# Unpredictable Tunneling of a Classical Wave-Particle Association

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A droplet bouncing on a vibrated bath becomes a "walker" moving at constant velocity on the interface when it couples to the surface wave it generates. Here the motion of a walker is investigated when it collides with barriers of various thicknesses. Surprisingly, it undergoes a form of tunneling: the reflection or transmission of a given incident walker is unpredictable. However, the crossing probability decreases exponentially with increasing barrier width. This shows that this wave-particle association has a nonlocality sufficient to generate a quantumlike tunneling at a macroscopic scale.

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Faraday discovered in 1831 that when a bath of a viscous liquid is vibrated vertically with an acceleration $\gamma = \gamma_m \cos(2\pi f_0 t)$, the liquid surface may destabilize spontaneously [1]: parametrically forced waves of frequency $f_0/2$ form and cover the whole interface. This phenomenon is observed when $\gamma_m$, the amplitude of the forcing, exceeds an onset value $\gamma_m^F$. It was found recently [2,3] that, below this instability threshold, a droplet can serve as a local oscillator and trigger the formation of a localized state of sustained Faraday waves. The structure consisting of the droplet and its associated wave is self-propelled, forming what we called [2,3] a "walker," which moves on the interface at a constant velocity $V$. The transition to walking results from a symmetry-breaking phenomenon. The bifurcation is supercritical [3] and characterized by its threshold $\gamma_m^W$ located below, but in the vicinity of $\gamma_m^F$. The dynamics of the walker relies on the interaction between the droplet and the surface waves. The waves are generated by the droplet, and the droplet moves because it falls on a surface distorted by the waves emitted by its previous bounces [3]. A walker is thus a symbiotic wave-particle association. It was previously shown that when a walker goes through a slit, a phenomenon of single-particle diffraction appears [4]. The droplet is deviated, and its deviation appears random; however, the determinism is recovered in a statistical sense. Surprisingly, a diffraction pattern is recovered in the histogram of the deviations of many successive crossings. This appears to be due to the walker's nonlocal character linked to the superposition of its own waves with those emitted in the past and reflected on the borders. In this Letter, we designed experiments aimed at probing both this duality and this nonlocality by investigating walkers' transmission through partially reflecting barriers.

Our experimental cell is square (180 × 180 mm) and filled with a liquid layer of depth $h_0 = 4.1$ mm. It is fixed on a vibration exciter driven by a low frequency generator. The cell is carefully set horizontal and perpendicular to the vibration axis. The liquid is a silicon oil, with viscosity $\mu = 20 \times 10^{-3}$ Pa · s, surface tension $\sigma = 0.0209$ N/m, and density $\rho = 965$ kg/m$^3$. The depth of the oil, in the cell and over the barriers that we have used, is carefully measured by mechanical means with an error smaller than 0.02 mm. We use a forcing frequency $f_0 = 80$ Hz. The measured Faraday wavelength $\lambda_F = 4.75$ mm corresponds to the value computed from the surface waves' dispersion relation: $\omega^2 = [gk + (\sigma/\rho)k^3]\tanh(kh)$.

Our experiments rely on the fluid depth dependence of the two thresholds $\gamma_m^F$ and $\gamma_m^W$. Both are shifted to larger acceleration when $h$ decreases [Fig. 1(a)]. We can first consider a situation where a cell is divided into two parts of different depths $h_0$ and $h_1$ ($h_0 > h_1$) with an abrupt edge

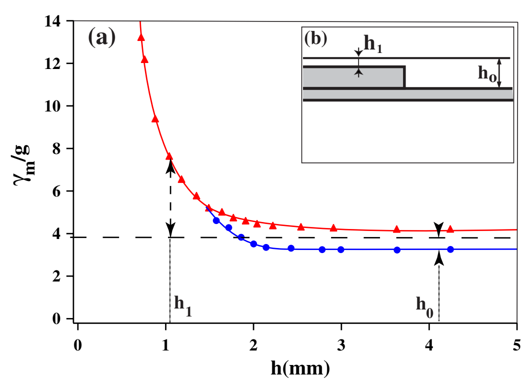


FIG. 1 (color online). (a) The measured thresholds for the Faraday instability $\gamma_m^F$ (triangles) and for the walking instability of $\gamma_m^W$ of a drop of diameter $D = 0.78$ mm (filled circles) as a function of the depth $h$ of the liquid bath. (b) In cells formed of two regions of depths $h_0 = 4.1$ mm and $h_1 = 1.1$ mm, when the system is tuned to a value $\gamma_m^W(h_0) > \gamma_m > \gamma_m^W(h_1)$ (e.g., $\gamma_m/g = 3.75$), a drop forms a walker in the regions of large depth but remains motionless in the shallow zone. The continuous lines are simple interpolations.

