the far wall.

"Erasing" the markings: II



Fringe interference pattern produced by +45° polarizer

"Erasing" the markings: III

- Each of the horizontal and vertical polarization states can be represented as a superposition of $+45°$ and $-45°$ polarization states. Just as the horizontal polarizer in front of slit 1 threw out the vertical component so we have no $|S1\rangle \otimes |V\rangle$ term in the superposition, so now the $+45°$ polarizer throws out the $-45°$ component of each of the $|H\rangle$ and $|V\rangle$ terms so the state transformation is:

$$\frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right]$$
$$\to \frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |+45°\rangle + |S2\rangle \otimes |+45°\rangle\right] =$$
$$\frac{1}{\sqrt{2}}\left(|S1\rangle + |S2\rangle\right) \otimes |+45°\rangle.$$

- It might be useful to again go through the calculation in some detail.

"Erasing" the markings: IV

1. $|H\rangle = (|+45^\circ\rangle\langle+45^\circ| + |-45^\circ\rangle\langle-45^\circ|)\,|H\rangle =$ $\langle+45^\circ|H\rangle\,|+45^\circ\rangle + \langle-45^\circ|H\rangle\,|-45^\circ\rangle$ and since a horizontal vector at 0° is the sum of the $+45^\circ$ vector and the -45° vector, $\langle+45^\circ|H\rangle = \langle-45^\circ|H\rangle = \frac{1}{\sqrt{2}}$ so that: $|H\rangle = \frac{1}{\sqrt{2}}\left[|+45^\circ\rangle + |-45^\circ\rangle\right]$.
2. $|V\rangle = (|+45^\circ\rangle\langle+45^\circ| + |-45^\circ\rangle\langle-45^\circ|)\,|V\rangle =$ $\langle+45^\circ|V\rangle\,|+45^\circ\rangle + \langle-45^\circ|V\rangle\,|-45^\circ\rangle$ and since a vertical vector at 90° is the sum of the $+45^\circ$ vector and the negative of the -45° vector, $\langle+45^\circ|V\rangle = \frac{1}{\sqrt{2}}$ and $\langle-45^\circ|V\rangle = -\frac{1}{\sqrt{2}}$ so that: $|V\rangle = \frac{1}{\sqrt{2}}\left[|+45^\circ\rangle - |-45^\circ\rangle\right]$.

- Hence making the substitutions gives:

$$\frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[\begin{array}{c} |S1\rangle \otimes \frac{1}{\sqrt{2}}\left[|+45^\circ\rangle + |-45^\circ\rangle\right] \\ +|S2\rangle \otimes \frac{1}{\sqrt{2}}\left[|+45^\circ\rangle - |-45^\circ\rangle\right] \end{array}\right].$$

"Erasing" the markings: V

- We then regroup the terms according to the measurement being made by the 45° polarizer:

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} \left[|S1\rangle \otimes |+45^\circ\rangle + |S2\rangle \otimes |+45^\circ\rangle\right] \\ +\frac{1}{\sqrt{2}} \left[|S1\rangle \otimes |-45^\circ\rangle - |S2\rangle \otimes |-45^\circ\rangle\right] \end{bmatrix}$$

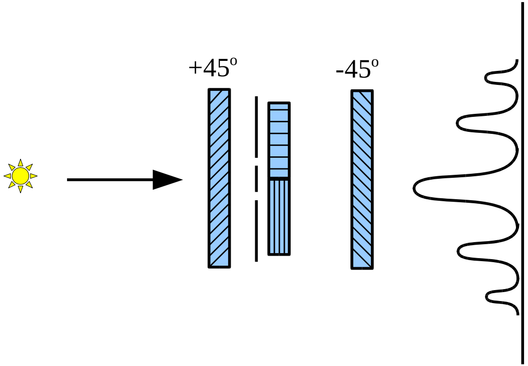
$$= \tfrac{1}{2} \left(|S1\rangle + |S2\rangle\right) \otimes |+45^\circ\rangle + \tfrac{1}{2} \left(|S1\rangle - |S2\rangle\right) \otimes |-45^\circ\rangle.$$