 

[Fig. 1(b)]. If the imposed acceleration is such that $\gamma_m^W(h_0) < \gamma_m^W < \gamma_m^W(h_1)$, a droplet forms a walker in the region of large depth but remains motionless in the shallow region. When a walker located in the deep part of the cell impinges on the border of the shallow region, it is reflected away [3]. This repulsion results from the droplet's interaction with the waves reflected by the boundary. We used the technique of free-surface synthetic schlieren [5,6] to measure the wave field of the walker. When it comes close to the boundary of the extended shallow region, an exponentially damped wave is observed over it. When the shallow zone forms a barrier with a finite horizontal extent $e$ [Fig. 2(b)], a wave of weak amplitude is transmitted on the other side [Fig. 2(a)]. Its measured amplitude [Fig. 2(c)] shows that the exponential decay over the barrier has a characteristic length scale $\lambda_W = 2.9 \pm 0.2$ mm. This means that, near the barrier, the wave field surrounding the walker depends on the barrier thickness. Two questions thus arise: is the reflection of a walker affected, and can the walker be transmitted through the barrier? We first address these questions by confining a walker inside a thin-walled square billiard in which it undergoes repeated collisions with the boundaries. A walker is trapped inside a cavity formed by an immersed square frame [7,8] [Fig. 3(a)] of inner side $L = 55$ mm. Above a wall, the fluid depth is reduced to $h_1 = 1.30 \pm 0.02$ mm, while it is $h_0 = 4.1 \pm 0.02$ mm elsewhere. Ten different frames were used, with horizontal thickness $e$ ranging from 1.5 mm to 8 mm. With thick walls (e.g., $e = 8$ mm), a walker remains confined inside the cavity. For thinner walls escapes are observed, and they become increasingly frequent with decreasing $e$.

During its motion inside the cavity the walker undergoes repeated collisions with the walls. We measure the number $N$ of these collisions before an escape. This is a measure of lifetime $\tau$ inside the trap [9]. Two important points can be noticed. First, by placing $n$ times the same walker inside the same frame, we find that $N$ has widely scattered values. Each realization turns out to have an unpredictable output.

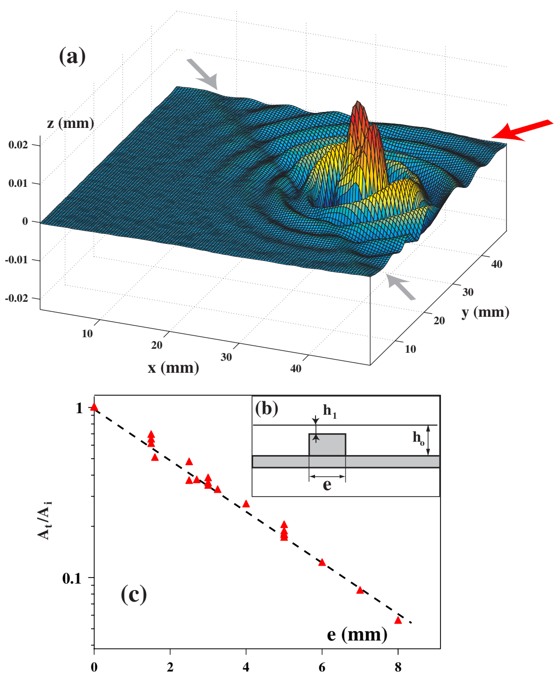


FIG. 2 (color online). (a) The wave field surrounding a walker of velocity $V = 13.7 \pm 0.1$ mm/s as it approaches a barrier. A cross section of this barrier, of finite thickness $e = 2$ mm and over which the fluid depth is $h_1 = 1.1$ mm, is shown in the insert (b). In (a) the arrow on the right shows the direction of motion of the walker, and the barrier is located between the light grey arrows. The topography of the liquid interface is measured using the free-surface synthetic schlieren technique [5], which provides a vertical accuracy $5 \times 10^{-4}$ mm. A transmitted wave of weak amplitude can clearly be observed. (c) Measured ratio $A_t/A_i$ of the amplitude of the transmitted wave over that of the incident wave as a function of the barrier thickness. The dashed line is the result of the exponential fit with a characteristic length $\lambda_W = 2.9 \pm 0.2$ mm.

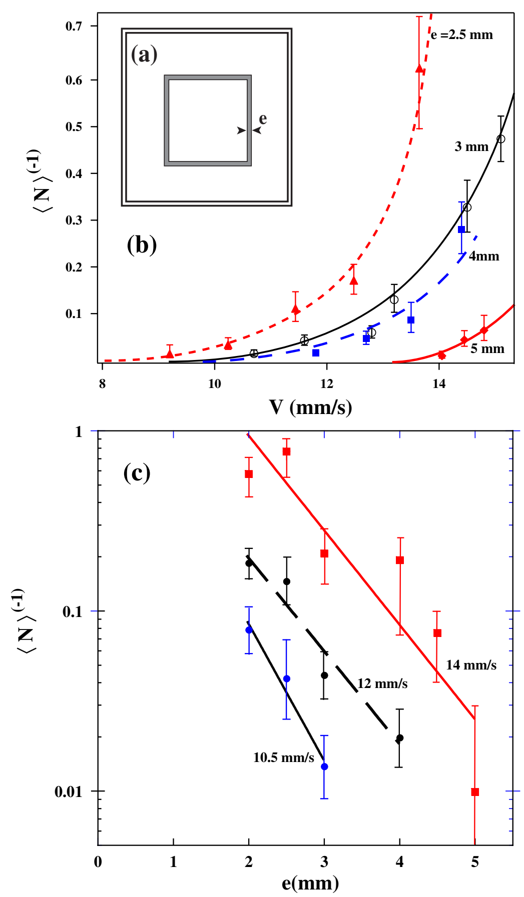


FIG. 3 (color online). (a) Sketch of the square frame of width $e$ seen from above. The fluid depth above the barrier (in grey) is reduced to $h_1 = 1.30 \pm 0.02$ mm. (b) Probability of escape of walkers out of cavities with walls of thickness $e = 2.5$, 3, 4 and 5 mm, respectively, as a function of their velocity. The continuous curves are simple interpolations. (c) Semilogarithmic plot showing, for three different velocities ($V = 10.5$ mm/s, $V = 12$ mm/s, $V = 14$ mm/s), the probability of escape of the walkers as a function of barrier thickness $e$. The lines are exponential fits with characteristic lengths $\lambda_\tau = 0.65$, 0.85, and 0.8 mm, respectively.

Second, the probability of crossing $P = 1/\langle N\rangle$, where $\langle N\rangle$ is the average number of collisions before an escape, is found to have a smooth dependence on the velocity and the wall thickness. The statistical sampling is done by gathering data from walkers having velocities in a range $V \pm \Delta V$ with $\Delta V/V = 5\%$. Each value of $P$ is computed with more than a thousand collisions. We find that $P$ increases with increasing velocity [Fig. 3(b)]. For walkers of a given speed, $P$ decreases with increasing wall thickness [Fig. 3(c)]. It can be fitted by an exponential with a characteristic decay length $\lambda_\tau = 0.8 \pm 0.1$ mm, for small values of $P$.

Using a particle-tracking software we record the walker's trajectories inside the cavity. Figures 4(a)–4(d) show four examples of the observed trajectories. After an initial transient, a walker placed inside a cavity with thick walls (e.g., $e = 8$ mm) reaches a stable trajectory similar to the basic mode of a particle in a square billiard [Fig. 4(a)]. It is slightly tilted because incidence and reflection angles are not equal [3]. For thinner walls or faster walker, the trajectories become progressively unstable. A slow drift of the trajectories is first observed [Fig. 4(b)], which leads to scarce intermittent bursts of disorder. Correlatively the direction of rotation in the cavity can be reversed. This phenomenon becomes more and more frequent and the time spent by the walker in the limit cycle becomes shorter as the wall thickness is decreased [Fig. 4(c)]. Ultimately the limit cycle is never reached [Fig. 4(d)] [10–12]. The trajectories inside the cavity and the escapes are correlated. As long as the droplet follows the limit cycle, no escape is observed. When the trajectories become disordered, the collisions with the walls show a larger variety of angles. Those with near normal incidence can lead to an escape. In this first experiment the probabilistic output of the collisions with the barrier is related to the scatter in the collision angles. We can wonder if the output, reflection, or crossing is still probabilistic for a given angle. In order to address this issue, we designed another experimental geometry. A walker is confined inside a submerged rhomb-shaped frame [Figs. 5(a) and 5(b)], of inner length $L_1 = 120$ mm and shorter diagonal $L_2 = 45$ mm. When it moves towards one of the rhomb vertices, it bounces back and forth between the converging boundaries. Close to the vertex, it reverses its mean direction and, guided by its waves, moves along the bisector axis. A barrier is placed transversely, along the small diagonal of the rhomb. The fluid depth above it is $h_1 = 1.10 \pm 0.02$ mm, and its horizontal thickness $e$ ranges from 1.6 mm to 6 mm. In this geometry, the same walker impinges repeatedly on the barrier with normal incidence. We observe that the crossing of the barrier remains random. Figure 5 shows examples of recorded trajectories. We measure the velocity in $A$ and $A'$ located 4 cm away from the barrier. At this