- Then with probability $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}$, the $+45^\circ$ polarization measure passes the state $\left(|S1\rangle + |S2\rangle\right) \otimes |+45^\circ\rangle$ and blocks the state $\left(|S1\rangle - |S2\rangle\right) \otimes |-45^\circ\rangle$. Hence the normalized state that evolves is: $\frac{1}{\sqrt{2}} \left(|S1\rangle + |S2\rangle\right) \otimes |+45^\circ\rangle$, as indicated above.

"Erasing" the markings: VI

- Then at the wall, the positional measurement $P_{\Delta y}$ of the first component is the evolved superposition $|S1\rangle + |S2\rangle$ which again shows an interference pattern. But it is not the same as the original interference pattern before H, V or $+45°$ polarizers were inserted. This "shifted" interference pattern is called the *fringe* pattern.
- Alternatively we could insert a $-45°$ polarizer which would transform the state $\frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right]$ into $\frac{1}{\sqrt{2}}\left(|S1\rangle - |S2\rangle\right) \otimes |-45°\rangle$ which produces the interference pattern from the "other half" of the photons and which is called the *anti-fringe* pattern.
- The all-the-photons sum of the fringe and anti-fringe patterns reproduces the "mush" non-interference pattern.

"Erasing" the markings: VII



Anti-fringe interference pattern produced by $-45°$ polarizer

Interpreting the Quantum Eraser: I

- This is one of the simplest examples of a quantum eraser experiment.
- But there is a **mistaken interpretation** of the quantum eraser experiment that leads one to infer that there is *retrocausality*. The incorrect reasoning is as follows:

Interpreting the Quantum Eraser: II

1. The markings by insertion of the horizontal and vertical polarizers creates the half-half mixture where each photon is reduced to either a horizontally polarized photon going through slit 1 or a vertically polarized photon going through slit 2. Hence the photon "goes through one slit or the other." [This is the separation fallacy]

2. The insertion of the $+45°$ polarizer erases that which-slot information so interference reappears which means that the photon had to "go through both slits."

*3. Hence the delayed choice to insert or not insert the $+45°$ polarizer–after the photons have traversed the screen and H, V polarizers–**retrocauses** the photons to either:*

3.a. Go through both slits, or

3.b. Go through only one slit or the other.

Interpreting the Quantum Eraser: III

- Now we can see the importance of realizing that prior to inserting the second $+45°$ polarizer, the photons were in the *superposition* state $\frac{1}{\sqrt{2}}\left[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle\right]$, not a half-half mixture of the reduced states $|S1\rangle \otimes |H\rangle$ or $|S2\rangle \otimes |V\rangle$.
- The **proof** that the system was not in that mixture is obtained by inserting the $+45°$ polarizer which yields the (fringe) interference pattern.
    1. If a photon had been, say, in the state $|S1\rangle \otimes |H\rangle$ then, with 50% probability, the photon would have passed through the filter in the state $|S1\rangle \otimes |+45°\rangle$, but that would not yield any interference pattern at the wall since their was no contribution from slit 2.

Interpreting the Quantum Eraser: IV

2. And similarly if a photon in the state $|S2\rangle \otimes |V\rangle$ hits the $+45°$ polarizer.

- The fact that the insertion of the $+45°$ polarizer yielded interference *proved* that the incident photons were in a superposition state $\frac{1}{\sqrt{2}}[|S1\rangle \otimes |H\rangle + |S2\rangle \otimes |V\rangle]$ which, in turn, means there was no "going through one slit or the other" in case the second $+45°$ polarizer had not been inserted.
- Thus a correct interpretation of the quantum eraser experiment removes any inference of *retrocausality* and fully accounts for the experimentally verified facts given in the figures. For more, see my mathblog:

Interpreting the Quantum Eraser: V

http://www.mathblog.ellerman.org/2011/11/a-common-qm-fallacy/.