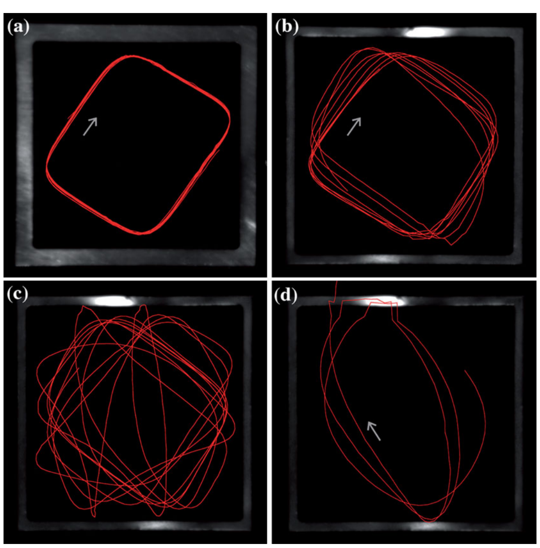


FIG. 4 (color online). The recorded trajectories of the walker inside the square trap of side $L = 55$ mm. In (a) $e = 4.5$ mm and $V = 9.95$ mm/s. In (b) $e = 2.5$ mm and $V = 9$ mm/s. The probability of escape $P$ is of the order of 1%. In (c) $e = 2.5$ mm and $V = 11.8$ mm/s. $P \simeq 10\%$. In (d) $e = 2.5$ mm and $V = 13.2$ mm/s. $P \simeq 30\%$.

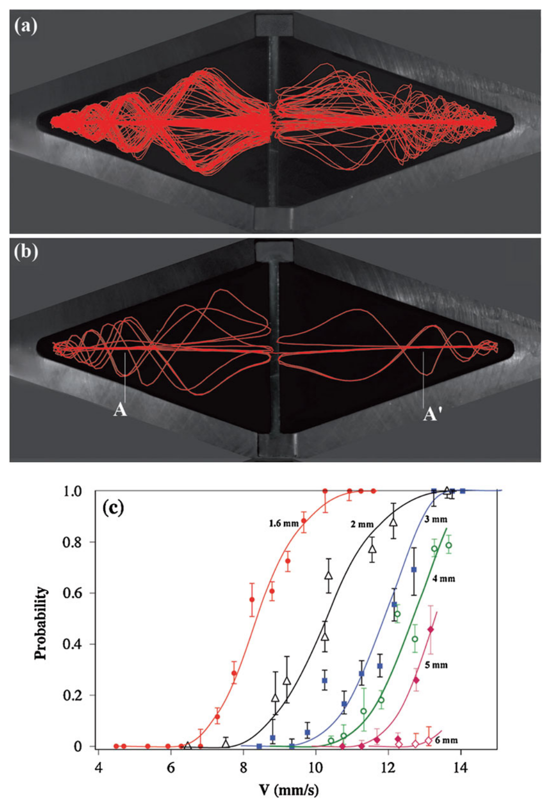


FIG. 5 (color online). (a) Superposition of the trajectories of a walker of velocity $V = 12$ mm/s producing 110 collisions with a barrier with $e = 3$ mm leading to $n = 14$ crossings. (b) Detail of 10 consecutive collisions of the previous recording. $A$ and $A'$ are the points where we measure the walker's velocity $V$ before each collision and where all trajectories coincide. (c) Probability of crossing for walkers impinging with normal incidence on barriers of thicknesses 1.6, 2, 3, 4, 5 and 6 mm, respectively, as a function of their velocity. The continuous curves are simple interpolations.

distance from the barrier, we cannot observe any deviation from the bisector axis. We gather collisions having the same velocity in order to evaluate the probability of crossing. Out of $N$ collisions with the barrier, the walker gets across $n$ times. Figure 5(c) presents the probability $P_{\perp} = n/N$ as a function of the impact velocity $V$ for different barrier thicknesses $e$. The probability increases with increasing $V$ and decreases with $e$. Despite the normal incidence angle far away from the barrier, the nonlocal interaction with the reflected waves leads to diverging trajectories. The impact of a walker on a barrier can be analyzed as its collision with a mirror image. It induces a hyperbolic divergence during the reflection process and yields a random output. Only walkers retaining a normal incidence have eventually a chance to cross. On the barrier, the droplet undergoes a slow and erratic motion since it does not generate the waves necessary to its self-propulsion. The droplet is still moving because of a complex superposition of evanescent waves produced by the previous bounces in the deep region. Should a droplet be deposited directly on a barrier, it would remain motionless. As such, a walker does not exist in the barrier region. When the droplet reaches the other side, the walker reappears.

From this point of view, the escape of a trapped droplet studied in the first set of experiments is the analog of an alpha particle tunneling out of a nucleus [8]. Naturally our system is different from a tunneling experiment, not only because of its macroscopic scale [13,14], but also because it is dissipative and sustained. Obviously the formalism of conservative systems does not apply, but the terms of a comparison can still be examined. In our experiment, the escape probability depends on velocity and barrier thickness (Figs. 4 and 5). These parameters are the counterparts of energy and thickness in quantum systems. The velocity $V$ of a walker is an order parameter of the system controlled by the distance to the walking threshold. In the deep region, $V$ scales as $[\gamma_m - \gamma_m^W(h_0)]^{1/2}$. Over the barrier, $\gamma_m < \gamma_m^W(h_1)$ and $V$ decreases exponentially [15] on a time scale given by $[\gamma_m^W(h_1) - \gamma_m]^{-1}$. When $h_1$ is increased, the crossing probability increases as well. The difference between $\gamma_m$ and the local value of $\gamma_m^W(h)$ thus controls the velocity $V$ and plays a role comparable to the energy difference between the particle and the barrier in the quantum effect.

In this Letter, we have tested the dual behavior of a walker and demonstrated that the partial transmission of its waves through thin barriers results in a probabilistic crossing of the whole structure. Although the interplay between the droplet and its wave is complex, it is noteworthy that both the characteristic transmission length $\lambda_\tau$ and the decay length of the waves $\lambda_W$ are of the same order of magnitude. This experiment, together with the results on diffraction [4], clearly shows that the dual character of a walker results in both individual trajectory unpredictability and deterministic statistical behavior. More work is needed to understand in depth the relation between the trajectories of the droplet and its waves. Of particular interest is the link between the memory effect due to the superposition of past waves and the observed uncertainty. Although our experiment is foreign to the quantum world, the similarity of the observed behaviors is intriguing.

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[1] M. Faraday, Philos. Trans. R. Soc. London **121**, 299 (1831).
[2] Y. Couder, S. Protiere, E. Fort, and A. Boudaoud, Nature (London) **437**, 208 (2005).
[3] S. Protiere, A. Boudaoud, and Y. Couder, J. Fluid Mech. **554**, 85 (2006).
[4] Y. Couder and E. Fort, Phys. Rev. Lett. **97**, 154101 (2006).
[5] F. Moisy, M. Rabaud, and K. Salsac, Exp. Fluids **46**, 1021 (2009).
[6] See EPAPS Document No. E-PRLTAO-103-065952 for details about surface measurements. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.
[7] The escape of a walker out of a confining trap is a situation inspired by the escape of an alpha particle out of a nucleus modeled by Gamow [8].
[8] G. Gamow, Nature (London) **122**, 805 (1928).
[9] The typical trajectory inside the frame is a cycle of length $L\sqrt{2}$ with 4 collisions. The average lifetime $\tau$ during which the walker is confined is thus related to the probability of exit $P$ by $\langle\tau\rangle \approx L(2\sqrt{2}VP)^{-1}$.
[10] See EPAPS Document No. E-PRLTAO-103-065952 for visualization of the trajectory of a walker with $V = 13.3$ mm/s in a frame of thickness $e = 6$ mm. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.
[11] See EPAPS Document No. E-PRLTAO-103-065952 for visualization of the trajectory of a walker with $V = 11.8$ mm/s in a frame of thickness $e = 2.5$ mm. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.
[12] See EPAPS Document No. E-PRLTAO-103-065952 for visualization of the trajectory of a walker with $V = 12.5$ mm/s in a frame of thickness $e = 2.5$ mm. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.
[13] The possibility of crossing potential barriers by objects of insufficient energy has been shown [14] to exist in classical systems, provided the object is extended so that not all its parts are involved simultaneously in the crossing. This is the situation of the Fosbury flop in high jump. The output is deterministic so that it is entirely different from the present system.
[14] M. Rabinowitz and A. Cohn, Int. J. Theor. Phys. **29**, 215 (1990).
[15] M. McCracken and J. E. Marsden, *The Hopf Bifurcation and Its Applications* (Springer-Verlag, New York, 1976).